What does localization mean in interacting systems?

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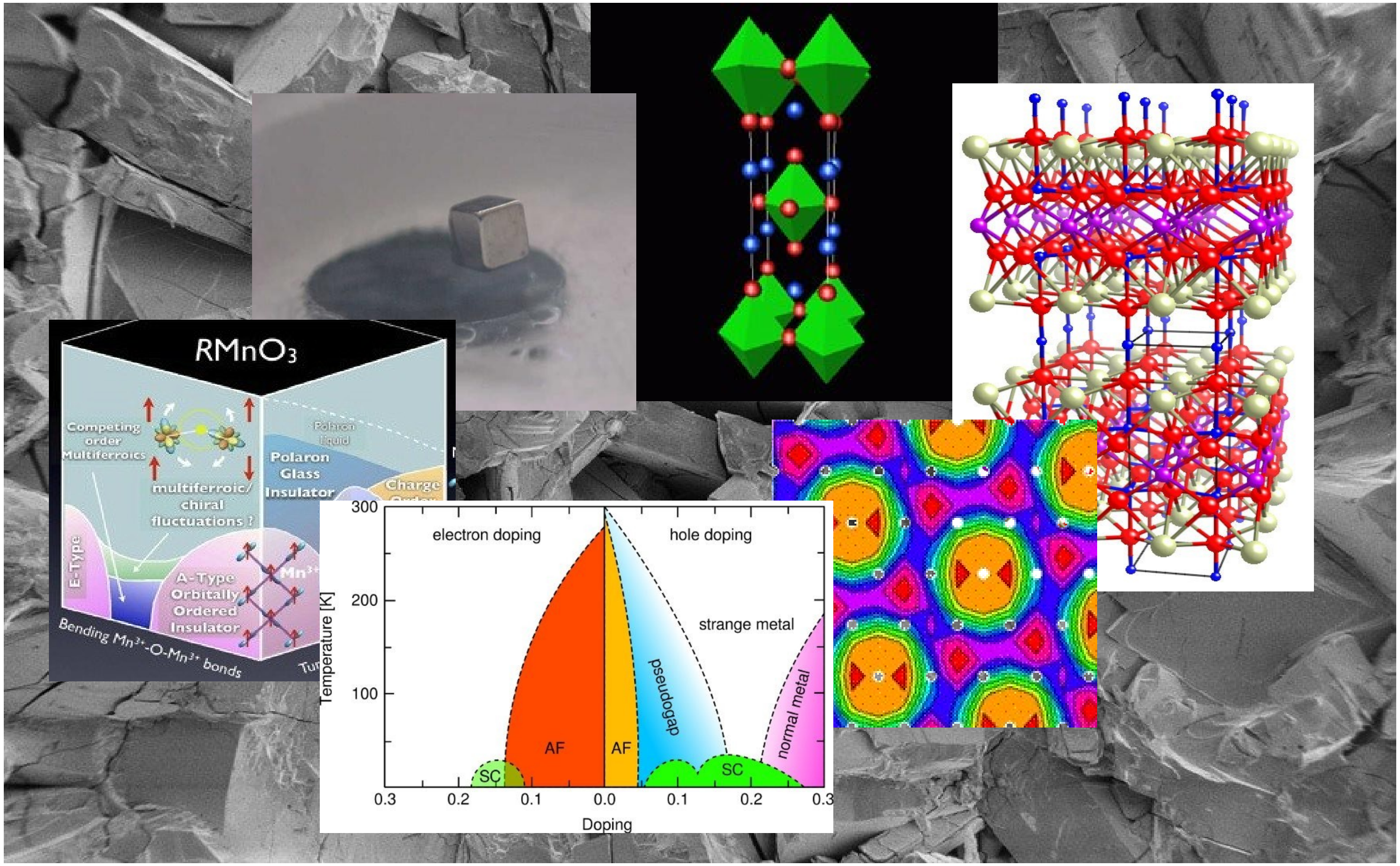
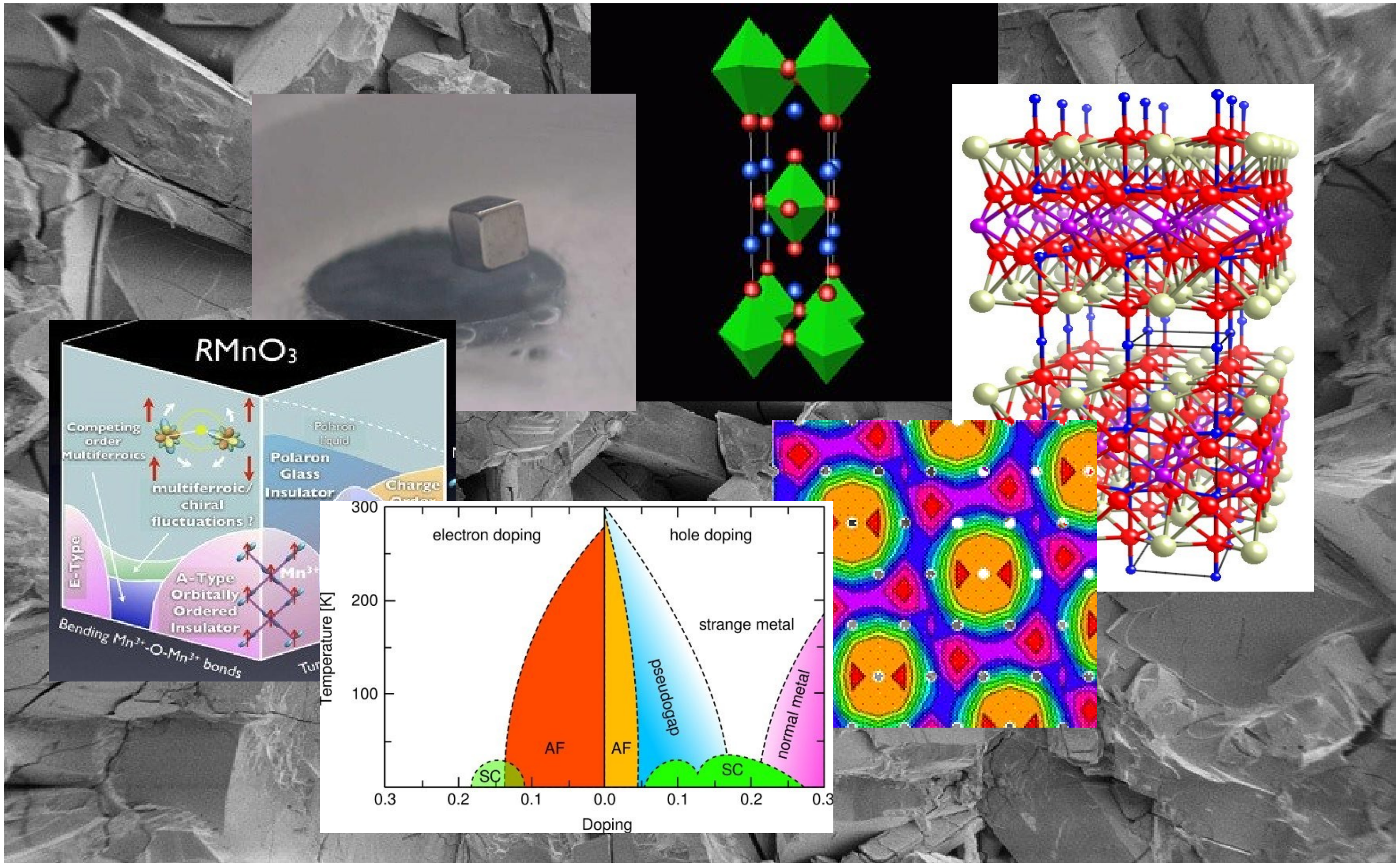
SHARCNET™

Transition metal oxides



What does disorder do to strongly correlated systems?

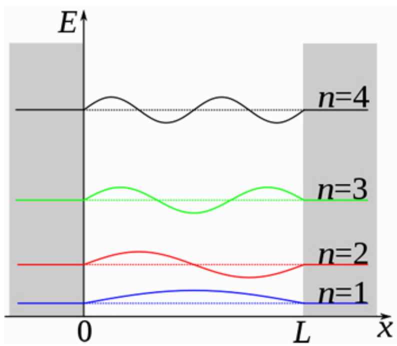
Transition metal oxides

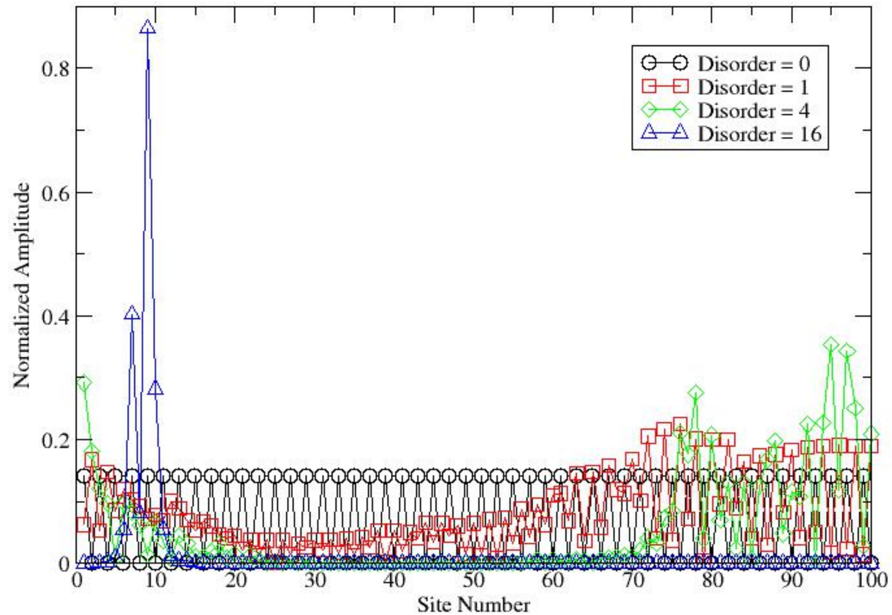


~~What does disorder do to strongly correlated systems?~~

What do interactions do to disordered systems?

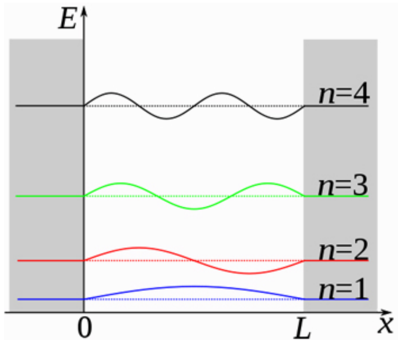
Anderson localization

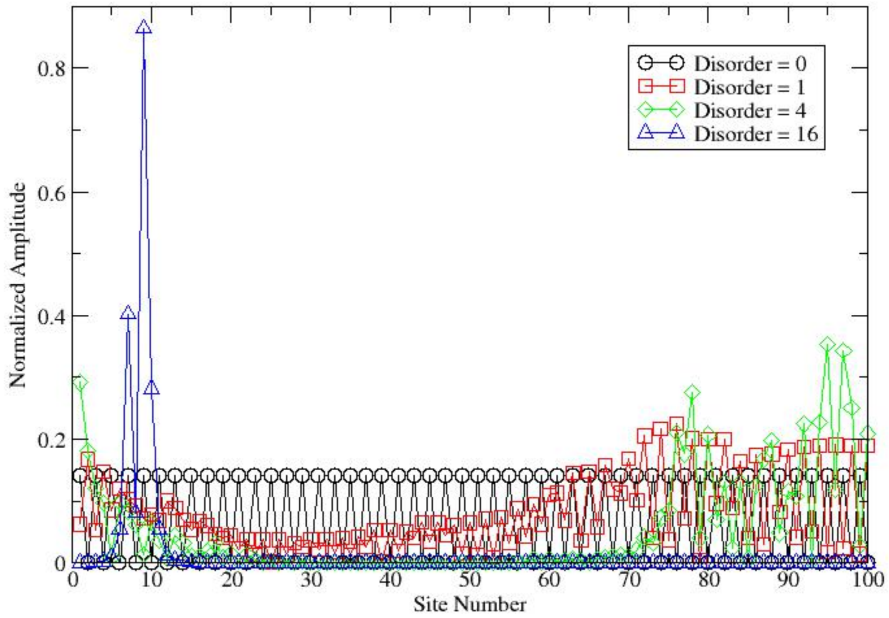




Anderson, Phys. Rev. **109** 1492 (1958) *Absence of diffusion in certain random lattices*

Anderson localization





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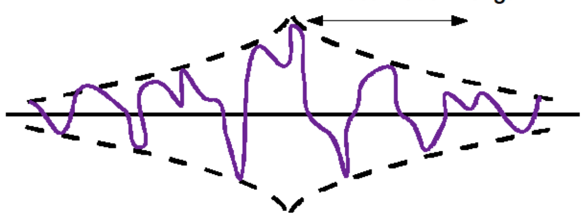
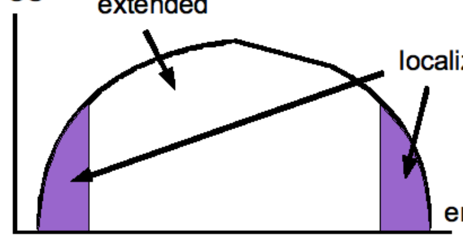
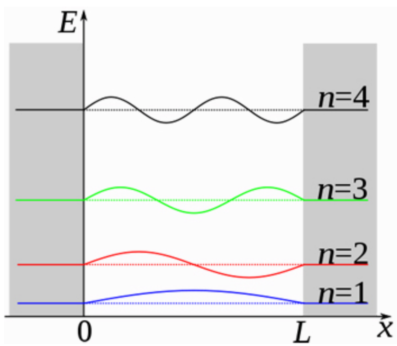
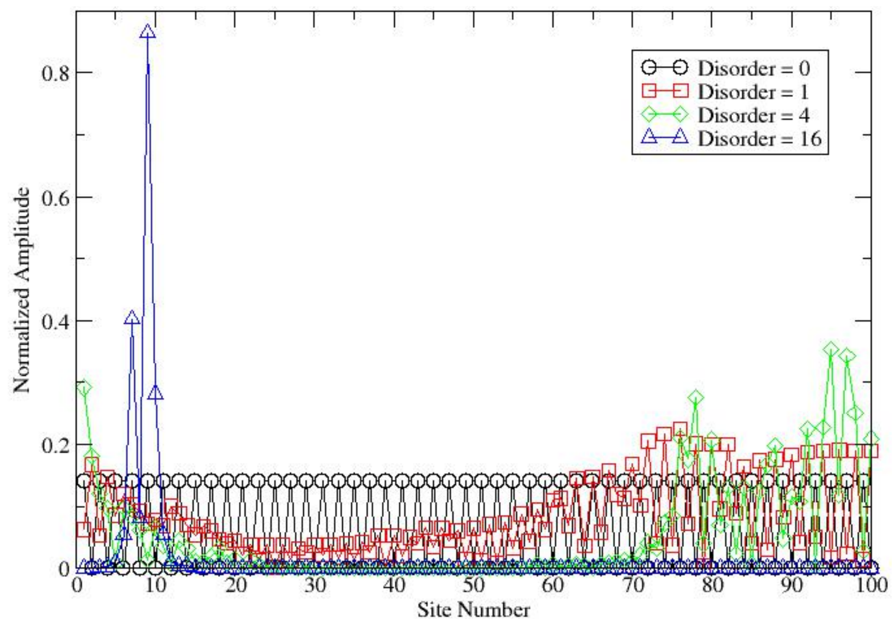
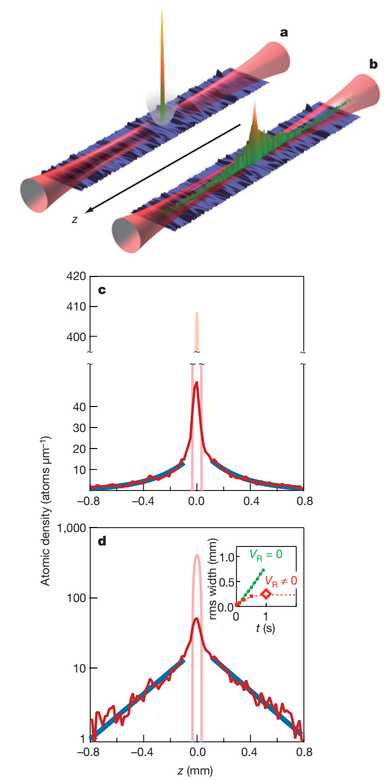


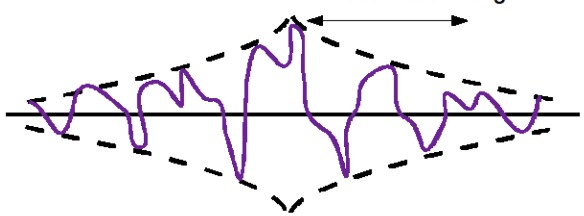


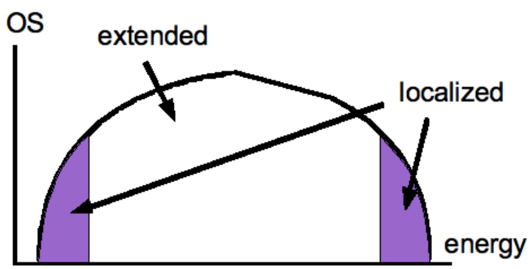
Anderson localization











Billy, et al, Nature **453** 891 (2008)

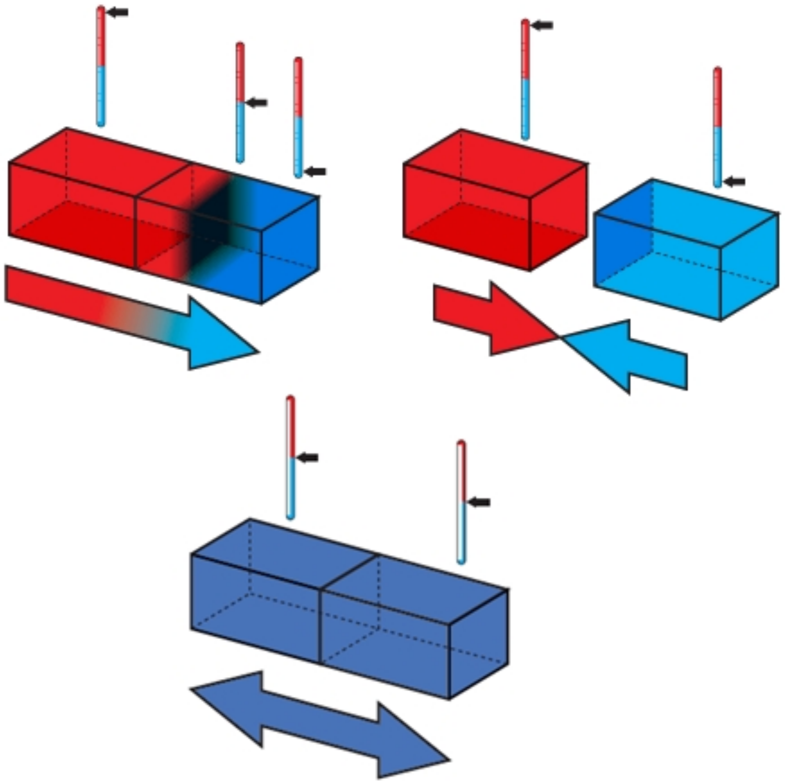
Outline

What is thermal equilibrium and how is it reached?

What is many-body localization?

Our work on the Anderson-Hubbard model

What is thermal equilibrium?



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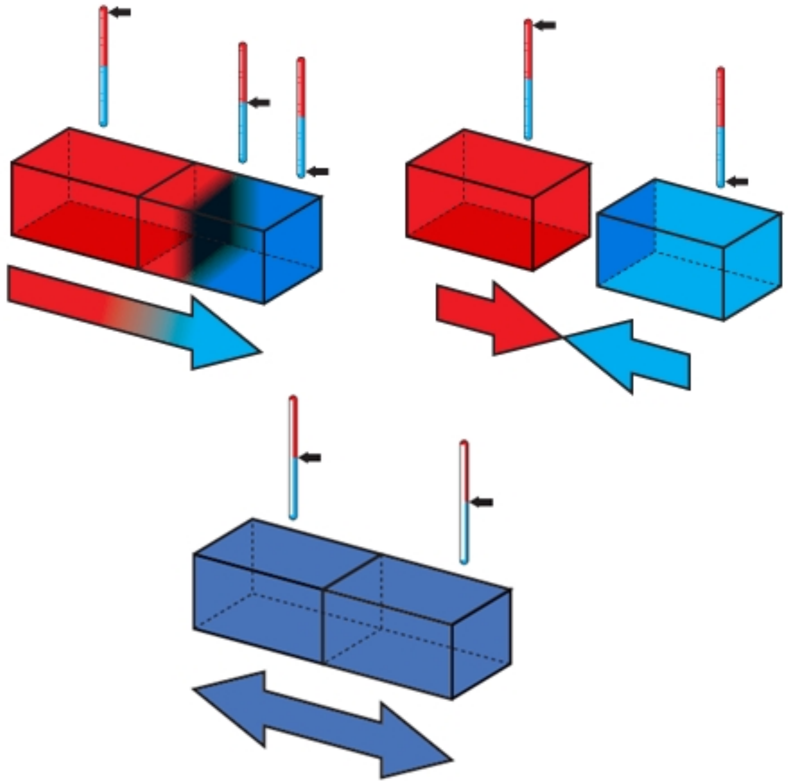
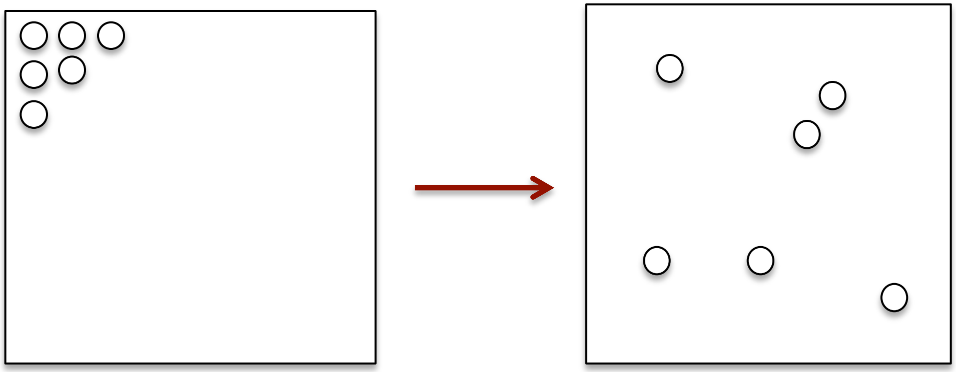


At fixed energy and particle number,
all accessible states are equally likely.

How does a system reach equilibrium?



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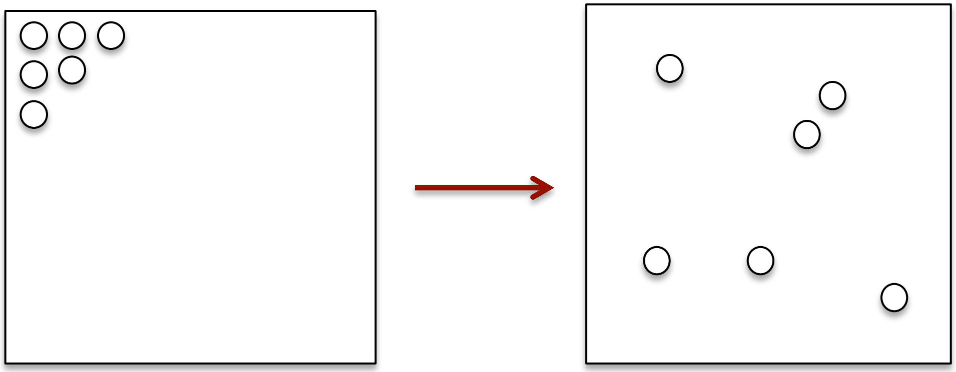

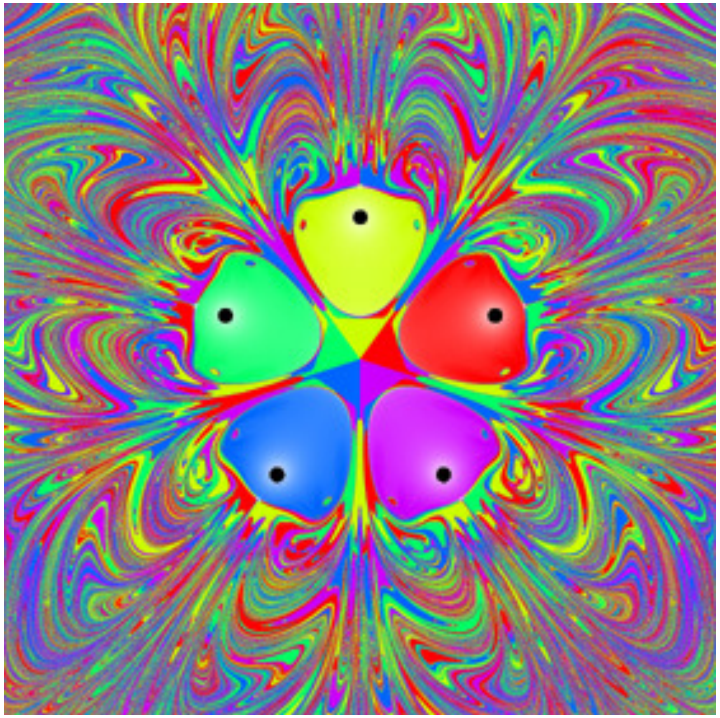

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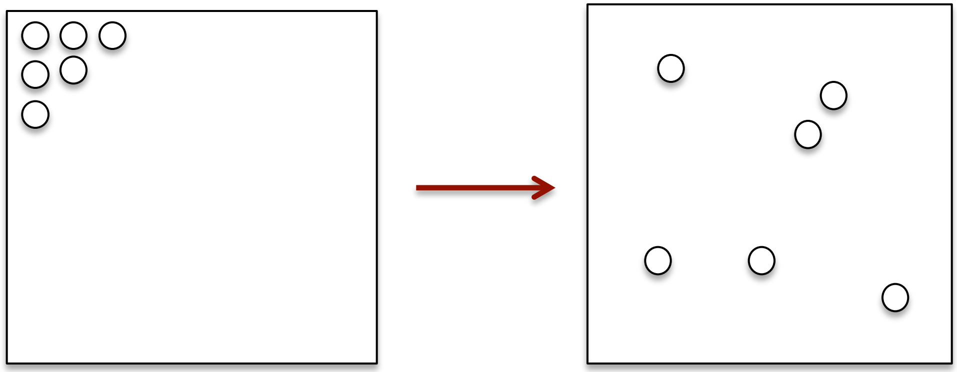
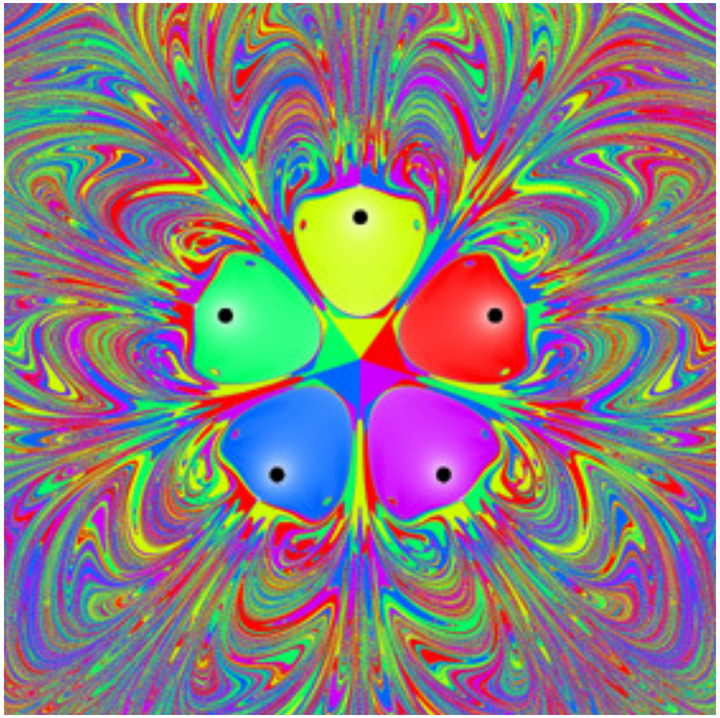
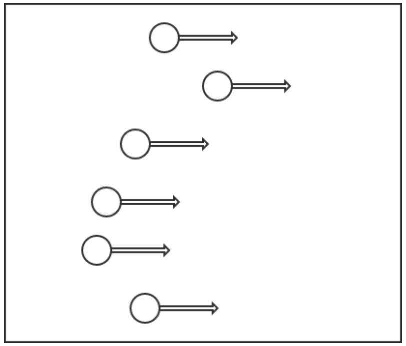






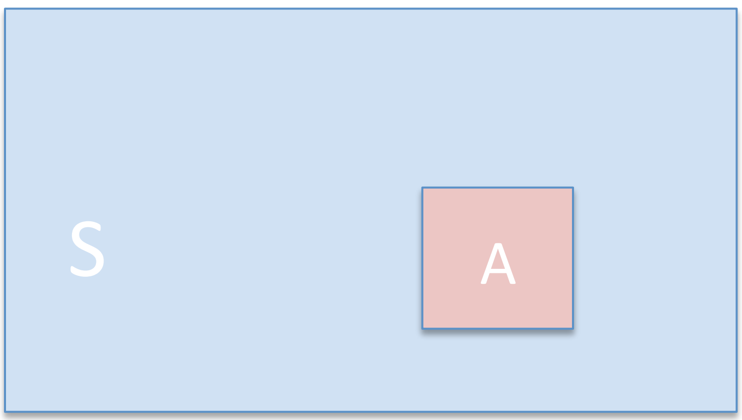
non-ergodic

What about isolated quantum systems?

Schrodinger equation linear → no chaotic dynamics

The ergodic hypothesis: $\langle \hat{O} \rangle_{time} = \langle \hat{O} \rangle_{ensemble}$

What does the ergodic hypothesis imply in quantum systems?
[Rigol, et al, Nature **452** 854 (2008)]



$$P_{Ai} \propto e^{-\epsilon_{Ai}/k_B T}$$

$$\text{where } T \leftrightarrow E_s$$

The rest of the system acts as
a thermal bath for the subsystem.

What happens when interactions are added to disordered systems?

Basko, Aleiner & Altshuler,
Annals of Physics **321** 1126 (2006)

Pal & Huse, PRB **82** 174411 (2010)

Bardarson, Pollmann & Moore
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Serbyn, Papic and Abanin,
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Huse, Nandkishore and Oganesyan,
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... and many more

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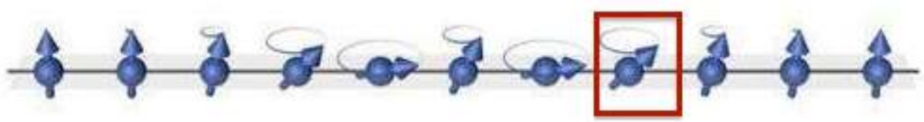
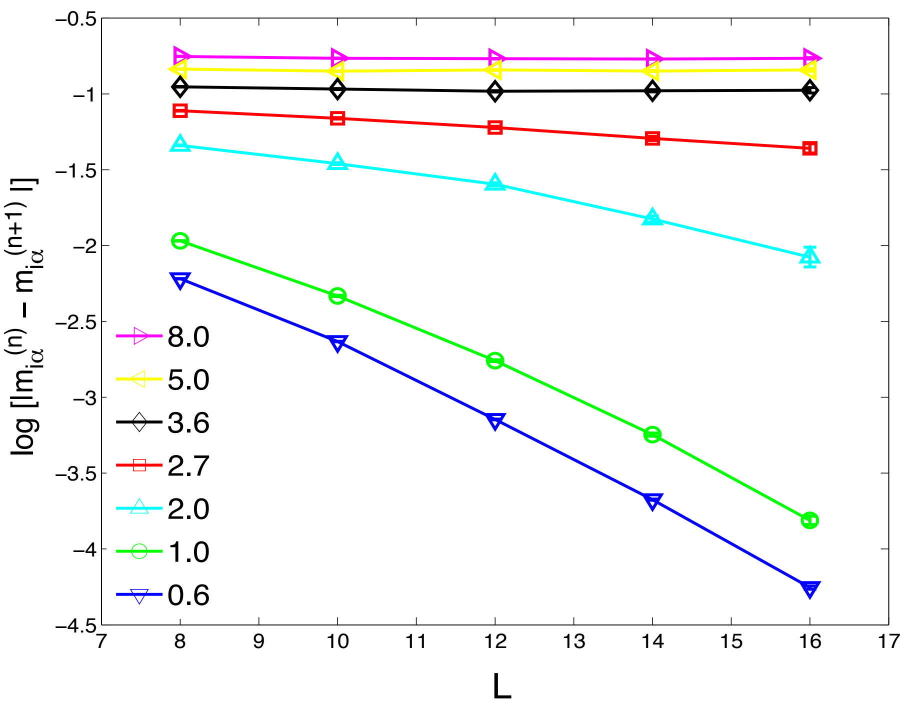
Many-body localized system does not act as a thermal bath for a subsystem.





Measures of many-body localization

Does not thermalize (non-ergodic)
No transport
Poisson statistics of level spacing
Many conserved local quantities
Localization in Fock space
Discrete local spectrum
Entanglement entropy satisfies area law
Logarithmic growth of the entanglement entropy
Dephasing without dissipation
Memory of initial conditions

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. . . what about length scale?

The spin Hamiltonian

$$H = \sum_{i} h_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \sum_{i,j,k} K_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

Huse, Nandkishore & Oganesyan,
PRB **90**, 174202 (2014)

Questions

What does the Anderson-Hubbard model look like written in terms of spins?

$$H = -t \sum_{\langle i,j \rangle,\sigma} (\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + \hat{c}^\dagger_{j\sigma}\hat{c}_{i\sigma}) + \sum_{i\sigma} \epsilon_i \hat{n}_{i\sigma} + U \sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$

What's the nature of the spins?

Can we use this to make connections between the different measures?

Why is a spin Hamiltonian always possible for Hilbert-space dimension 2^n ?

Consider Hilbert-space dimension 4

$$E_1, \; E_2, \; E_3, \; E_4$$

$$E_i \;=\; c_0 + c_1\sigma_{1i} + c_2\sigma_{2i} + c_3\sigma_{1i}\sigma_{2i}$$

i	σ_{1i}	σ_{2i}	E_i
1	-1	-1	$c_0 - c_1 - c_2 + c_3$
2	-1	$+1$	$c_0 - c_1 + c_2 - c_3$
3	$+1$	-1	$c_0 + c_1 - c_2 - c_3$
4	$+1$	$+1$	$c_0 + c_1 + c_2 + c_3$

Constructing a spin Hamiltonian for the 2-site Anderson-Hubbard model

$$H = -t\sum \sigma(\hat{c}^\dagger_{1\sigma}\hat{c}_{2\sigma} + \hat{c}^\dagger_{2\sigma}\hat{c}_{1\sigma}) + \sum_\sigma \epsilon_1 \hat{n}_{1\sigma} + \sum_\sigma \epsilon_2 \hat{n}_{2\sigma} + U\hat{n}_{1\uparrow}\hat{n}_{1\downarrow} + U\hat{n}_{2\uparrow}\hat{n}_{2\downarrow}$$

eigenstate	τ state
$\|0\rangle$	$\|----\rangle$
$\|m\uparrow\rangle$	$\tau_1^+\|----\rangle = \|+---\rangle$
$\|p\uparrow\rangle$	$\tau_2^+\|----\rangle = \|-+--\rangle$
$\|m\downarrow\rangle$	$\tau_3^+\|----\rangle = \|--+-\rangle$
$\|p\downarrow\rangle$	$\tau_4^+\|----\rangle = \|---+\rangle$
$\|t\uparrow\rangle$	$\tau_2^+\tau_1^+\|----\rangle = \|++--\rangle$
$\|t0\rangle$	$\tau_4^+\tau_1^+\|----\rangle = \|+--+\rangle$
$\|t\downarrow\rangle$	$\tau_4^+\tau_3^+\|----\rangle = \|--++\rangle$
$\|u1\rangle$	$\tau_3^+\tau_1^+\|----\rangle = \|+-+-\rangle$
$\|u2\rangle$	$\tau_3^+\tau_2^+\|----\rangle = \|-++-\rangle$
$\|u3\rangle$	$\tau_4^+\tau_2^+\|----\rangle = \|-+-+\rangle$
$\|3m\uparrow\rangle$	$\tau_3^+\tau_2^+\tau_1^+\|----\rangle = \|+++-\rangle$
$\|3p\uparrow\rangle$	$\tau_4^+\tau_2^+\tau_1^+\|----\rangle = \|++-+\rangle$
$\|3m\downarrow\rangle$	$\tau_4^+\tau_3^+\tau_1^+\|----\rangle = \|+-++\rangle$
$\|3p\downarrow\rangle$	$\tau_4^+\tau_3^+\tau_2^+\|----\rangle = \|-+++\rangle$
$\|4\rangle$	$\tau_4^+\tau_3^+\tau_2^+\tau_1^+\|----\rangle = \|++++\rangle$

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$\|4\rangle$	$\tau_4^+\tau_3^+\tau_2^+\tau_1^+\|----\rangle = \|++++\rangle$

$$E_4\ \tau_4^+\tau_3^+\tau_2^+\tau_1^+\tau_1^-\tau_2^-\tau_3^-\tau_4^-$$

$$\begin{aligned} E_{1m\uparrow}\Big(& \tau_1^+\tau_1^- - \tau_2^+\tau_1^+\tau_1^-\tau_2^- \\ & -\tau_4^+\tau_1^+\tau_1^-\tau_4^- \\ & -\tau_3^+\tau_1^+\tau_1^-\tau_3^- \\ & +\tau_3^+\tau_2^+\tau_1^+\tau_1^-\tau_2^-\tau_3^- \\ & +\tau_4^+\tau_2^+\tau_1^+\tau_1^-\tau_2^-\tau_4^- \\ & +\tau_4^+\tau_3^+\tau_1^+\tau_1^-\tau_3^-\tau_4^- \\ & -\tau_4^+\tau_3^+\tau_2^+\tau_1^+\tau_1^-\tau_2^-\tau_3^-\tau_4^- \Big) \end{aligned}$$

$$\tau_i^z = \tau_i^+\tau_i^- - \frac{1}{2}$$

Spin Hamiltonian for the 2-site Anderson-Hubbard model

$$
\begin{aligned}
H &= C_0 + C_1\tau_1^z + C_2\tau_2^z + C_3\tau_3^z + C_4\tau_4^z \\
&\quad + C_{12}\tau_1^z\tau_2^z + C_{13}\tau_1^z\tau_3^z + C_{14}\tau_1^z\tau_4^z \\
&\quad + C_{23}\tau_2^z\tau_3^z + C_{24}\tau_2^z\tau_4^z + C_{34}\tau_3^z\tau_4^z \\
&\quad + C_{123}\tau_1^z\tau_2^z\tau_3^z + C_{124}\tau_1^z\tau_2^z\tau_4^z \\
&\quad + C_{134}\tau_1^z\tau_3^z\tau_4^z + C_{234}\tau_2^z\tau_3^z\tau_4^z \\
&\quad + C_{1234}\tau_1^z\tau_2^z\tau_3^z\tau_4^z
\end{aligned}
$$

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& + C_{1234}\tau_1^z\tau_2^z\tau_3^z\tau_4^z
\end{aligned}
$$

atomic, non-interacting: $H = \epsilon_1\tau_1^z + \epsilon_2\tau_2^z + \epsilon_1\tau_3^z + \epsilon_2\tau_4^z$

interacting, atomic limit:

$$
H = \left(\epsilon_1 + \frac{U}{2}\right)(\tau_1^z + \tau_3^z) + \left(\epsilon_2 + \frac{U}{2}\right)(\tau_2^z + \tau_4^z) + U\tau_1^z\tau_3^z + U\tau_2^z\tau_4^z + \epsilon_1 + \epsilon_2 + \frac{U}{2}
$$

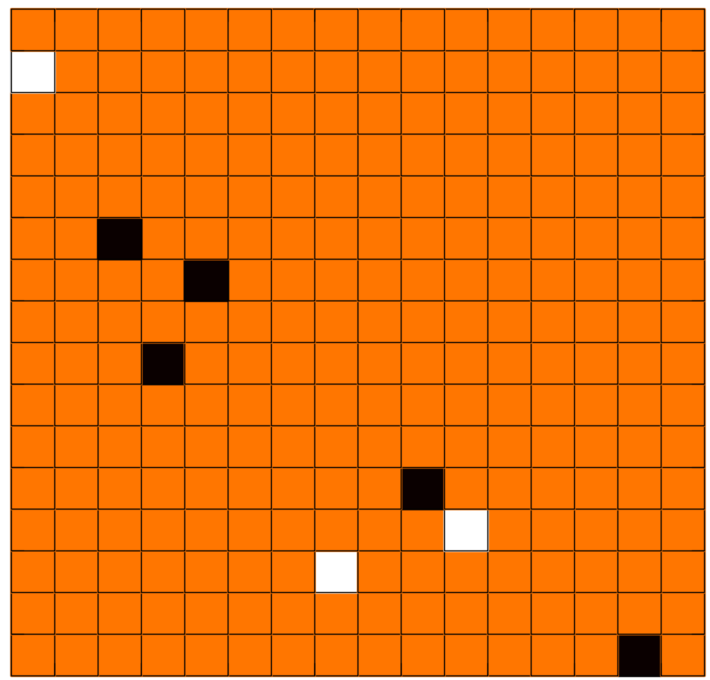
clean, non-interacting: $H = -t\tau_1^z + t\tau_2^z - t\tau_3^z + t\tau_4^z$

The spin operators

$\hat{\tau}_1^+$

t/W=0

U/W=0



$|00\rangle$

$|\uparrow 0\rangle$

$|0\uparrow\rangle$

$|\downarrow 0\rangle$

$|0\downarrow\rangle$

$|\uparrow\uparrow\rangle$

$(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$

$|\downarrow\downarrow\rangle$

$|20\rangle$

$(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$|02\rangle$

$|2\uparrow\rangle$

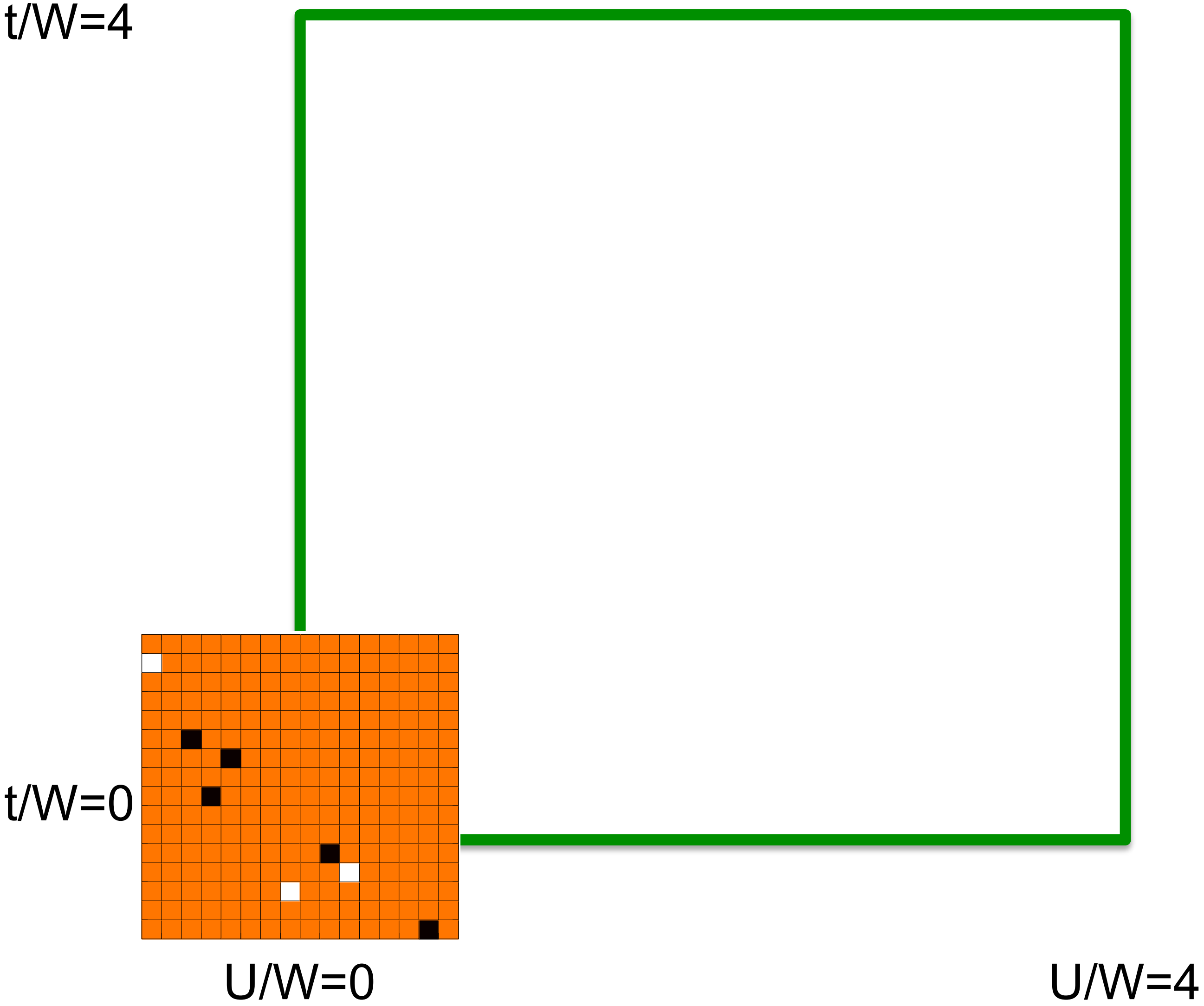
$|\uparrow 2\rangle$

$|2\downarrow\rangle$

$|\downarrow 2\rangle$

$|22\rangle$

The spin operators



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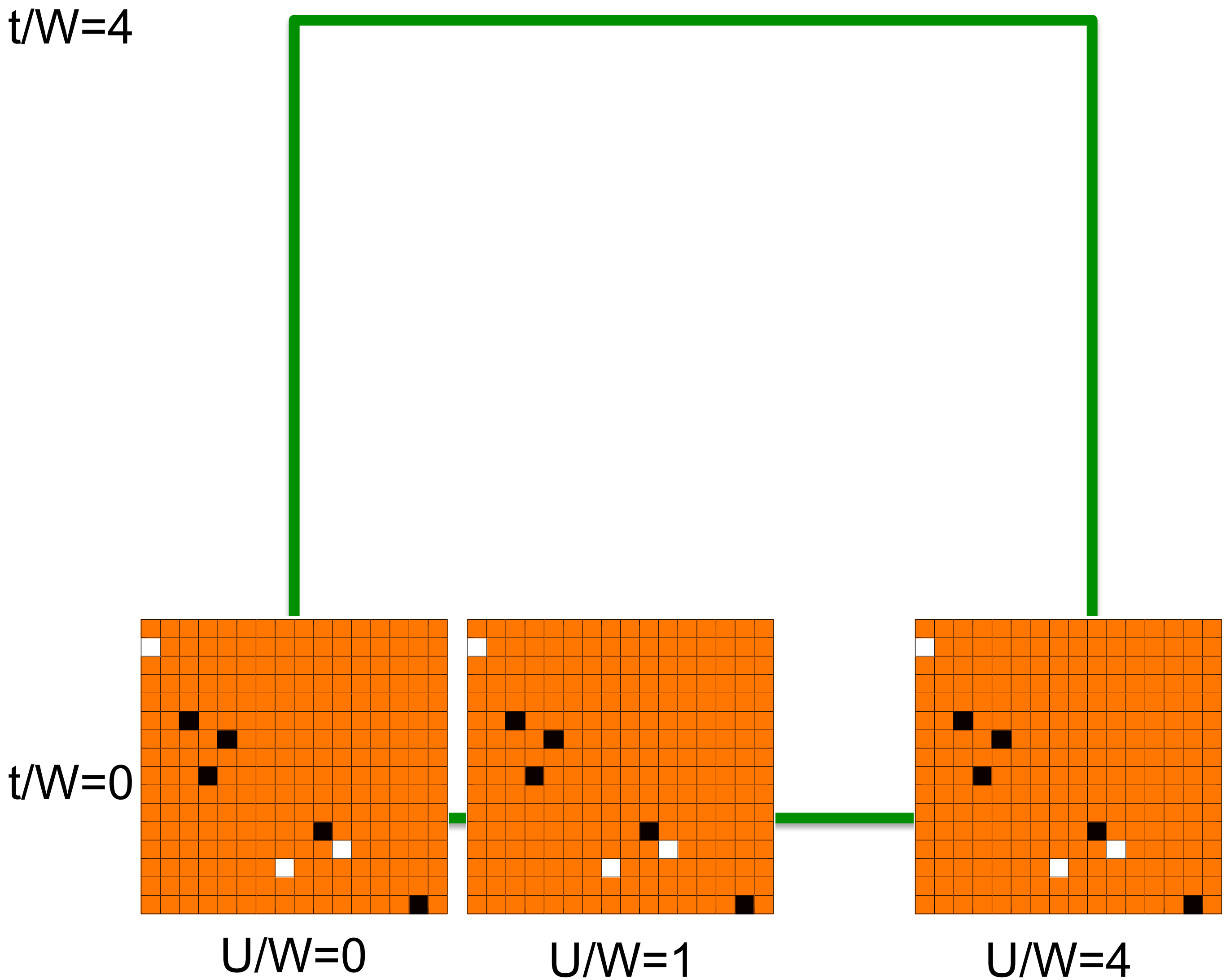

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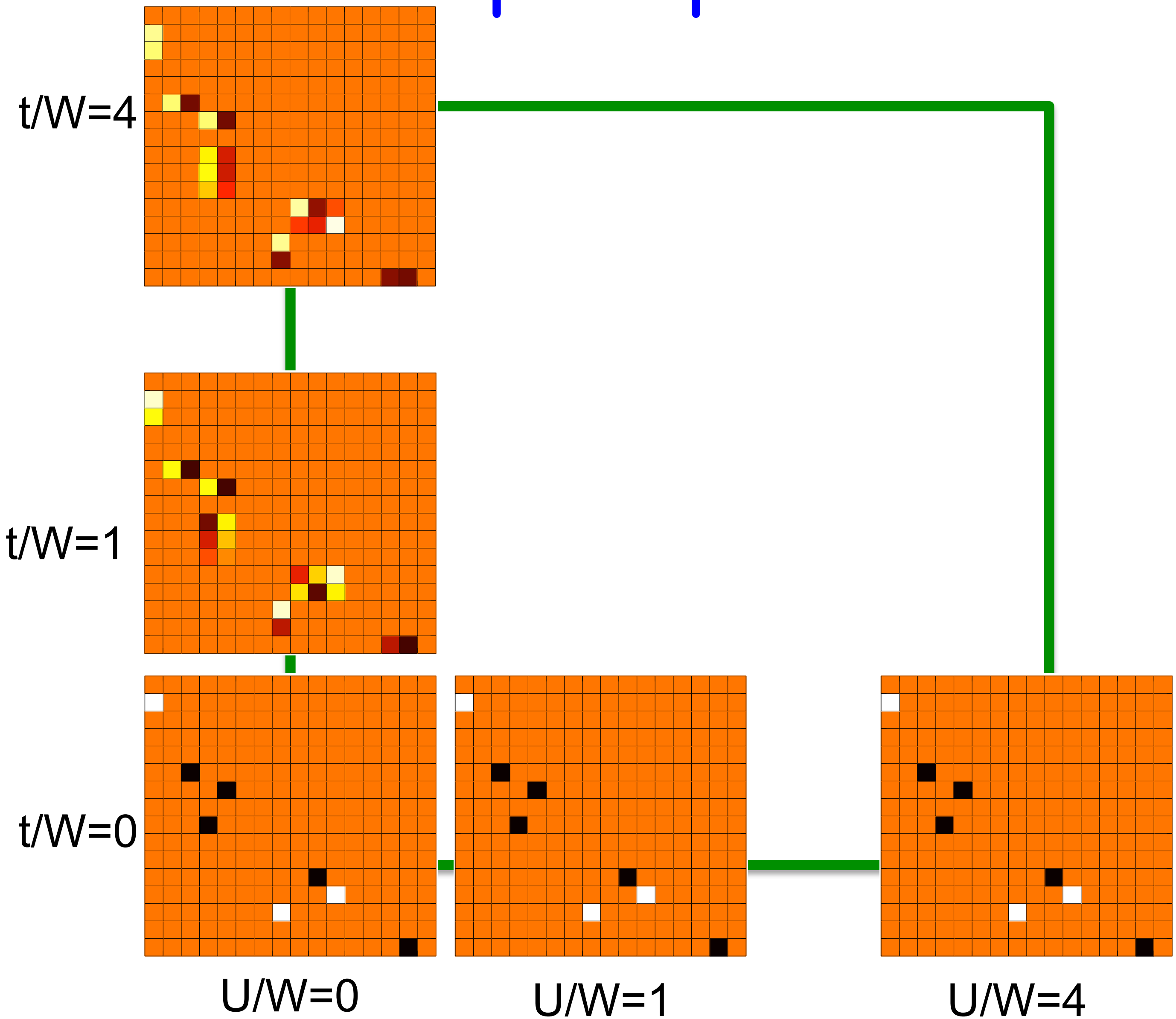

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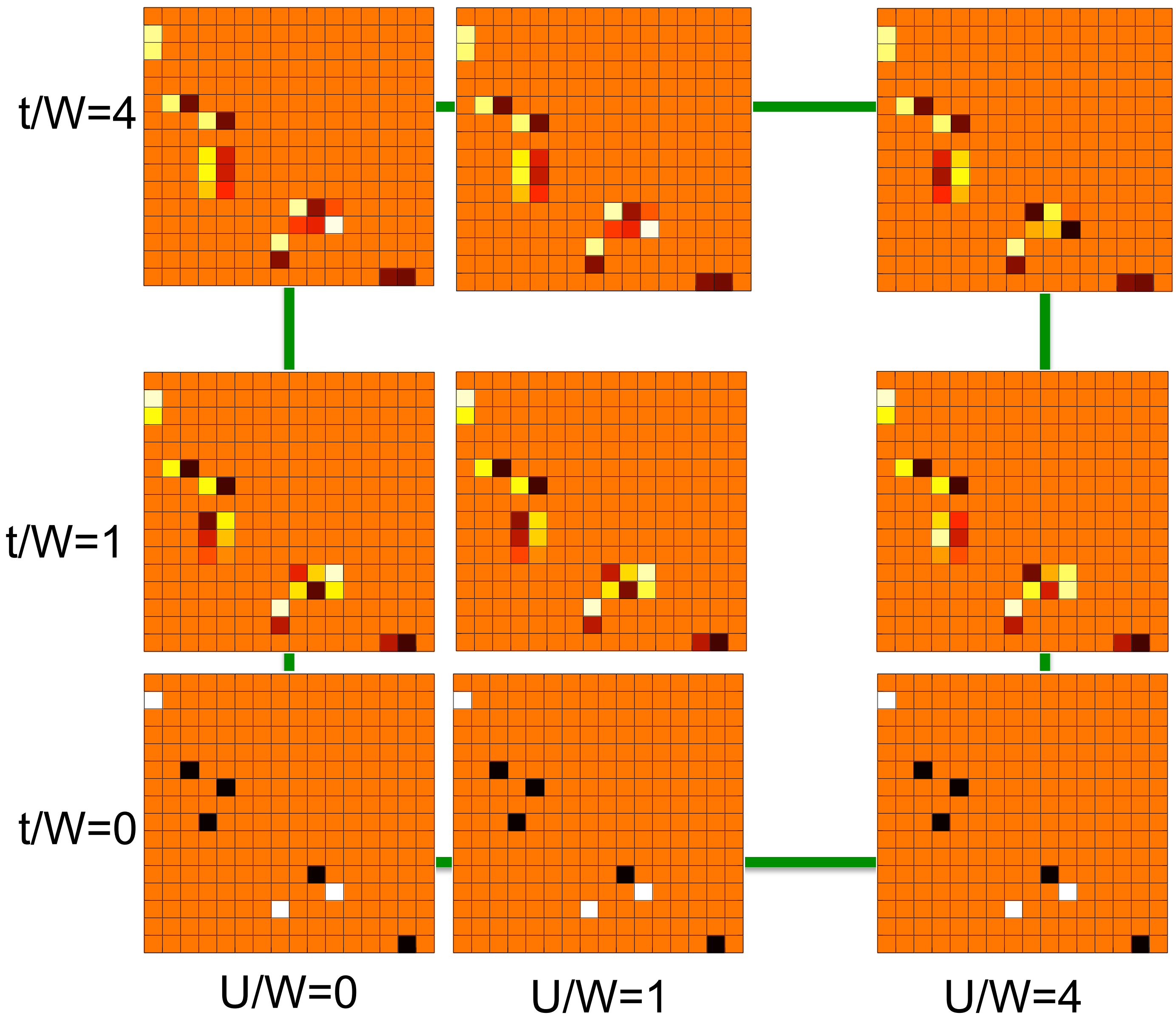
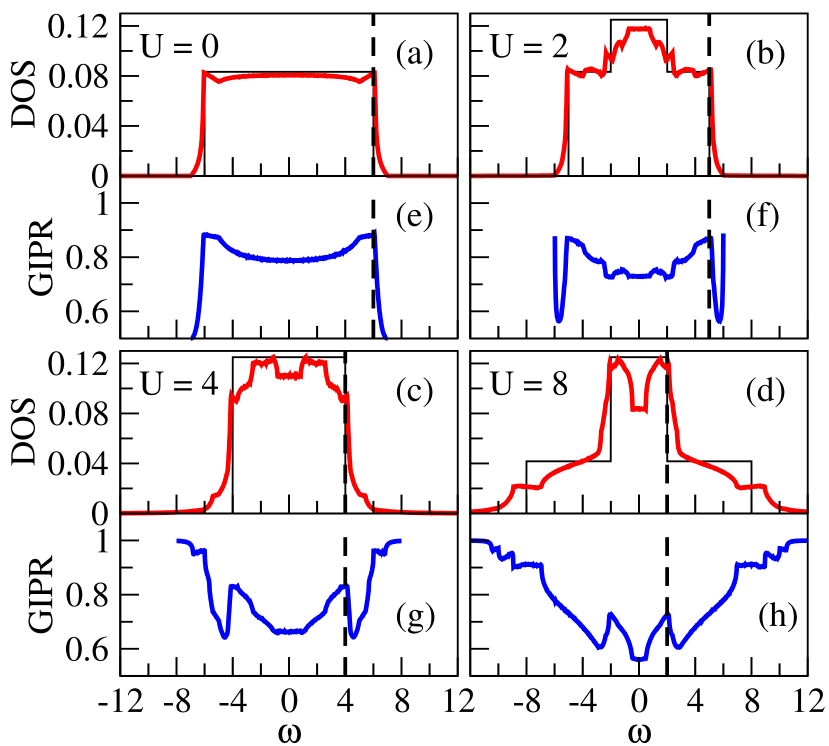
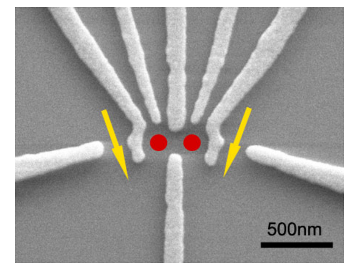
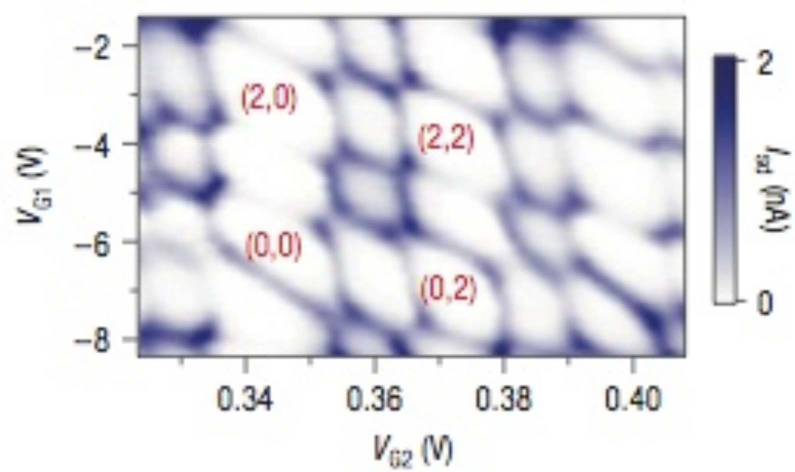


Seeing spins in experiments

The generalized inverse participation ratio measures the size of these spins.







Jorgensen, et al,
Nature Physics **4** 536 (2008)

Summary and next steps

A many-body localized system is non-ergodic.
Many measures of many-body localization have been proposed, but it's not clear they all measure the same thing.

Can the Anderson-Hubbard model be expressed in terms of Ising spins? Yes, and we've done it for the 2-site case.

Can examining the spins and their coefficients help clarify the connections between proposed measures?

Can the spin form of the 2-site Anderson-Hubbard model contribute to a renormalization group approach?