Modifications of Heisenberg's Uncertainty Principle Motivated by Quantum Gravity

2015 CAP Congress
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Based upon: *Generalized Uncertainty Principle Corrections to the Simple Harmonic Oscillator in Phase Space*

arXiv: 1412.6467 [gr-qc]

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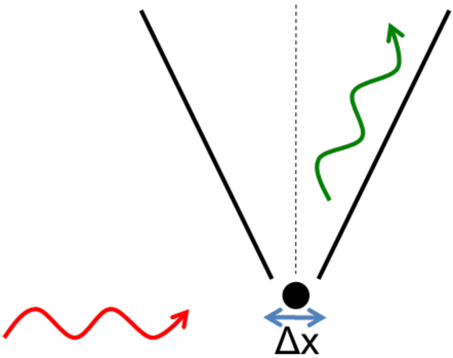
Outline

Motivations



Heisenberg's microscope: $\Delta x \Delta p \geq \frac{\hbar}{2}$

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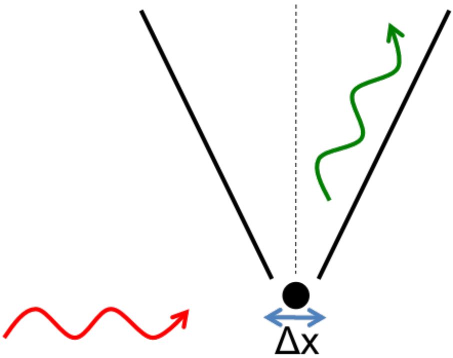

Heisenberg's microscope: $\Delta x \Delta p \geq \frac{\hbar}{2}$

Heisenberg's microscope + Black Hole: $\Delta x \Delta p \geq \frac{\hbar}{2}\left[1 + \beta_0 \frac{l_{Pl}^2}{\hbar^2}(\Delta p)^2\right]$

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Generalized Uncertainty Principle (GUP)

- Commutation Relations
  - Heisenberg: $[x, p] = i\hbar$

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    $f(p) = 1 + \beta p^2; \beta \sim l_{Pl}^2$
    $f(p) = 1 + \alpha p + \beta p^2; \alpha \sim l_{Pl}$

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- Predictions
  - Maximum uncertainty in momentum: $\Delta p \leq (\Delta p)_{max} \approx \frac{M_{Pl}c}{\alpha_0}$
  - Minimum uncertainty in length: $\Delta x \geq (\Delta x)_{min} \approx \alpha_0 l_{Pl}$

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Phase Space Quantum Mechanics and the Wigner Function

- Alternate formulation of quantum mechanics

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- Alternate formulation of quantum mechanics
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  - Quasiprobability distribution

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- Wigner function calculated from the momentum space wave function, ϕ:

$$W(x,p) = \frac{1}{\hbar}\int_{-\infty}^{\infty} \phi^*\left(p + u/2\right)\,\phi\left(p - u/2\right)\, e^{-ixu/\hbar}\, du$$

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  3. Probability for momentum: $P(p) = |\phi(p)|^2 = \int W(x,p)\, dx$
  4. Normalization: $\int W(x,p)\, dx\, dp = 1$

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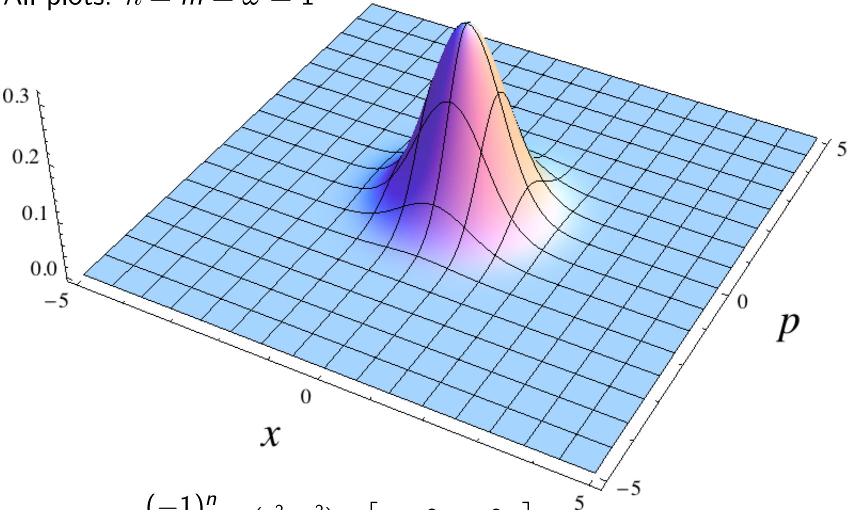
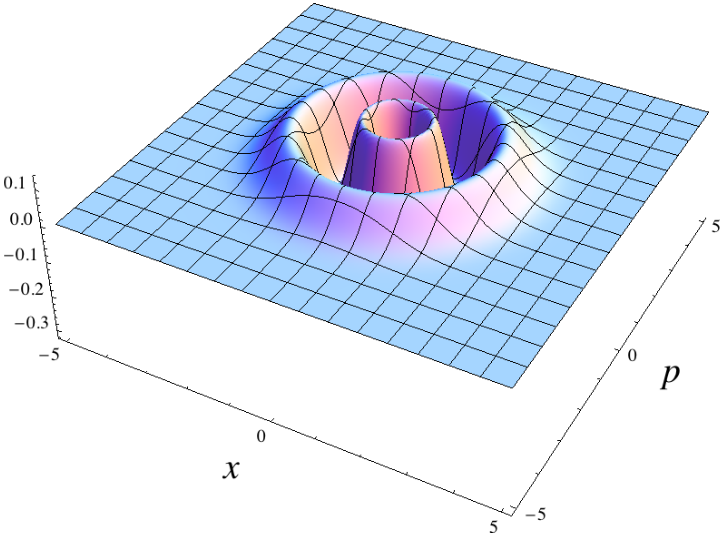
Wigner Function of the SHO (n=0)

All plots: $\hbar = m = \omega = 1$

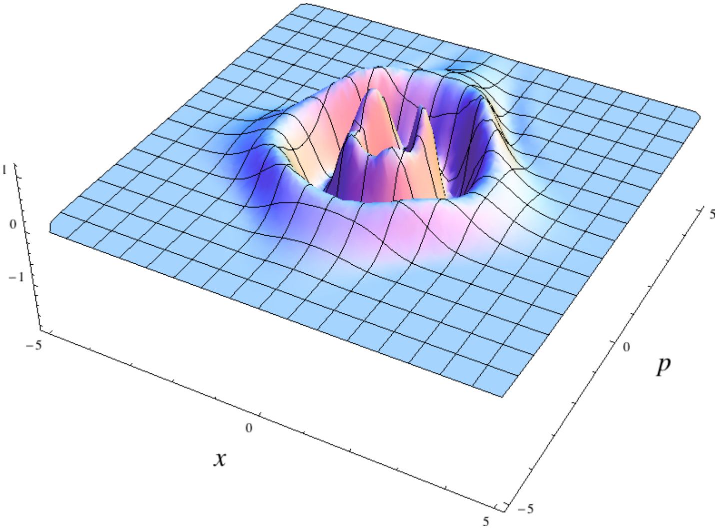


$$W(x,p) = \frac{(-1)^n}{\pi} e^{-(x^2+p^2)} L_n\left[2\left(x^2+p^2\right)\right]$$

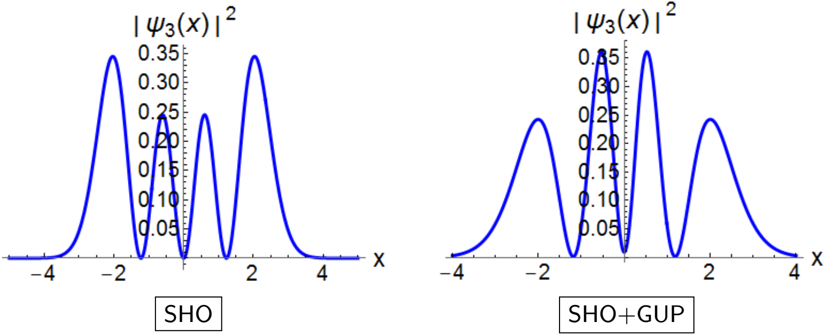
Wigner Function of the SHO (n=3)



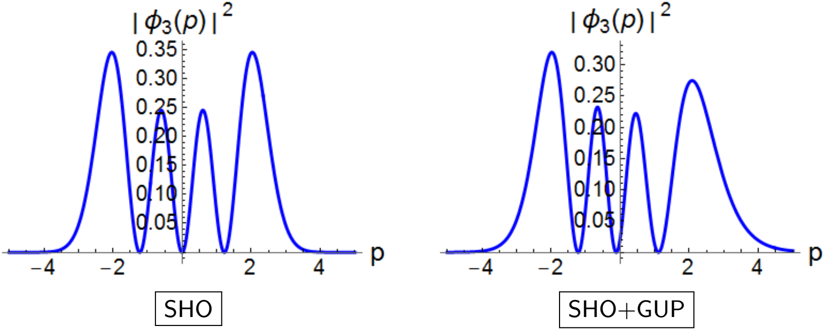
Wigner Function of SHO with $[x, p] = i\hbar\left(1 + \alpha p + \beta p^2\right)$ $n = 3$; $\alpha = 0.15$, $\beta = 0.1$



Comparison of Probabilities for Position: n=3

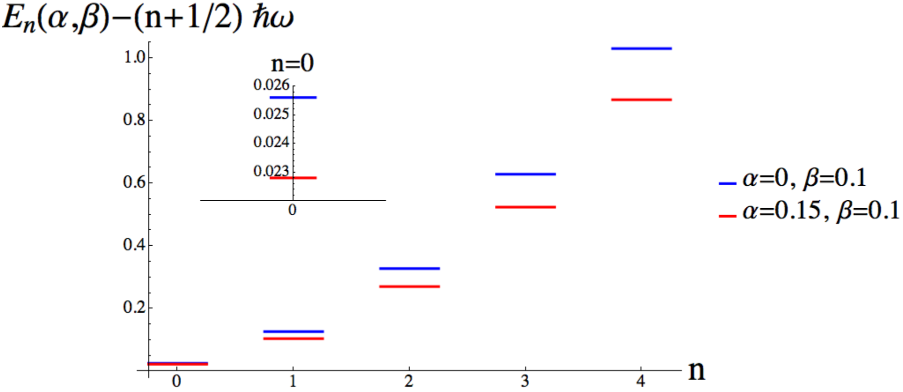


Comparison of Probabilities for Momentum: n=3



Energy Levels of GUP-corrected SHO

$$f(p) = 1 + \alpha p + \beta p^2$$



Summary and Conclusions

- GUP models quantum gravity corrections at low energies
- Able to visualize GUP effects using Wigner Functions
- These effects cause small perturbations in the position and momentum probabilities, as well as the energy levels of the SHO
- Values of GUP parameters (α, β) determine size of perturbations