Representation Theory for Multiphoton Interferometry

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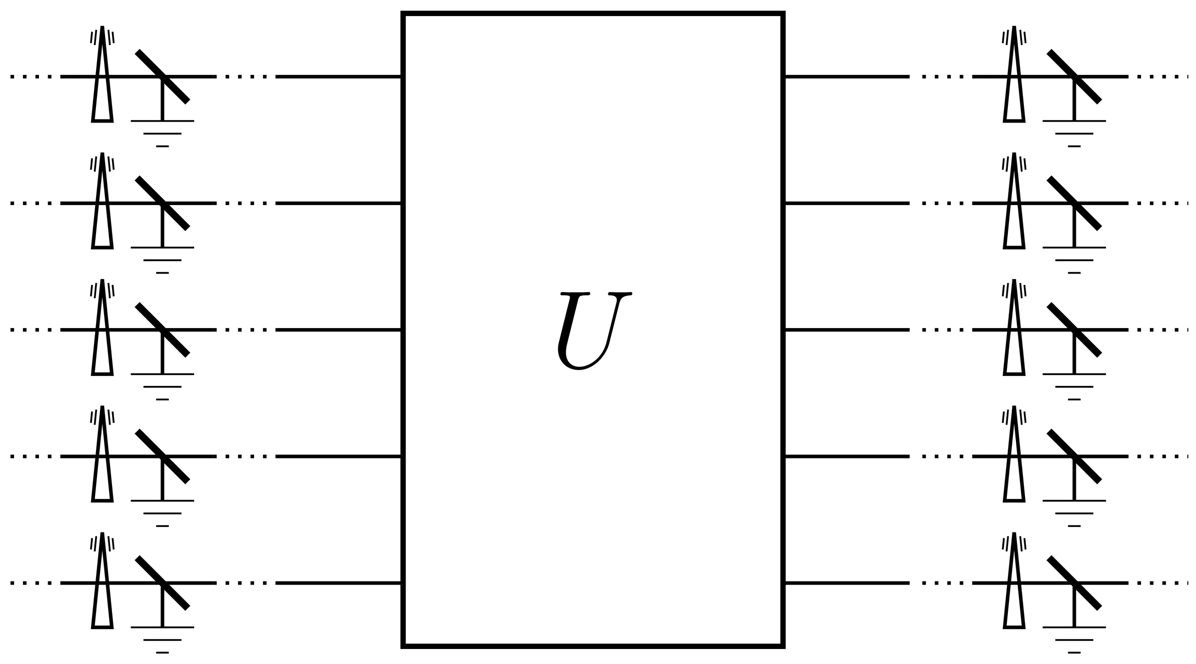
Multichannel Linear Optical Interferometer



Why Linear Optics? Theory and Experiment

Linear optics is powerful for quantum computing and control:

- Linear optics + detectors is universal for quantum computing,
- Sampling linear optics outputs is classically hard,
- Optical quantum walk nonclassical behaviour.

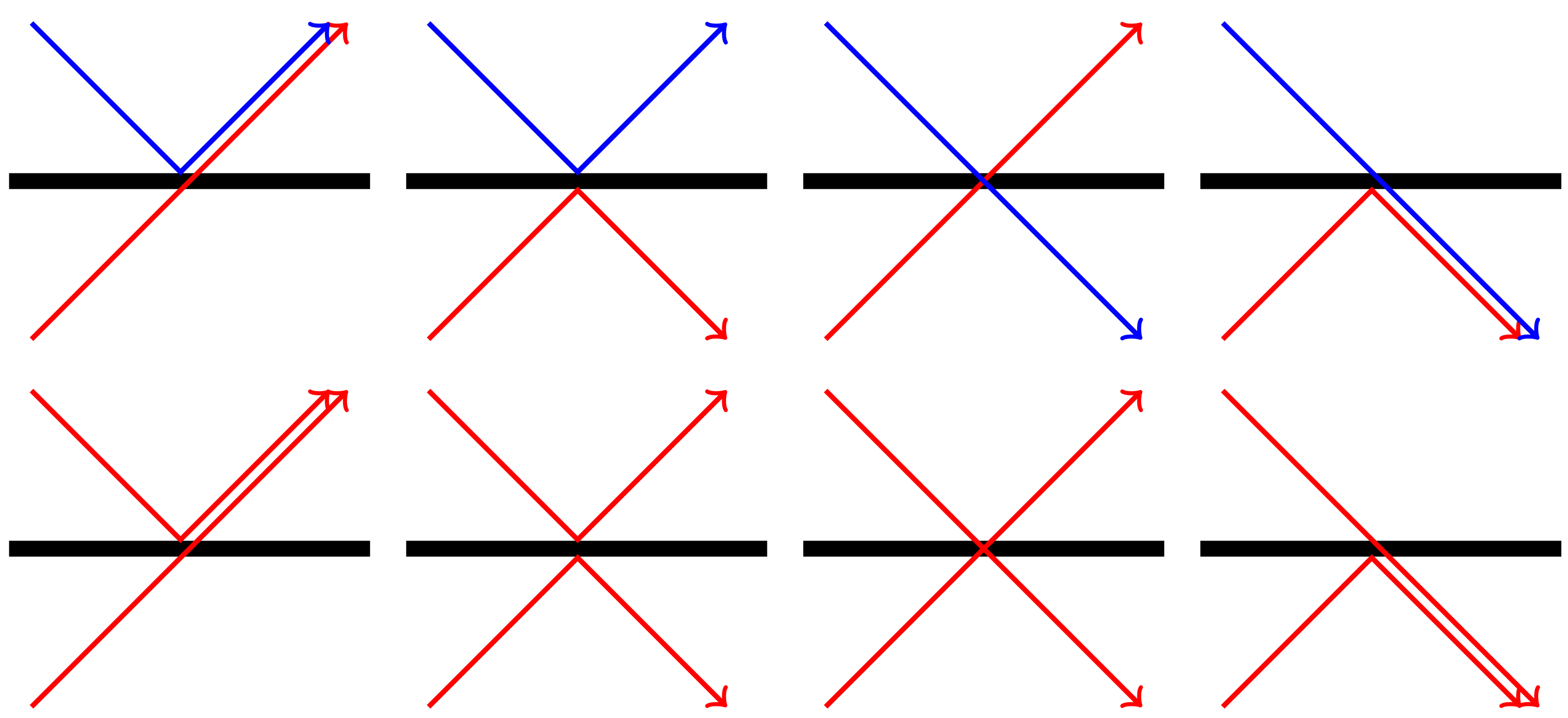
Rapid advance in experimental implementation:

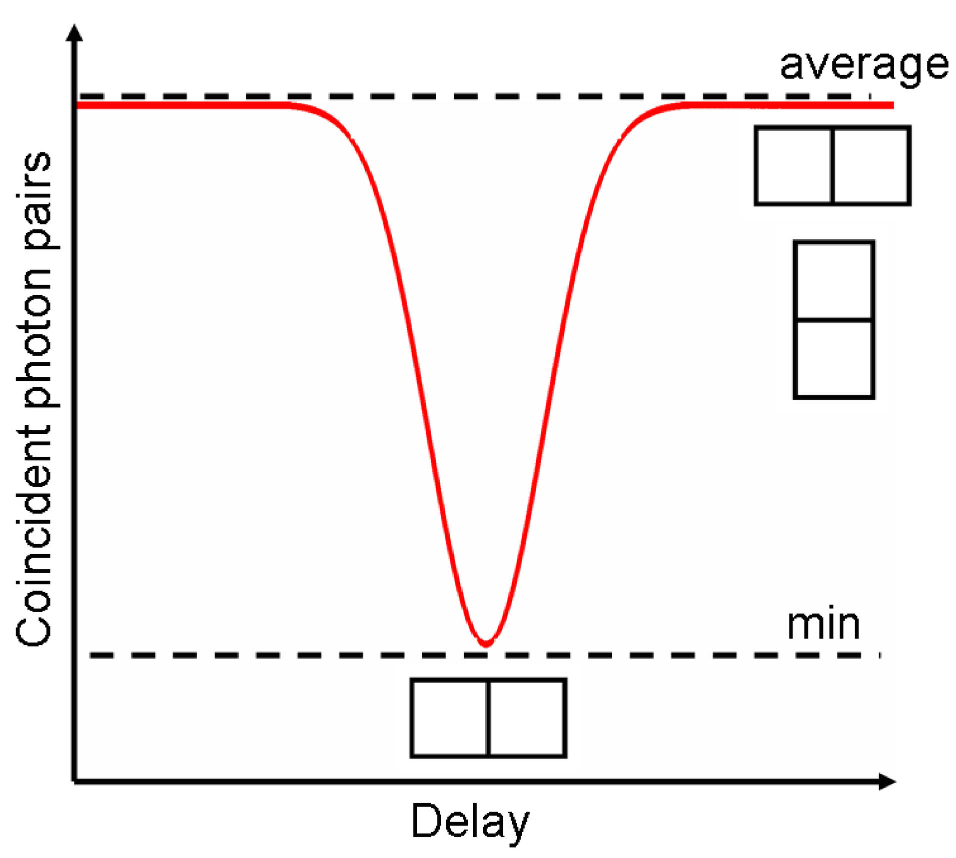
- On-demand single-photon sources,
- Efficient number-resolving detectors with low dark counts,
- Tunable photonic integrated circuits to implement arbitrary linear optics protocol.

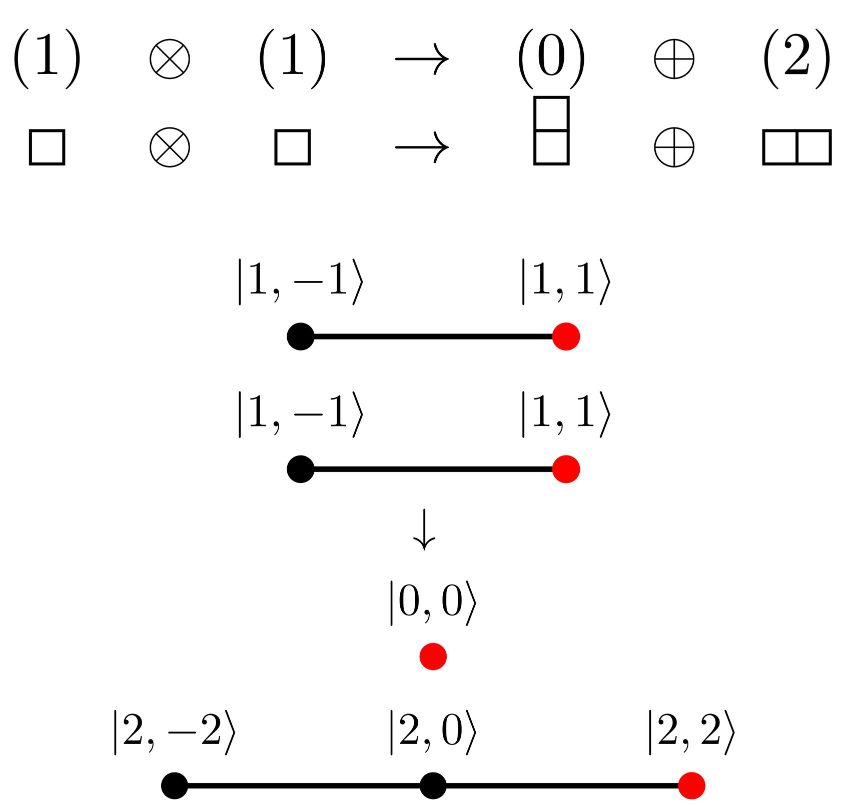
Problem

The current procedure for simulation of multi-photon multi-channel interferometers is suboptimal and not intuitive.

Enter Group Theory (Two-Photon Interference)







Brute Force Interferometer Simulation

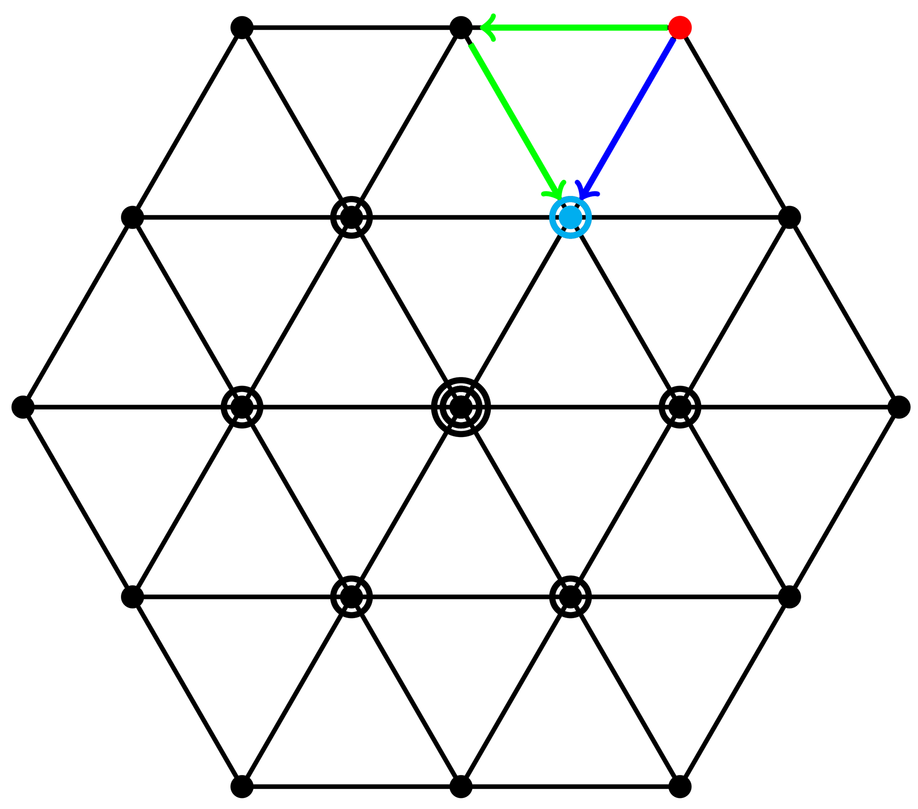
- n photons in m channels:

$$a_1^\dagger \to \sum_{j=1}^{m} U_{1j} a_j^\dagger$$
$$a_2^\dagger \to \sum_{j=1}^{m} U_{2j} a_j^\dagger$$
$$\dots$$
$$a_n^\dagger \to \sum_{j=1}^{m} U_{nj} a_j^\dagger.$$

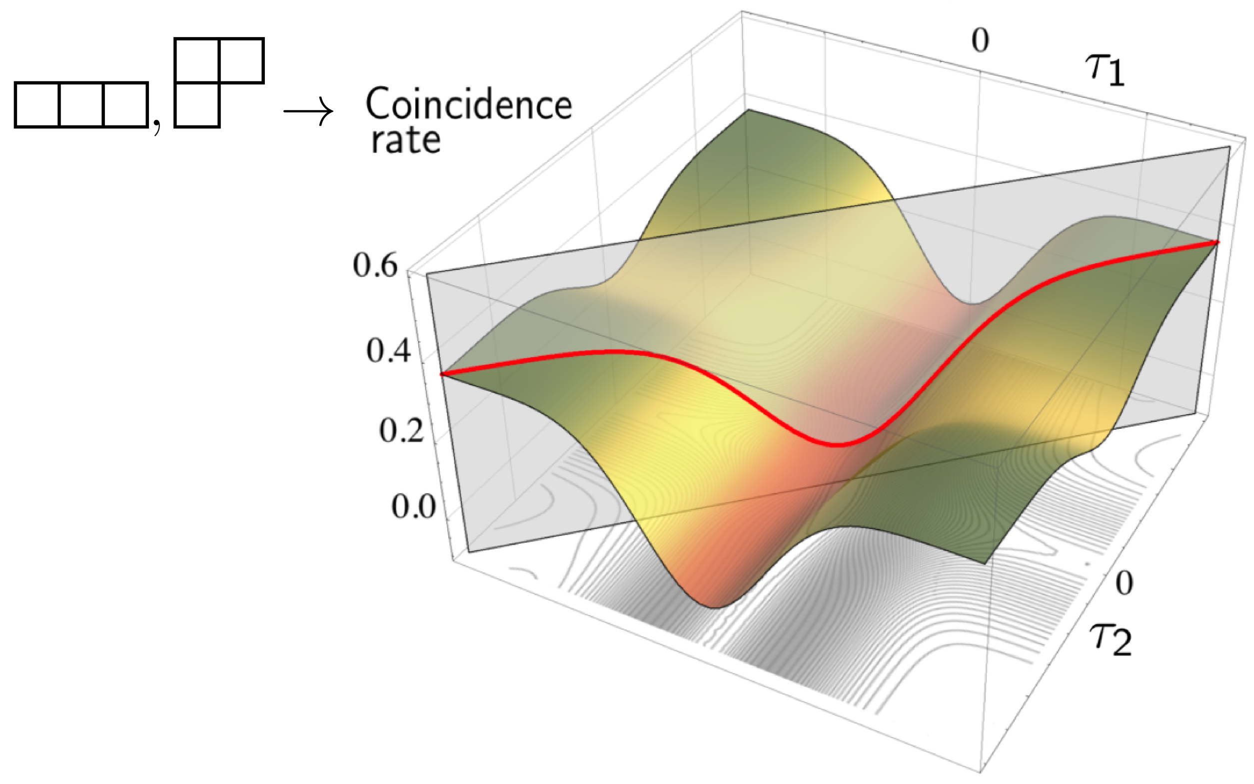
- Not intuitive for interpreting outputs.
- Expensive. $O(n!^2)$ cost of computing outputs.

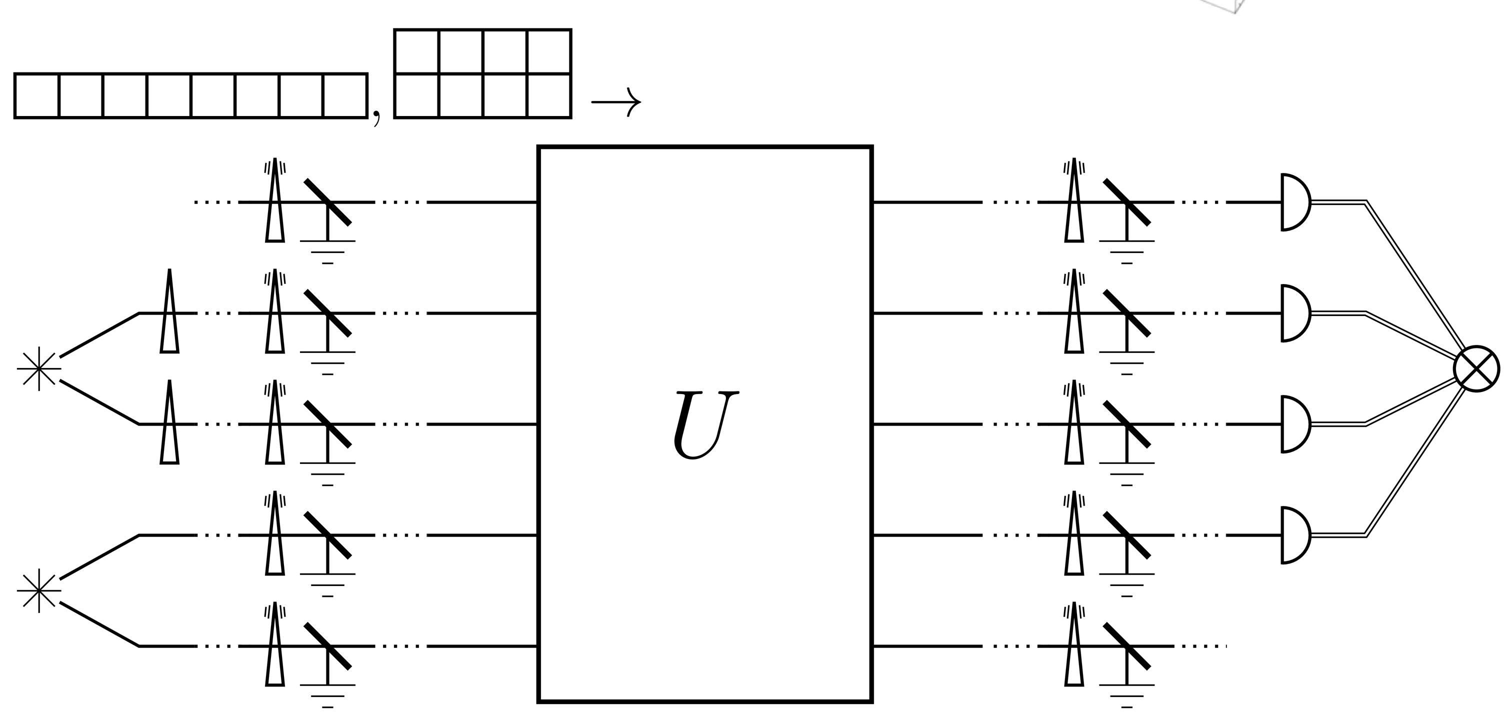
Results: Algorithms for Boson Realizations, $\mathcal{D}$-Functions and Interferometry

- New graph-theoretic algorithm for computing boson realizations and $\mathcal{D}$-functions of SU(n) irreps for arbitrary n.
- New algorithms for performing interferometry using $\mathcal{D}$-functions.



Results: Faster Interferometer Simulation





$O(n!)$ Simulation time.

Representation theory enables us to exploit permutation symmetries inherent in bosons to effect a reduction in the computational cost of simulating interferometers.