Tunneling decay of false kinks

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Introduction

The semiclassical theory of false vacuum decay by Coleman [1] tells us that, for a given quantum potential with false and true vacua, the tunneling decay width per unit volume is given by

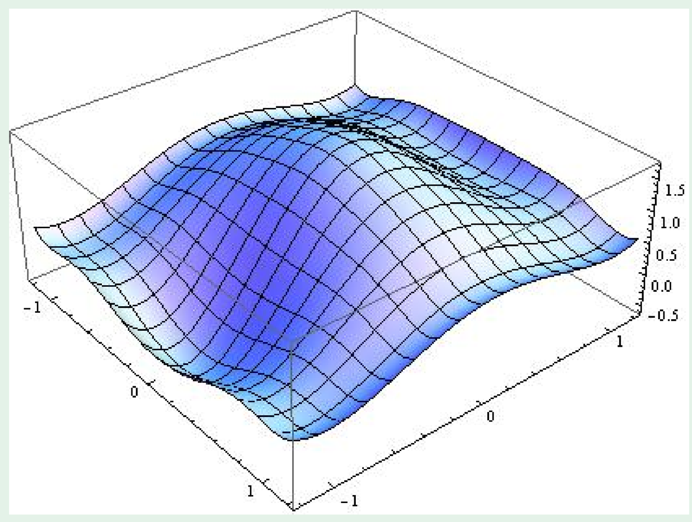
$$Ae^{-B/\hbar}$$

where B is the Euclidean action of the bounce (instanton) configuration studied. The decay of false kinks, or other solitons, which are formed by breaking a discrete symmetry, can also be studied in the same way.

Objectives

There are two objectives: 1) studying the existence and stability of false kinks and 2) determine whether the presence of kinks in 1+1 dimensions can significantly enhance false vacuum decay in the very early universe. We study a three-parameter model with two real scalar fields ϕ, χ with potential:

$$V = V_1 + V_2$$
$$V_1 = (\phi^2 - 1)^2(\phi^2 - \delta_1)$$
$$V_2 = \frac{(\chi^2 - 1)^2 - \frac{\delta_2}{4}(\chi - 2)(\chi + 1)^2}{\phi^2 + \gamma}$$

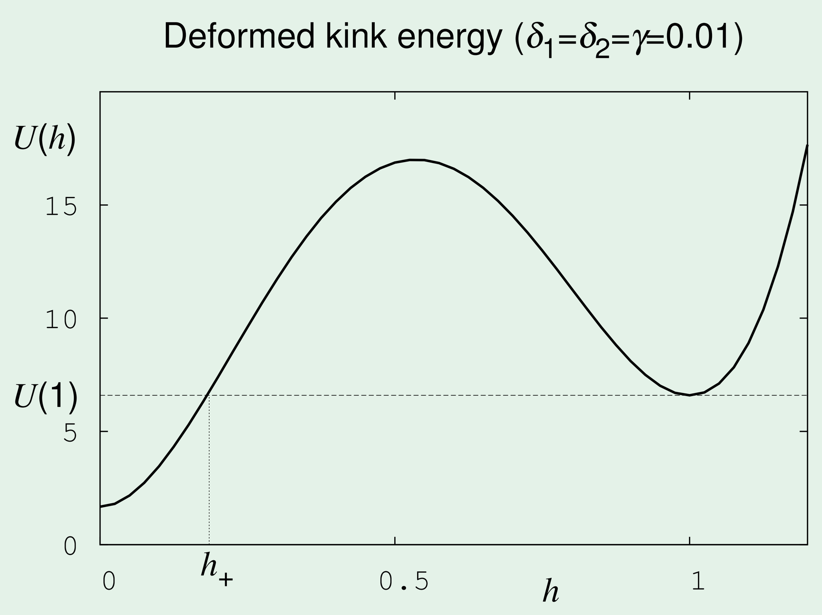


The potential for $\gamma = \delta_1 = 0.6, \delta_2 = 0.7$

Methods

The relaxation method was used to obtain the static solitons (ϕ_0, χ_0) from the equations of motion. To study tunneling, we devised a one-parameter family of configurations: $\chi = h(t)(\chi_0(x) + 1) - 1$. The bounce interpolates between $h = h_+$ (as shown in the figure below) and $h = 1$. The bounce action is:

$$S_k = \int_{h_+}^{1} \mathrm{d}h\sqrt{8MU}$$

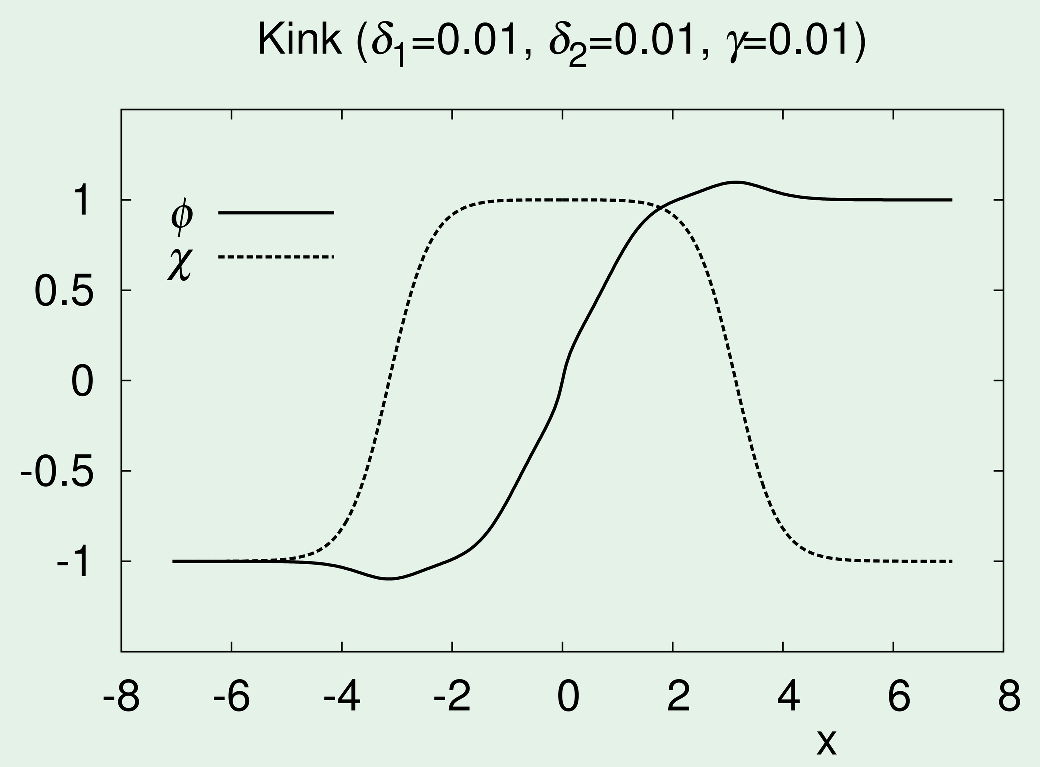


The kink energy as a function of h

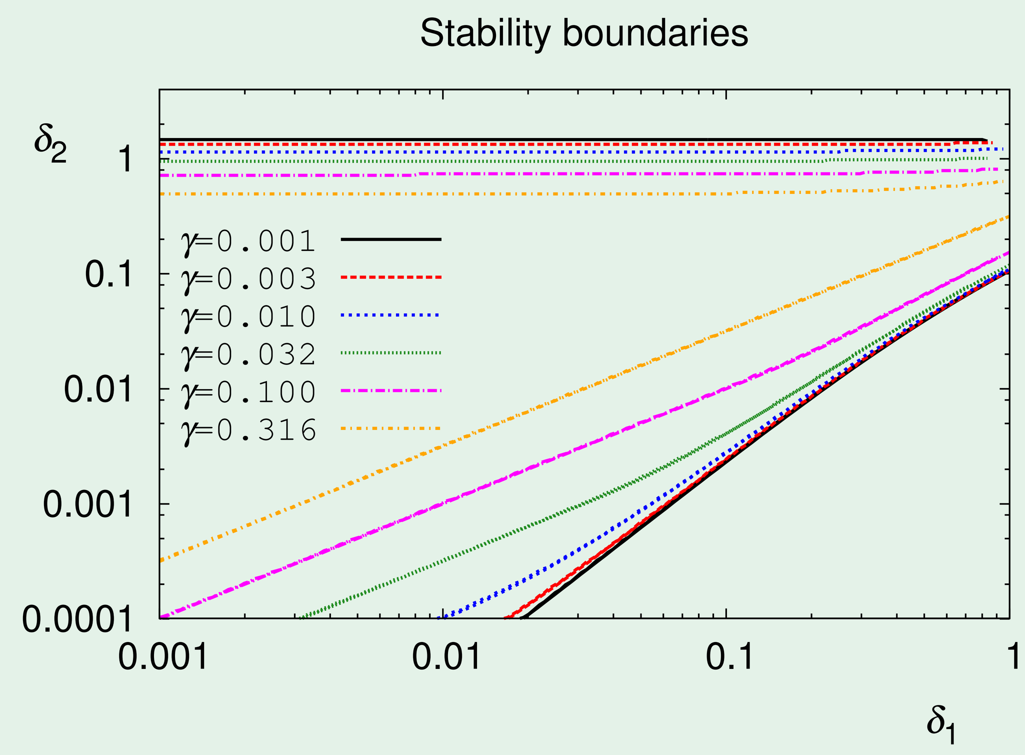
where M represents the kink mass and U the kink energy.

Results 1

The kink solution obtained by the relaxation method had the following characteristics: ϕ is antisymmetric while χ is symmetric. The solutions are found to decrease in spatial width as δ_2 increases.



The kink profile for $\gamma = \delta_1 = \delta_2 = 0.01$



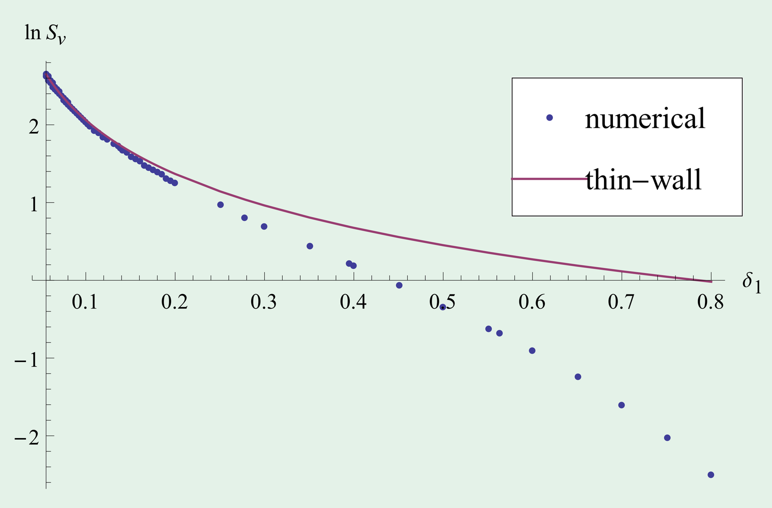
The dissociation zones

We found that there are two boundaries for the stability zones in parameter space. Approaching the the lower boundary, the kink decays to another false vacuum, while approaching the upper boundary, the kink decays to the true vacuum. Beyond the boundaries, no stable kinks exist.

Results 2

The Euclidean action of the kinks is found to depend mostly on δ_2, while the bounce action is found to depend entirely on δ_1. The bounce action is, in the thin-wall approximation ($\delta_1 \ll 1$):

$$S_v = \frac{\pi}{4\delta_1} \tag{1}$$



The bounce action, analytical and numerical

Conclusion

It was found that the ability of kinks to enhance false vacuum decay by quantum tunneling rises dramatically when they reach some point in parameter space where they become unstable. The false vacuum can be made arbitrarily stable even though it may not stabilize the kink under the considered deformation. Two dissociation zones were found to exist, whose locations were confirmed by other methods, including, but not limited to, the second variation [2].
Finally, one can define a critical kink density where the total kink decay width would be of the same order as that of the bounce. In the semiclassical limit, this gives:

$$\rho_c = \frac{A_v}{A_k} e^{S_k - S_v} = M_c e^{S_k - S_v}$$

where $M_c = A_v/A_k$ is a typical mass scale of the problem, on the order of the particle masses. When $\rho > \rho_c$, the kinks significantly catalyze vacuum decay.

References

[1] Sidney Coleman. Fate of the false vacuum: semiclassical theory. *Physical Review D*, 15(10):2929–2936, May 1977.

[2] E. Dupuis et al. Tunneling decay of false kinks. *pending*.

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Future research

Future research will examine the form of the A coefficient for both the kink and the bounce with greater sophistication.