Scale Invariance and Conformal Invariance in Quantum Field Theory

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mostly based on
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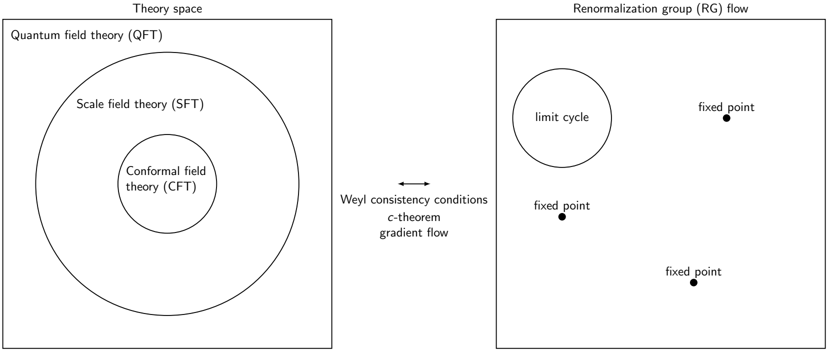
with Benjamín Grinstein and Andreas Stergiou

The big picture



Why is it interesting ?

QFT phases

- Infrared (IR) free
  - With mass gap ⇒ Exponentially-decaying correlation functions (e.g. Higgs phase)
  - Without mass gap ⇒ Trivial power-law correlation functions (e.g. Abelian Coulomb phase)
- IR interacting
  - CFTs ⇒ Power-law correlation functions (e.g. non-Abelian Coulomb phase)
  - SFTs ⇒ ?

Possible types of RG flows

- Strong coupling
- Weak coupling
  - Fixed points (e.g. Banks-Zaks fixed point Banks, Zaks (1982))
  - Recurrent behaviors (e.g. limit cycles or ergodic behaviors)

Outline

Preliminaries ($d > 2$)

- Dilatation current   Wess (1960)
  - $\mathcal{D}^\mu(x) = x^\nu T_\nu{}^\mu(x) - V^\mu(x)$
  - $T_\nu{}^\mu(x)$ any symmetric EM tensor following from spacetime nature of scale transformations
  - $V^\mu(x)$ local operator (virial current) contributing to scale dimensions of fields
  - Freedom in choice of $T_\nu{}^\mu(x)$ compensated by freedom in choice of $V^\mu(x)$
- Scale invariance $\Rightarrow$ $T_\mu{}^\mu(x) = \partial_\mu V^\mu(x)$

- Conformal current Wess (1960)
  - $\mathcal{C}_\nu{}^\mu(x) = v_\nu{}^\lambda(x) T_\lambda{}^\mu(x) - (\partial_\lambda v_\nu{}^\lambda)(x) V'^\mu(x) + (\partial_\rho \partial_\lambda v_\nu{}^\lambda)(x) L^{\rho\mu}(x)$
  - $T_\lambda{}^\mu(x)$ any symmetric EM tensor following from spacetime nature of conformal transformations
  - $V'^\mu(x)$ local operator corresponding to ambiguity in choice of dilatation current
  - $L^{\rho\mu}(x)$ local symmetric operator correcting position dependence of scale factor
  - $(\partial_\lambda v_\nu{}^\lambda)(x)$ scale factor (general linear function of x_ν)
  - Freedom in choice of $T_\lambda{}^\mu(x)$ compensated by freedom in choice of $V'^\mu(x)$ and $L^{\rho\mu}(x)$
- Conformal invariance $\Rightarrow$ $T_\mu{}^\mu(x) = \partial_\mu V'^\mu(x) = \partial_\mu \partial_\nu L^{\nu\mu}(x)$
- Conformal invariance $\Rightarrow$ Existence of symmetric traceless energy-momentum tensor Polchinski (1988)

Scale without conformal invariance

Non-conformal scale-invariant QFTs Polchinski (1988)

- Scale invariance $\Rightarrow T_\mu{}^\mu(x) = \partial_\mu V^\mu(x)$
- Conformal invariance $\Rightarrow T_\mu{}^\mu(x) = \partial_\mu \partial_\nu L^{\nu\mu}(x)$
- Scale without conformal invariance

  $\Rightarrow$ $T_\mu{}^\mu(x) = \partial_\mu V^\mu(x)$ where $V^\mu(x) \neq J^\mu(x) + \partial_\nu L^{\nu\mu}(x)$ with $\partial_\mu J^\mu(x) = 0$
- Constraints on possible virial current candidates
  - Gauge invariant (spatial integral)
  - Fixed $d-1$ scale dimension in d spacetime dimensions
- No suitable virial current $\Rightarrow$ Scale invariance implies conformal invariance (examples: ϕ^p in $d = n - \epsilon$ for $(p, n) = (6, 3), (4, 4)$ and $(3, 6)$)

Virial current candidates ($d = 4$)

Most general classically scale-invariant renormalizable theory in $d = 4 - \epsilon$ spacetime dimensions Jack, Osborn (1985)

$$
\begin{aligned}
\mathcal{L} = &- \mu^{-\epsilon} Z_A \frac{1}{4g_A^2} F^A_{\mu\nu} F^{A\mu\nu} + \frac{1}{2} Z^{\frac{1}{2}}_{ab} Z^{\frac{1}{2}}_{ac} D_\mu \phi_b D^\mu \phi_c \\
&+ \frac{1}{2} Z^{\frac{1}{2}*}_{ij} Z^{\frac{1}{2}}_{ik} \bar{\psi}_j i \bar{\sigma}^\mu D_\mu \psi_k - \frac{1}{2} Z^{\frac{1}{2}*}_{ij} Z^{\frac{1}{2}}_{ik} D_\mu \bar{\psi}_j i \bar{\sigma}^\mu \psi_k \\
&- \frac{1}{4!} \mu^\epsilon (\lambda Z^\lambda)_{abcd} \phi_a \phi_b \phi_c \phi_d \\
&- \frac{1}{2} \mu^{\frac{\epsilon}{2}} (y Z^y)_{a|ij} \phi_a \psi_i \psi_j - \frac{1}{2} \mu^{\frac{\epsilon}{2}} (y Z^y)^*_{a|ij} \phi_a \bar{\psi}_i \bar{\psi}_j
\end{aligned}
$$

- $\phi_a(x)$ real scalar fields
- $\psi_i^\alpha(x)$ Weyl fermions
- $A_\mu^A(x)$ gauge fields
- Dimensional regularization with minimal subtraction

Virial current candidates and new improved EM tensor

- Virial current $V^\mu(x) = Q_{ab}\phi_a D^\mu \phi_b - P_{ij}\bar{\psi}_i i\bar{\sigma}^\mu \psi_j$
  - $Q_{ba} = -Q_{ab}$
  - $P^*_{ji} = -P_{ij}$
- New improved energy-momentum tensor $[\Theta_\nu{}^\mu(x)]$ Callan, Coleman, Jackiw (1970)
  - Finite and not renormalized (vanishing anomalous dimension)
  - Anomalous trace Osborn (1989,1991) & Jack, Osborn (1990)

$$[\Theta_\mu{}^\mu(x)] = \frac{\mathrm{B}_A}{2g_A^3}[F^A_{\mu\nu}F^{A\mu\nu}] - \frac{1}{4!}\mathrm{B}_{abcd}[\phi_a\phi_b\phi_c\phi_d] - \frac{1}{2}\left(\mathrm{B}_{a|ij}[\phi_a\psi_i\psi_j] + \mathrm{h.c.}\right) - ((\delta+\Gamma)f)\cdot\frac{\delta}{\delta f}\mathcal{A}$$

- Anomalous trace

$$[\Theta_\mu{}^\mu(x)] = \mathrm{B}^I[\mathcal{O}_I(x)] - ((\delta+\Gamma)f)\cdot\frac{\delta}{\delta f}A$$

- Conserved dilatation current $\partial_\mu \mathcal{D}^\mu(x) = 0$ (up to EOMs)

$$\mathrm{B}^I = \mathcal{Q}^I \equiv -(gQ)^I$$

- Conserved conformal current $\partial_\mu \mathcal{C}_\nu{}^\mu(x) = 0$ (up to EOMs)

$$\mathrm{B}^I = 0$$

$\Rightarrow$ Both SFT ($Q \neq 0$) and CFT ($Q = 0$) can be treated simultaneously

Virial current and unitarity bounds

- New improved energy-momentum tensor $\Rightarrow$ Finite and not renormalized Callan, Coleman, Jackiw (1970)
- Operators related to EOMs $\Rightarrow$ Finite and not renormalized Politzer (1980) & Robertson (1991)
- Virial current $\Rightarrow$ Finite and not renormalized
  - Unconserved current with scale dimension exactly 3
- Unitarity bounds for conformal versus scale-invariant QFTs Grinstein, Intriligator, Rothstein (2008)
- Non-trivial virial current $\Rightarrow$ Non-conformal scale-invariant QFTs

RG flows along scale-invariant trajectories

Scale-invariant solution $(\lambda_{abcd}, y_{a|ij}, g_A) \Rightarrow$ RG trajectory

$$\begin{aligned}
\bar{\lambda}_{abcd}(t) &= \widehat{Z}_{a'a}(t)\widehat{Z}_{b'b}(t)\widehat{Z}_{c'c}(t)\widehat{Z}_{d'd}(t)\lambda_{a'b'c'd'}\\
\bar{y}_{a|ij}(t) &= \widehat{Z}_{a'a}(t)\widehat{Z}_{i'i}(t)\widehat{Z}_{j'j}(t)y_{a'|i'j'}\\
\bar{g}_A(t) &= g_A
\end{aligned}$$

$$\left.\begin{aligned}
\widehat{Z}_{a'a}(t) &= \left(e^{Qt}\right)_{a'a}\\
\widehat{Z}_{i'i}(t) &= \left(e^{Pt}\right)_{i'i}
\end{aligned}\right\} \quad t = \ln(\mu_0/\mu) \quad \text{(RG time)}$$

- $(\bar{\lambda}_{abcd}(t,g,\lambda,y), \bar{y}_{a|ij}(t,g,\lambda,y), \bar{g}_A(t,g,\lambda,y))$ also scale-invariant solution
- Q_{ab} and P_{ij} constant along RG trajectory
- $\widehat{Z}_{ab}(t)$ orthogonal and $\widehat{Z}_{ij}(t)$ unitary $\Rightarrow$ Always non-vanishing beta-functions along scale-invariant trajectory

Scale invariance and recurrent behaviors

RG flows along scale-invariant trajectories $\Rightarrow$ Recurrent behaviors !

Lorenz (1963,1964), Wilson (1971) & Kogut, Wilson (1974)

- Virial current $\Rightarrow$ Transformation in symmetry group of kinetic terms ($SO(N_S) \times U(N_F)$)
  - $\widehat{Z}_{ab}(t)$ and $\widehat{Z}_{ij}(t)$ in $SO(N_S) \times U(N_F)$
  - Q_{ab} antisymmetric and P_{ij} antihermitian $\Rightarrow$ Purely imaginary eigenvalues

$\Rightarrow$ Periodic (limit cycle) or quasi-periodic (ergodicity) scale-invariant trajectories

Recurrent behaviors

Intuition from $\mathcal{D}^\mu(x) = x^\nu T_\nu{}^\mu(x) - V^\mu(x)$

- RG flow $\Rightarrow$ Generated by scale transformation $(x^\nu T_\nu{}^\mu(x))$
- RG flow $\Rightarrow$ Related to virial current through conservation of dilatation current
- Virial current $\Rightarrow$ Generates internal transformation of the fields
  - Internal transformation in compact group $SO(N_S) \times U(N_F)$

  $\Rightarrow$ Rotate back to or close to identity
- RG flow return back to or close to identity $\Rightarrow$ Recurrent behavior

Why dilatation generators generate dilatations

Dilatation generators do not generate dilatations in non-scale-invariant QFTs Coleman, Jackiw (1971)

- Quantum anomalies at low orders
  - Anomalous dimensions
  - ⇒ Possible to absorb into redefinition of scale dimensions of fields
  - ✔ Preserve scale invariance
- Quantum anomalies at high orders
  - Beta-functions
  - ⇒ Not possible to absorb
  - ✗ Break scale invariance

Why dilatation generators generate dilatations in scale-invariant QFTs ?

- Beta-functions on scale-invariant trajectories
  - Both vertex correction and wavefunction renormalization contributions
  - Very specific form for vertex correction contribution
  - Equivalent in form to wavefunction renormalization contribution (redundant operators)

  $\Rightarrow$ Also possible to absorb into redefinition of scale dimensions of fields

  ✔ Preserve scale invariance !

- Beta-functions from vertex corrections and wavefunction renormalizations ($d = 4$ spacetime dimensions)

$$
\begin{aligned}
\mathrm{B}_{abcd} &= -\frac{d\lambda_{abcd}}{dt} \\
&= -(\lambda\gamma^{\lambda})_{abcd} + \lambda_{a'bcd}\Gamma_{a'a} + \lambda_{ab'cd}\Gamma_{b'b} + \lambda_{abc'd}\Gamma_{c'c} + \lambda_{abcd'}\Gamma_{d'd} \\
\mathrm{B}_{a|ij} &= -\frac{dy_{a|ij}}{dt} = -(y\gamma^{y})_{a|ij} + y_{a'|ij}\Gamma_{a'a} + y_{a|i'j}\Gamma_{i'i} + y_{a|ij'}\Gamma_{j'j} \\
\mathrm{B}_{A} &= -\frac{dg_A}{dt} = \gamma_A g_A \quad \text{(no sum)}
\end{aligned}
$$

- Beta-functions on scale-invariant trajectories

$$
\begin{aligned}
\mathrm{B}_{abcd} &= -\lambda_{a'bcd}Q_{a'a} - \lambda_{ab'cd}Q_{b'b} - \lambda_{abc'd}Q_{c'c} - \lambda_{abcd'}Q_{d'd} \\
\mathrm{B}_{a|ij} &= -y_{a'|ij}Q_{a'a} - y_{a|i'j}P_{i'i} - y_{a|ij'}P_{j'j} \\
\mathrm{B}_{A} &= 0
\end{aligned}
$$

Ward identity for scale invariance

Callan-Symanzik equation for effective action Callan (1970) & Symanzik (1970)

$$\left[M\frac{\partial}{\partial M}+B^I\frac{\partial}{\partial g^I}+\Gamma_J^I\int d^4x\, f_I(x)\frac{\delta}{\delta f_J(x)}\right]\Gamma[f(x),g,M]=0$$

- In non-scale-invariant QFTs
  - ✔ Anomalous dimensions
  - ✗ Beta-functions

$$\left[M\frac{\partial}{\partial M}+(\Gamma+Q)_J^I\int d^4x\, f_I(x)\frac{\delta}{\delta f_J(x)}\right]\Gamma[f(x),g,M]=0$$

- In scale-invariant QFTs
  - ✔ Anomalous dimensions
  - ✔ Beta-functions (redundant operators)

Poincaré algebra augmented with dilatation charge

- Beta-functions on scale-invariant trajectories
  - Quantum-mechanical generation of scale dimensions
  - Appropriate scale dimensions required by virial current

  $\Rightarrow$ Conserved dilatation current $\mathcal{D}^\mu(x)$

- Poincaré algebra with dilatation charge $D = \int d^3x\, \mathcal{D}^0(x)$

$$\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\sigma}M_{\nu\rho}) \\
[M_{\mu\nu}, P_\rho] &= -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu) \\
[D, P_\mu] &= -iP_\mu
\end{aligned}$$

- Algebra action on fields $\mathcal{O}_I(x)$

$$\begin{aligned}
[M_{\mu\nu}, \mathcal{O}_I(x)] &= -i(x_\mu\partial_\nu - x_\nu\partial_\mu + \Sigma_{\mu\nu})\mathcal{O}_I(x) \\
[P_\mu, \mathcal{O}_I(x)] &= -i\partial_\mu\mathcal{O}_I(x) \\
[D, \mathcal{O}_I(x)] &= -i(x\cdot\partial + \Delta)\mathcal{O}_I(x)
\end{aligned}$$

- New classical scale dimensions of fields due to virial current

$$[D,\phi_a(x)] = -i(x\cdot\partial+1)\phi_a(x) - iQ_{ab}\phi_b(x)$$
$$[D,\psi_i(x)] = -i(x\cdot\partial+\tfrac{3}{2})\psi_i(x) - iP_{ij}\psi_j(x)$$

  - How do non-conformal scale-invariant QFTs know about new scale dimensions ?

  $\Rightarrow$ Generated by beta-functions !

- Quantum-mechanical scale dimensions of fields

$$\Delta_{ab} = \delta_{ab} + \Gamma_{ab} + Q_{ab}$$
$$\Delta_{ij} = \tfrac{3}{2}\delta_{ij} + \Gamma_{ij} + P_{ij}$$

c-theorem

c-theorem Barnes, Intriligator, Wecht, Wright (2004)

- RG flow $\Rightarrow$ Irreversible process (integrating out DOFs)
- $c(g) \sim$ measure of number of massless DOFs
- c-theorem and implications for SFT
  - weak ($c_{IR} < c_{UV}$) Komargodski, Schwimmer (2011) & Luty, Polchinski, Rattazzi (2012)
  - stronger ($\frac{dc}{dt} \leq 0$) Osborn (1989,1991) & Jack, Osborn (1990)
  - ~~strongest~~ (RG flows as gradient flows)

Gradient flow

- Gradient flow

$$\mathrm{B}^I(g) = -\frac{dg^I}{dt} = G^{IJ}(g)\frac{\partial c(g)}{\partial g^J}$$

  - G^{IJ} positive-definite metric
  - Potential $c(g)$ function of couplings

- Potential $c(g)$ monotonically decreasing along RG trajectory

$$\frac{dc(g(t))}{dt} = -G_{IJ}(g)\,\mathrm{B}^I\,\mathrm{B}^J \leq 0$$

  - Recurrent behaviors (scale-invariant trajectories) $\not\Leftrightarrow$ Gradient flows (scale implies conformal invariance) Wallace, Zia (1975)

$\Rightarrow$ Another way to prove scale implies conformal invariance

  - Different than proof for $d = 2$ unitarity interacting QFTs with well-defined correlation functions Zamolodchikov (1986) & Polchinski (1988)

c-theorem and gradient flow at weak coupling

- Weyl consistency conditions Osborn (1989,1991) & Jack, Osborn (1990)

$$\frac{\partial c(g)}{\partial g^I} = (G_{IJ} + A_{IJ})\beta^J \Rightarrow \frac{dc(g(t))}{dt} = -\beta^I G_{IJ}(g)\beta^J$$

  - Curved spacetime $\Rightarrow$ Background metric with spacetime-dependent couplings

$\Rightarrow$ (Weak-coupling) RG flow recurrent behaviors forbidden at all loops

Local and global renormalized operators

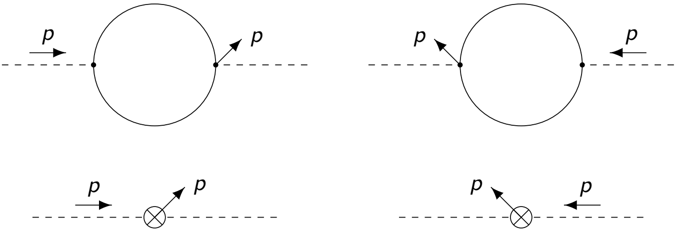
Global renormalized operator $\mathcal{O}_I(x) = \partial\mathcal{L}(x)/\partial g^I$

- Finite global insertion in Green functions $\Rightarrow$ $-i\partial\langle\ldots\rangle/\partial g^I = \langle\int\; d^dx\,\mathcal{O}_I(x)\ldots\rangle$
- Infinite local insertion in Green functions $\Rightarrow$ $\langle\mathcal{O}_I(x)\ldots\rangle$

Local renormalized operator $[\mathcal{O}_I(x)] = \delta\mathcal{A}/\delta g^I(x)$

- Finite local insertion in Green functions $\Rightarrow$ $\langle[\mathcal{O}_I(x)]\ldots\rangle = \langle(\mathcal{O}_I(x) - \partial_\mu J_I^\mu(x))\ldots\rangle$
- Infinite current $J_I^\mu(x) = -(N_I)_{ab}\phi_a D^\mu\phi_b + (M_I)_{ij}\bar{\psi}_i i\bar{\sigma}^\mu\psi_j$
  - $(N_I)_{ba} = -(N_I)_{ab}$ and $(M_I)^*_{ji} = -(M_I)_{ij}$
  - $N_I = \sum_{i\geq 1}\frac{N_I^{(i)}}{\epsilon^i}$ and $M_I = \sum_{i\geq 1}\frac{M_I^{(i)}}{\epsilon^i}$

Computations of new divergences



$$(N_{c|ij})_{ab} = -\frac{1}{16\pi^2\epsilon}\,\frac{1}{2}(y^*_{a|ij}\delta_{bc} - y^*_{b|ij}\delta_{ac}) + \text{h.c.} + \text{finite}$$

Finite contributions to EM tensor

Anomalous trace Osborn (1989,1991) & Jack, Osborn (1990)

$$[\Theta_\mu{}^\mu(x)] = \beta^I[\mathcal{O}_I] - D_\mu[S_{ab}\phi_a D^\mu\phi_b - R_{ij}\bar{\psi}_i i\bar{\sigma}^\mu\psi_j] - ((\delta+\gamma)f)\cdot\frac{\delta}{\delta f}\mathcal{A}$$

$$f_0 = \mu^{(\frac{1}{2}-\delta)\epsilon} Z^{\frac{1}{2}}(g) f \qquad g_0^I = \mu^{k_I\epsilon}(g^I + L^I(g))$$

$$\hat{\gamma} = (\tfrac{1}{2}-\delta)\epsilon - k_I g^I \partial_I Z^{\frac{1}{2}(1)} \qquad \hat{\beta}^I = -k_I g^I\epsilon - k_I L^{I(1)} + k_J g^J \partial_J L^{I(1)}$$

$$S = -k_I g^I N_I^{(1)} \qquad R = -k_I g^I M_I^{(1)}$$

Ambiguities in RG functions

Relevant quantities Osborn (1989,1991) & Jack, Osborn (1990)

- Square root of wavefunction renormalization $Z^{\frac{1}{2}}$
  - Freedom $Z^{\frac{1}{2}} \to \tilde{Z}^{\frac{1}{2}} = OZ^{\frac{1}{2}}$ with $Z = Z^{\frac{1}{2}T} Z^{\frac{1}{2}} \to Z^{\frac{1}{2}T} O^T O Z^{\frac{1}{2}}$
  - $O^T O = 1$ and $O = 1 + \sum_{i\geq 1} \frac{O^{(i)}}{\epsilon^i}$

- Extra freedom with $\omega = k_I g^I \partial_I O^{(1)}$

$$Z^{\frac{1}{2}(1)} \to Z^{\frac{1}{2}(1)} + O^{(1)} \quad L^{I(1)} \to L^{I(1)} - (gO^{(1)})^I \quad N_I^{(1)} \to N_I^{(1)} - \partial_I O^{(1)}$$

$$\hat{\gamma} \to \hat{\gamma} - \omega \qquad \hat{\beta}^I \to \hat{\beta}^I - (g\omega)^I \qquad S \to S + \omega$$

- Invariant anomalous trace

$$[\Theta_\mu{}^\mu(x)] = (\beta^I + (gS)^I)[\mathcal{O}_I] - ((\delta + \gamma + S)f) \cdot \frac{\delta}{\delta f}\mathcal{A}$$

$$= \mathrm{B}^I[\mathcal{O}_I] - ((\delta + \Gamma)f) \cdot \frac{\delta}{\delta f}\mathcal{A}$$

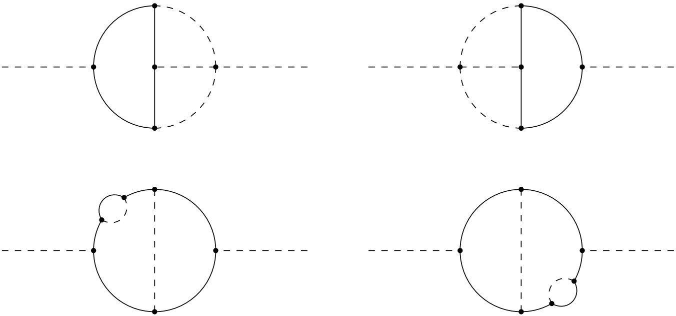
Scale and conformal invariance

- "Correct" RG flow $\Rightarrow$ $\mathrm{B}^I = \beta^I + (gS)^I = -(gQ)^I$
  - SFTs ($Q \neq 0$) $\Rightarrow$ limit cycles ($B^I = -(gQ)^I \neq 0$)
  - CFTs ($Q = 0$) $\Rightarrow$ fixed points ($B^I = 0$)
- "Old" RG flow $\Rightarrow$ $\beta^I = -(g(S+Q))^I$
  - SFTs ($Q \neq 0$) $\Rightarrow$ fixed points ($S = -Q$) and limit cycles ($S \neq -Q$)
  - CFTs ($Q = 0$) $\Rightarrow$ fixed points ($S = 0$) and limit cycles ($S \neq 0$)

$\Rightarrow$ Systematic understanding of SFTs and CFTs through "correct" RG flow (unless S vanishes identically)

Computation of S

$S^{(\text{one-loop})} = S^{(\text{two-loop})} = 0$ due to symmetry of contributions to N_I



$$(16\pi^2)^3 S_{ab} = \tfrac{5}{8}\mathrm{tr}(y_a\, y_c^*\, y_d\, y_e^*)\lambda_{bcde} + \tfrac{3}{8}\mathrm{tr}(y_a\, y_c^*\, y_d\, y_d^*\, y_b\, y_c^*) - \{a \leftrightarrow b\} + \text{h.c.}$$

$S \neq 0 \Rightarrow$ Examples of CFTs with $S \neq 0$ exist JFF, Grinstein, Stergiou (2012)

Generalized c-theorem

- Weyl consistency conditions and local current conservation
  Osborn (1989,1991) & Jack, Osborn (1990)

$$\frac{\partial c(g)}{\partial g^I} = (G_{IJ} + A_{IJ})\,\mathrm{B}^J \Rightarrow \frac{dc(g(t))}{dt} = -\,\mathrm{B}^I\,G_{IJ}\,\mathrm{B}^J$$

  - Curved spacetime ⇒ Background metric with spacetime-dependent couplings
  - Spin-one operator of dimension 3 ⇒ Background gauge fields with gauge-dependent couplings

  ⇒ (Weak-coupling) RG flow recurrent behaviors allowed at all loops

- Scale invariance implies conformal invariance JFF, Grinstein, Stergiou (2012) & Luty, Polchinski, Rattazzi (2012)

Features and future work

Features of SFTs and CFTs

- Correct RG flow
  - SFTs $\Rightarrow$ Recurrent behaviors
  - CFTs $\Rightarrow$ Fixed points
- Generalized c-theorem $\Rightarrow$ Only CFTs allowed
  - $\Rightarrow$ Scale invariance implies conformal invariance
  - Unexpected CFTs with expected behaviors

Future work

- Proof at strong coupling Farnsworth, Luty, Prelipina (2013)
- 6d analysis Grinstein, Stergiou, Stone, Zhong (2014,2015)

Thank you !