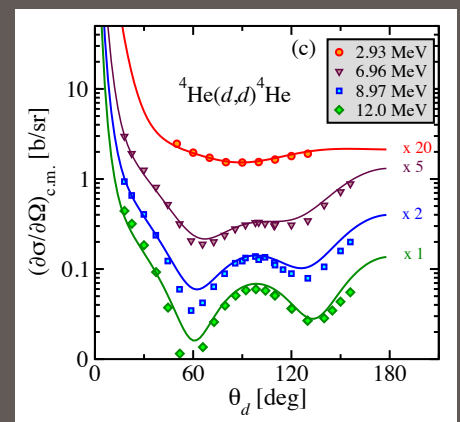
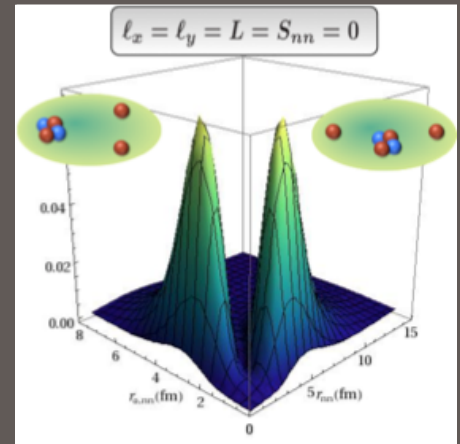


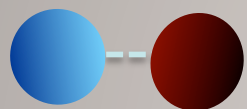
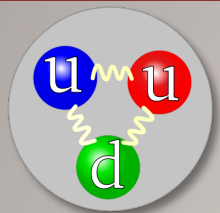
Ab Initio calculations of Nuclear Structure and Reactions

2015 CAP Congress
 Edmonton, Alberta
 June 13–19, 2015

Francesco Raimondi | TRIUMF
 Petr Navratil, Jeremy Dohet Eraly, Angelo Calci | TRIUMF
 Sofia Quaglioni, Carolina Romero-Redondo | LLNL
 Guillaume Hupin | CNRS



From QCD to nuclei

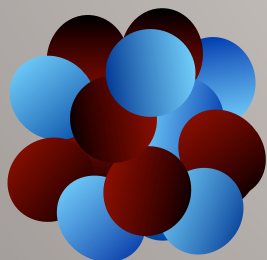


Low-energy QCD



NN+3N interactions
from chiral EFT

...or accurate
meson-exchange
potentials



Nuclear structure and reactions

Chiral Effective Field Theory

- **First principles for Nuclear Physics:**

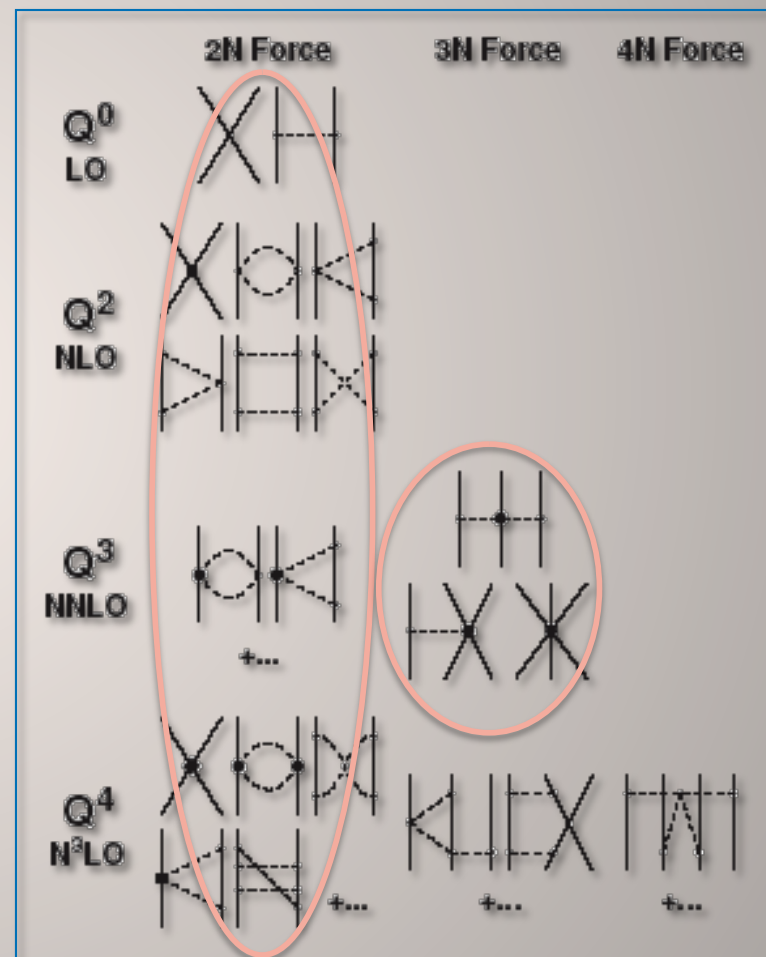
- **QCD**

- Non-perturbative at low energies
 - Lattice QCD in the future

- ***For now a good place to start:***

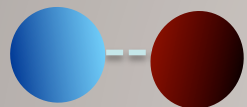
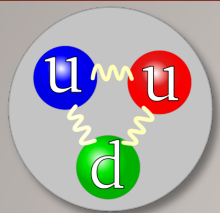
- **Inter-nucleon forces from chiral effective field theory**

- Based on the symmetries of QCD
 - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
 - Degrees of freedom: nucleons + pions
 - Systematic low-momentum expansion to a given order (Q/Λ_χ)
 - Hierarchy
 - Consistency
 - Low energy constants (LEC)
 - Fitted to data
 - Can be calculated by lattice QCD



$\Lambda_\chi \sim 1 \text{ GeV}$:
Chiral symmetry breaking scale

From QCD to nuclei

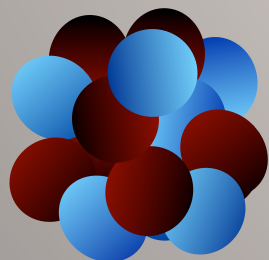


Low-energy QCD

NN+3N interactions
from chiral EFT

...or accurate
meson-exchange
potentials

$$H|\Psi\rangle = E|\Psi\rangle$$



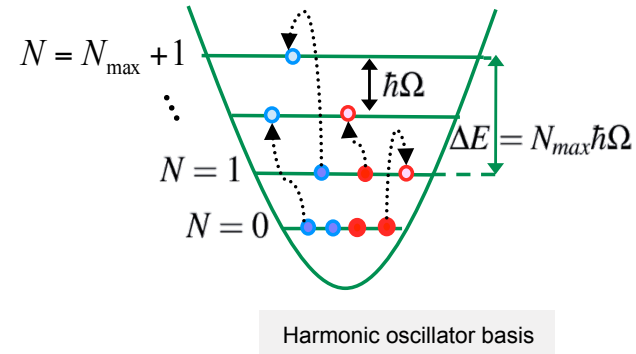
Many-Body methods

NCSM, NCSM/RGM,
NCSMC, CCM, GFMC,
HH, Nuclear Lattice
EFT...


Nuclear structure and reactions

Unified approach to bound & continuum states; to nuclear structure & reactions

- *Ab initio* no-core shell model
 - Short- and medium range correlations
 - Bound-states, narrow resonances

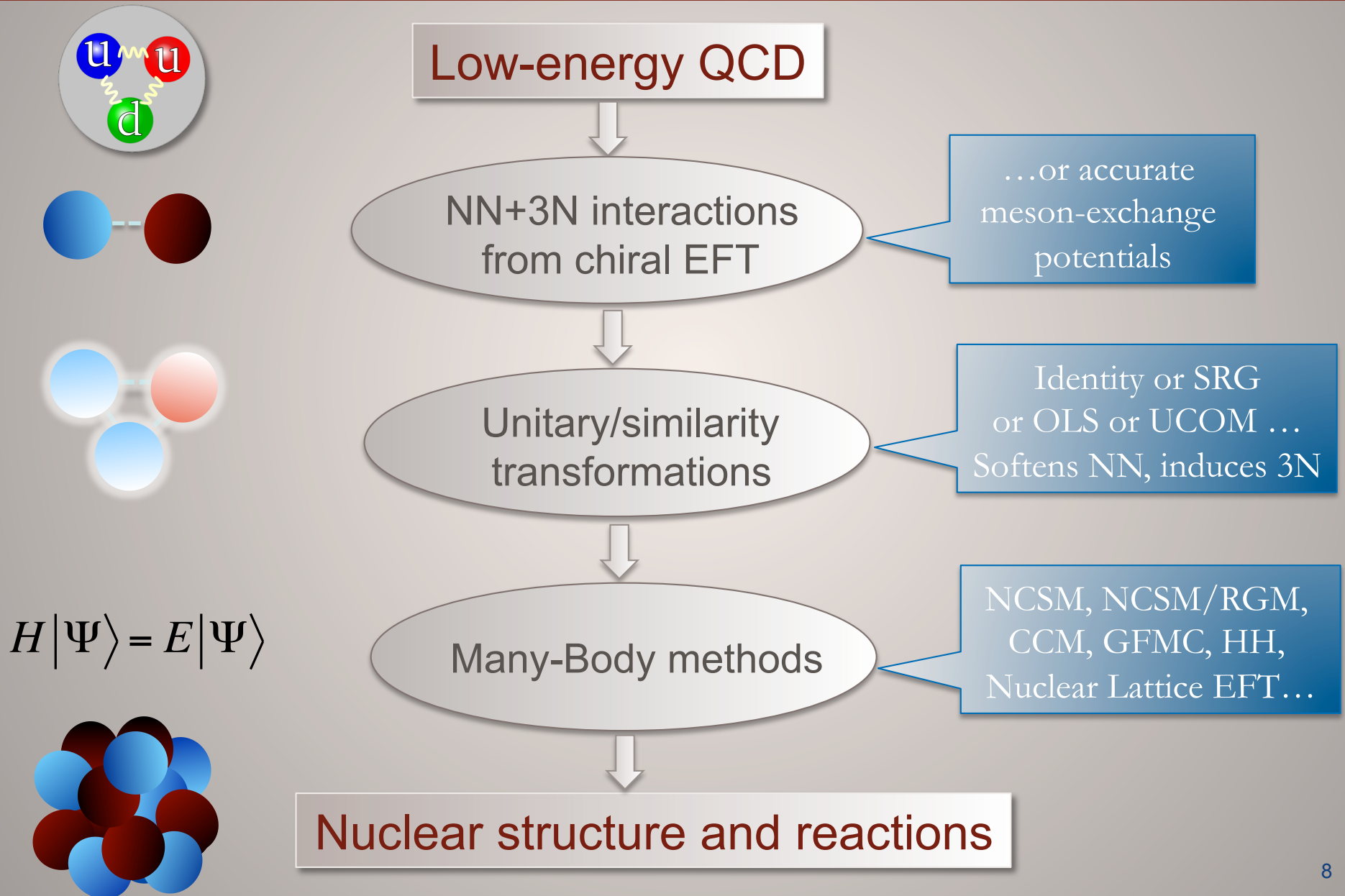


NCSM

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| (A) \text{  , \lambda \right\rangle$$

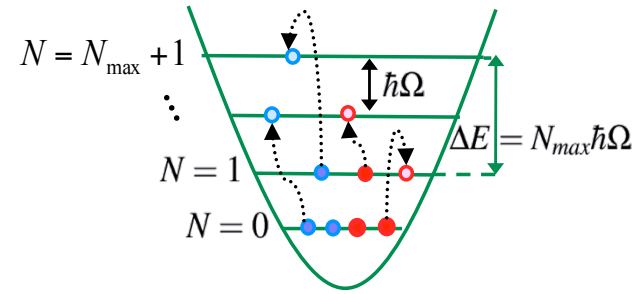
Unknowns

From QCD to nuclei



Unified approach to bound & continuum states; to nuclear structure & reactions

- *Ab initio* no-core shell model
 - Short- and medium range correlations
 - Bound-states, narrow resonances

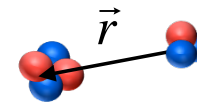


Harmonic oscillator basis



NCSM

- ...with resonating group method
 - Bound & scattering states, reactions
 - Cluster dynamics, long-range correlations



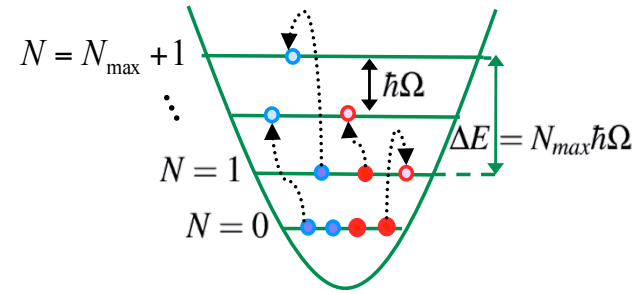
NCSM/RGM

$$\Psi^{(A)} = \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left[\begin{array}{c} \text{NCSM/RGM} \\ \text{channel states} \\ \left| \begin{array}{c} \vec{r} \\ (A-a) \quad (a) \end{array} \right\rangle, \nu \end{array} \right]$$

Unknowns →

Unified approach to bound & continuum states; to nuclear structure & reactions

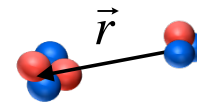
- *Ab initio* no-core shell model
 - Short- and medium range correlations
 - Bound-states, narrow resonances



NCSM

Harmonic oscillator basis

- ...with resonating group method
 - Bound & scattering states, reactions
 - Cluster dynamics, long-range correlations



NCSM/RGM

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

- Most efficient: *ab initio* no-core shell model with continuum

NCSMC

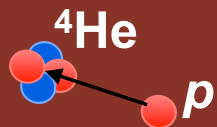
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left[\begin{array}{c} \text{NCSM eigenstates} \\ \left| \begin{array}{c} (A) \\ \text{NCSM} \\ \lambda \end{array} \right\rangle \right] + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left[\begin{array}{c} \text{NCSM/RGM} \\ \text{channel states} \\ \left| \begin{array}{c} (A-a) \quad (a) \\ \text{NCSM/RGM} \\ \nu \end{array} \right\rangle \right] \end{array} \right.$$

Unknowns

Coupled NCSMC equations

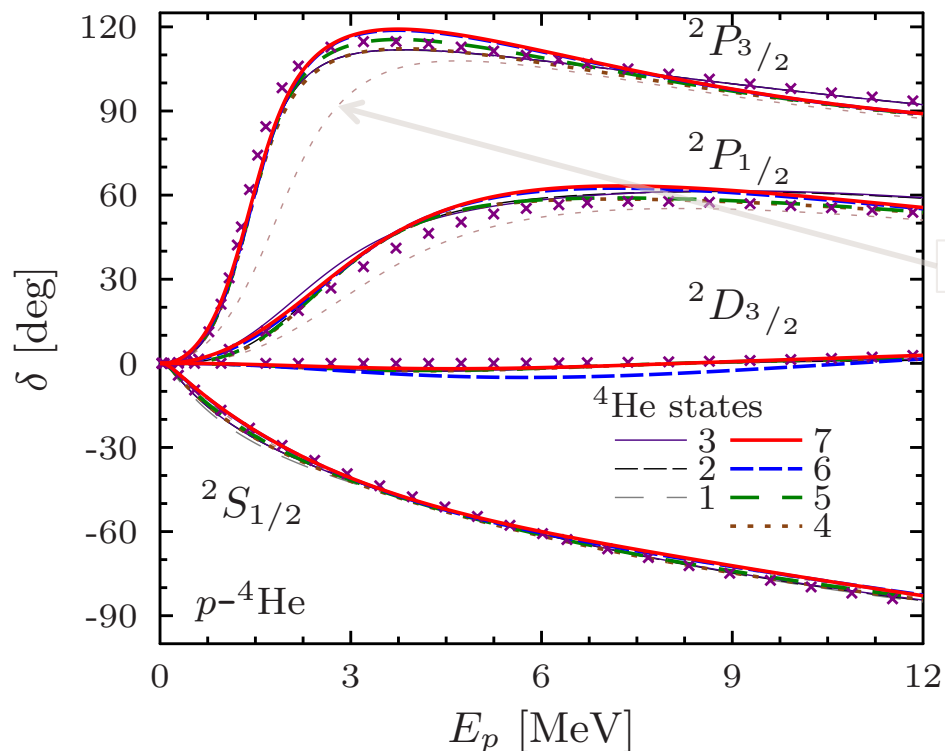
$$\begin{array}{c}
 \begin{array}{c}
 \boxed{E_{\lambda}^{NCSM} \delta_{\lambda\lambda'}} \\
 \downarrow \text{blue} \\
 \begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix}
 \end{array} \\
 \begin{array}{c}
 \boxed{\langle (A) \left| H \hat{A}_v \right| (a) (A-a) \rangle} \\
 \downarrow \text{green} \\
 \begin{pmatrix} c \\ \gamma \end{pmatrix} \\
 \uparrow \text{red} \\
 \boxed{\langle (A-a) (a) \left| \hat{A}_{v'} H \hat{A}_v \right| (a) (A-a) \rangle}
 \end{array}
 \end{array}
 = E
 \begin{array}{c}
 \begin{array}{c}
 \boxed{\delta_{\lambda\lambda'}} \\
 \downarrow \text{blue} \\
 \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix}
 \end{array} \\
 \begin{array}{c}
 \boxed{\langle (A) \left| \hat{A}_v \right| (a) (A-a) \rangle} \\
 \downarrow \text{green} \\
 \begin{pmatrix} c \\ \gamma \end{pmatrix} \\
 \uparrow \text{red} \\
 \boxed{\langle (A-a) (a) \left| \hat{A}_{v'} \hat{A}_v \right| (a) (A-a) \rangle}
 \end{array}
 \end{array}
 \end{array}$$

Scattering matrix (and observables) from matching solutions to known asymptotic with microscopic R -matrix on Lagrange mesh



p - ^4He scattering within NCSMC

p - ^4He scattering phase-shifts for NN+3N potential:
Convergence



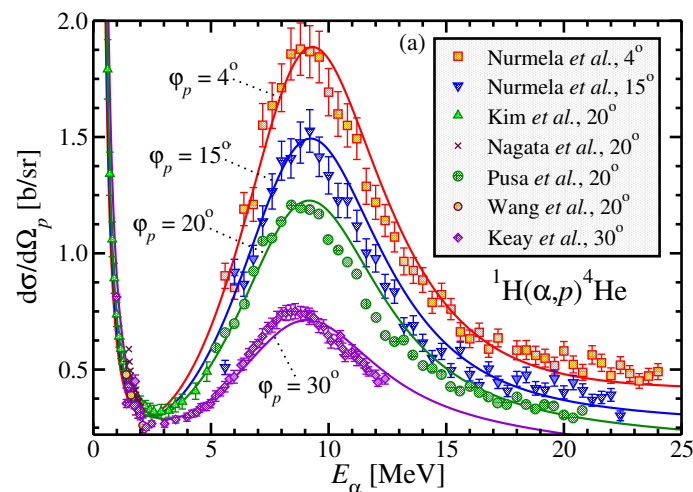
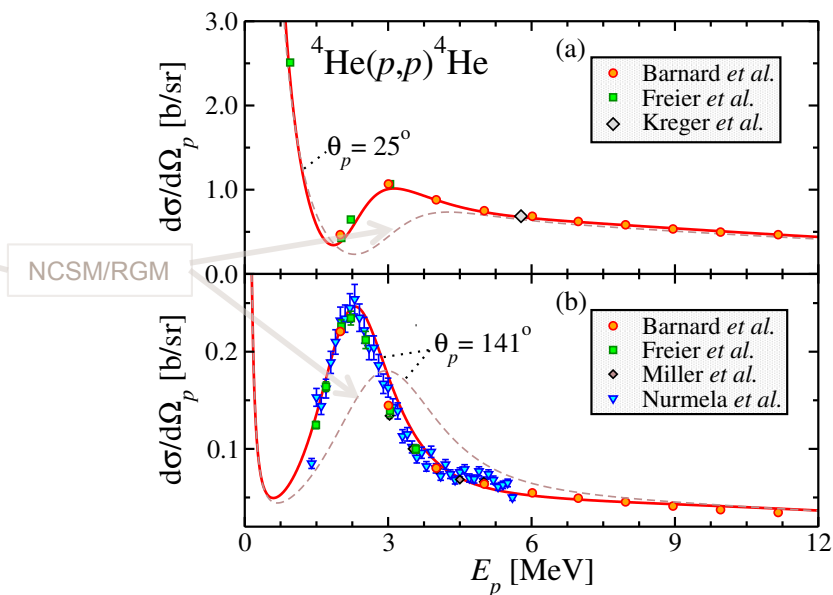
Predictive power in the $3/2^-$ resonance region:
Applications to material science

PHYSICAL REVIEW C **90**, 061601(R) (2014)

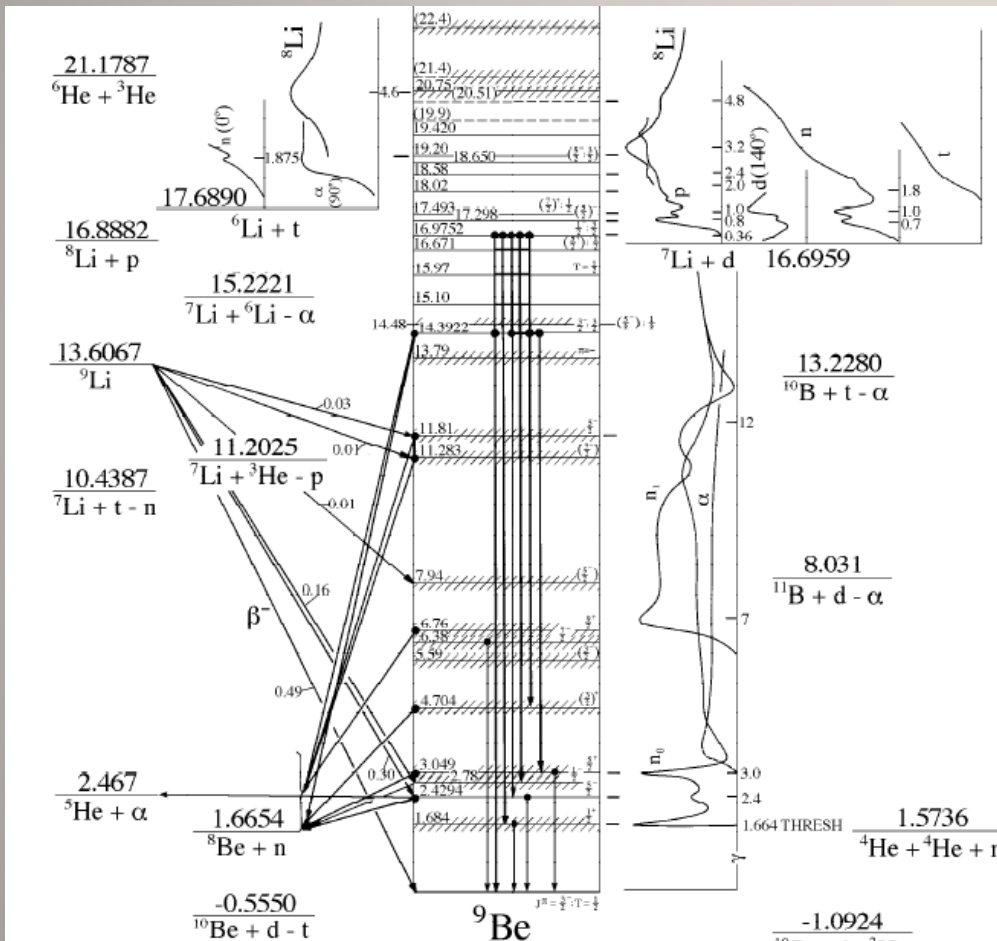
Predictive theory for elastic scattering and recoil of protons from ^4He

Guillaume Hupin,^{1,*} Sofia Quaglioni,^{1,†} and Petr Navrátil^{2,‡}

Differential p - ^4He cross section with NN+3N potentials



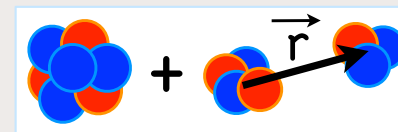
Structure of ${}^9\text{Be}$



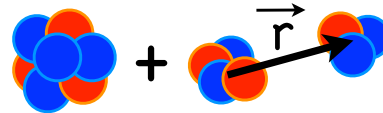
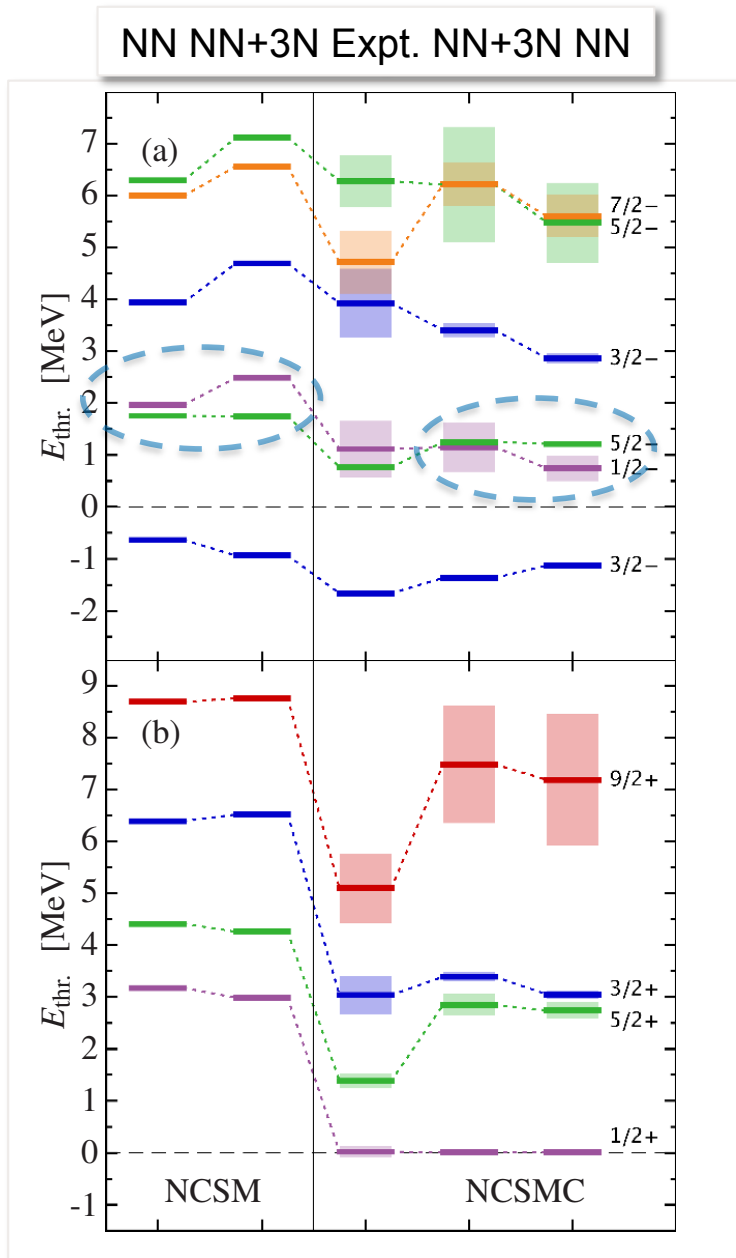
${}^9\text{Be}$ is a stable nucleus
 ... but all its excited states unbound
 A proper description requires to include
 effects of continuum

The lowest threshold: n - ${}^8\text{Be}$ (n - α - α)

Optimal description:
 Square-integrable ${}^9\text{Be}$ basis + n - ${}^8\text{Be}$ clusters



NCSMC with chiral NN+3N: Structure of ${}^9\text{Be}$



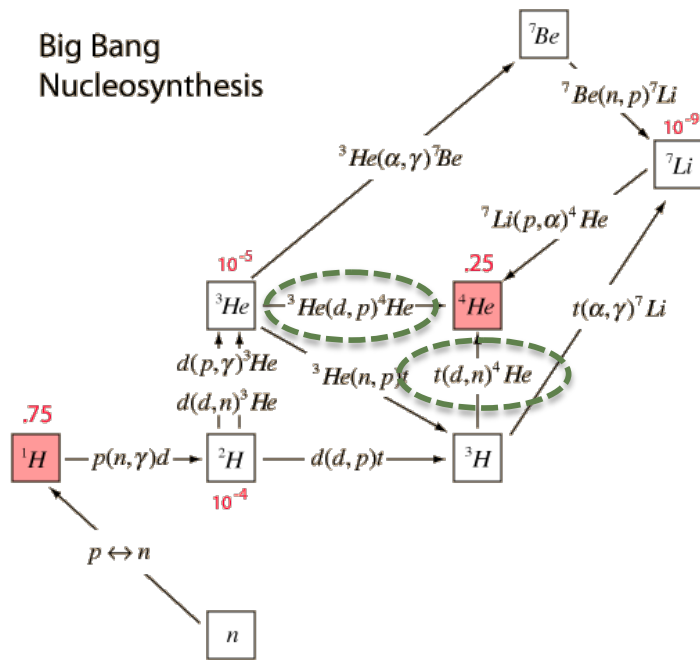
${}^9\text{Be}$ is a stable nucleus
 ... but all its excited states unbound
 A proper description requires to include
 effects of continuum

Three-nucleon interaction *and* continuum
 improve agreement with experiment for
 negative parity states

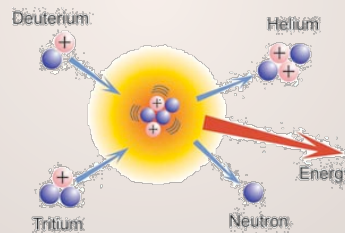
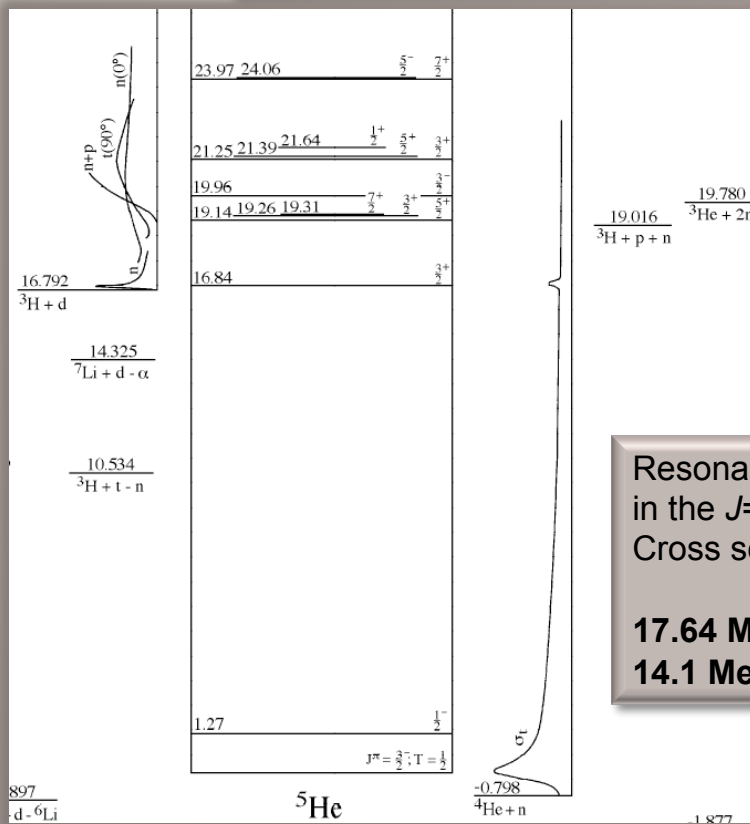
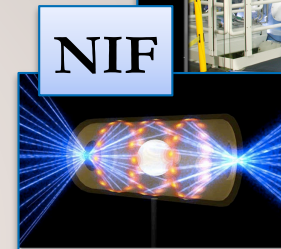
Continuum crucial for the description of
 positive-parity states



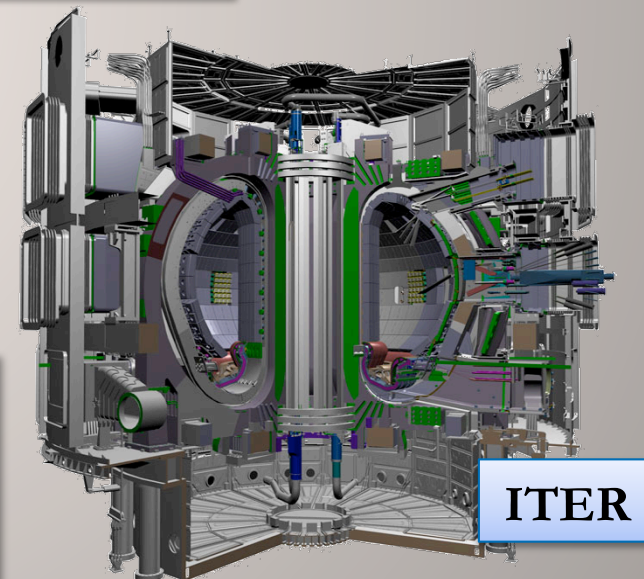
Big Bang Nucleosynthesis



Deuterium-Tritium fusion



Resonance at $E_{cm} = 48 \text{ keV}$ ($E_d = 105 \text{ keV}$)
 in the $J = 3/2^+$ channel
 Cross section at the peak: 4.88 b
17.64 MeV energy released:
14.1 MeV neutron and 3.5 MeV alpha

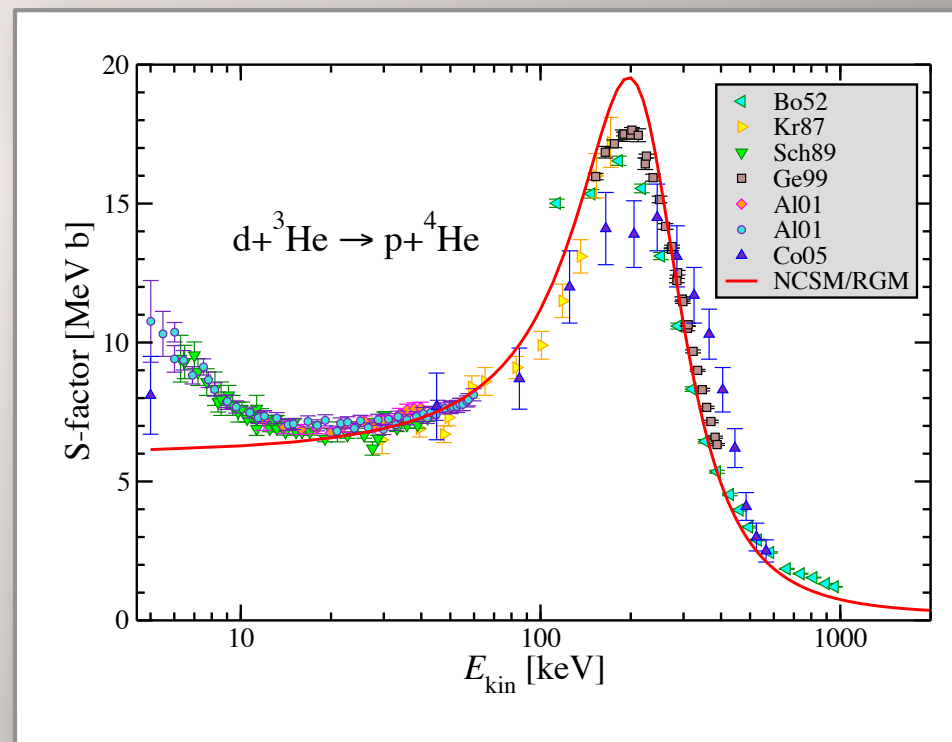


NCSM/RGM calculations of transfer reactions

$$\int dr r^2 \begin{pmatrix} \left\langle \begin{array}{c} \mathbf{r}' \\ n \end{array} \middle| \hat{A}_1 (H - E) \hat{A}_1 \middle| \begin{array}{c} \mathbf{r} \\ \alpha \end{array} \right\rangle & \left\langle \begin{array}{c} \mathbf{r}' \\ n \end{array} \middle| \hat{A}_1 (H - E) \hat{A}_2 \middle| \begin{array}{c} \mathbf{r} \\ d \end{array} \right\rangle \\ \left\langle \begin{array}{c} \mathbf{r}' \\ d \end{array} \middle| \hat{A}_2 (H - E) \hat{A}_1 \middle| \begin{array}{c} \mathbf{r} \\ \alpha \end{array} \right\rangle & \left\langle \begin{array}{c} \mathbf{r}' \\ d \end{array} \middle| \hat{A}_2 (H - E) \hat{A}_2 \middle| \begin{array}{c} \mathbf{r} \\ d \end{array} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$

Straightforward to couple different mass partitions in the NCSM/RGM formalism

Applications to (d,p) and (d,n) reactions
Example: ${}^3\text{He}(d,p){}^4\text{He}$

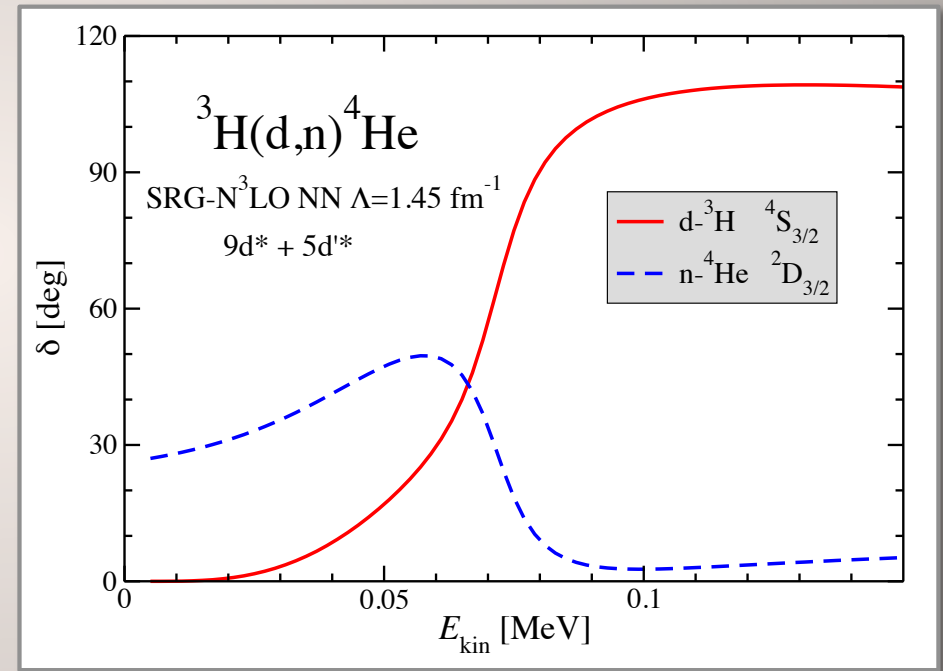
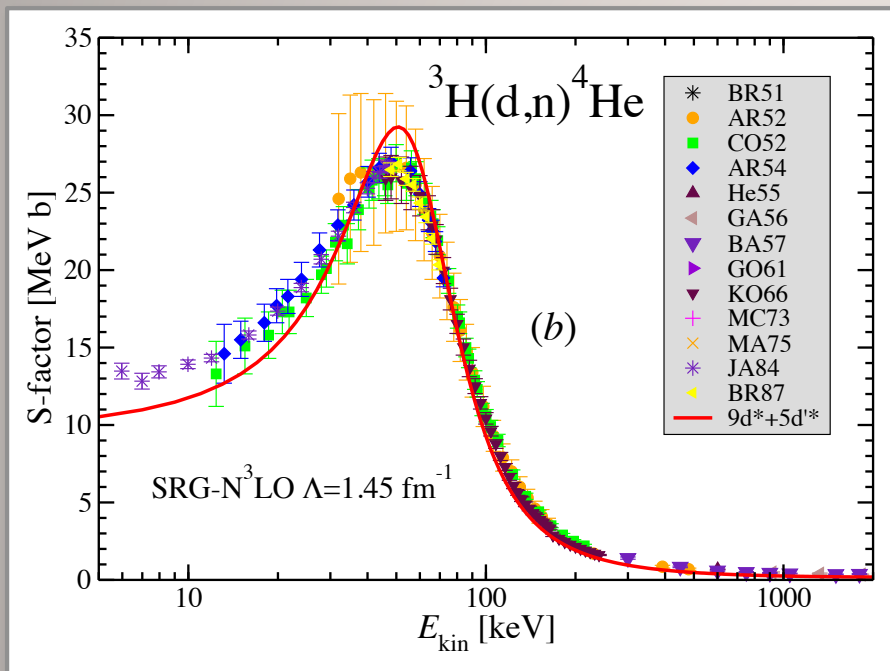


$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a} Z_a e^2 / \hbar v_{A-a,a}$$

${}^3\text{H}(d,n){}^4\text{He}$ cross section

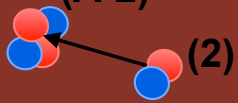
- SRG- N^3LO ($\Lambda=1.45 \text{ fm}^{-1}$) NN potential
 - Position of the resonance matches experiment



S-factor narrower than the data

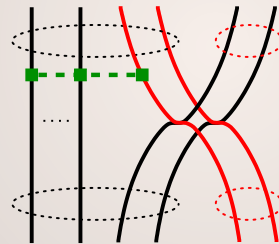
Resonance in the d - ${}^3\text{H}$ ${}^4\text{S}_{1/2}$ partial wave

n - ${}^4\text{He}$ ${}^2\text{D}_{3/2}$ decreasing, does not cross 90 degrees



The deuteron-projectile formalism: Three-nucleon interaction and/or $A_1 > 4$ target

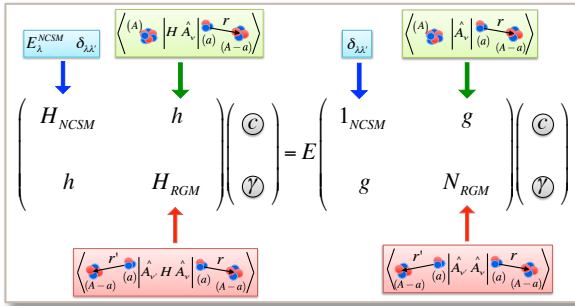
$$\begin{array}{c}
 \begin{array}{c}
 \langle \psi_{\lambda}^{NCSM} | \delta_{\lambda\lambda} \rangle \\
 \downarrow \\
 H_{NCSM} \\
 \uparrow \\
 \langle \psi_{\lambda}^{NCSM} | \delta_{\lambda\lambda} \rangle
 \end{array}
 \quad
 \begin{array}{c}
 \langle \psi_{\lambda}^{(A)} | H \hat{A}_\nu | \psi_{\lambda}^{(A-a)} \rangle \\
 \downarrow \\
 h \\
 \uparrow \\
 \langle \psi_{\lambda}^{(A)} | \hat{A}_\nu H \hat{A}_\nu | \psi_{\lambda}^{(A-a)} \rangle
 \end{array}
 \end{array}
 \begin{pmatrix} \textcircled{C} \\ \textcircled{V} \end{pmatrix}
 = E
 \begin{array}{c}
 \begin{array}{c}
 \langle \psi_{\lambda}^{(A)} | \delta_{\lambda\lambda} \rangle \\
 \downarrow \\
 1_{NCSM} \\
 \uparrow \\
 \langle \psi_{\lambda}^{(A)} | \delta_{\lambda\lambda} \rangle
 \end{array}
 \quad
 \begin{array}{c}
 \langle \psi_{\lambda}^{(A)} | \hat{A}_\nu | \psi_{\lambda}^{(A-a)} \rangle \\
 \downarrow \\
 g \\
 \uparrow \\
 \langle \psi_{\lambda}^{(A)} | \hat{A}_\nu \hat{A}_\nu | \psi_{\lambda}^{(A-a)} \rangle
 \end{array}
 \end{array}
 \begin{pmatrix} \textcircled{C} \\ \textcircled{V} \end{pmatrix}$$



$$\text{SD} \left\langle \psi_{\mu_1}^{(A-2)} \left| a^+ a^+ a^+ a^+ a a a a \right| \psi_{\nu_1}^{(A-2)} \right\rangle_{\text{SD}}$$

For $A=6$ use completeness
For $A>6$ a sophisticated algorithm required

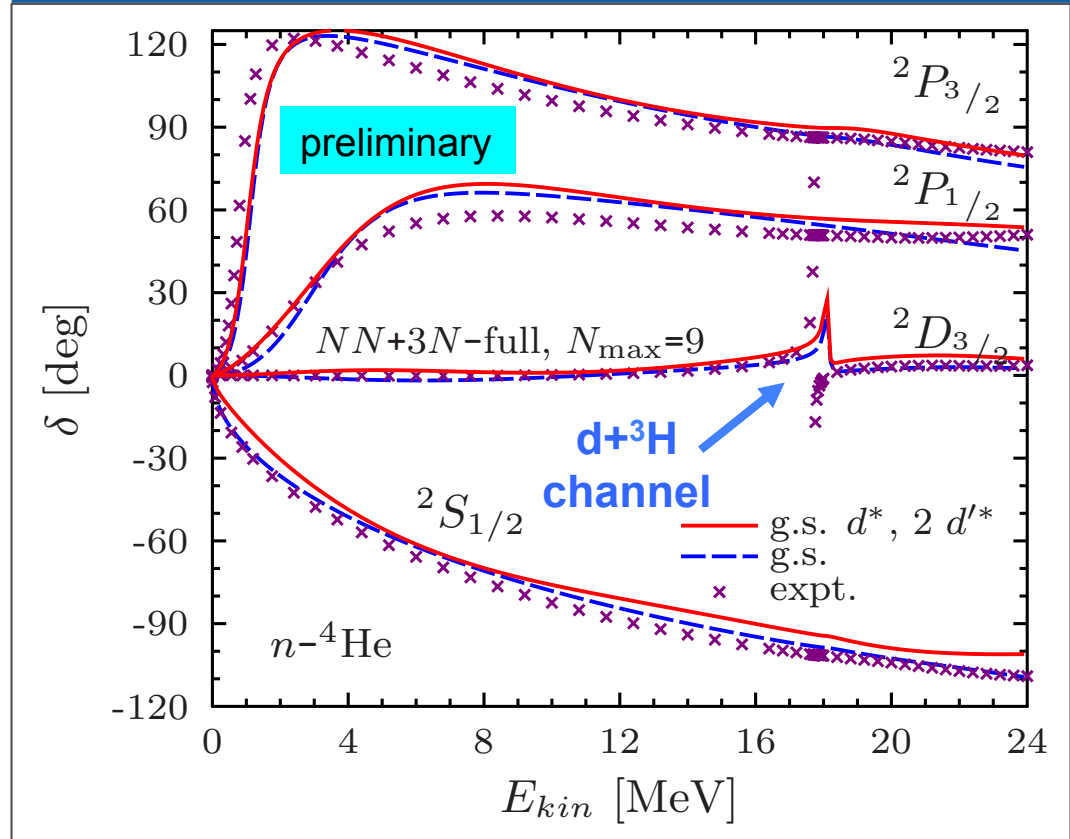
${}^3\text{H}(d,n){}^4\text{He}$ fusion with chiral NN+3N



NCSMC

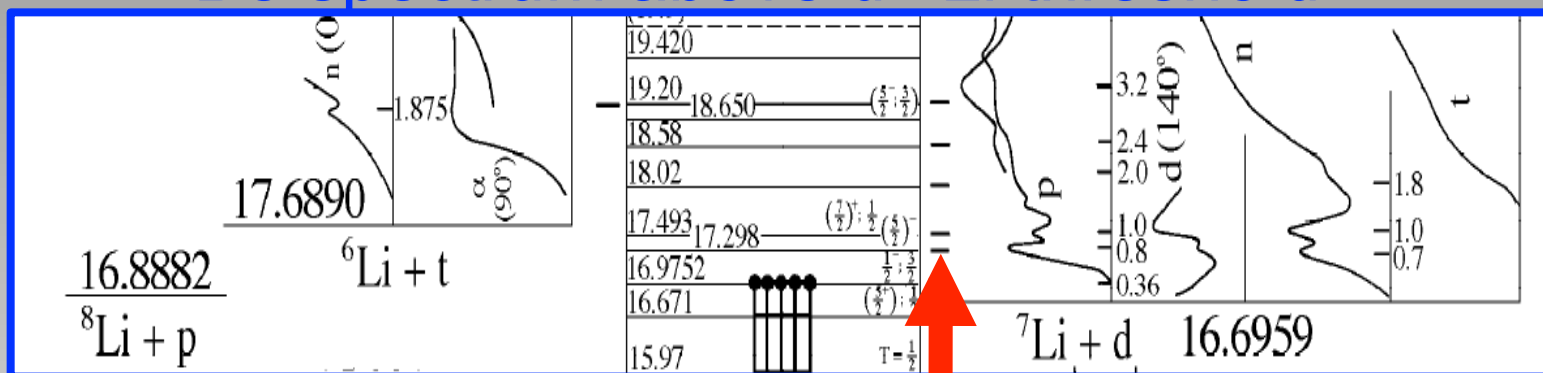
- Towards first ab initio calculation of ${}^3\text{H}(d,n){}^4\text{He}$ fusion with 3N forces
 - $N_{\text{max}} = 9$ model space
 - $n+{}^4\text{He}$ & $d+{}^3\text{H}$ continuum channels
 - Up to 14 ${}^5\text{He}$ states
 - Only g.s. of ${}^4\text{He}$ and ${}^3\text{H}$: effect of target excitation described by ${}^5\text{He}$ states
 - 3-body dynamics approximated above deuteron breakup

$n-{}^4\text{He}$ scattering phase shifts



Spin-parity assignment of 0.78 MeV resonance of ${}^9\text{Be}$

${}^9\text{Be}$ spectrum above d - ${}^7\text{Li}$ threshold

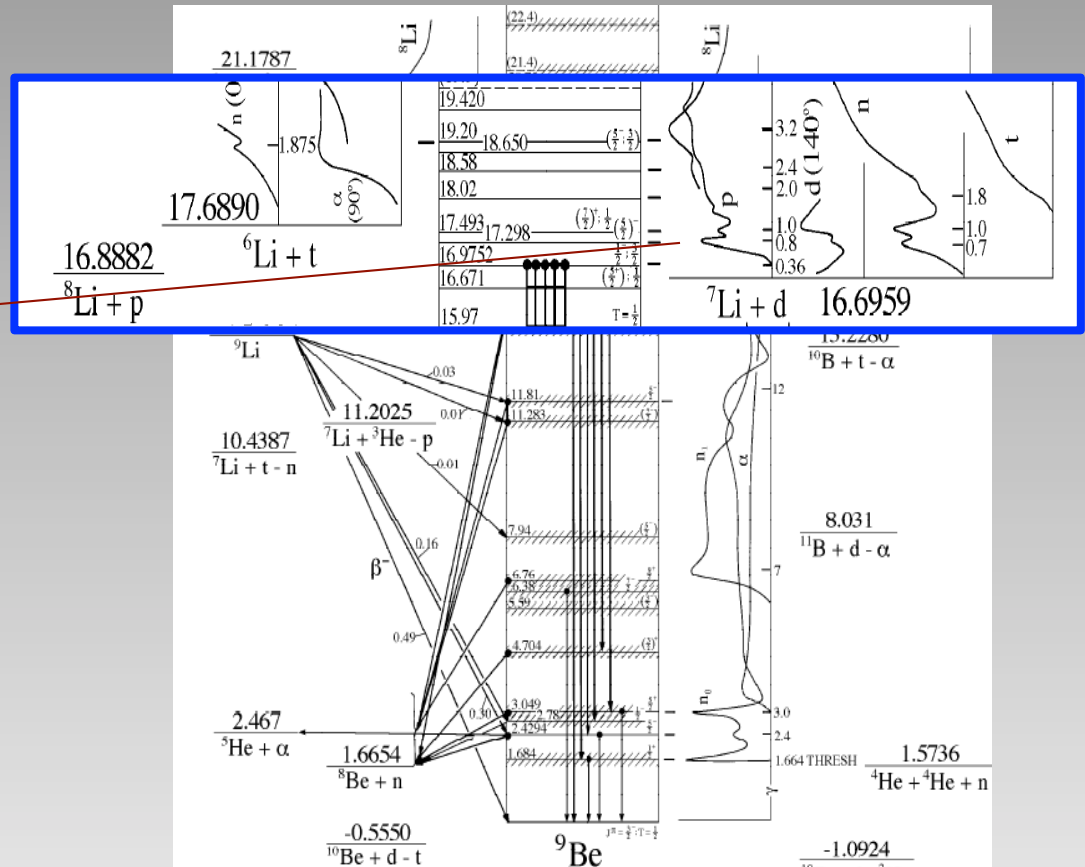
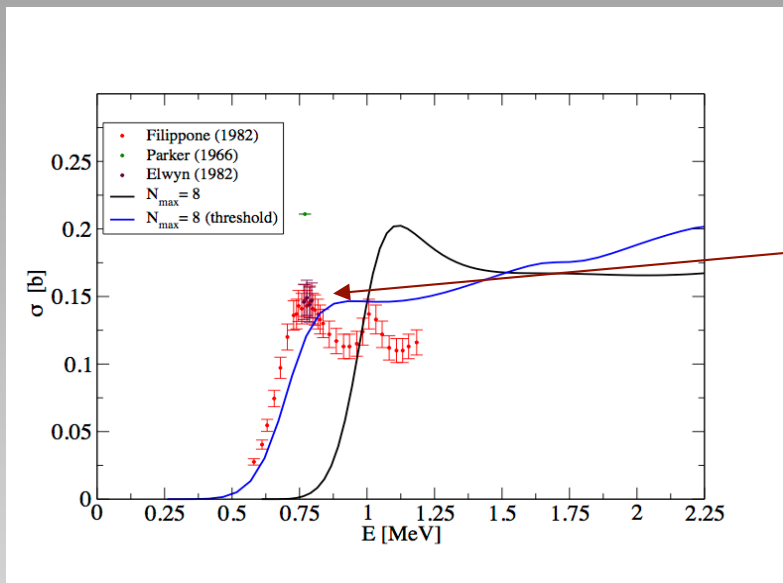


Low peak in the experimental total cross section:
 $E(5/2^-) \sim 0.78$ MeV above the threshold
 (Uncertain spin-parity assignment)

◆ Calibration reaction for astrophysical process: ${}^7\text{Li}(d,p){}^8\text{Li}$ as target calibration for ${}^7\text{Be}(p,\gamma){}^8\text{B}$ (Solar abundance problem)

◆ Possible mechanism of destruction of ${}^7\text{Li}$ in the context of baryon-inhomogeneous models of the primordial nucleosynthesis (Primordial Lithium abundance problem)

${}^7\text{Li}(d,p){}^8\text{Li}$ scattering results (NCSM-RGM)



Included channels:

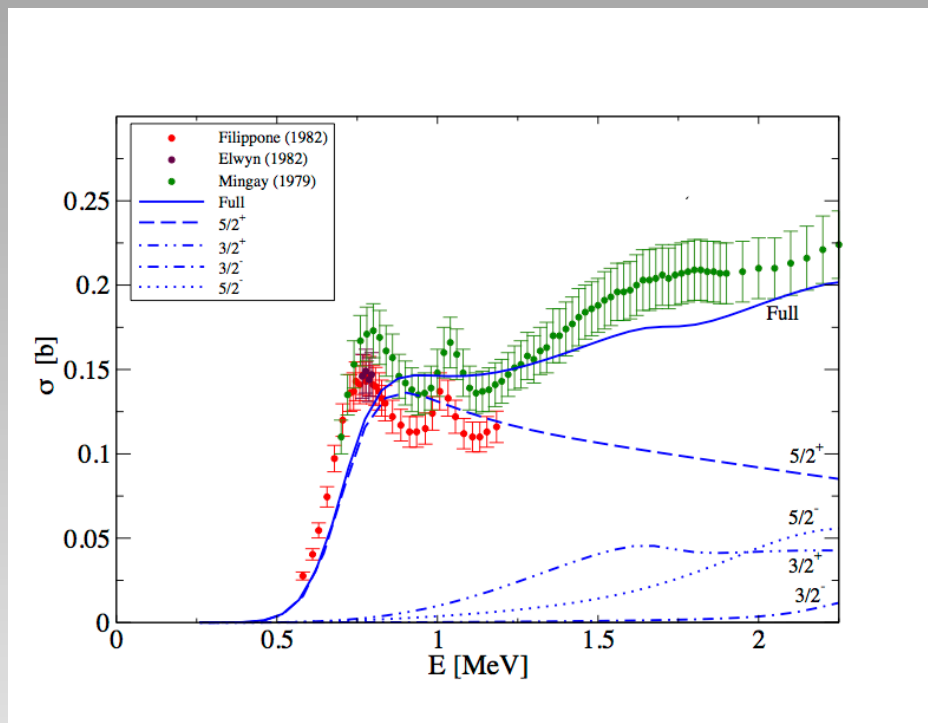
- (1) $p, {}^8\text{Li}$
- (2) $d, {}^7\text{Li}$
- (3) coupling (d,p)
- (4) virtual breakup of d

Not-included channels: (1) ${}^8\text{Be}, n$ (2) ${}^6\text{Li}, t$

NCSM-RGM calculations with SRG-evolved ($\lambda=2.02 \text{ fm}^{-1}$) chiral N^3LO NN potentials
 4 eigenstates of ${}^8\text{Li}$, 2 eigenstates of ${}^7\text{Li}$ and 5 pseudostates of deuteron
 $N_{\text{max}}=8, h\Omega=20 \text{ MeV}$

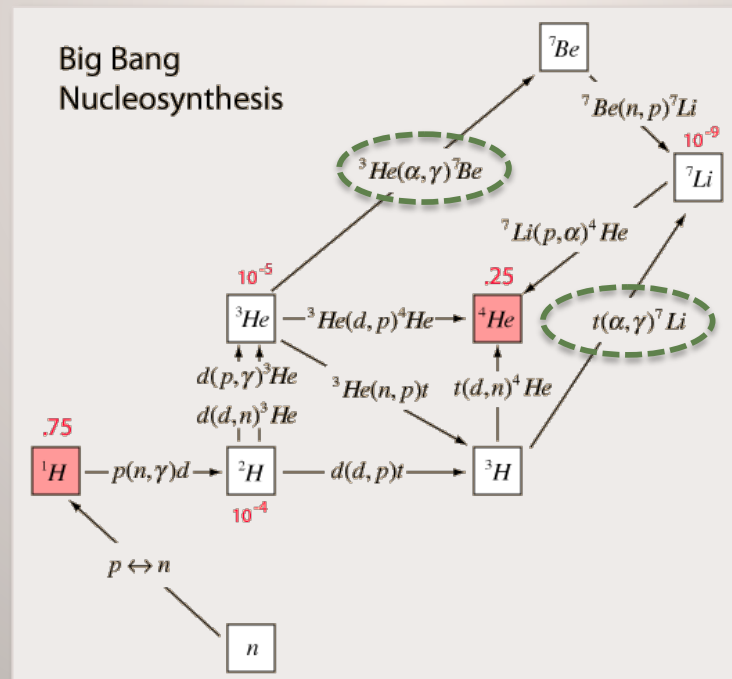
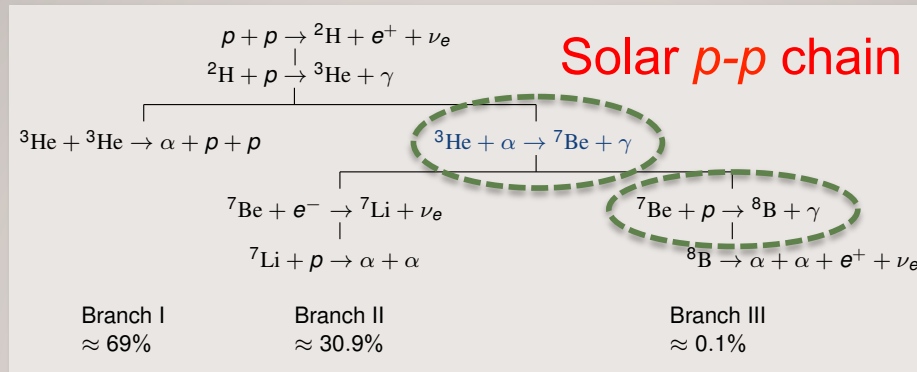
Impact of different partial waves (NCSM-RGM)

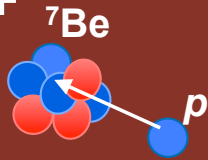
${}^7\text{Li}(d,p){}^8\text{Li}$ cross section



- Position of the first resonant peak slightly overestimated
- Increasing trend up to deuteron break-up fairly well reproduced (contribution from $3/2^+$ partial wave)
- Double-peak structure at low energy not resolved

Capture reactions important for astrophysics

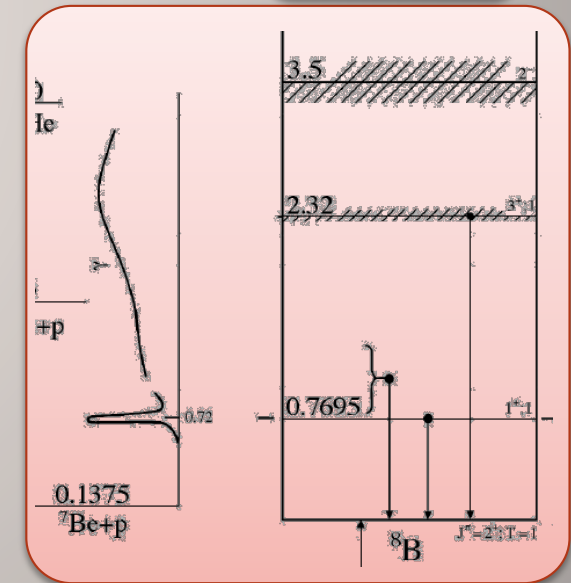
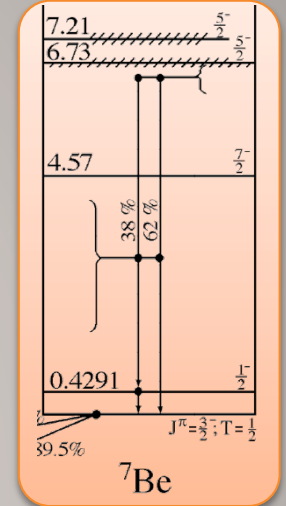
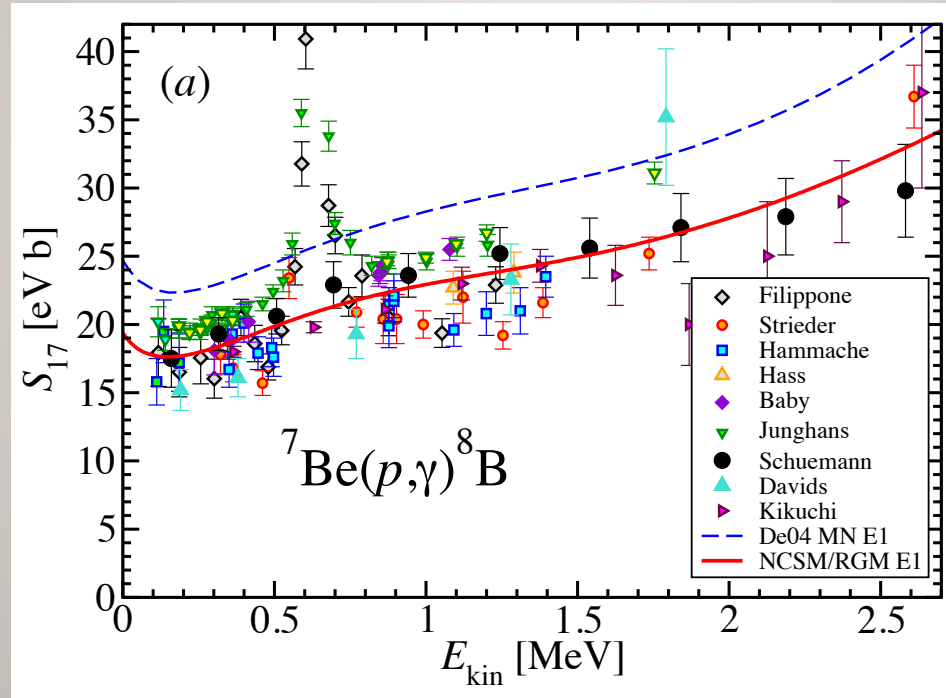


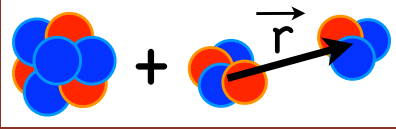


${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture

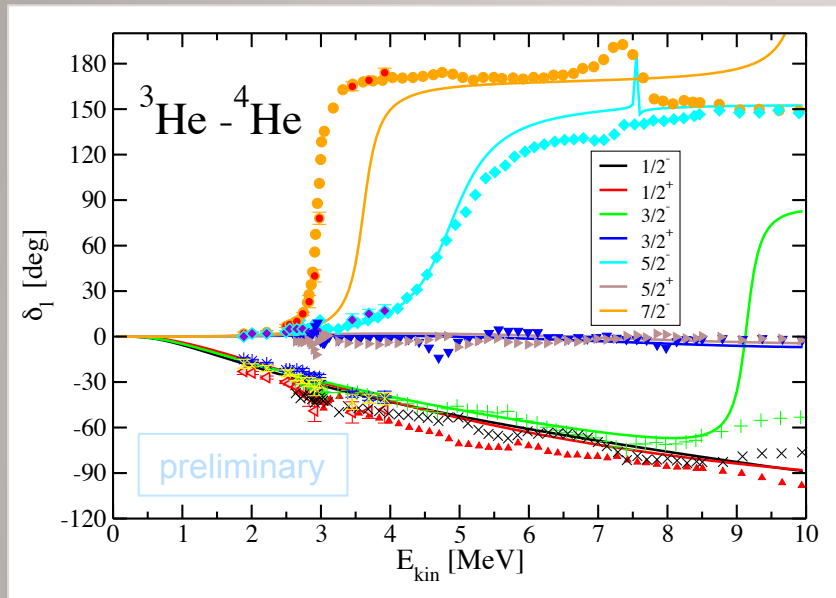
- NCSM/RGM calculation of ${}^7\text{Be}(p,\gamma){}^8\text{B}$ radiative capture
 - ${}^7\text{Be}$ states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
 - Soft NN potential (SRG-N³LO with $\Lambda = 1.86 \text{ fm}^{-1}$)

${}^8\text{B}$ 2^+ g.s. bound by 136 keV
 (expt. 137 keV)
 $S(0) \sim 19.4(0.7) \text{ eV b}$
 Data evaluation:
 $S(0) = 20.8(2.1) \text{ eV b}$





${}^3\text{He}$ - ${}^4\text{He}$ and ${}^3\text{H}$ - ${}^4\text{He}$ scattering

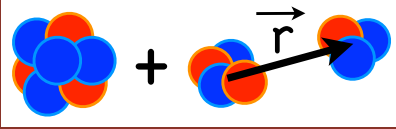


	${}^7\text{Be}$		${}^7\text{Li}$	
	NCSMC	Expt.	NCSMC	Expt.
$E_{3/2^-}$ [MeV]	-1.52	-1.586	-2.43	-2.467
$E_{1/2^-}$ [MeV]	-1.26	-1.157	-2.15	-1.989
r_{ch} [fm]	2.62	2.647(17)	2.42	2.390(30)
Q [$e \text{ fm}^2$]	-6.14		-3.72	-4.00(3)
μ [μ_N]	-1.20	-1.3995(5)	+3.08	+3.256

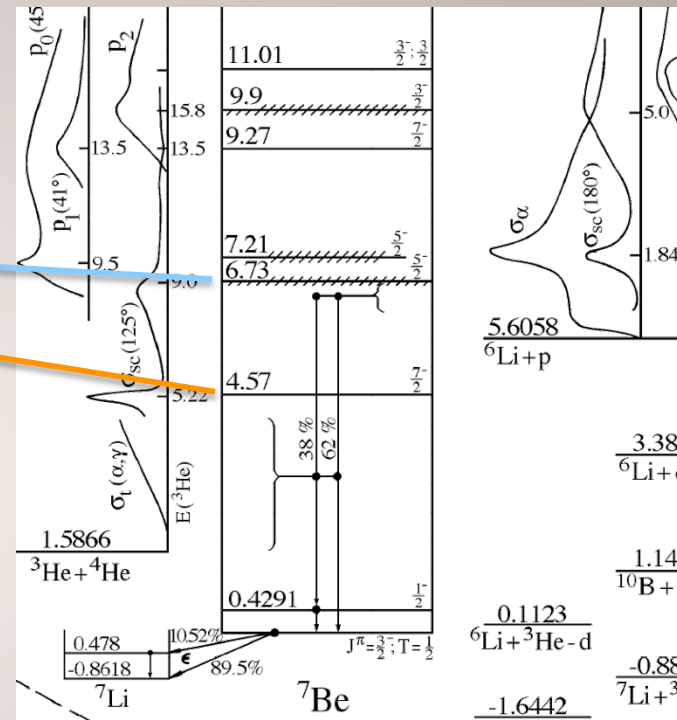
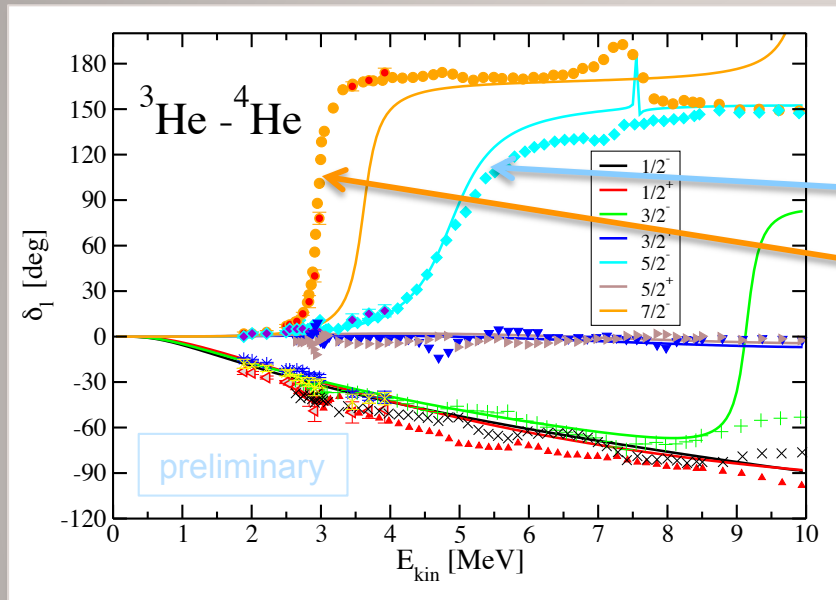
NCSMC calculations with chiral SRG- N^3LO NN potential ($\lambda=2.15 \text{ fm}^{-1}$)

${}^3\text{He}$, ${}^3\text{H}$, ${}^4\text{He}$ ground state, $8(\pi^-) + 6(\pi^+)$ eigenstates of ${}^7\text{Be}$ and ${}^7\text{Li}$

Preliminary: $N_{\text{max}}=12$, $\hbar\Omega=20 \text{ MeV}$



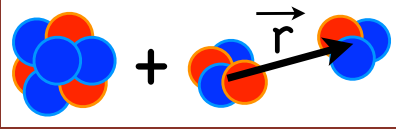
^3He - ^4He and ^3H - ^4He scattering



NCSMC calculations with chiral SRG- $N^3\text{LO}$ NN potential ($\lambda=2.15 \text{ fm}^{-1}$)

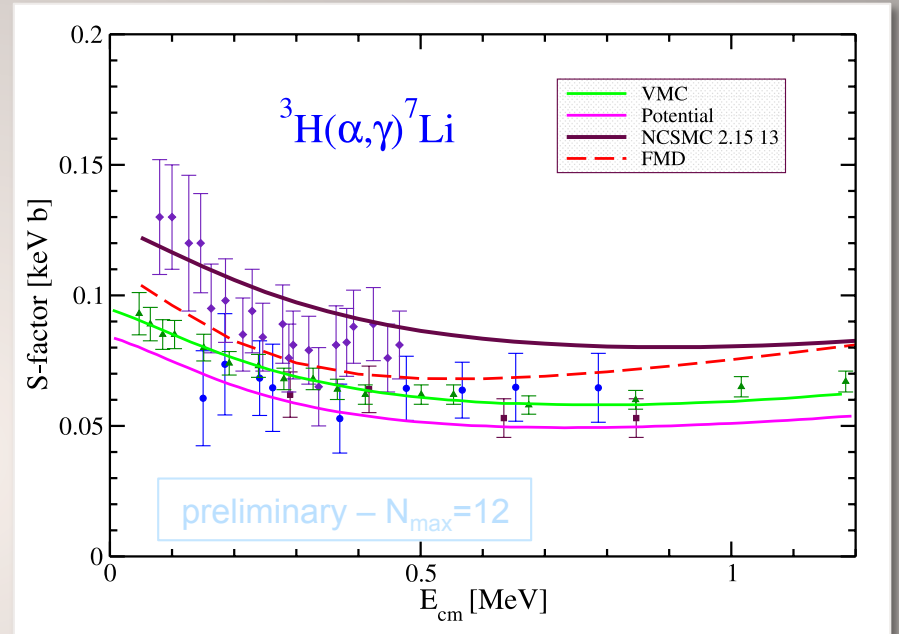
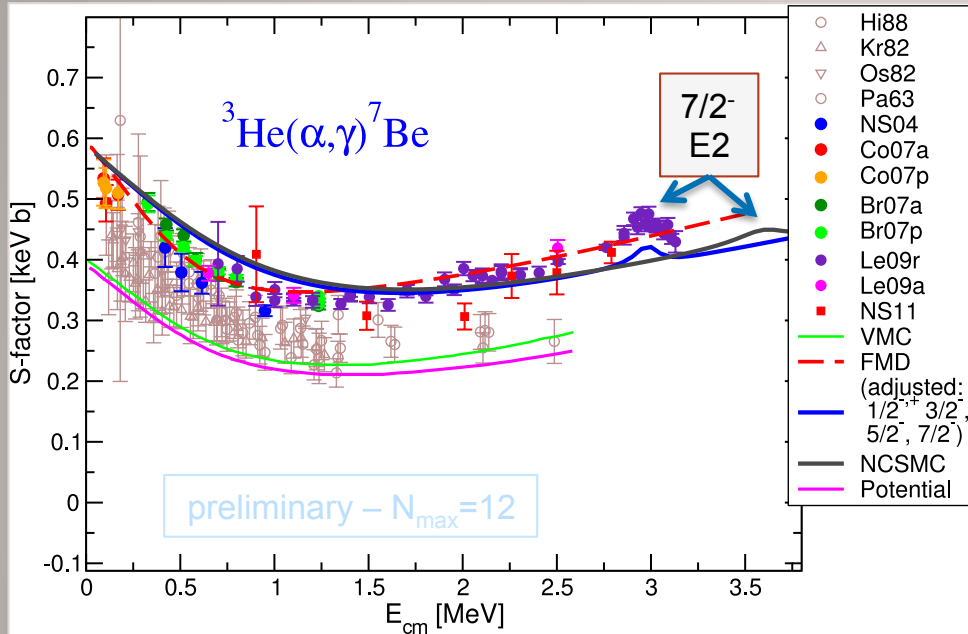
^3He , ^3H , ^4He ground state, $8(\pi^-) + 6(\pi^+)$ eigenstates of ^7Be and ^7Li

Preliminary: $N_{\text{max}}=12$, $\hbar\Omega=20 \text{ MeV}$



^3He - ^4He and ^3H - ^4He capture

E1 radiative capture with small E2 contribution at 7/2⁻ resonance



In progress

J. Dohet-Eraly, P.N., S. Quaglioni, W. Horiuchi, G. Hupin, F. Raimondi

NCSMC calculations with chiral SRG-N³LO NN potential ($\lambda=2.15 \text{ fm}^{-1}$)

^3He , ^3H , ^4He ground state, 8(π^-) + 6(π^+) eigenstates of ^7Be and ^7Li

Preliminary: $N_{\text{max}}=12$, $\hbar\Omega=20 \text{ MeV}$

Conclusions and Outlook

- *Ab initio* calculations of nuclear structure and reactions is a dynamic field with significant advances
- We developed a new unified approach to nuclear bound and unbound states
 - Merging of the NCSM and the NCSM/RGM = **NCSMC**
 - Inclusion of three-nucleon interactions in reaction calculations for $A > 5$ systems
 - Extension to three-body clusters (${}^6\text{He} \sim {}^4\text{He} + n + n$): NCSMC in progress
- Ongoing projects:
 - Transfer reactions
 - Applications to capture reactions important for astrophysics
 - Bremsstrahlung
- Outlook
 - Alpha-clustering (${}^4\text{He}$ projectile)
 - ${}^{12}\text{C}$ and Hoyle state: ${}^8\text{Be} + {}^4\text{He}$
 - ${}^{16}\text{O}$: ${}^{12}\text{C} + {}^4\text{He}$