SPIN-ORBIT COUPLED DILUTE 2D ELECTRON GAS

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Outline

1) Objective

2) Rashba 2DEG model

3) Two-electron case

4) Interactions

5) Summary/ Future work

Objective

2D electrons + SOC

↙ ↘

Solids:

Spin degeneracy = time-reversal + spatial inversion symmetry

SOC breaks inversion symmetry, but preserves TR

↓

Spin-split spectrum

e.g. Oxide interfaces

Ultracold atoms:

2-photon Raman transitions (momentum-dependent spin transitions)

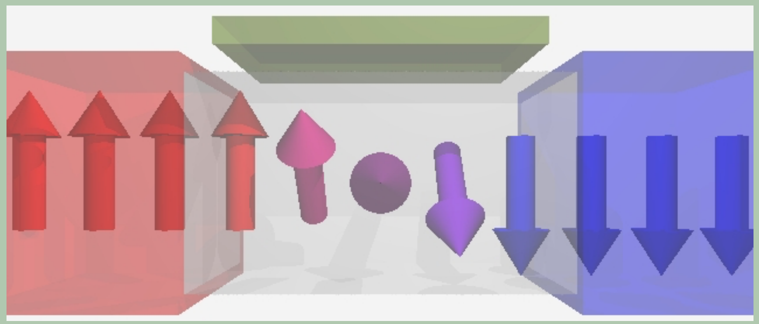
↓

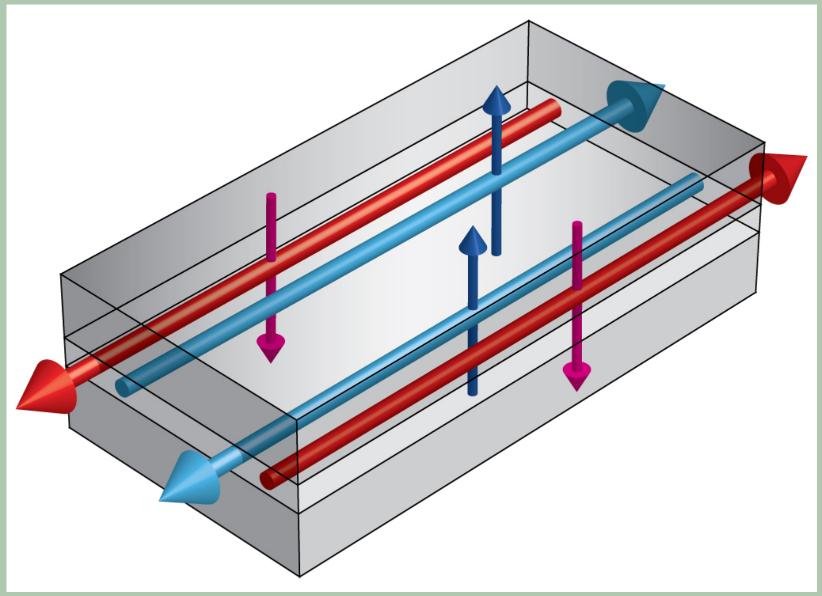
Tunable SOC

Objective

Applications

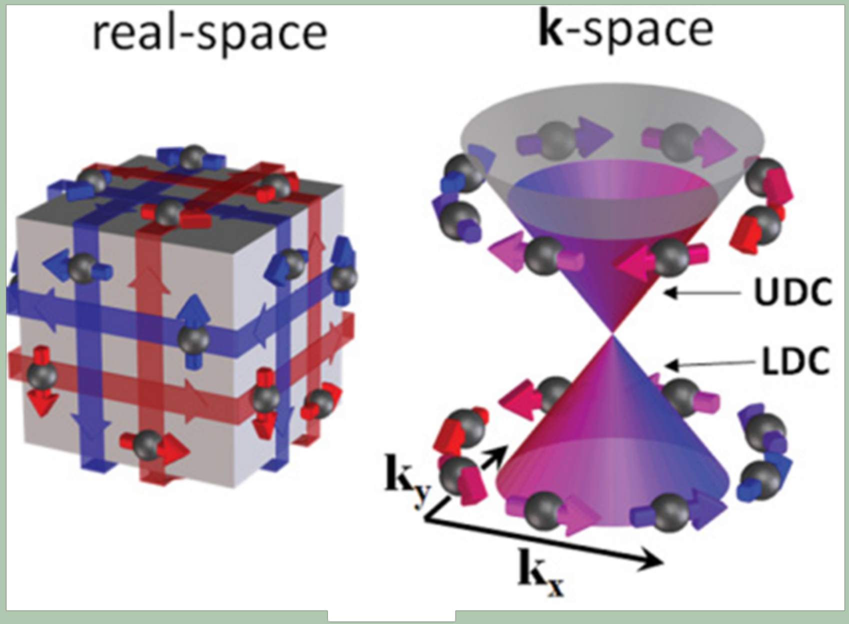
Spintronics: Manipulate spin without magnetic field





Quantum spin Hall effect

Topological Insulators



Objective

2D electron gas + SOC + Repulsive interactions

What kind of order (symmetry breaking) occurs in dilute limit of this system?

Kinetic energy scales as $1/r^2$

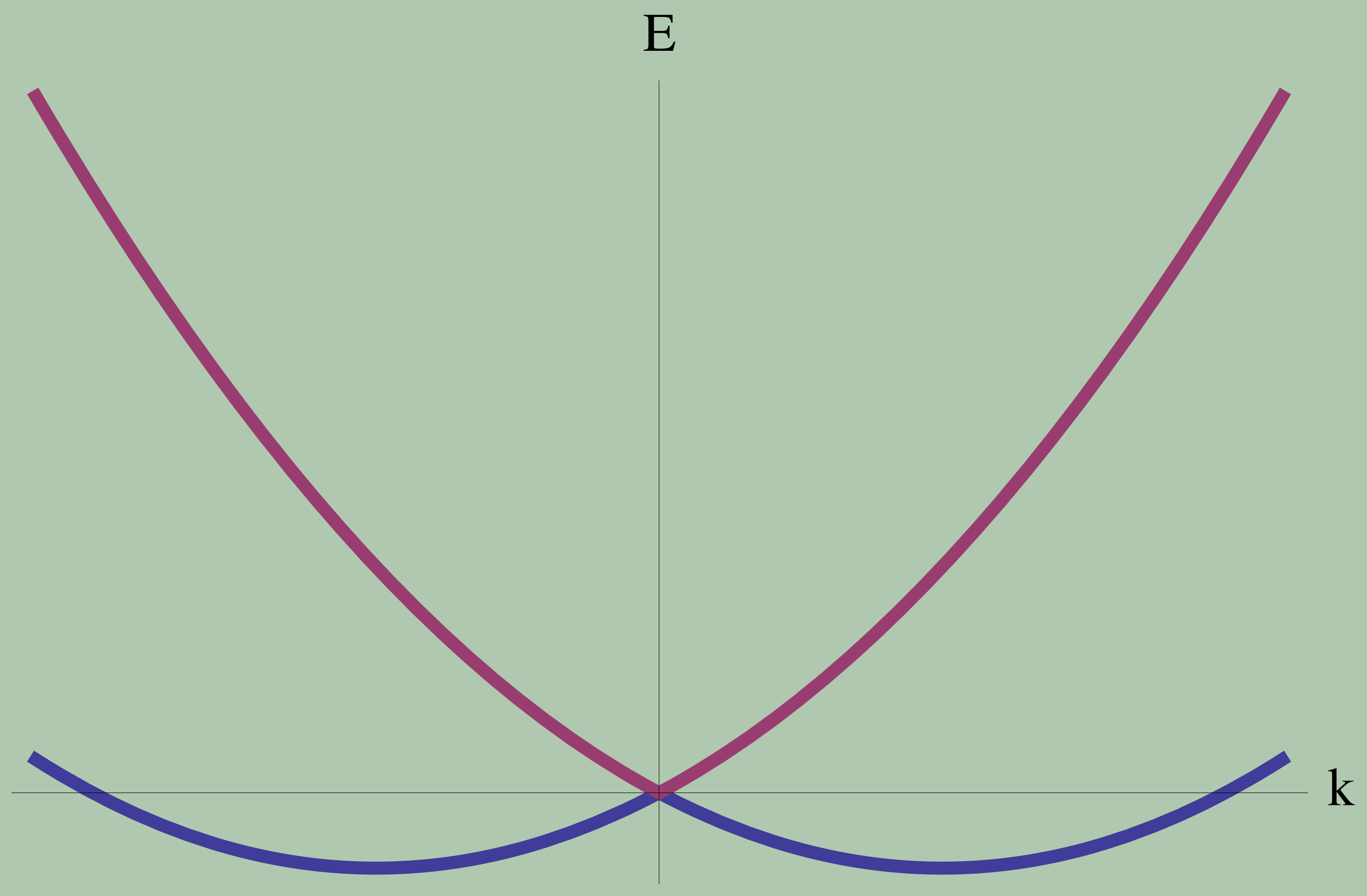
Coulomb energy scales as $1/r$

So interactions might induce order at low densities (Wigner Crystal). But what about more general interactions?

Rashba 2DEG Model

Single particle Hamiltonian: $$h(\boldsymbol{k}) = \frac{\boldsymbol{k}^2}{2m} + \lambda \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{k})$$

Rashba term "spin-splits" the free particle spectrum



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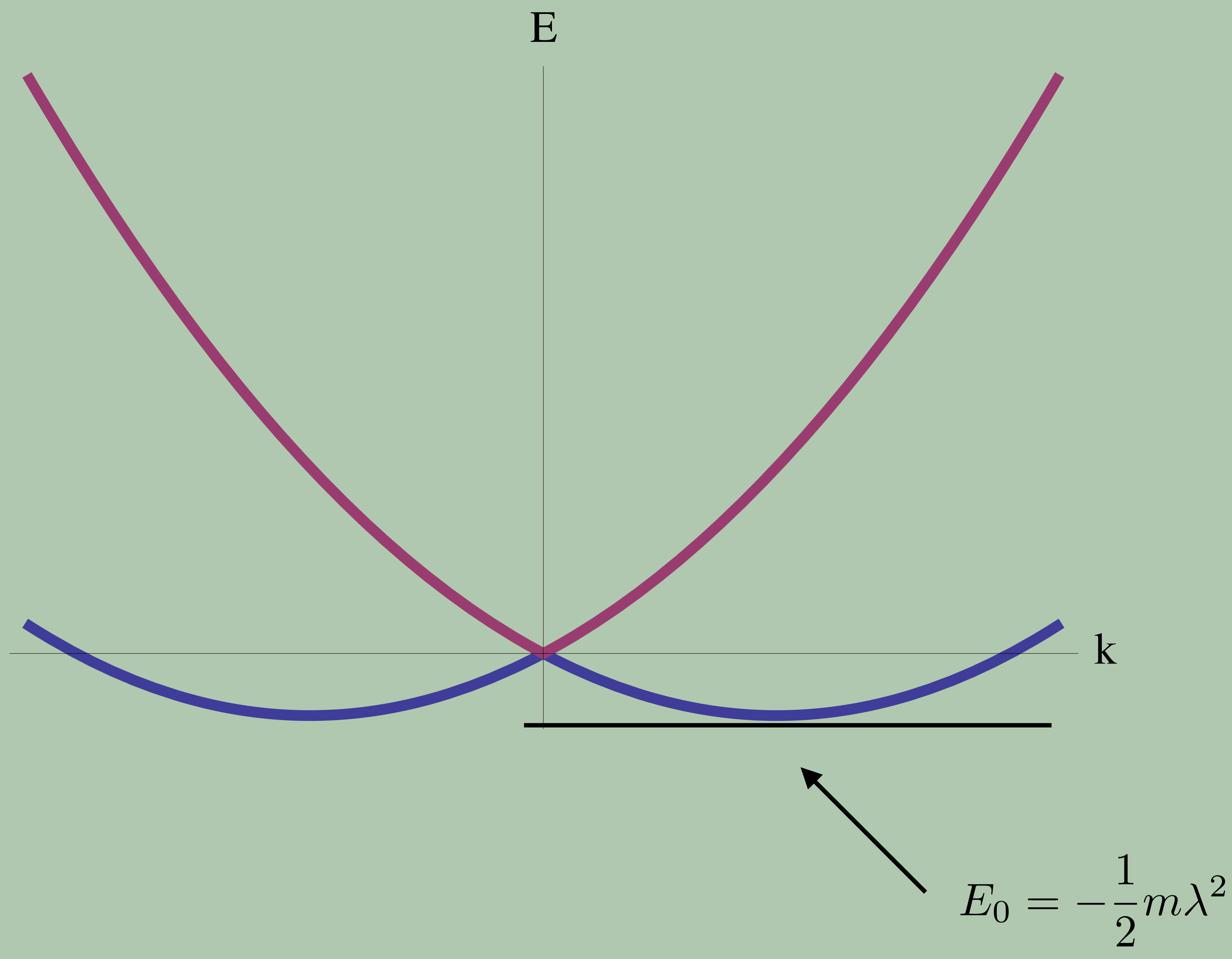
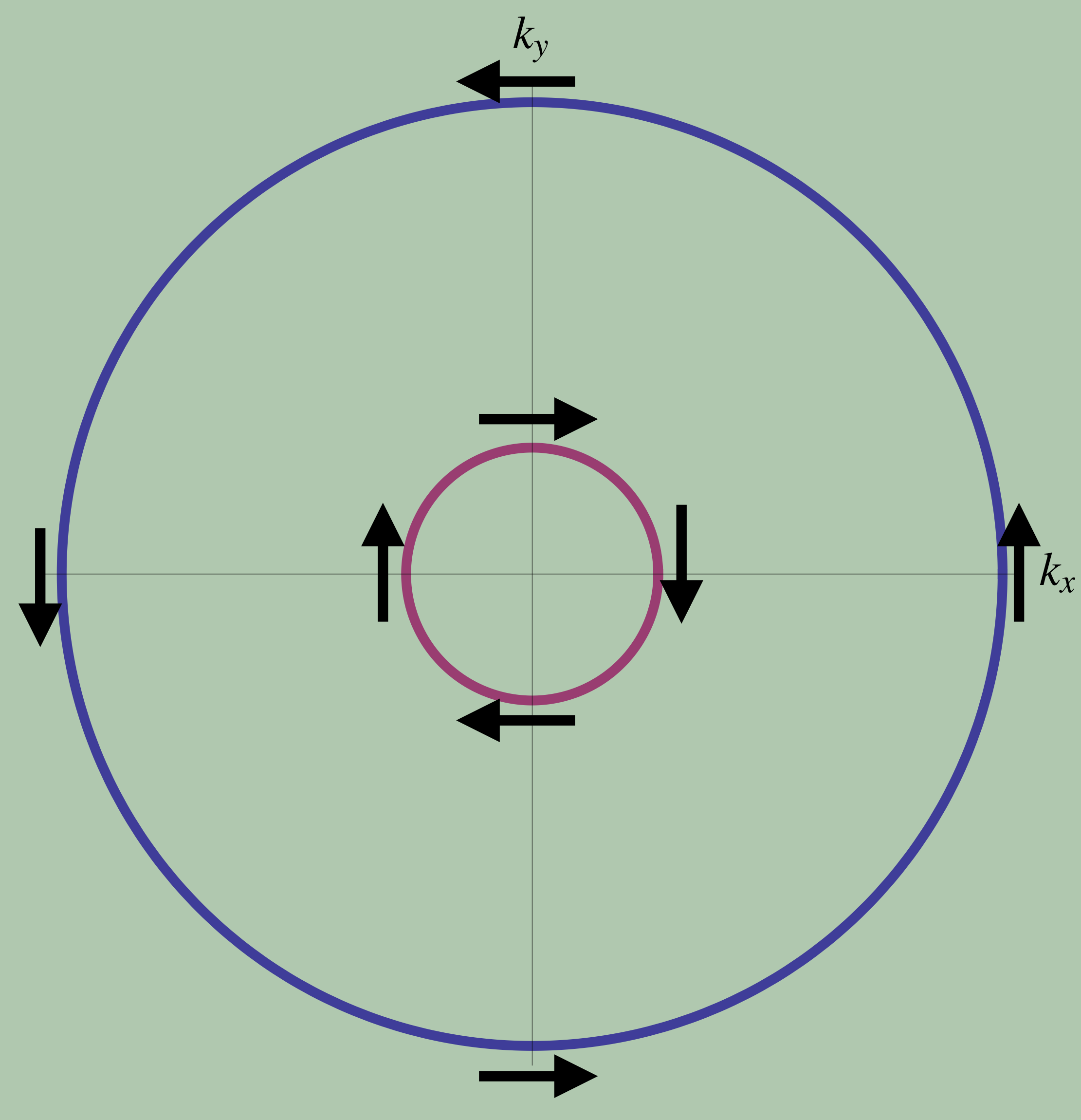


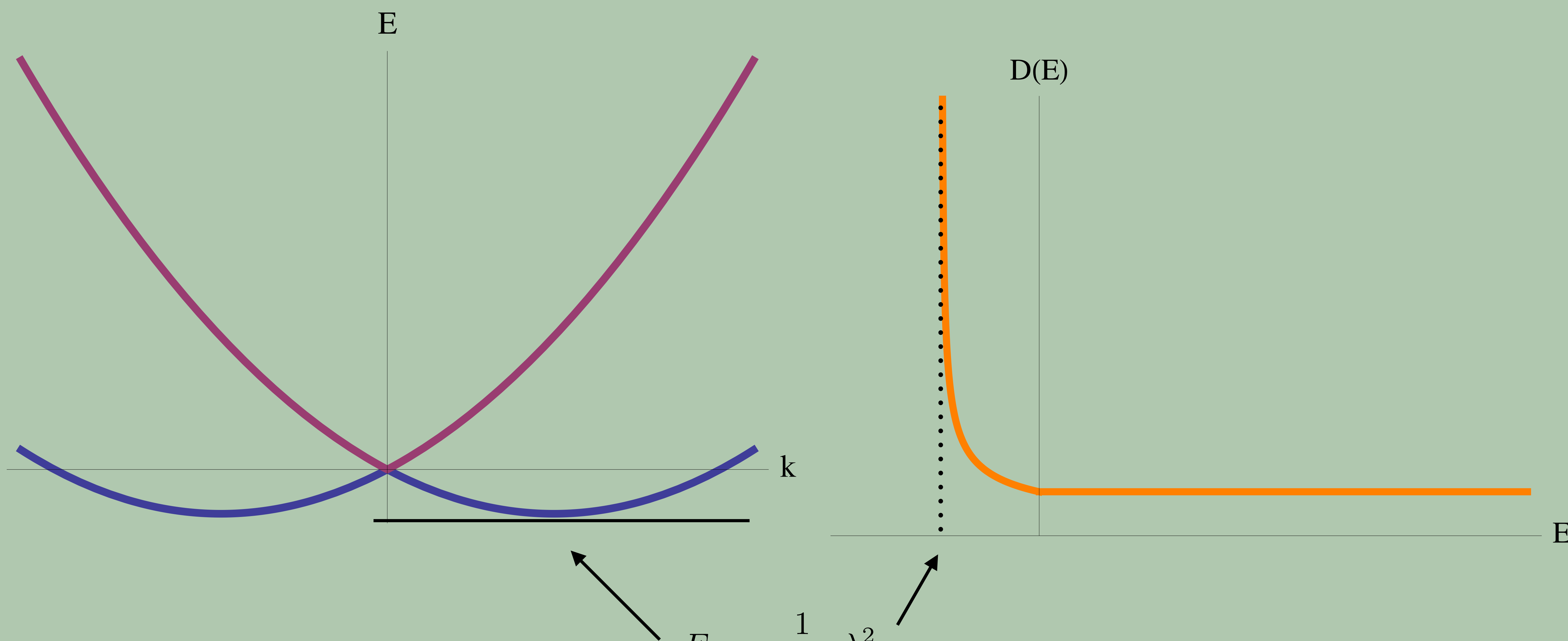


Rashba 2DEG Model

Density of states exhibits Van Hove singularity at band bottom

$$E > 0 \qquad D(E) = 4\pi m$$

$$E < 0 \qquad D(E) = \frac{4\pi m \lambda}{\sqrt{\lambda^2 + 2E/m}}$$

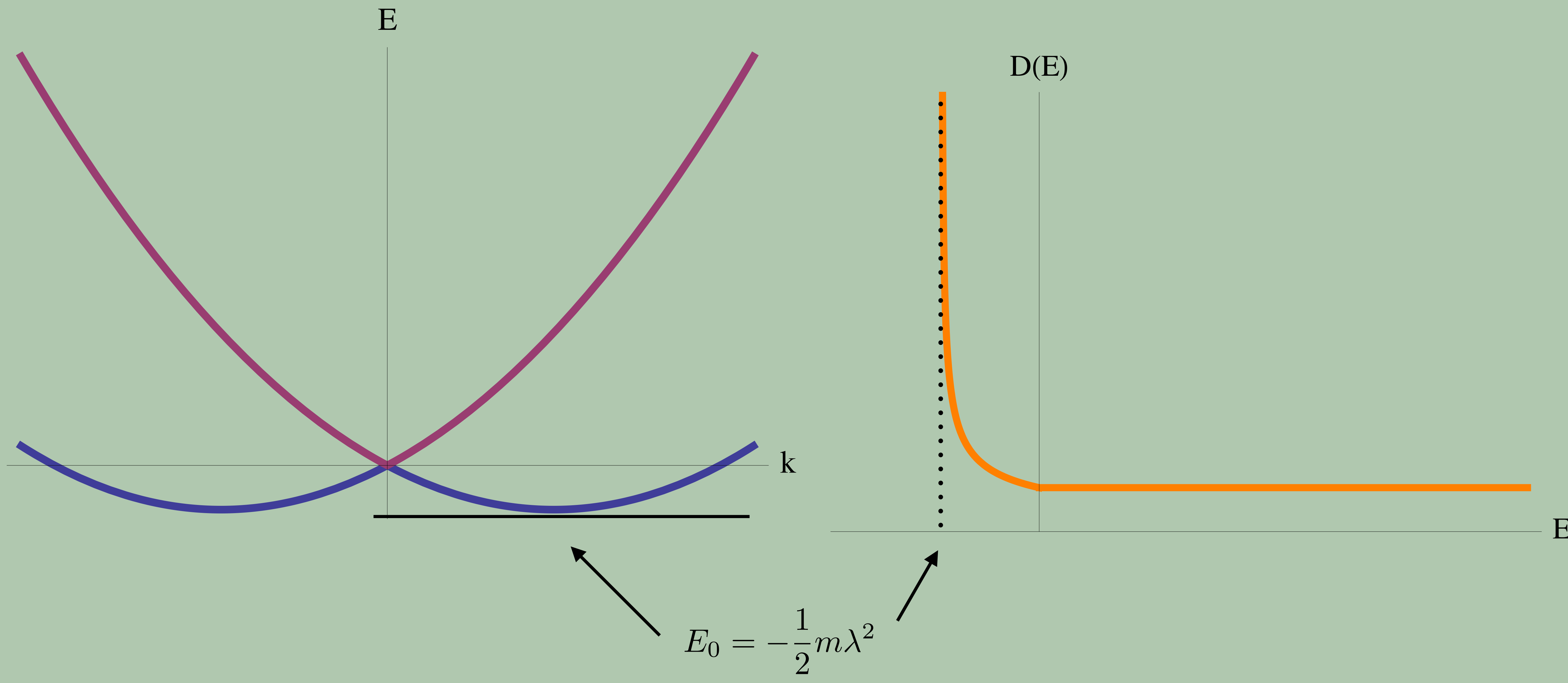


$$E_0 = -\frac{1}{2} m \lambda^2$$

Rashba 2DEG Model

Enhanced low energy DOS suggests instability to formation of new phases.

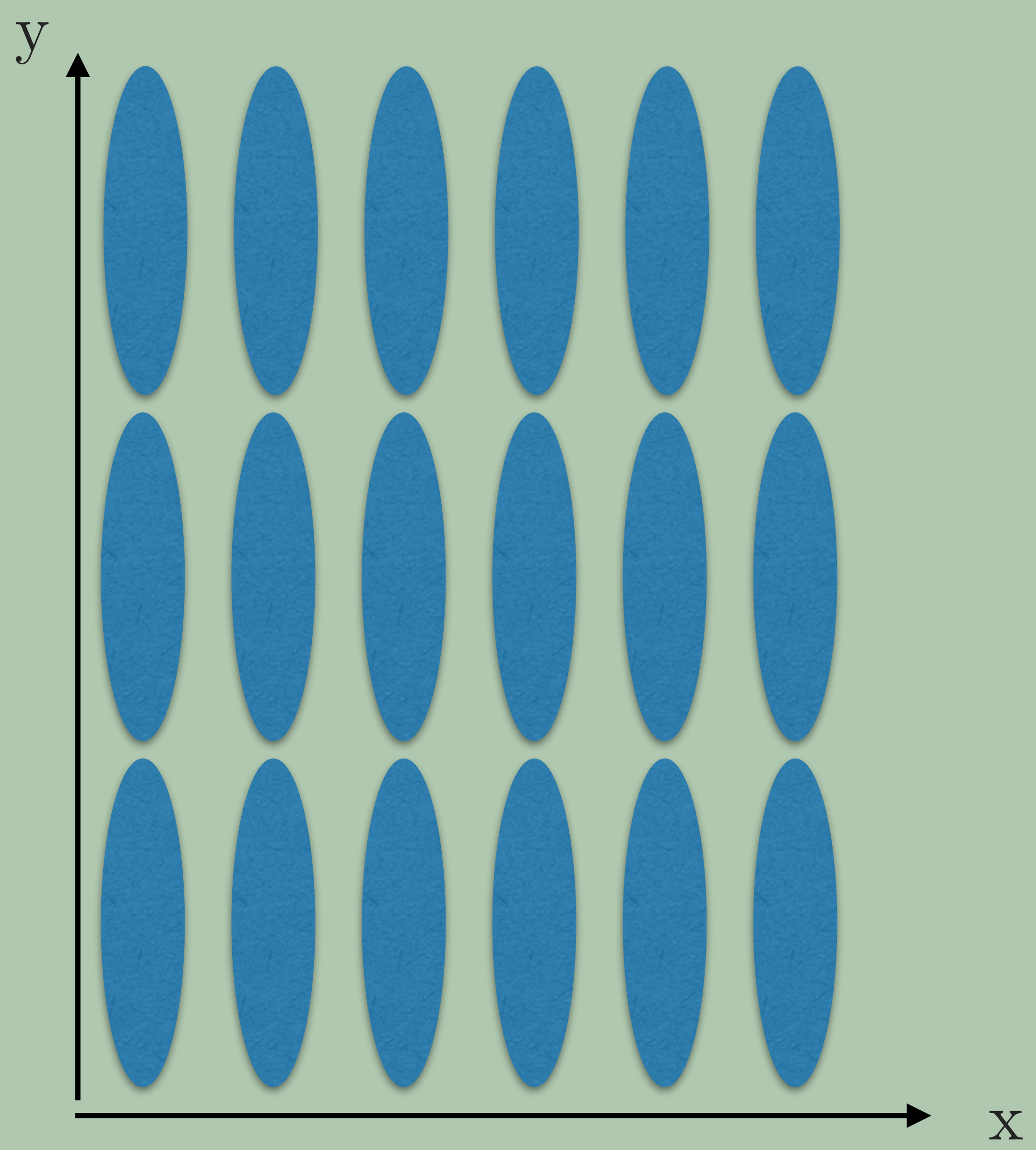
Perhaps we can get ordered phases even with short range interactions?



Rashba 2DEG Model

Enhanced interactions can cause ordering in the vibrational motion of electrons

E.g.



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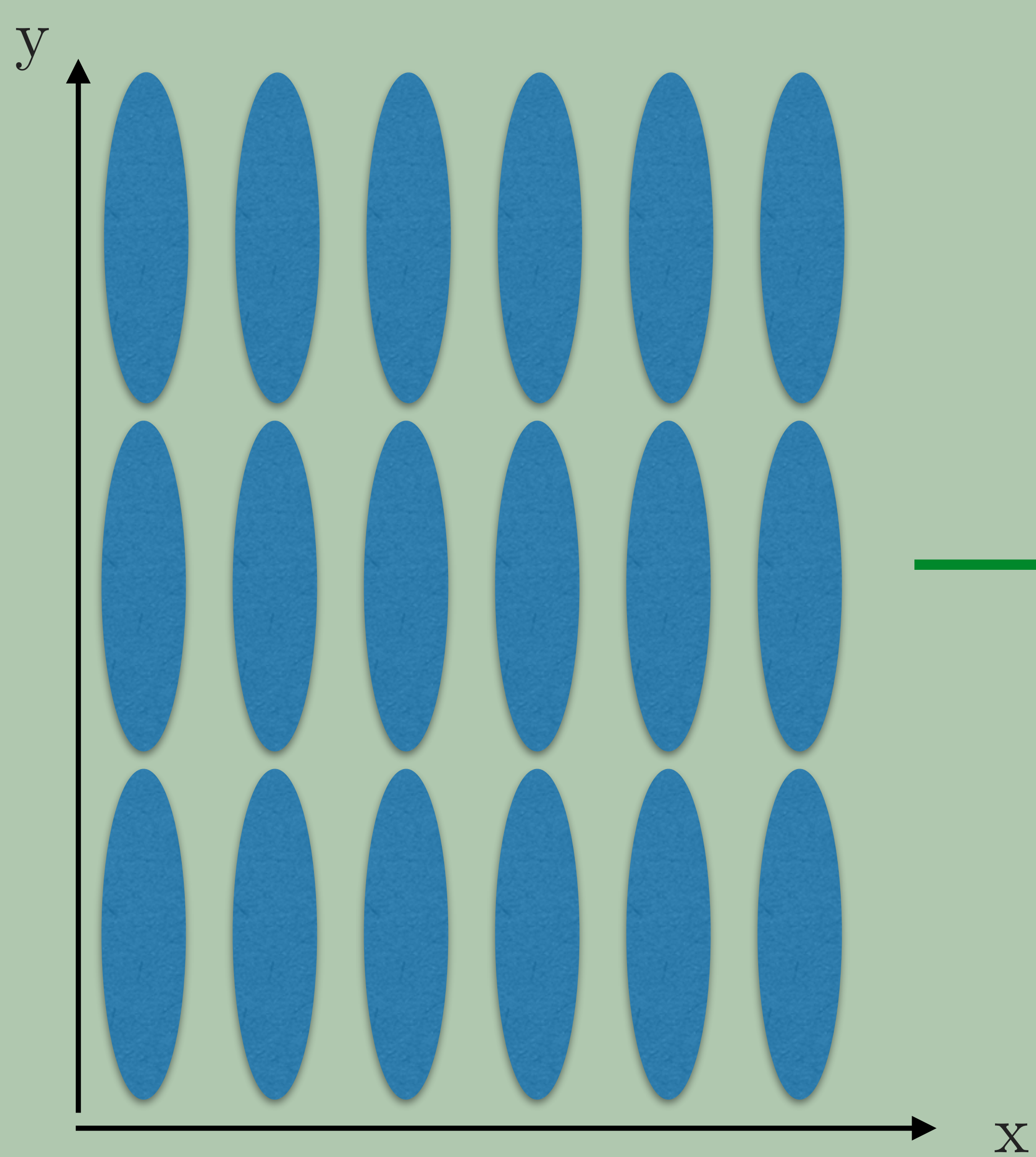
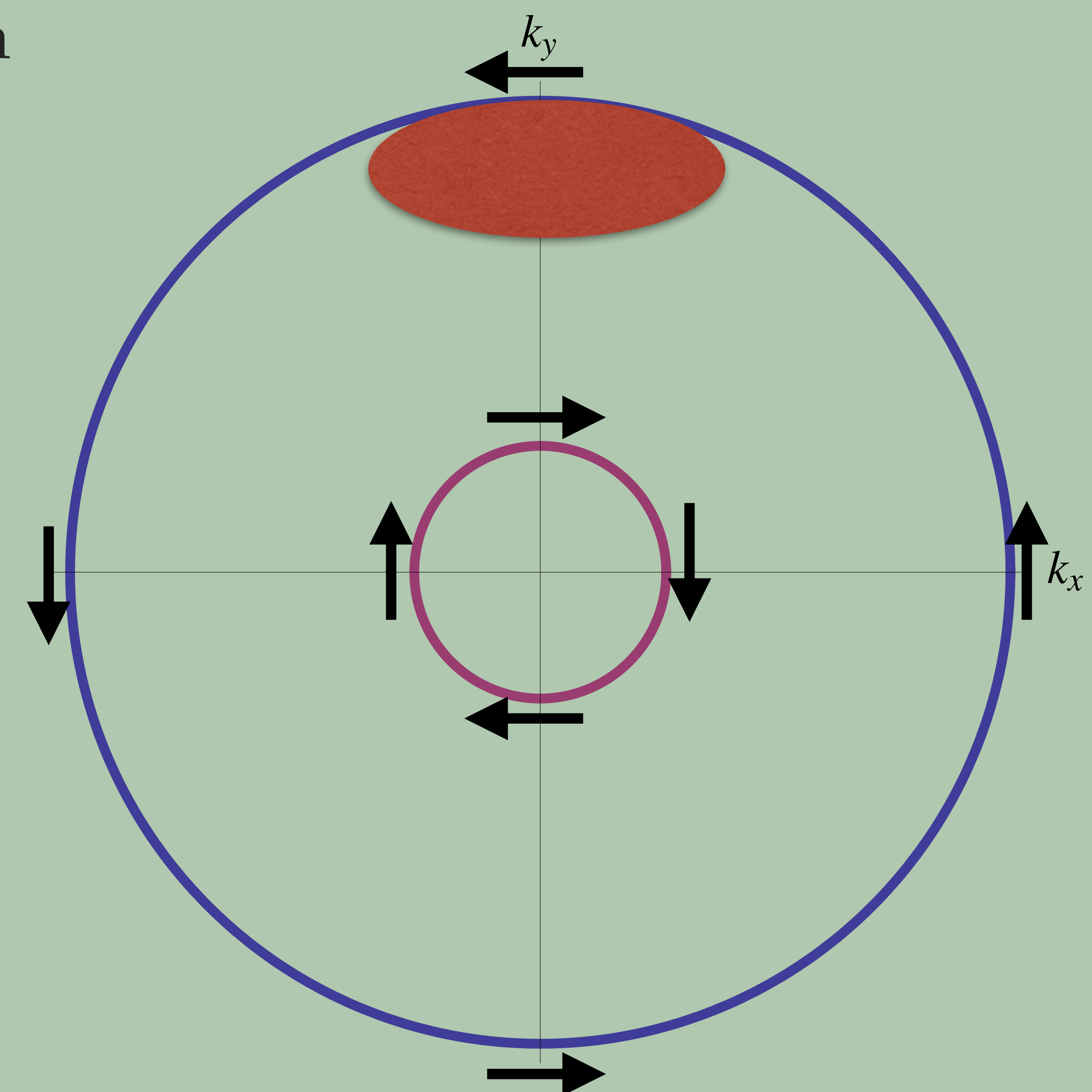


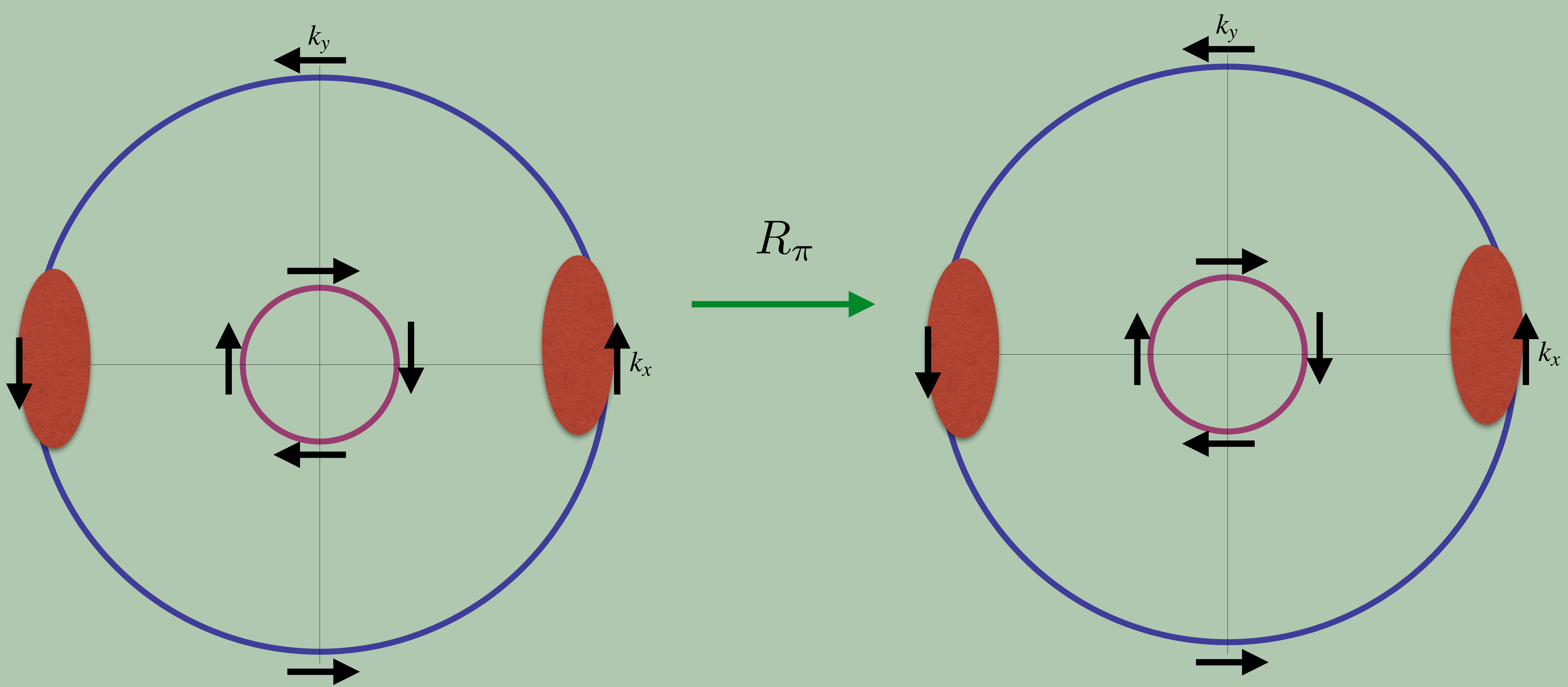
which means the ground state shows clustering around wave vectors in the band bottom



Rashba 2DEG Model

Two possibilities of interest.

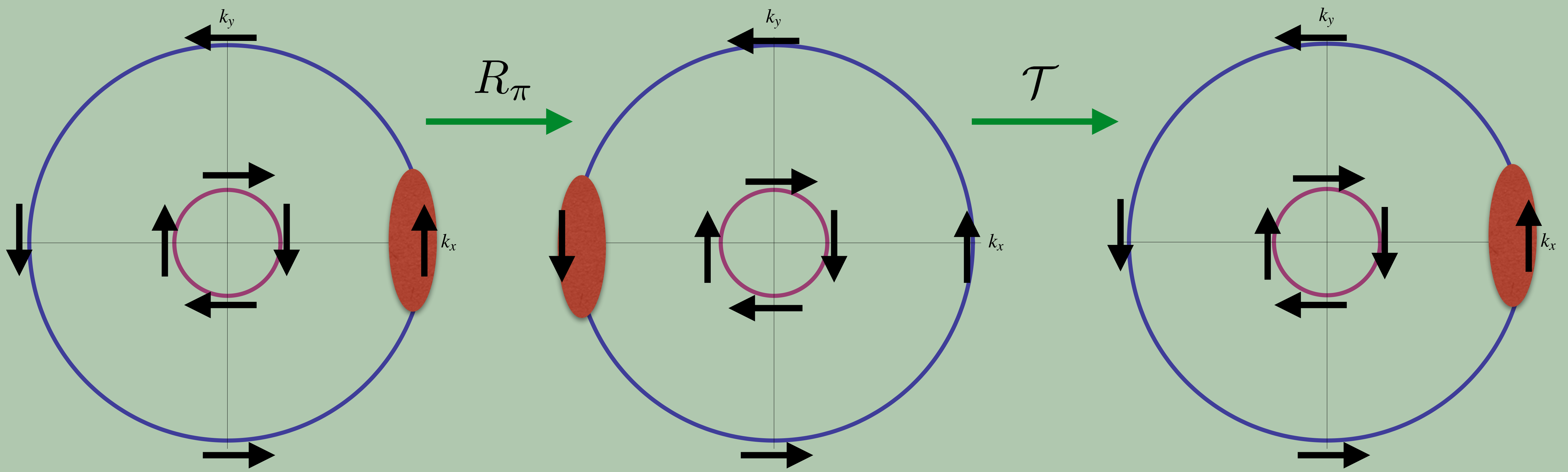
1) Nematic: Invariant under R_π



Rashba 2DEG Model

Two possibilities of interest.

2) Ferromagnetic Nematic: Invariant under $\mathcal{T} R_\pi$



Rashba 2DEG Model

The full Hamiltonian is diagonal in j_z.

Should be able to characterize symmetry breaking according to j_z eigenvalues.

Nematic: $j_z = 2$

Ferromagnetic nematic: $j_z = 1$

Isotropic: $j_z = 0$

2-electron case

Perhaps the physics of the ultra-dilute regime is captured by a 2-electron system.

Want to describe with j_z eigenstates in singlet-triplet basis

$$|p, j_z, s, s_z\rangle \equiv \int_0^{2\pi} \frac{d\theta_p}{2\pi} \sqrt{\pi p} \sum_{\sigma\sigma'} C^s_{\sigma\sigma' s_z} e^{i(j_z - s_z)\theta_p} |\mathbf{p}\sigma\rangle | -\mathbf{p}\sigma'\rangle.$$

p is now the relative momentum

2-electron case

Antisymmetry of the wavefunction splits even and odd j_z states.

$$|\Phi\rangle = \int_0^\infty dp\, p \Big[\sum_{j_z \text{odd}} \phi_{10}^{j_z}(p)|p, j_z, 1, 0\rangle + \sum_{j_z \text{even}} \Big(\phi_{00}^{j_z}(p)|p, j_z, 0, 0\rangle + \phi_{1-1}^{j_z}(p)|p, j_z, 1, -1\rangle + \phi_{11}^{j_z}(p)|p, j_z, 1, 1\rangle \Big) \Big],$$

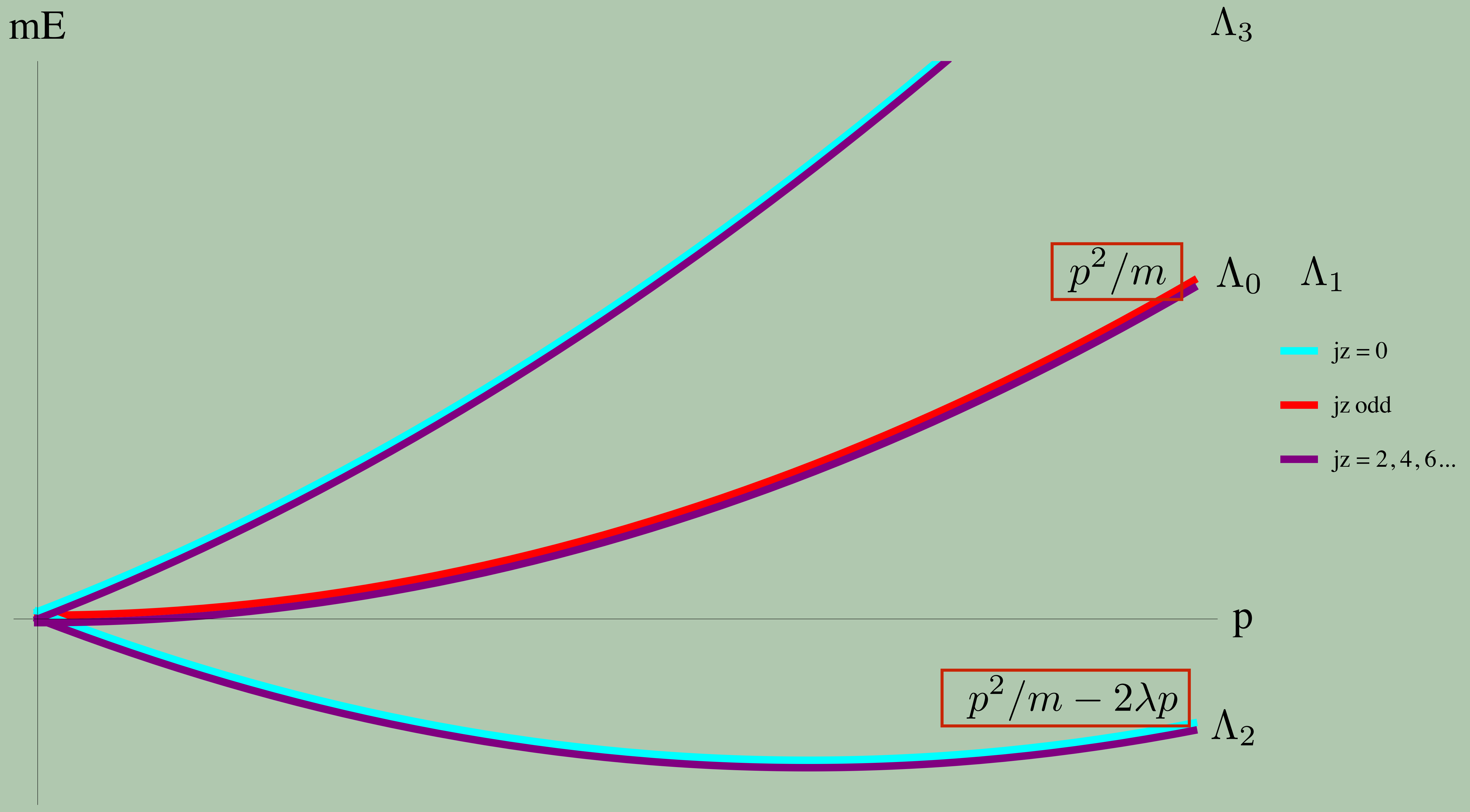
The Hamiltonian in this basis has even and odd j_z sectors:

$$\langle H \rangle = p^2/m\delta(p-p')\delta_{j_z j_z'} \longrightarrow$$ odd j_z s,s_z = (1,0)

$$\langle H \rangle = \delta(p-p')\delta_{j_z j_z'} \begin{pmatrix} p^2/m & -i\sqrt{2}\lambda & -i\sqrt{2}\lambda p \\ i\sqrt{2}\lambda p & p^2/m & 0 \\ i\sqrt{2}\lambda p & 0 & p^2/m \end{pmatrix} \longrightarrow$$ even j_z s,s_z = (0,0), (1,1), (1,-1)

2-electron case

2-particle spectrum



Interactions

What happens when we turn on repulsive interactions?

Should describe this in the framework of 2D scattering theory

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- Particles prepared in energy eigenstates
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$$\psi(r) \sim \frac{1}{\sqrt{\pi k r}} \cos(kr - l\pi/4 - \pi/4 + \delta^l)$$

Phase shift

Interactions

What are the asymptotic states in the 2-particle Rashba problem?

Eigenstates of the non-interacting position space Schrodinger equation:

<u>j_z odd</u>

$$-\frac{1}{m}\left(\partial_r^2 + \frac{(1/4 - j_z^2)}{r^2}\right)u(r) = Eu(r)$$

<u>j_z even</u>

$$\begin{pmatrix} -\frac{1}{m}\left(\partial_r^2 + \frac{(1/4-j_z^2)}{r^2}\right) & -\sqrt{2}\lambda\left(\partial_r + \frac{1/2-j_z}{r}\right) & -\sqrt{2}\lambda\left(\partial_r + \frac{j_z+1/2}{r}\right) \\ \sqrt{2}\lambda\left(\partial_r + \frac{j_z-1/2}{r}\right) & -\frac{1}{m}\left(\partial_r^2 + \frac{(1/4-(j_z-1)^2)}{r^2}\right) & 0 \\ \sqrt{2}\lambda\left(\partial_r - \frac{j_z+1/2}{r}\right) & 0 & -\frac{1}{m}\left(\partial_r^2 + \frac{(1/4-(j_z+1)^2)}{r^2}\right) \end{pmatrix} u(r) = Eu(r)$$

Interactions

Eigenfunctions are linear combinations of Bessel functions

j_z odd

$$u_{\Lambda_0}(r) = \sqrt{r}\left(a_{j_z} J_{j_z}(kr) + b_{j_z} N_{j_z}(kr)\right)$$

j_z even

$$u_{\Lambda_1}(r) = \sqrt{\frac{r}{2}}\left(a_{j_z}(J_{j_z-1}(kr) + J_{j_z+1}(kr)) + b_{j_z}(N_{j_z-1}(kr) + N_{j_z+1}(kr))\right)$$

$$u_{\Lambda_2}(r) = \frac{1}{2}\sqrt{\frac{r}{2}}\left(a_{j_z}(\sqrt{2}J_{j_z}(kr) - J_{j_z-1}(kr) + J_{j_z+1}(kr)) + b_{j_z}(\sqrt{2}N_{j_z}(kr) - N_{j_z-1}(kr) + N_{j_z+1}(kr))\right)$$

$$u_{\Lambda_3}(r) = \frac{1}{2}\sqrt{\frac{r}{2}}\left(a_{j_z}(-\sqrt{2}J_{j_z}(kr) - J_{j_z-1}(kr) + J_{j_z+1}(kr)) + b_{j_z}(-\sqrt{2}N_{j_z}(kr) - N_{j_z-1}(kr) + N_{j_z+1}(kr))\right)$$

Interactions

Compare large r expansions with the usual cosine form to get phase shifts

$\underline{\text{j}_\text{z}\text{ odd}}$

$$\tan \delta^{j_z}_{\Lambda_0}(E) = c^{j_z}(E)$$

$$c^{j_z}(E) = \frac{\beta_{j_z} N_{j_z}(kR) - kRN'_{j_z}(kR)}{-\beta_{j_z} J_{j_z}(kR) + kRJ'_{j_z}(kR)}$$

$\underline{\text{j}_\text{z}\text{ even}}$

$$\tan \delta^{j_z}_{\Lambda_1}(E) = c^{j_z}(E)$$

$$\tan \delta^{j_z}_{\Lambda_2}(E) = \left(\frac{c^{j_z}(E) - \sqrt{2}}{\sqrt{2}c^{j_z}(E) + 1} \right)$$

$$\tan \delta^{j_z}_{\Lambda_3}(E) = \left(\frac{c^{j_z}(E) + \sqrt{2}}{-\sqrt{2}c^{j_z}(E) + 1} \right)$$

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Interaction range

$$c^{j_z}(E) = \frac{\beta_{j_z} N_{j_z}(kR) - kR N'_{j_z}(kR)}{-\beta_{j_z} J_{j_z}(kR) + kR J'_{j_z}(kR)}$$

Logarithmic derivative of solution inside interaction range

Future Work/ Summary

The program now:

Phase shifts → T matrix → Vertex part → Susceptibility

Summary

- Spin orbit coupling can enhance effects of interactions, which may cause tendency to order at low energies.
- Nematic and Ferromagnetic nematic symmetry breaking can be characterized by j_z eigenvalues
- Fermion antisymmetry splits j_z even and odd sectors
- Scattering theory can be modified to find phase shifts for SOC asymptotic states

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