Chiral symmetry breaking and the Quantum Hall effect in monolayer graphene

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B. Roy, M. K., and S. Das Sarma, Phys. Rev. B **90**, 201409(R) (2014).

Motivation and Summary

- Integer Quantum Hall states are observed at fillings $\nu = 0$ and $\nu = \pm 1$ in graphene that cannot be explained within a picture of non-interacting electrons: unexpected fillings and gap too large to be Zeeman splitting

- We suggest that in a magnetic field, electron-electron interactions can induce ordered phases via "magnetic catalysis". The order parameters correspond to chiral (sublattice) symmetry breaking Dirac masses, leading to formation of gaps for $\nu = 0, \pm 1$

  $\nu = 0$: Coexisting easy-plane Neel order and easy-axis ferromagnetism

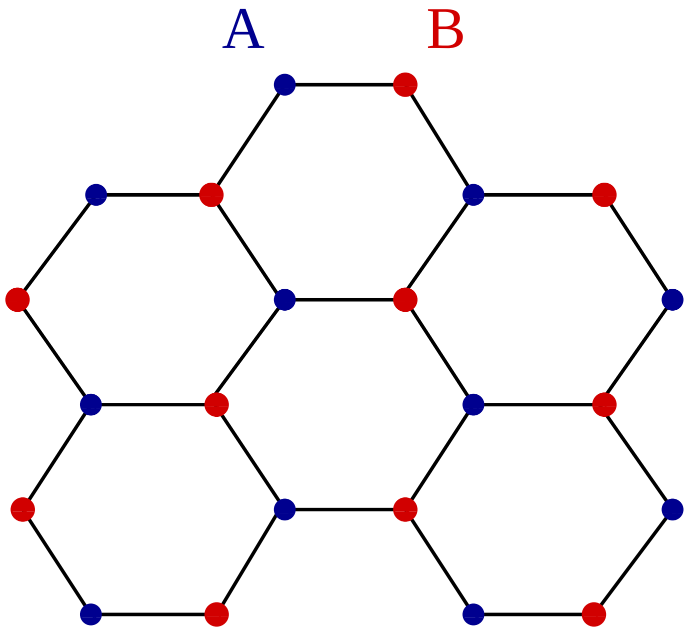
  $\nu = \pm 1$: Coexisting charge density wave and easy-axis Neel order

- We calculate gaps for these orders and find good agreement with experiment for both $\nu = 0$ and $\nu = \pm 1$

Graphene

- Two triangular sublattices:
  A and B

- One electron per site: half filling



- Tight binding model (Wallace, 1947)
  nearest neighbour hopping
  ($t = 2.5$ eV)

$$H_0 = -t \sum_{\mathbf{r}_i, j, \sigma} \left[u_\sigma^\dagger(\mathbf{r}_i) v_\sigma(\mathbf{r}_i + \mathbf{a}_j) + h.c. \right]$$

- Energy Spectrum

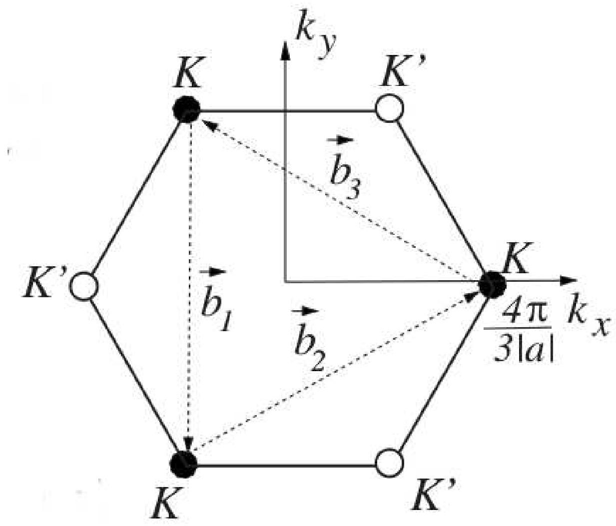
$$E(\mathbf{k}) = \pm t \left| \sum_j e^{i\mathbf{k}\cdot\mathbf{a}_j} \right|$$

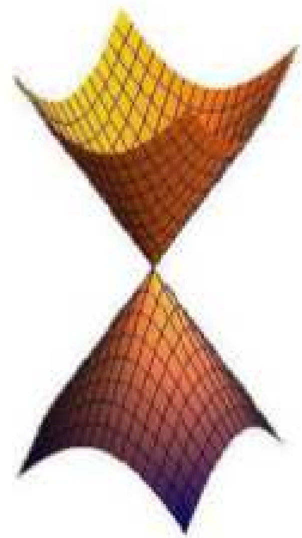
$\mathbf{a}_j$: nearest neighbour separations

- Lack of inversion symmetry around each site
  $\Longrightarrow$ Fermi points rather than a Fermi surface (unlike square lattice)

Emergent Dirac fermions

- Dirac cones centered on two inequivalent Dirac points: K and K'





- Low energy theory: Dirac Hamiltonian for 8 component Dirac fermions (2 spin, 2 valley, 2 sublattice)

$$H = \sigma_0 \otimes H_0, \qquad H_0 = \sum_{j=1}^{2} i\gamma_0\gamma_j \left(-i\partial_j - A_j\right)$$

- Gamma matrices $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, units with e, $\hbar$, v_F = 1 ($v_F \sim c/300$)
- Emergent chiral symmetry (for spinless fermions) under SU(2) generators

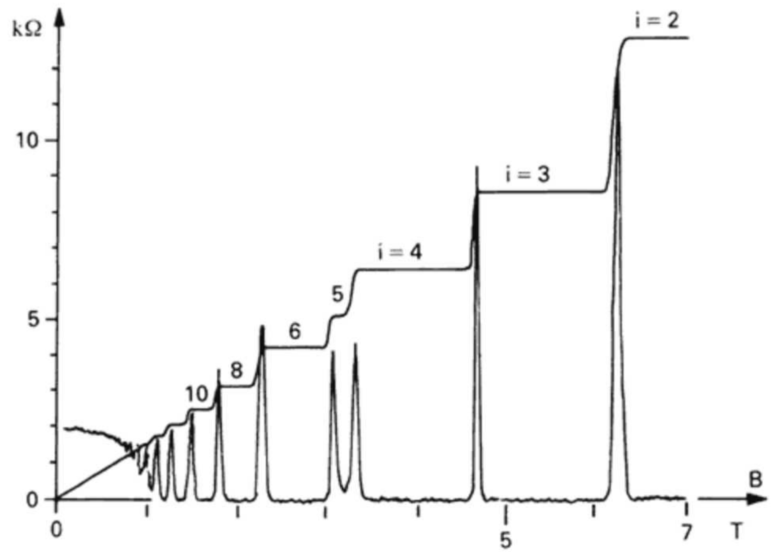
$$\{\gamma_3, \gamma_5, i\gamma_3\gamma_5\}$$

generator of translations: $i\gamma_3\gamma_5 = \tau_3 \otimes s_0$
(σ: spin, τ: valley pseudospin, s: sublattice pseudospin)

Integer Quantum Hall Effect

- Quantized Hall conductance

$$\sigma_{xy} = \nu \frac{e^2}{h}$$



Resistance vs field: GaAs [www.nobelprize.org]

- Landau levels

$$E_n = \hbar\omega_c \left(n + \frac{1}{2} \right)$$

- Degeneracy

$$eBA/h = \Phi/\Phi_0$$

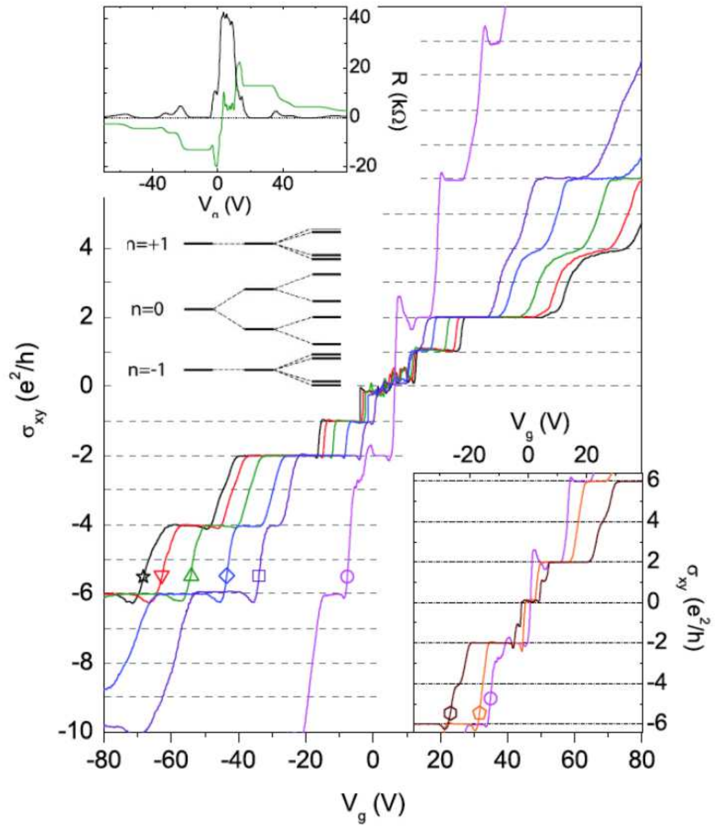
Integer QHE in Graphene

- Dirac fermions in a magnetic field: relativistic Landau levels:

$$E_n = \pm\sqrt{2nB}, \qquad n = 0, 1, 2, \ldots$$

- Experiment:

Y. Zhang *et al.*, PRL 2006



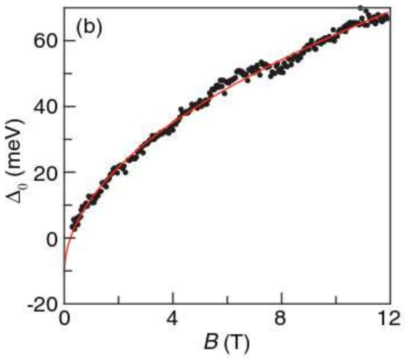
- Weak magnetic fields:
  $\nu = \pm(4n + 2)$ for $n = 1, 2, \ldots$
  Degeneracy:
  2 (valley) $\times$ 2 (spin) $\times$ Φ/Φ_0
  Gusynin and Sharapov, PRL 2005

- Strong magnetic fields
  – additional plateaux:
  i) $\nu = 0, \pm 1$
  $\Longrightarrow$ Interaction driven symmetry breaking

  ii) $\nu = \pm 4, \ldots$
  Zeeman splitting of higher LLs

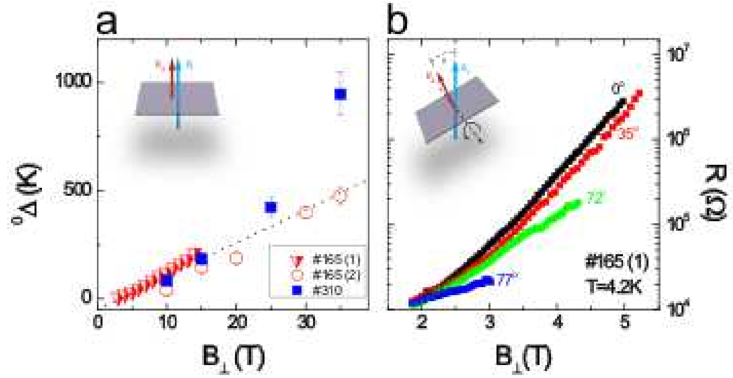
Gap for the $\nu = 0$ Hall state

Different groups techniques/substrates



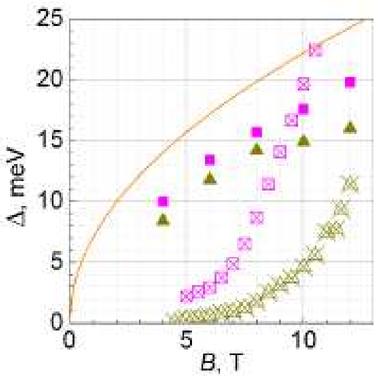
Compressibility/suspended

Abanin et al, PRB 88, 115407 (2013)



Transport/ boron nitride

Young $et\ al.$, Nat. Phys. 8, 550 (2012)



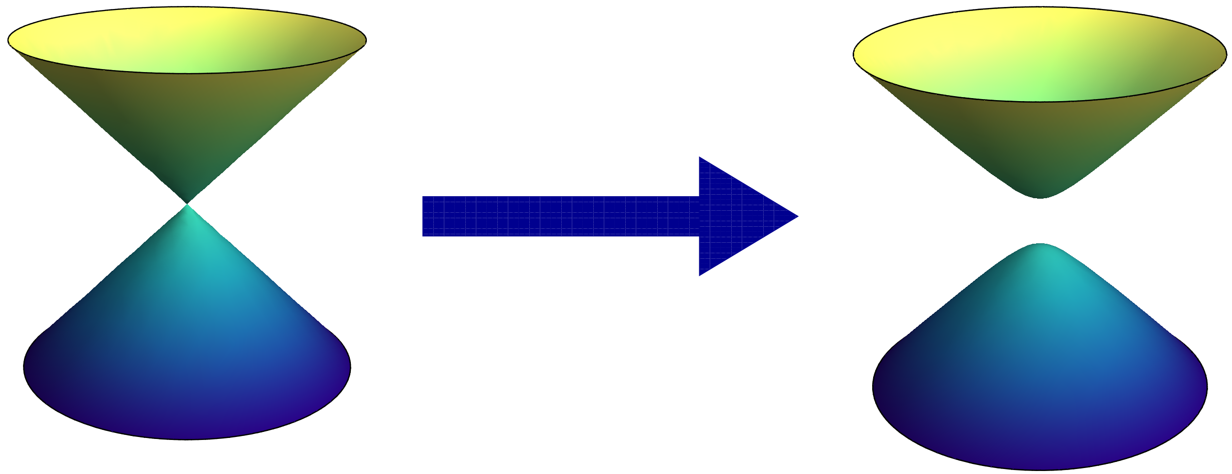
Capacitance/ boron nitride

Yu et al., PNAS 110, 3282 (2013)

- Gap for $\nu = 0$ Hall state shows crossover from linear to $\sim \sqrt{B}$ sub-linear dependence on magnetic field
- Resistance (R_{xx}) decreases with tilting of magnetic field (increasing $B_{\parallel}$ at fixed total field)

Interactions and symmetry breaking orders

- No magnetic field: strong interactions $\Longrightarrow$ Dirac fermion masses



- Short range electron-electron interactions: Hubbard U, V_1, V_2

- Order parameter Δ

$$\epsilon_{\pm}(\mathbf{p}) = \pm\sqrt{|p|^2 + |\Delta|^2}$$

Chiral symmetry breaking orders

CSB orders – break sublattice symmetry

$$H_{CSB} = \sigma_0 \otimes H_0 + m_j(\sigma_j \otimes \gamma_0), \qquad \gamma_0 = \tau_0 \otimes s_3$$

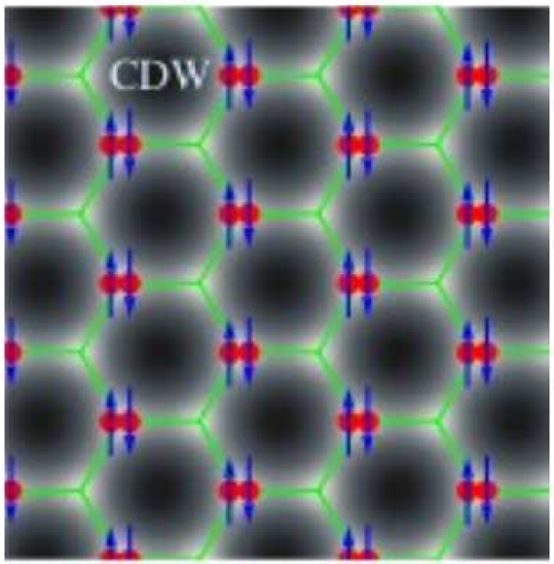
Antiferromagnetism ($j = 1, 2, 3$): Hubbard repulsion U

[I. Herbut, PRL 2006, Assaad and Herbut, PRX 2013]

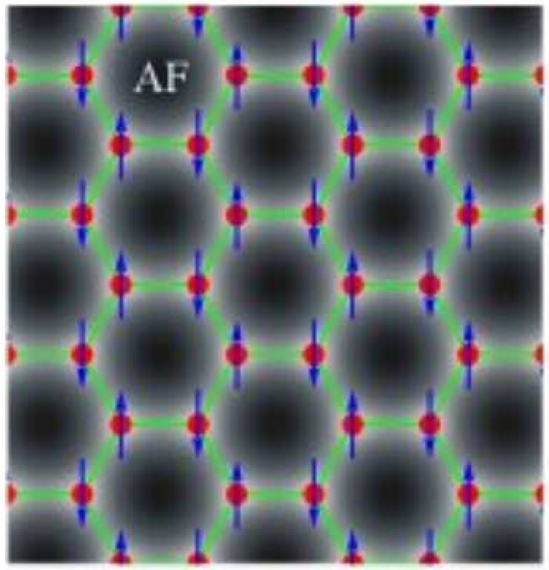
Charge density wave ($j = 0$): Nearest-neighbour repulsion V_1

[Herbut, PRL 2006; Herbut *et al*, PRB 2009; Weeks and Franz, PRB 2010]

Charge density wave



Antiferromagnetism



Topological orders

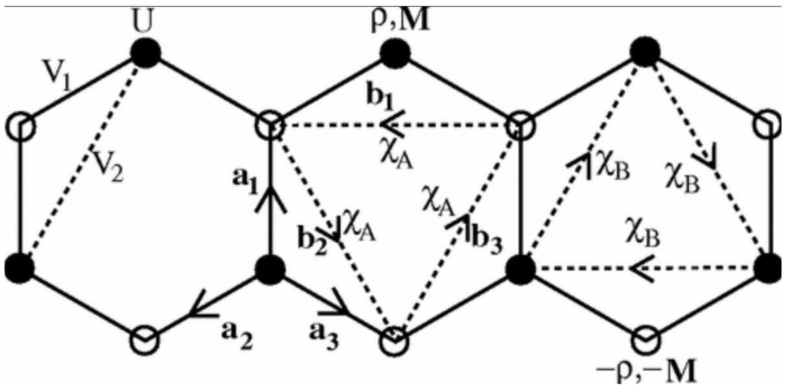
Broken time reversal symmetry

$$H_{TRSB} = \sigma_0 \otimes H_0 + \tilde{m}_j(\sigma_j \otimes i\gamma_1\gamma_2)$$

Quantum Anomalous Hall ($j = 0$)/ Spin Hall ($j = 1, 2, 3$) Insulators:

Next nearest-neighbour interactions V_2

[Raghu *et al.*, PRL 2008; Roy and Herbut, PRB 2013]



[Raghu *et al.*, PRL 2008]

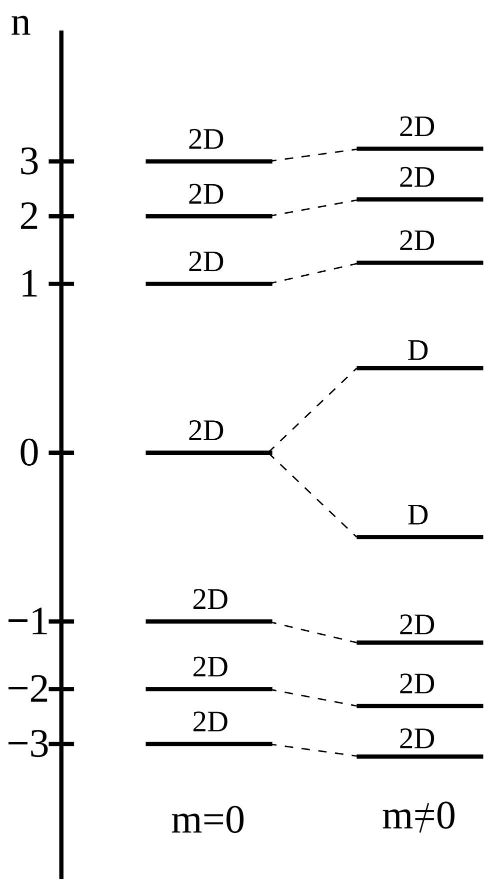
- In graphene: $U_{\rm crit} > U > V_1 > V_2$ [Wehling *et al.*, PRL 106, 236805 (2011)]
  – Interactions not strong enough to generate CSB orders when $B = 0$

- Magnetic fields can help CSB ordering at weak coupling

Magnetic Catalysis

- Magnetic fields quench kinetic energy $\Longrightarrow$ enhances interactions

- Formation of CSB orders at infinitesimal interaction in a magnetic field



Degeneracy $D = B/2\pi$

- Zeroth LL simultaneously sublattice/valley polarized

- Weak U, $V_1 \Longrightarrow$ AFM, CDW
  [I. Herbut, PRB 2007]

- Magnetic catalysis: magnetic field allows formation of CSB orders for subcritical zero field coupling
  [V. P. Gusynin *et al.*, PRL **73**, 3499 (1994)]

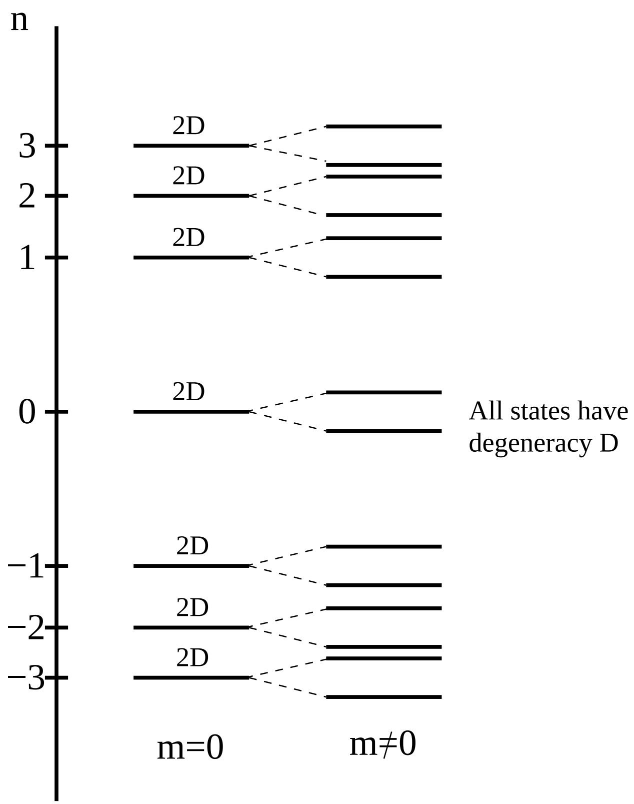
– Splits ZLL, pushes down all filled LLs $\Longrightarrow \nu = 0$ state

Filled LL with CSB order:

$$E_n = -\sqrt{2nB + \Delta_{CSB}^2}$$

Other proposals

- **Quantum Hall Ferromagnetism**



- QHFM: breaks valley degeneracy (sublattice symmetry not broken)

  [Barlas *et al.*, Nanotechnology (2012)]

$$\sigma_0(\text{spin}) \otimes \tau_3(\text{valley}) \otimes s_0(\text{sublattice})$$

- Splits all filled Landau levels:

$$E = -\sqrt{2nB} \pm \Delta_{QHFM}$$

- Gains energy only by splitting ZLL

c.f. CSB masses – all LLs contribute

- **Long range Coulomb interaction**

  Gap $\sim \sqrt{B}$: does not capture crossover in gap dependence on $B_\perp$

Self-consistent theory of $\nu = 0$ Hall state

- On-site repulsion $\Longrightarrow$ antiferromagnetism (CSB Dirac mass)
- Magnetic field: Zeeman coupling $(\lambda) \Longrightarrow$ ferromagnetism

Dirac LLs $\pm E_{n,\sigma}$:

$$E_{n\sigma} = \sqrt{N_\perp^2 + \left[\sqrt{N_3^2 + 2nB} + \sigma(m+\lambda)\right]^2}$$

Non-zero $\lambda \Longrightarrow N_3 = 0$: Easy-plane antiferromagnet

AFM order $\perp B$, FM order $\parallel B$ [I. Herbut, PRB **76**, 085432 (2007)]

- **Free energy**: in-plane antiferromagnetism $(N_\perp)$ and ferromagnetism (m)

$$F_0 = \frac{N_\perp^2}{4g_a} + \frac{m^2}{4g_f} - D\sum_{\sigma=\pm}\left[\frac{1}{2}E_{0,\sigma} + \sum_{n\geq 1}^{N_{\max}} E_{n,\sigma}\right]$$

Gap equations

Minimize F_0 with respect to $N_\perp$ and m:

$$\frac{1}{g_a} = \frac{B}{\pi} \sum_{\sigma=\pm} \left[\frac{1}{2E_{0,\sigma}} + \sum_{n\geq 1}^{N_{\max}} \frac{1}{E_{n,\sigma}} \right],$$

Displays UV divergence as $N_{\max} \to \infty$ (filled LLs pushed down)

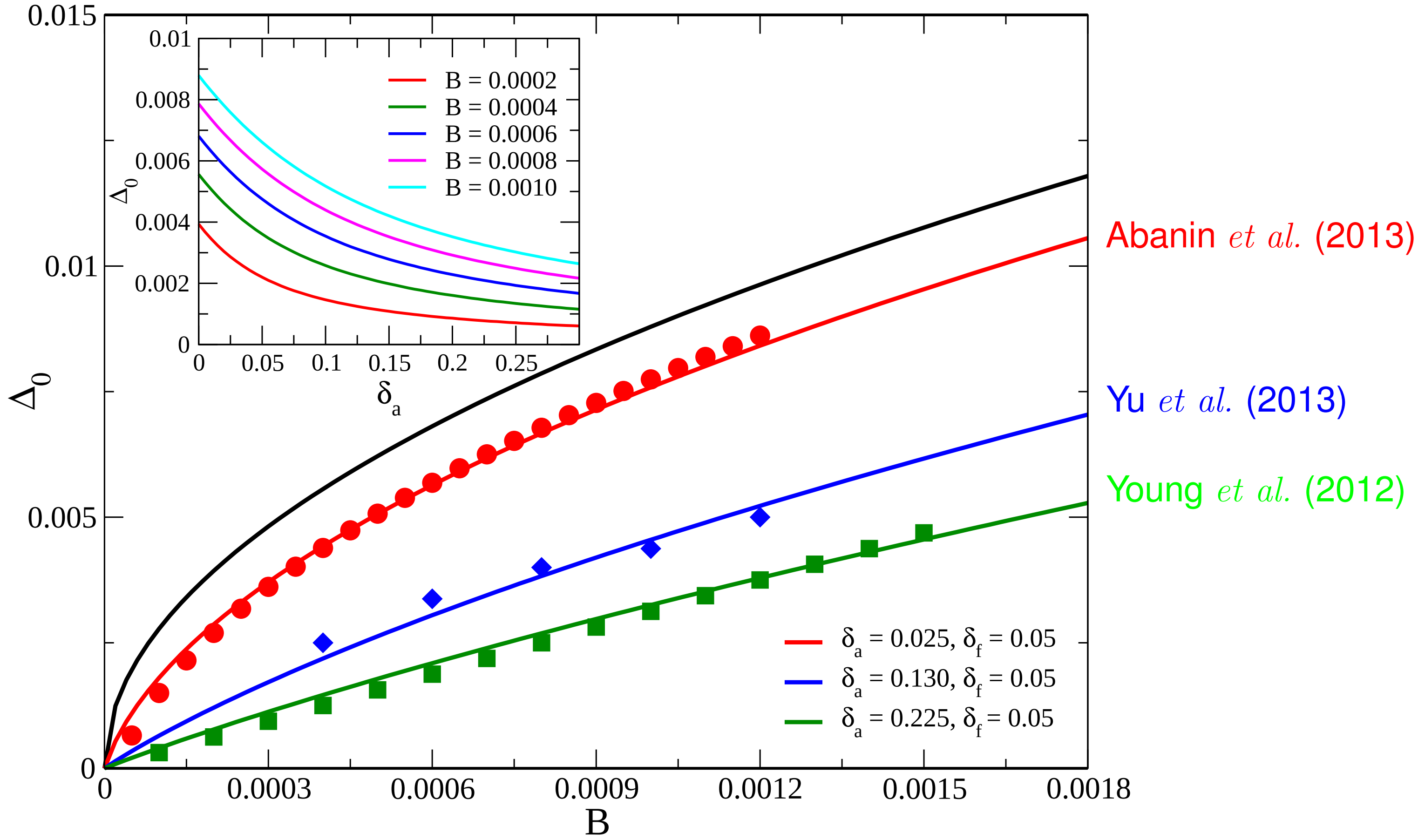
$$\frac{1}{g_f} = \frac{B}{\pi} \sum_{\sigma=\pm} \left[\frac{(m+\lambda)}{2E_{0,\sigma}} + \sum_{n\geq 1}^{N_{\max}} \frac{(m+\lambda)+\sigma\sqrt{2nB}}{E_{n,\sigma}} \right].$$

No UV divergence (filled LLs are split)

- Regularization: $\delta_a = \pi \left[\frac{1}{g_a \Lambda} - \frac{1}{g_a^c \Lambda} \right]$ with $(g_a^c)^{-1} = \int_{\Lambda^{-1}}^{\infty} ds/s^{\frac{3}{2}}$

  δ_a measures distance from zero field AFM quantum critical point g_a^c
  $\delta_a > 0$: subcritical interaction

Comparison with experiments at $\nu = 0$



- Two parameter fits to gap for easy plane antiferromagnetic ($N_\perp$) order. Ferromagnetism $m \ll N_\perp$.

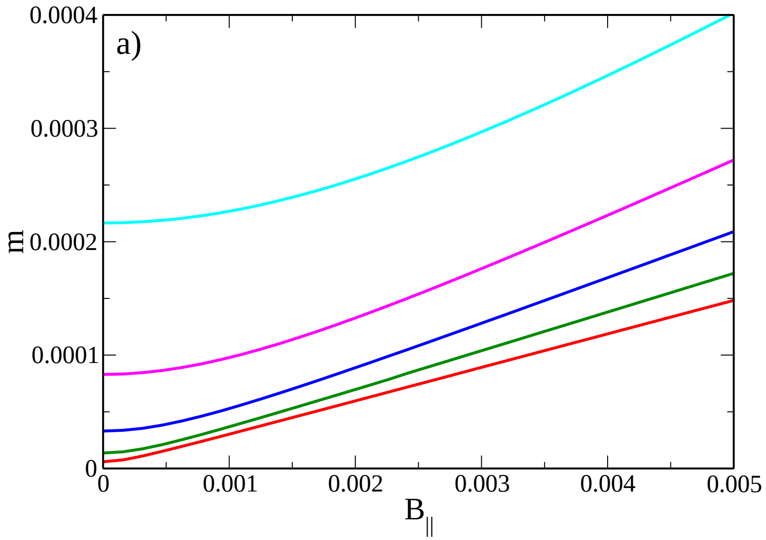
- Single parameter fits (with $\delta_f = 0$, $m = 0$) are equally good – keep m for consistency

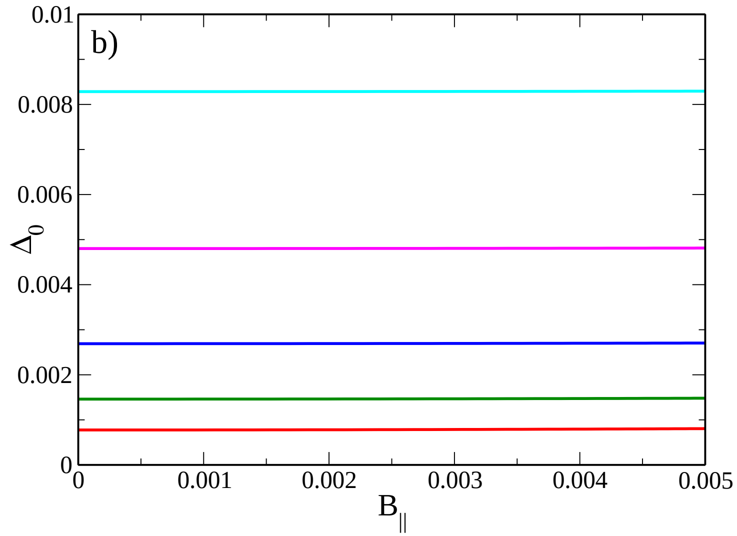
$\nu = 0$ Hall state in tilted magnetic fields

- $N_\perp$ and m scale with $B_\perp$ (Dirac LL driven orderings)

- Zeeman coupling λ scales with total magnetic field B_T not $B_\perp$

Ferromagnetic order, m, and gap Δ for 0 $\lesssim B_\parallel \lesssim 50$ T at fixed $B_\perp$



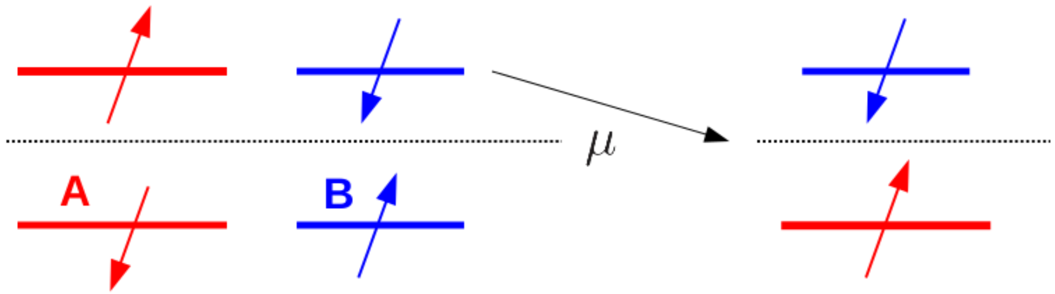


$B_\perp$ = 2, 4, 8, 16, 32 T

m grows linearly with $B_\parallel$ for $B_\parallel \gtrsim 2$ T

QHE at $\nu = \pm 1$

- Chemical potential close to first excited state



- Simultaneous lifting of sublattice and staggered spin degeneracy

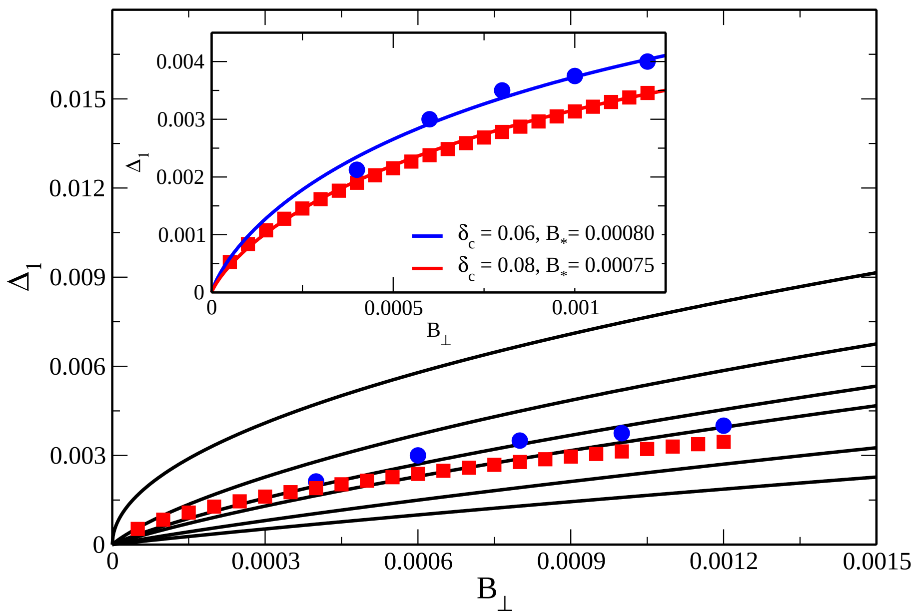
- Lifting of sublattice degeneracy $\Longrightarrow$ charge density wave (CDW)
  $\Longrightarrow$ charge gap $\Delta_1^C = C$ (scales with $B_\perp$)

- Lifting of staggered spin degeneracy $\Longrightarrow$ easy axis AFM (N_3)
  $\Longrightarrow$ spin gap $\Delta_1^S \propto (\lambda + m)$

- Coexistence of two chiral symmetry breaking orders

- $C \sim V_1$ (nearest-neighbour repulsion), hence $C \gg m + \lambda$, and $\Delta_1^C \gg \Delta_1^S$
  – gap in $\nu = \pm 1$ state primarily from C (CDW)

Comparison with experiments for $\nu = 1$



Yu *et al.* (2013)
Abanin *et al.* (2013)

- Self-consistent calculation of CDW gap similar to AFM gap for $\nu = 0$
- Long-range Coulomb interactions renormalize Fermi velocity

$$v_F = v_F^0 \left[1 + \frac{e^2}{8\epsilon v_F} \ln \left(\frac{B^*}{B} \right) \right]$$

where v_F^0 is the bare Fermi velocity, $B^* \sim 8$ T

[Theory: Herbut and Roy, PRB (2008); Experiment: Faugeras *et al.*, PRL (2015)]

$\Longrightarrow$ renormalizes $\nu = 1$ gap (in units of $v_F \Lambda$)

Summary

- Scenario for formation of gaps for $\nu = 0$: formation of CSB Dirac masses via magnetic catalysis
  - Microscopic origin: weak short range interactions
  - Candidate orders: Antiferromagnetism, CDW

- Observed scaling behaviour of the gap for $\nu = 0$ with magnetic field including both linear and sublinear ($\sim \sqrt{B}$) regimes can be explained with easy plane antiferromagnetism coexisting with weak easy-axis ferromagnetism

- The scaling of the gaps in the $\nu = 1$ state can be explained within the same framework: dominant CDW with weak easy axis antiferromagnetism

- A similar mechanism can be applied to other graphene-based layered systems, e.g. bilayer and trilayer graphene [B. Roy, PRB 2014] , Weyl semimetals [B. Roy, arXiv:1406.4501]

- Implications for Fractional Quantum Hall states?

B. Roy, M. K., and S. Das Sarma, Phys. Rev. B **90**, 201409(R) (2014).

$\nu = 1$ Hall state in a tilted magnetic field

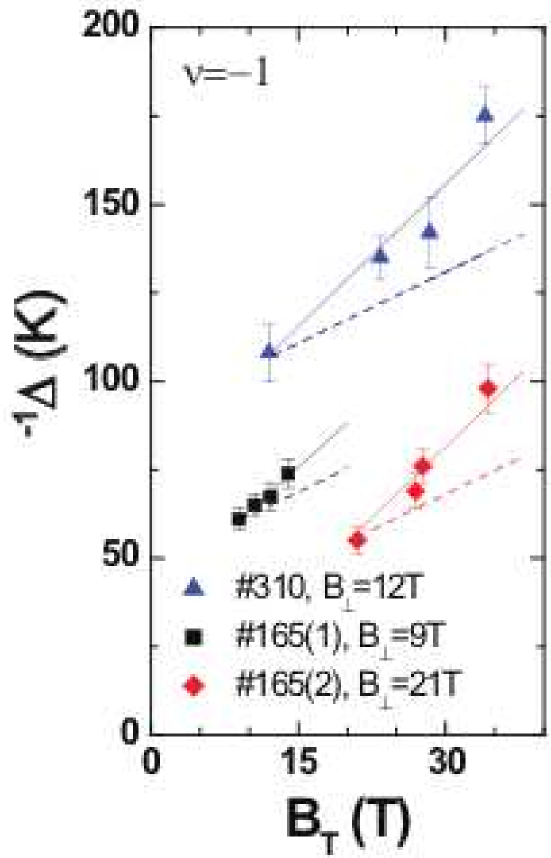
- Excitation gap
  (charge and spin contributions)

$$\Delta_1 \simeq \Delta_1^C + \Delta_1^S,$$

- Charge gap $\Delta_1^C = C$ (scales as $B_\perp$)

- Spin gap $\Delta_1^S = 2(m+\lambda)N_3/\Delta_0$
  – N_3, Δ_0 scale as $B_\perp$
  – $(m+\lambda)$ scales as B_T

- Experimentally, gap scales with B_T but with larger slope than for single particle Zeeman coupling (λ): presence of a spin gap provides an explanation for the enhanced slope



Young, *et al.*, Nat. Phys. (2012).