Microscopic simulations with modern nuclear forces

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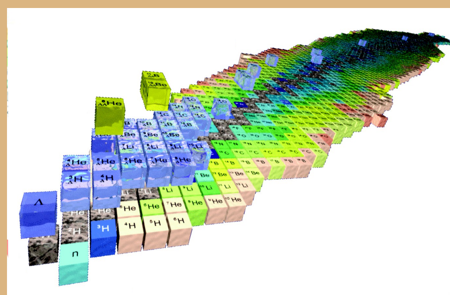
Getting the TLAs out of the way

QCD = Quantum Chromodynamics

EFT = Effective Field Theory

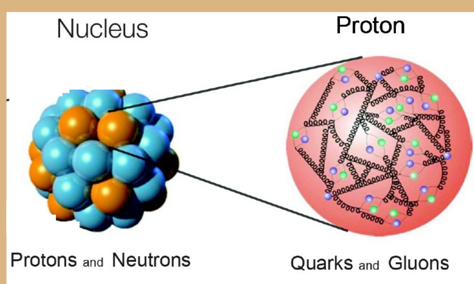
QMC = Quantum Monte Carlo

Outline



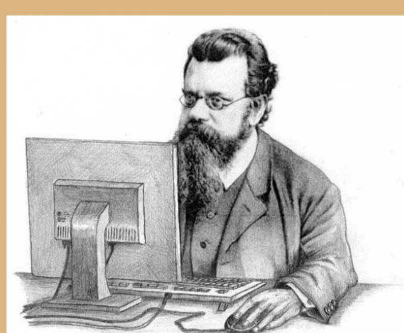
Many-nucleon problem

- Chart of nuclides
- Neutron stars
- Perturbative vs non-perturbative



Nuclear forces

- Chiral Effective Field Theory (EFT)
- Local chiral EFT



Credit: Bernhard Reischl

Few- and many-body results

- Pure neutron matter
- Light nuclei

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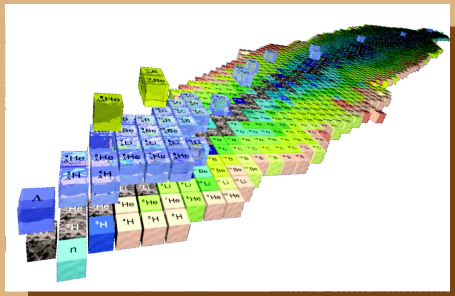

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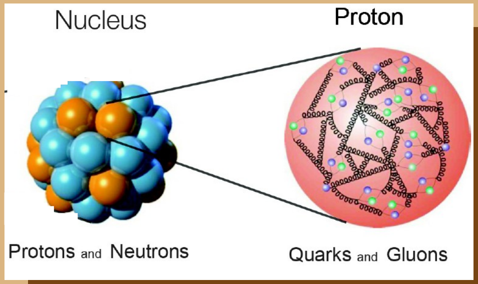

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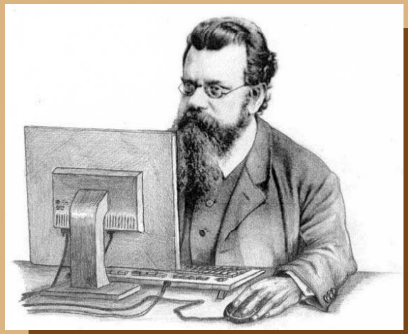

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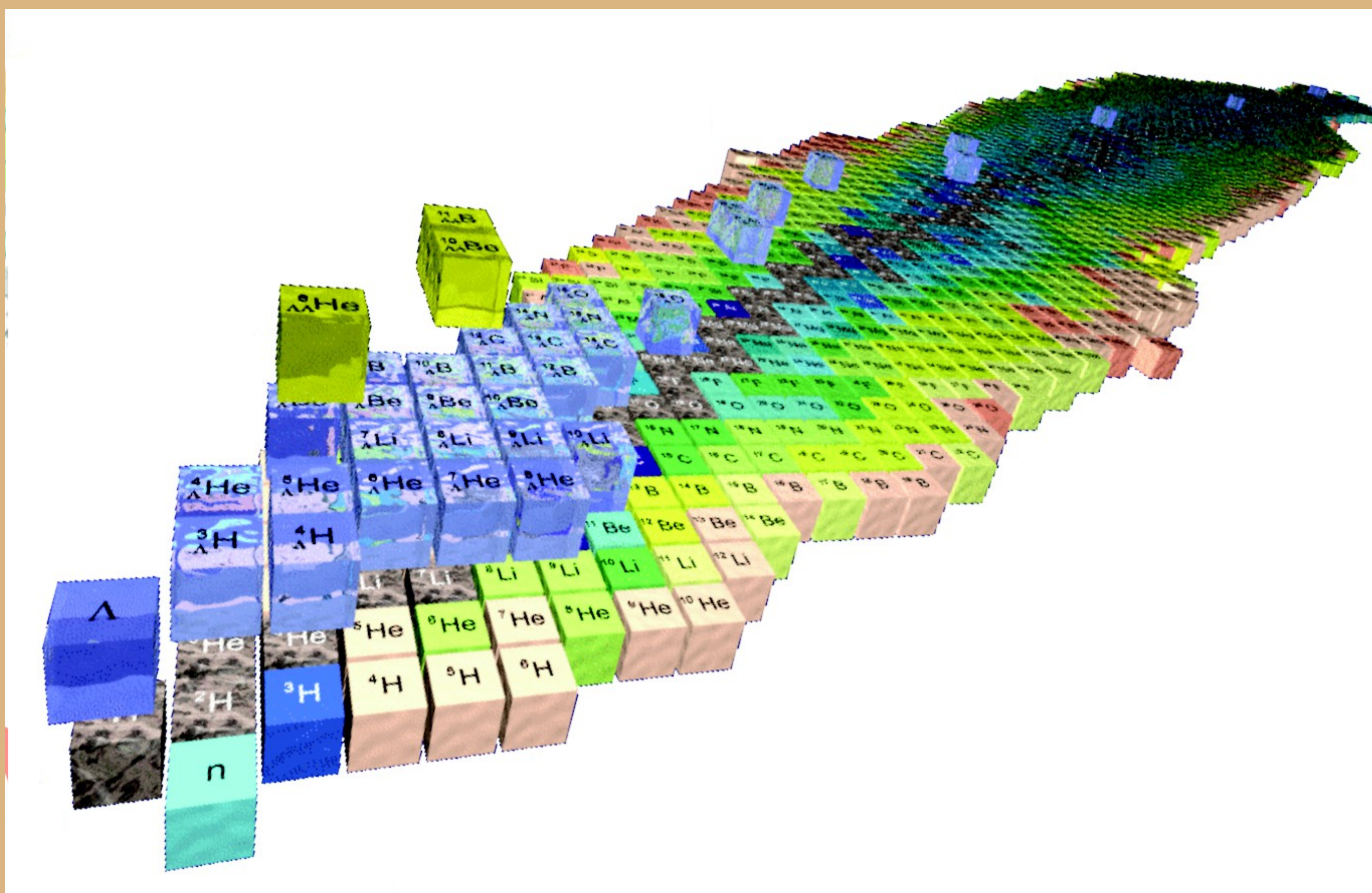
Goals: terrestrial

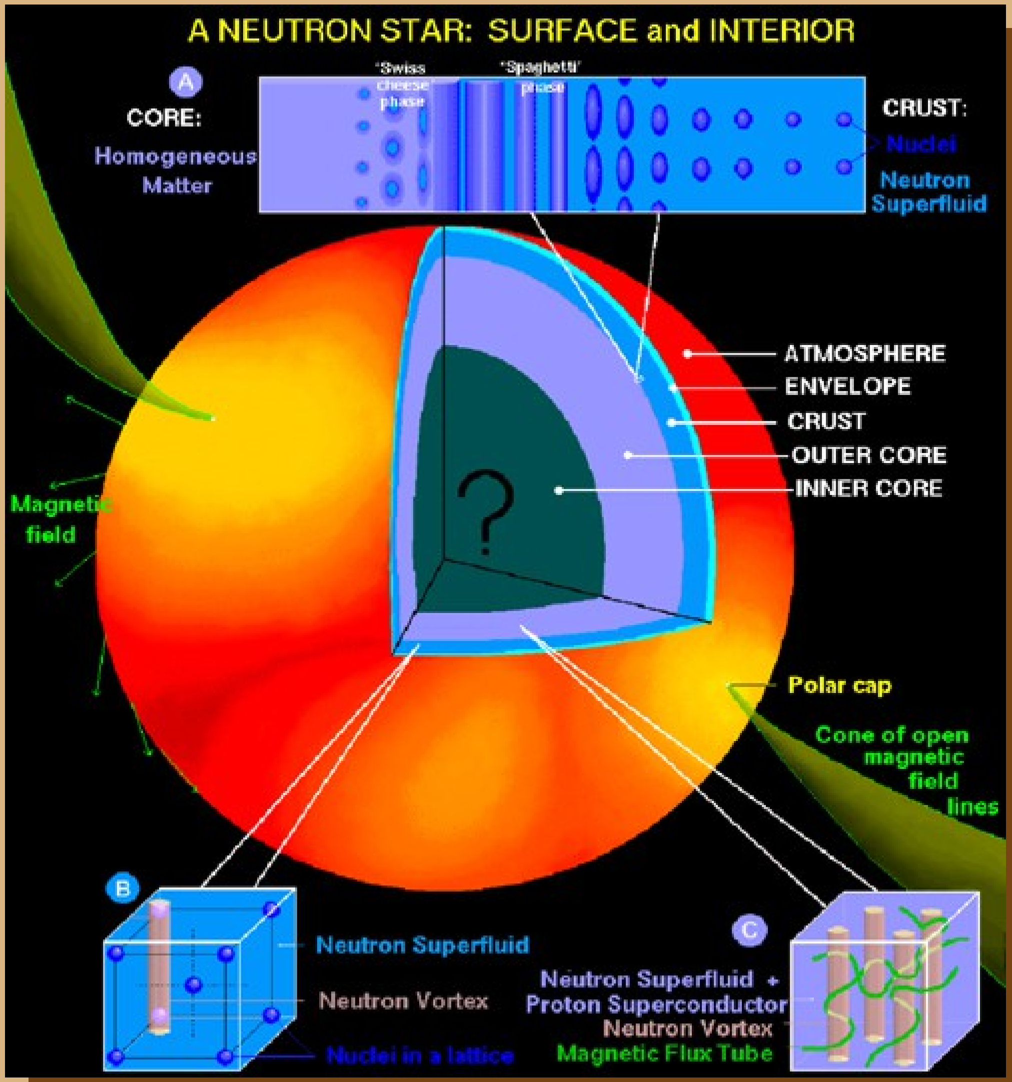
Chart of nuclides



- Nuclear physics is developed and tested on earth
- Using complicated many-body methods we can try to “build nuclei from scratch”
- We then extrapolate that knowledge to more exotic systems
- Can we describe nuclei *from first principles*?
- What does *first* mean?

Goals: astrophysical

Neutron stars as ultra-dense matter laboratories



Credit: Dany Page

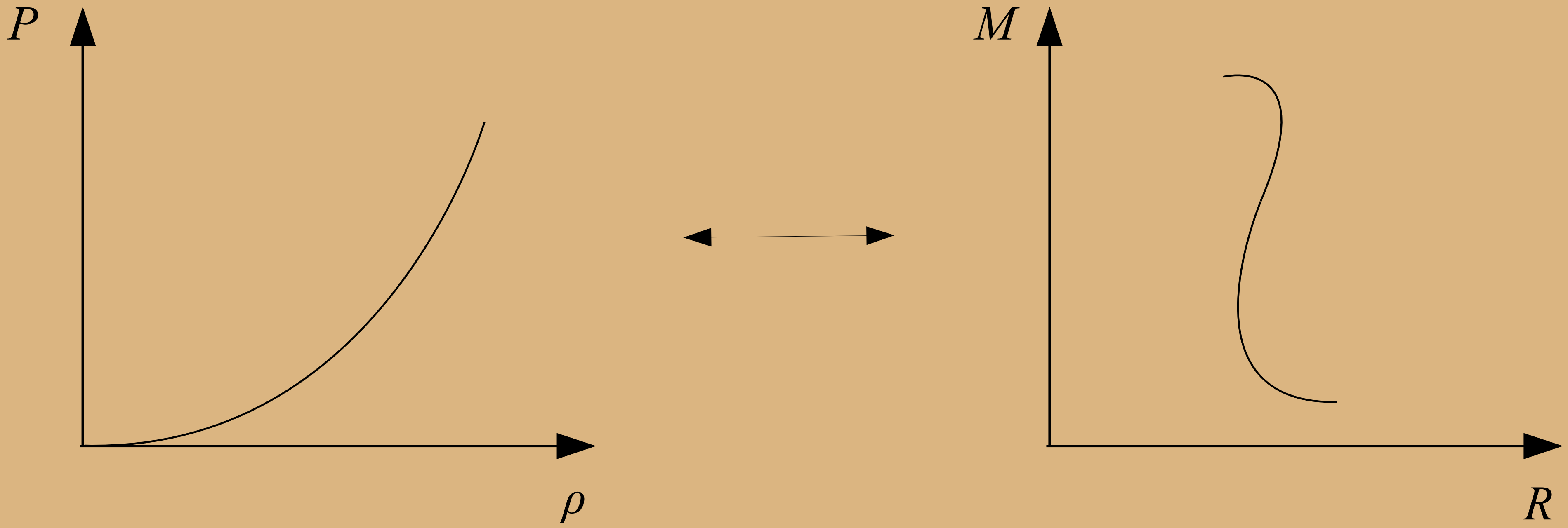
- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, i.e., not experimentally accessible
- Can we describe neutron-star matter *from first principles*?
- What does *first* mean?

Neutron stars: micro-macro

TOV equations
(or Hartle-Thorne, etc)

Pressure vs density

Mass vs radius



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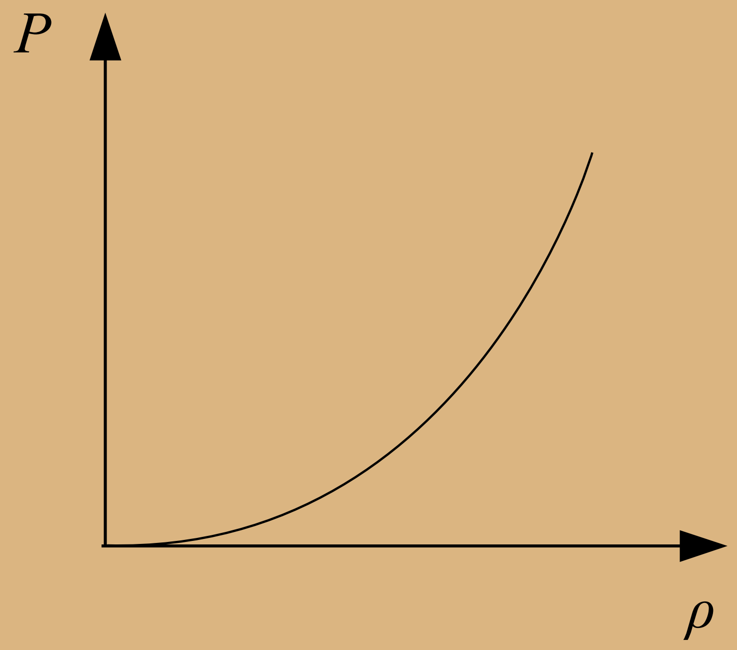

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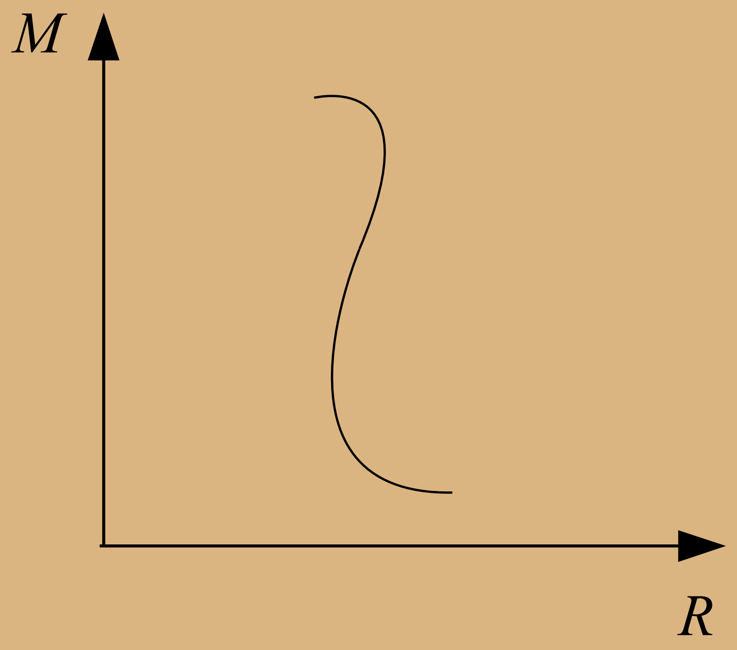
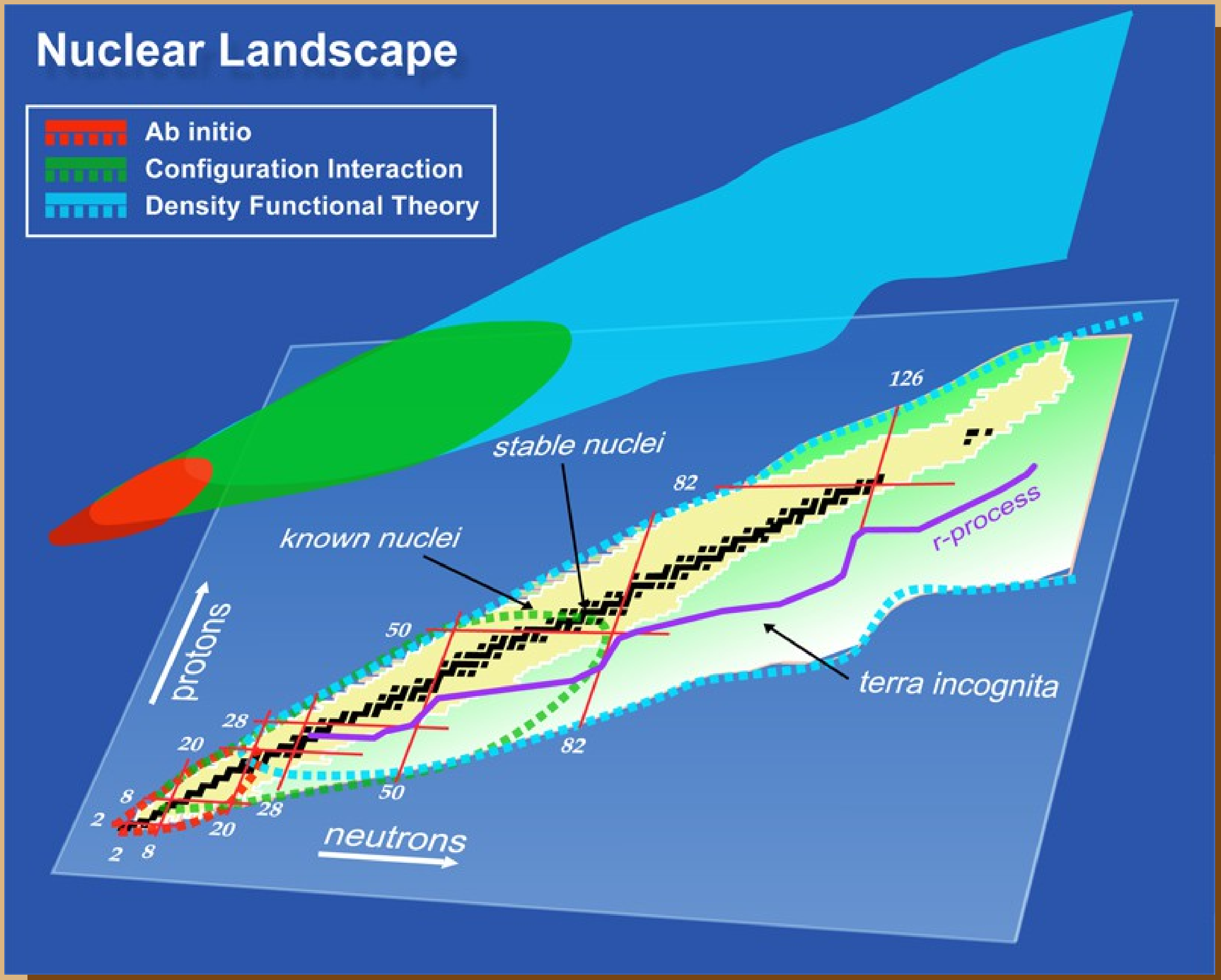


Modern goal: systematic theoretical error bars

Many-nucleon problem: methods



- No universal method exists (yet?)
- A lot to be learned if the degrees of freedom are actual particles and there are no free parameters
- Regions of overlap between different methods are crucial
- Is it possible to work at the level of nucleons & pions but still connect to the underlying level?

Many-nucleon problem

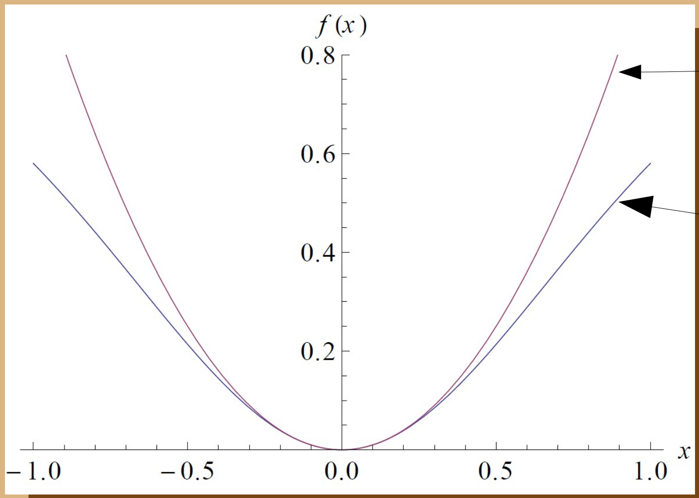
Lesson from history of physics:
Find a small expansion parameter α and use it to organize the different contributions:

$$\alpha + \alpha^2 + \alpha^3 + \cdots$$

This is called perturbation theory.

However, some problems are non-perturbative.

Perturbative vs non-perturbative



$$1 - \left(\frac{1}{\cosh(x)}\right)^2 = x^2 + \text{something small} \quad ?$$

Maybe

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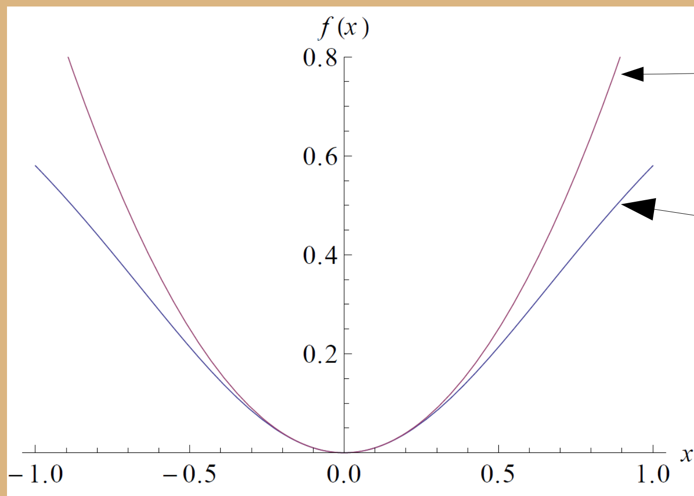

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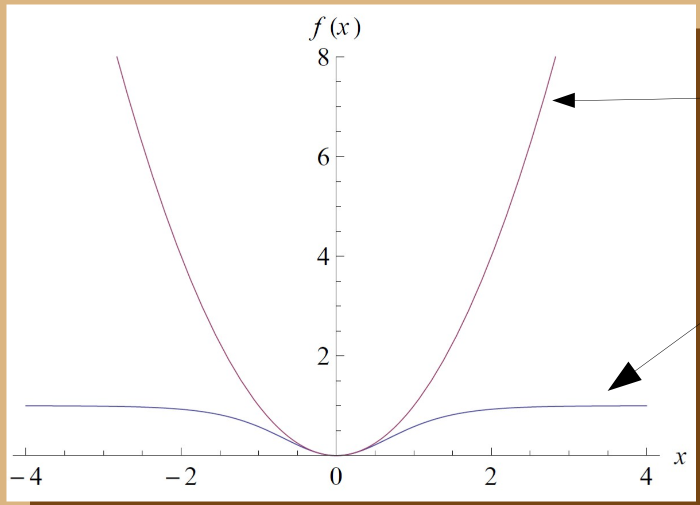

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Maybe not

Many-body problem: QMC

Quantum Monte Carlo: stochastically solve the many-body Schrödinger equation in a fully non-perturbative manner

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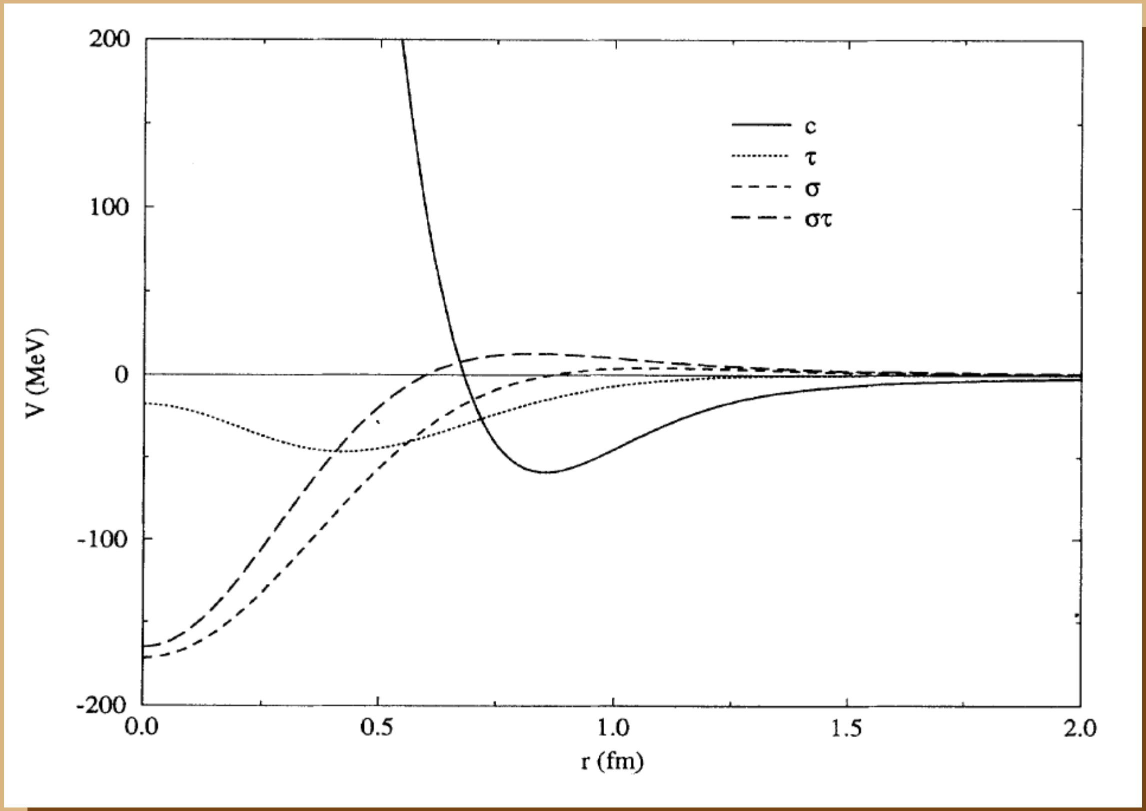
Rudiments of Diffusion Monte Carlo:

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H}-E_T)\tau}\Psi_V$$

$$\to \alpha_0 e^{-(E_0-E_T)\tau}\Psi_0$$

Many-body problem: QMC

Quantum Monte Carlo was, however, limited to the use of (local) phenomenological nucleon-nucleon potentials.



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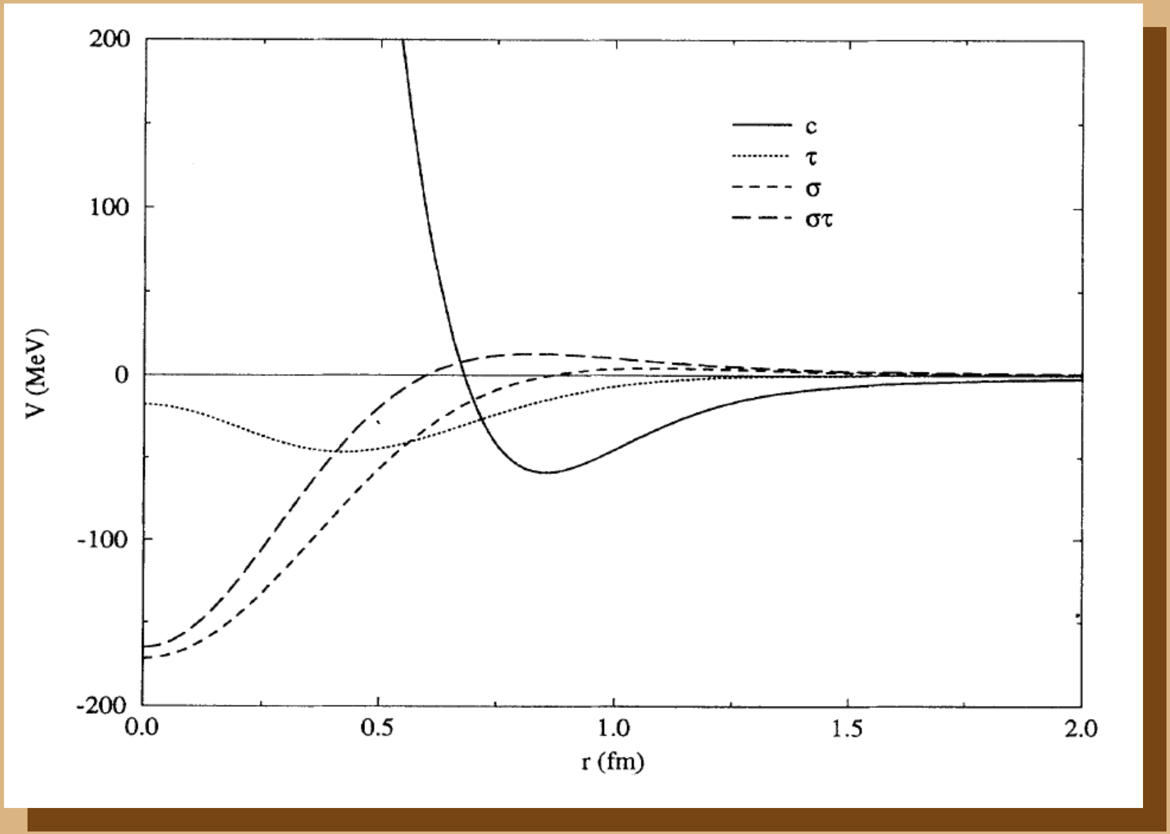

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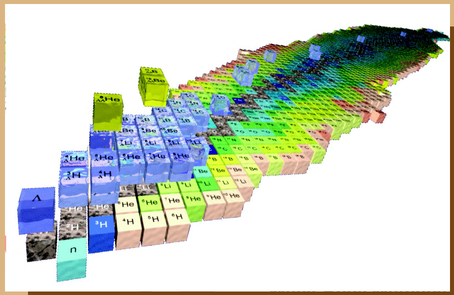
Such potentials are hard, making them non-perturbative at the many-body level (which isn't a problem for QMC, but is one for almost every other method).

Outline



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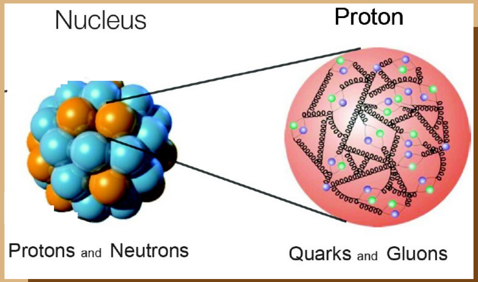

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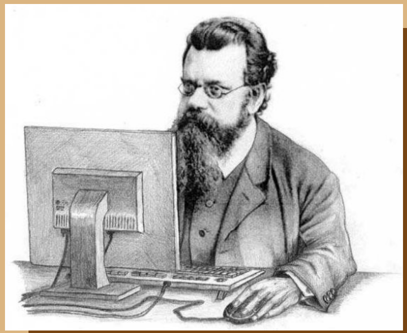

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Few- and many-body results

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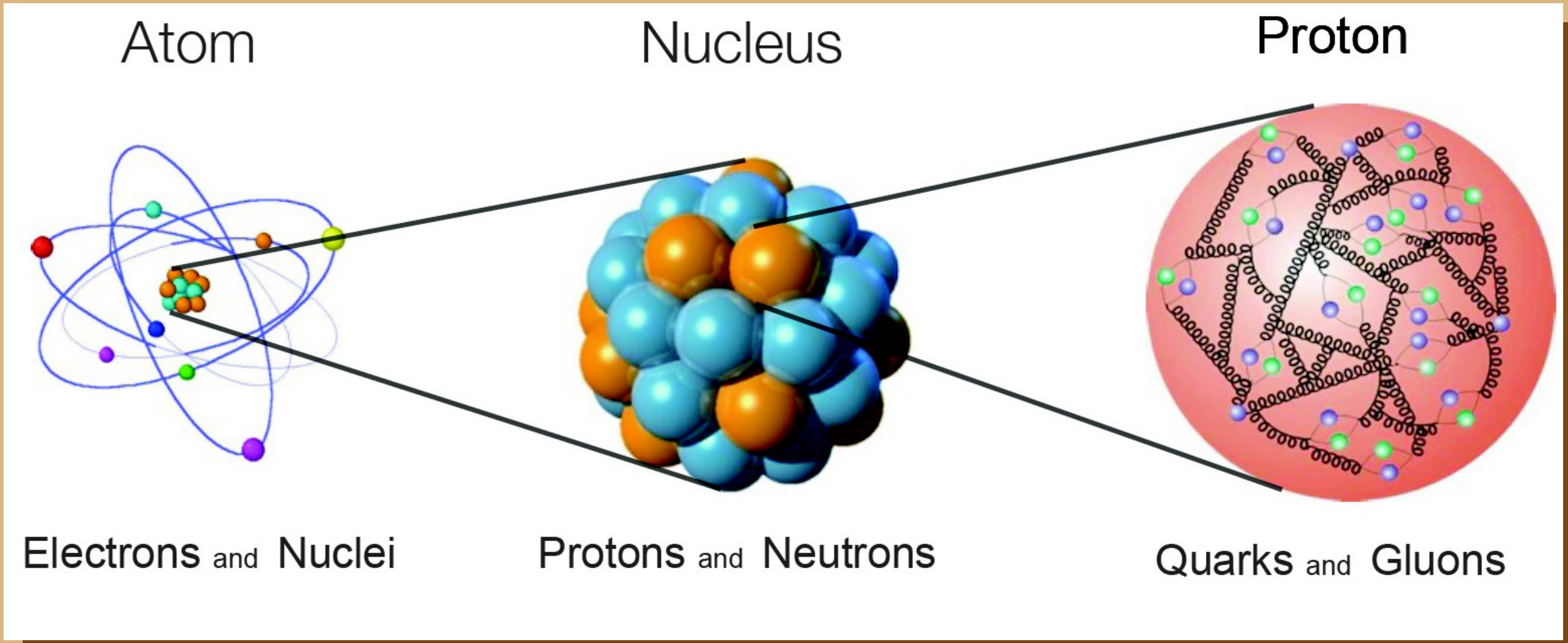
Nuclear forces: degrees of freedom



Lesson from history of physics

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Quantum Electrodynamics (QED)

$$\alpha = \frac{1}{137.036}$$

for most practical purposes

(see Gell-Mann & Low paper with 770 citations, not Gell-Mann & Low paper with 630 citations)

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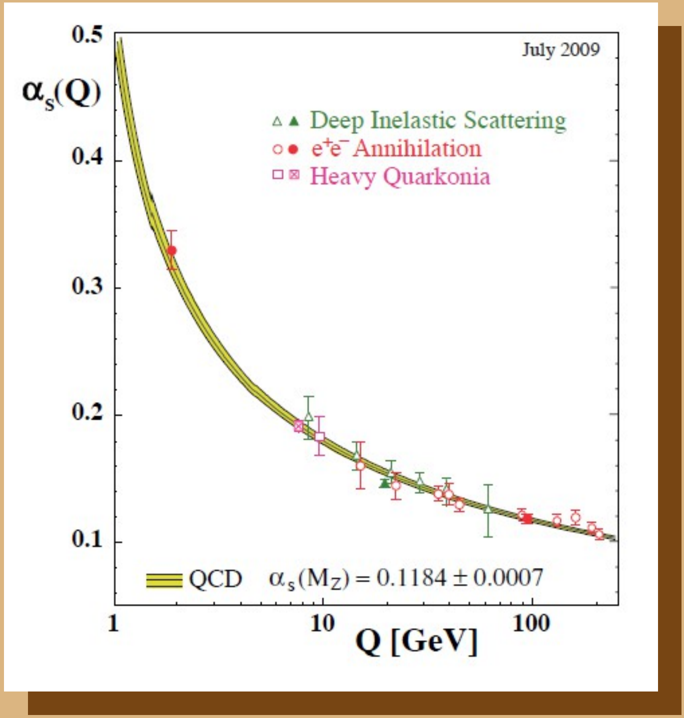
Quantum Chromodynamics (QCD)



Credit: Particle Data Group

Perturbative and non-perturbative

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Perhaps a non-perturbative method should also be used for nuclear forces (lattice QCD).

For now, many-nucleon studies are limited to nuclear forces that follow from nucleons and pions (but it would be nice not to ignore the existence of the underlying level).

Nuclear Hamiltonian: chiral EFT

How to build on QCD in a systematic manner?

Exploit separation of scales:

$$a_{^1S_0} = (11\ \mathrm{MeV})^{-1}$$

$$m_\pi = 140\ \mathrm{MeV}$$

$$\Lambda_\chi \approx m_\rho \approx\ 800\ \mathrm{MeV}$$

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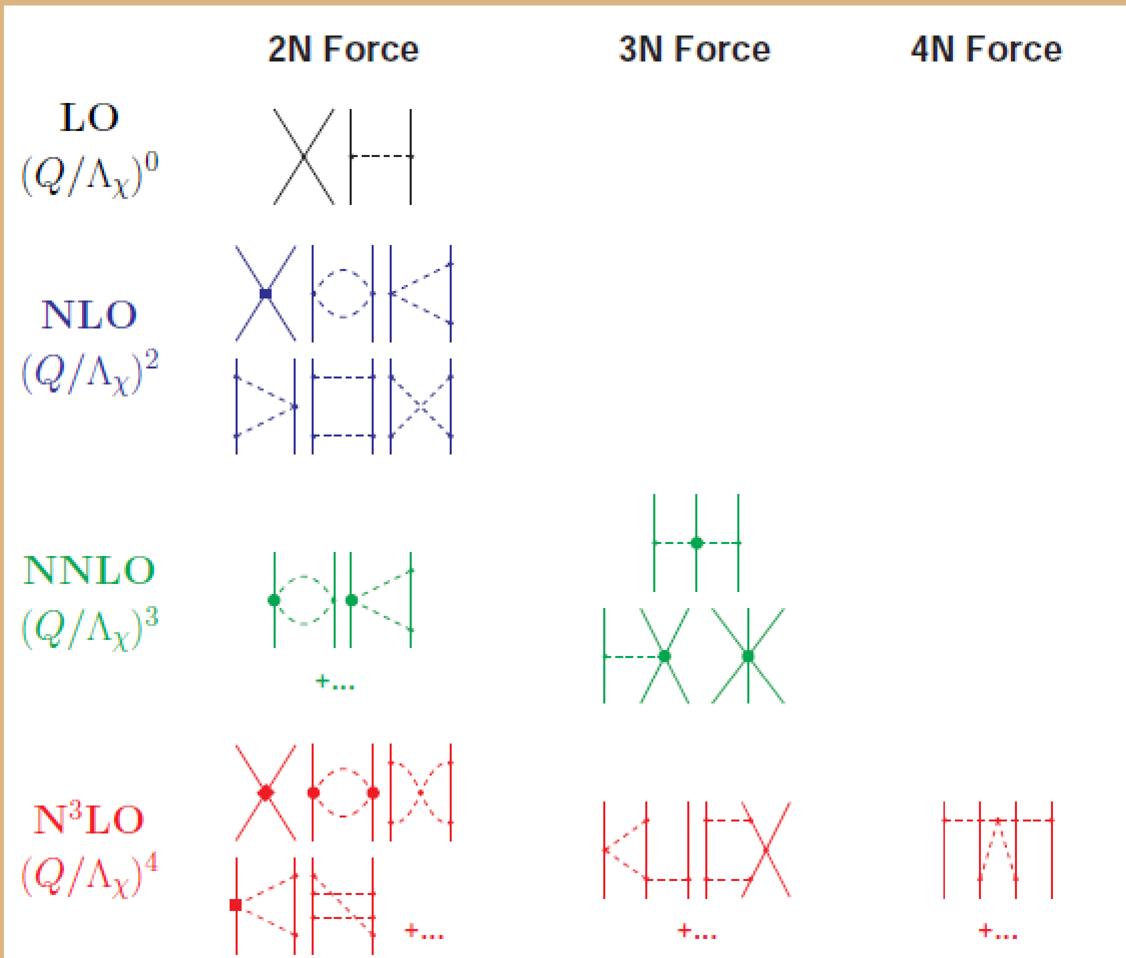
Chiral Effective Field Theory approach:

Use nucleons and pions as degrees of freedom

Systematically expand in $\dfrac{Q}{\Lambda_\chi}$

Program introduced by S. Weinberg, now taken over by the nuclear community

Nuclear Hamiltonian: chiral EFT



- Attempts to connect with underlying theory (QCD)
- Systematic low-momentum expansion
- Consistent many-body forces
- Low-energy constants from experiment or lattice QCD
- Until now non-local in coordinate space, so unused in continuum QMC
- Power counting's relation to renormalization still an open question

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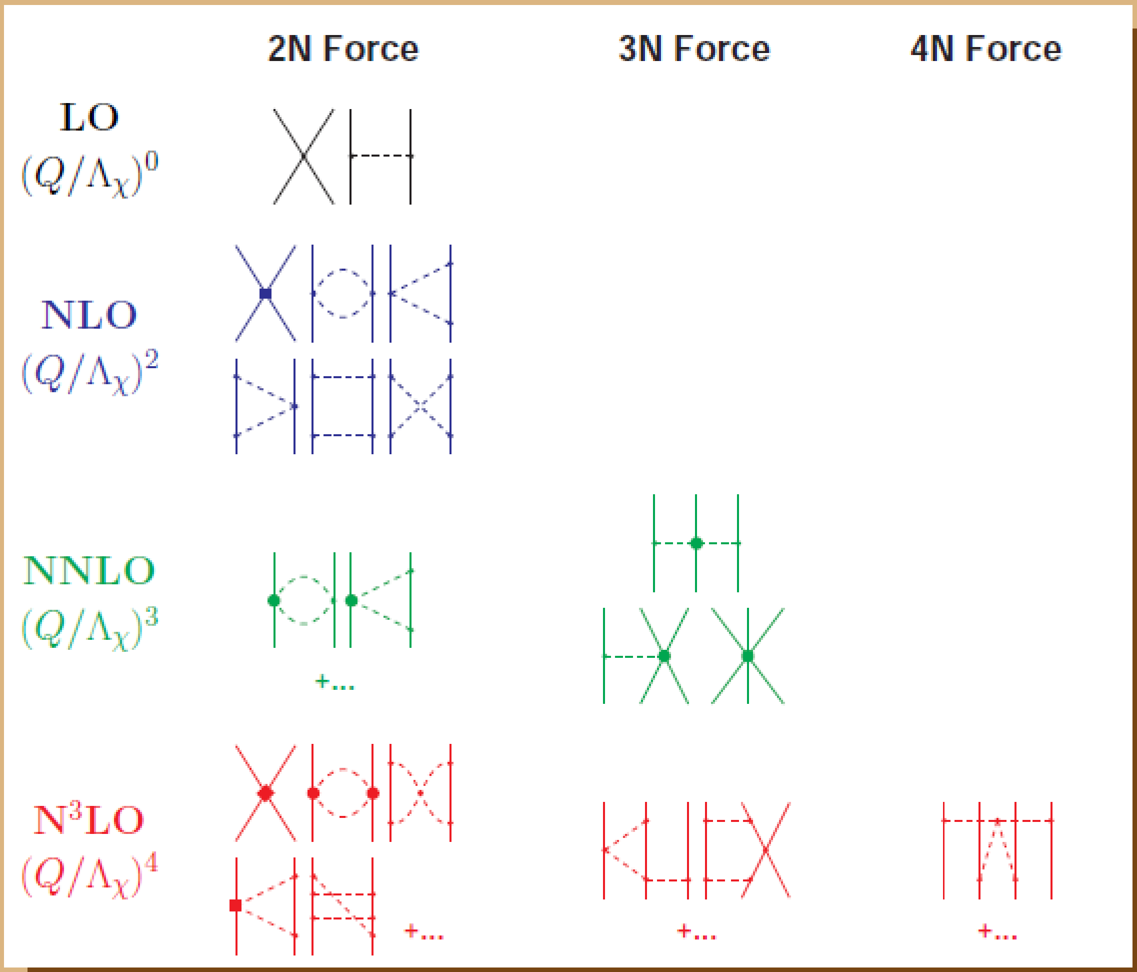

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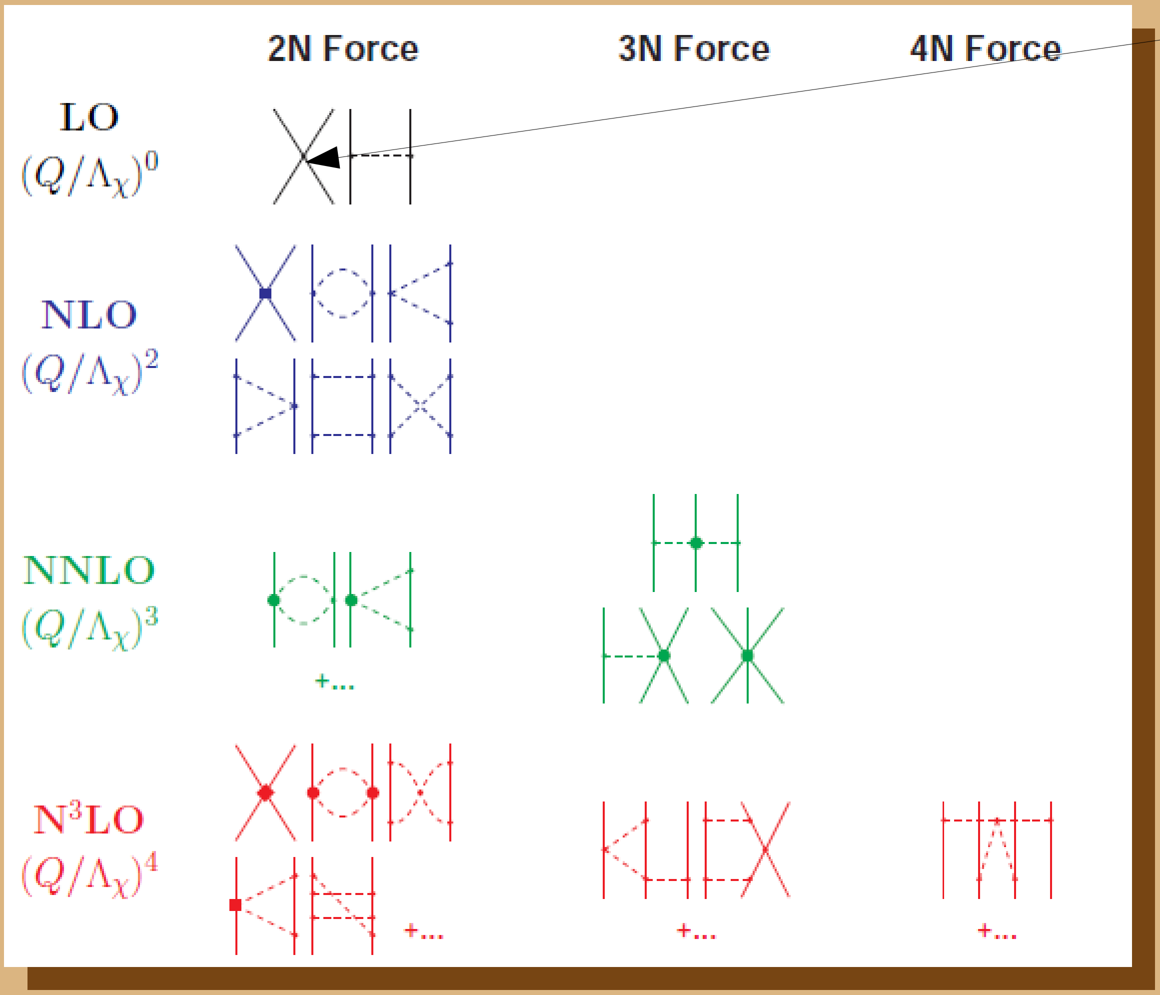

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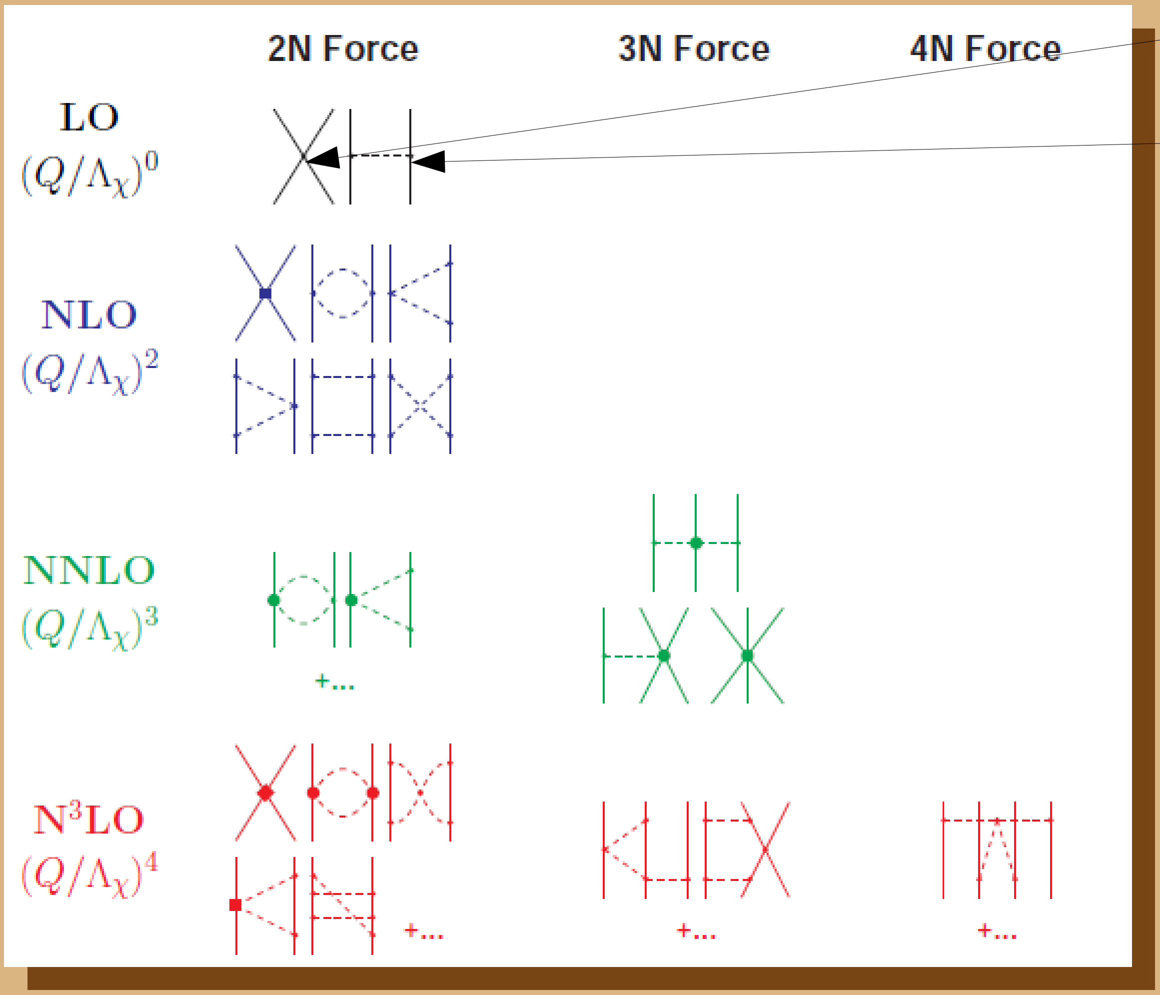

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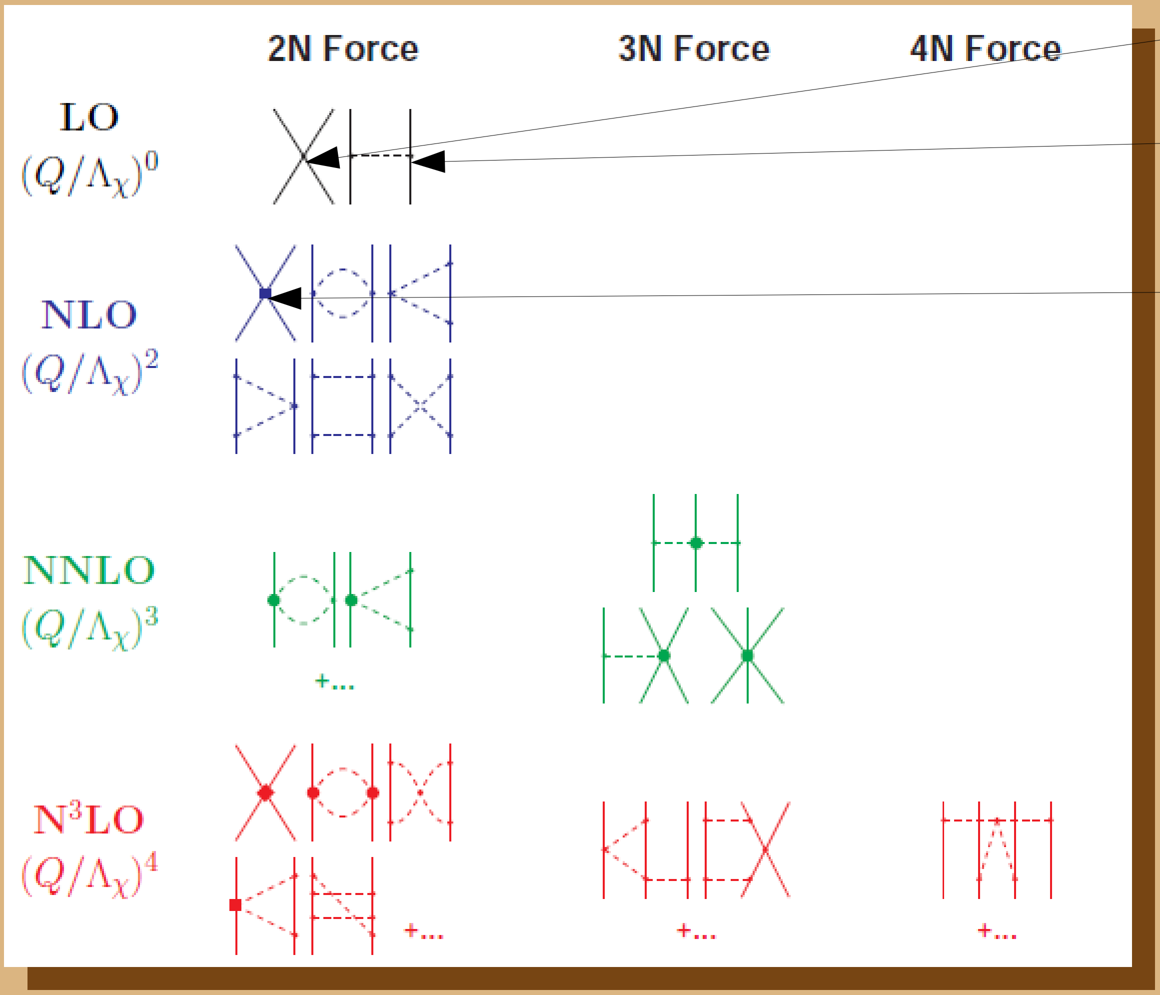

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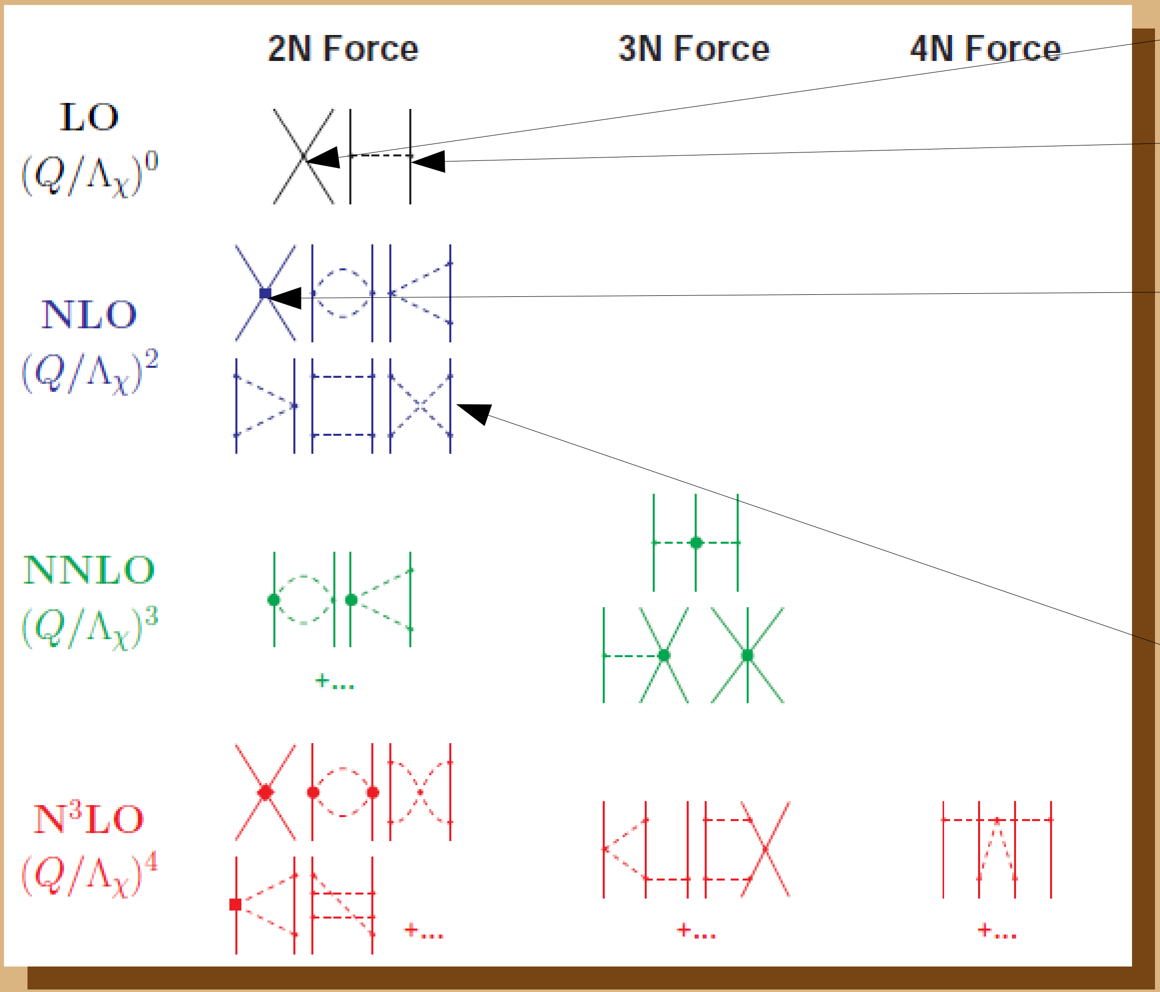

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Long-studied
two-pion exchange

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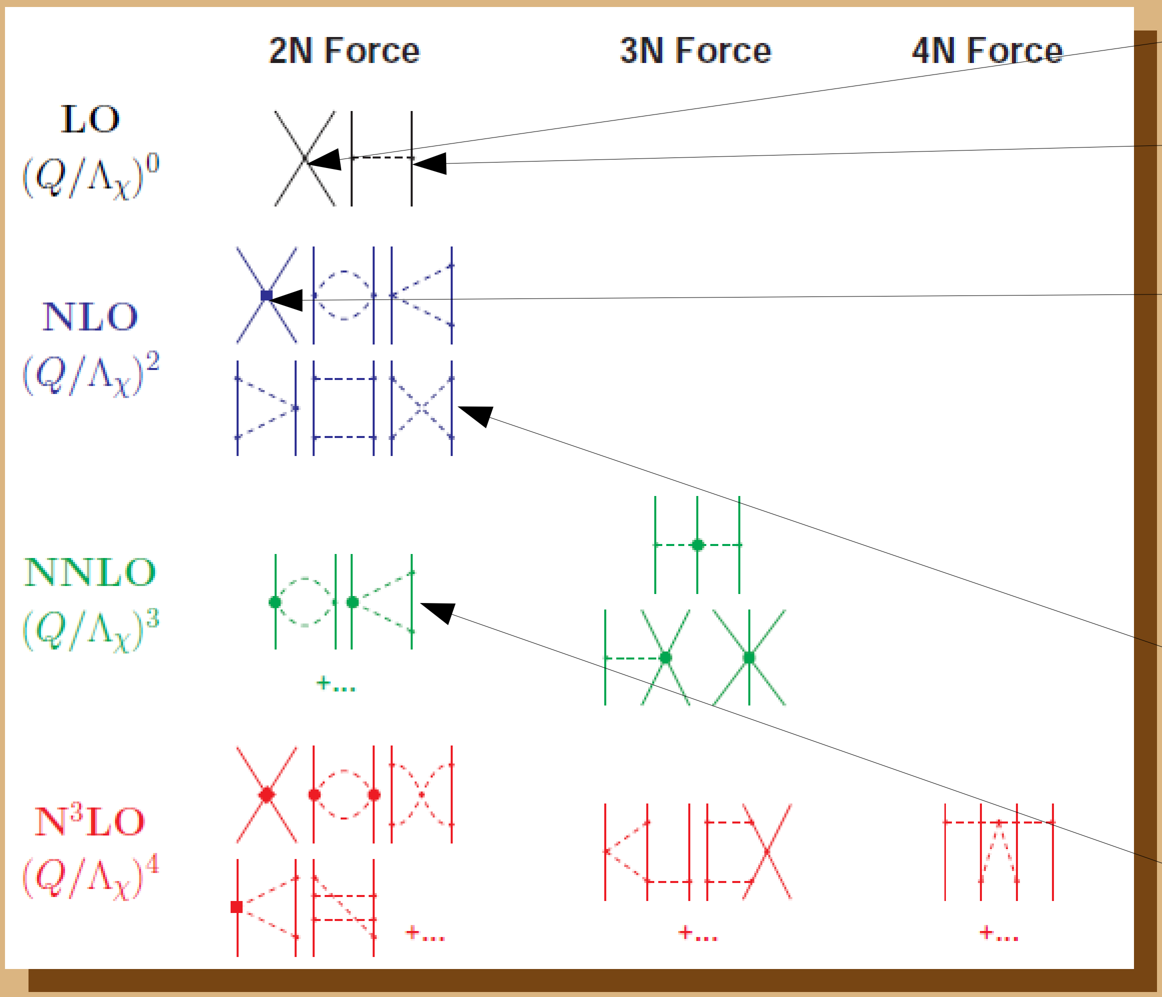

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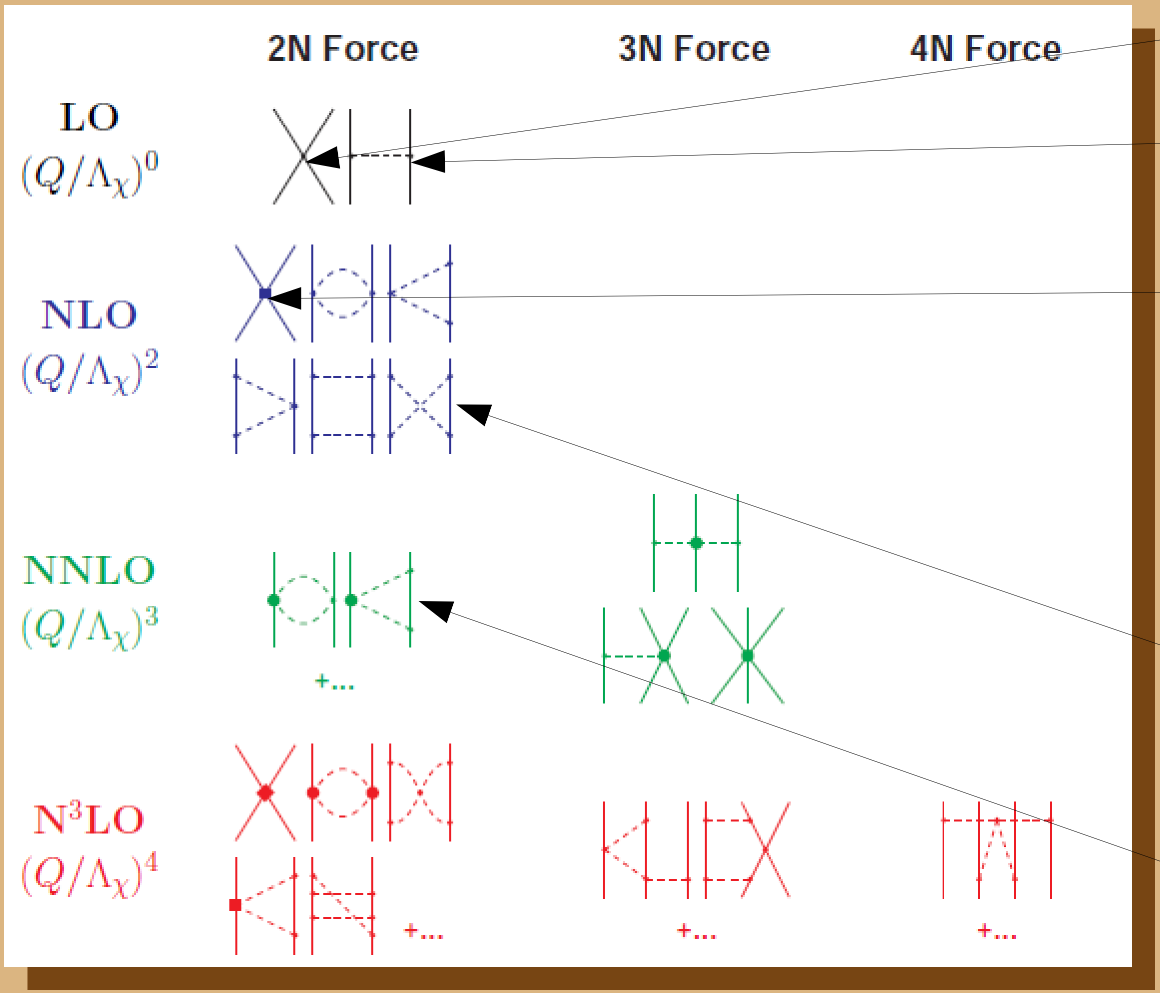
Contains couplings
from πN scattering

Nuclear Hamiltonian: chiral EFT



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Contains couplings
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q means local

k means non-local

Nuclear forces: summary

Local high-quality phenomenology is hard

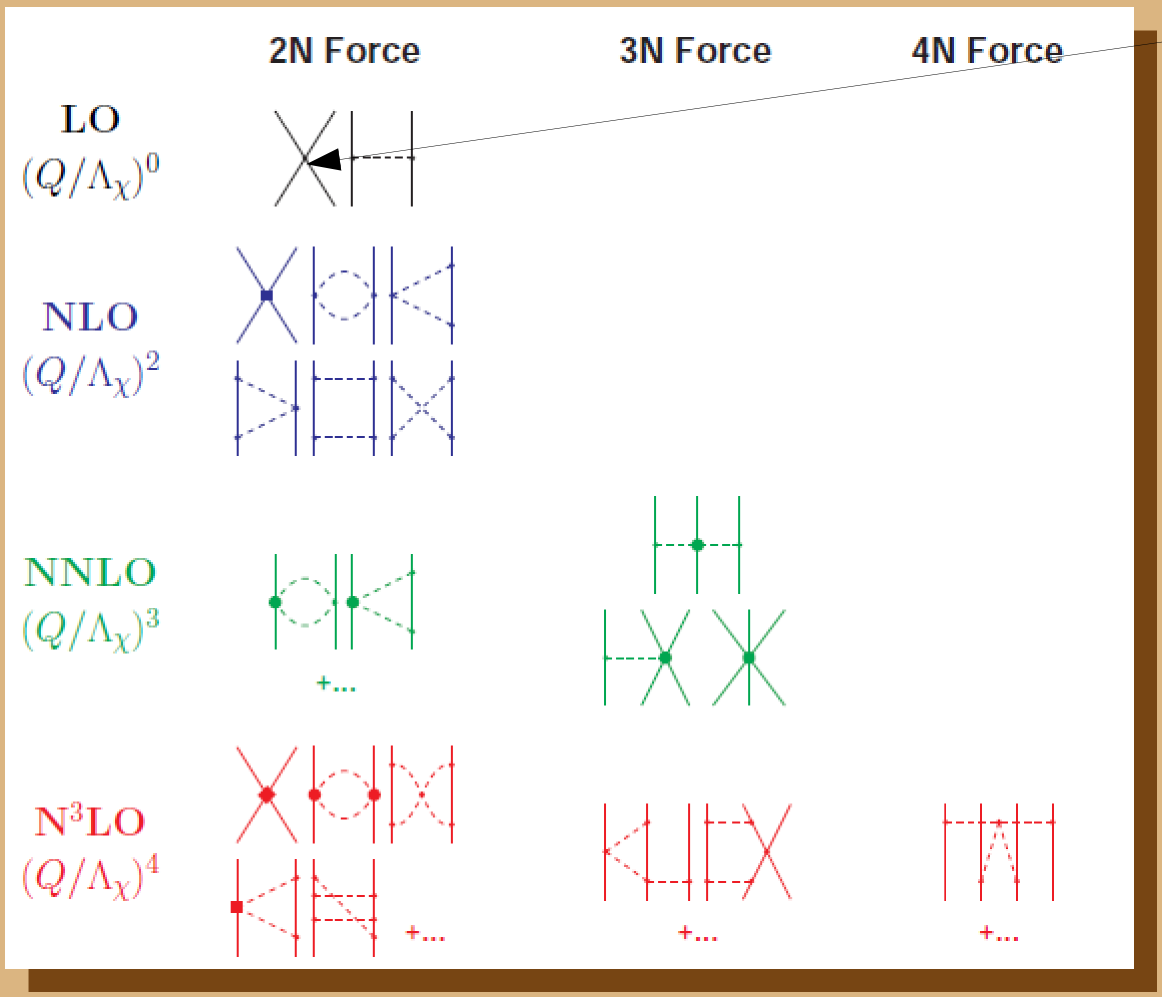
Consubstantial with the successes of nuclear QMC,
difficult to use in most other many-body methods

Chiral EFT
a) is connected to symmetries of QCD
b) has consistent many-body forces, and
c) allows us to produce systematic uncertainty bands
also happens to be non-local (such are the *sumbebekota*)

Heavily used in other methods, but not used in nuclear QMC

Turning to the resolution

Nuclear Hamiltonian: chiral EFT



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Merely the standard choice.

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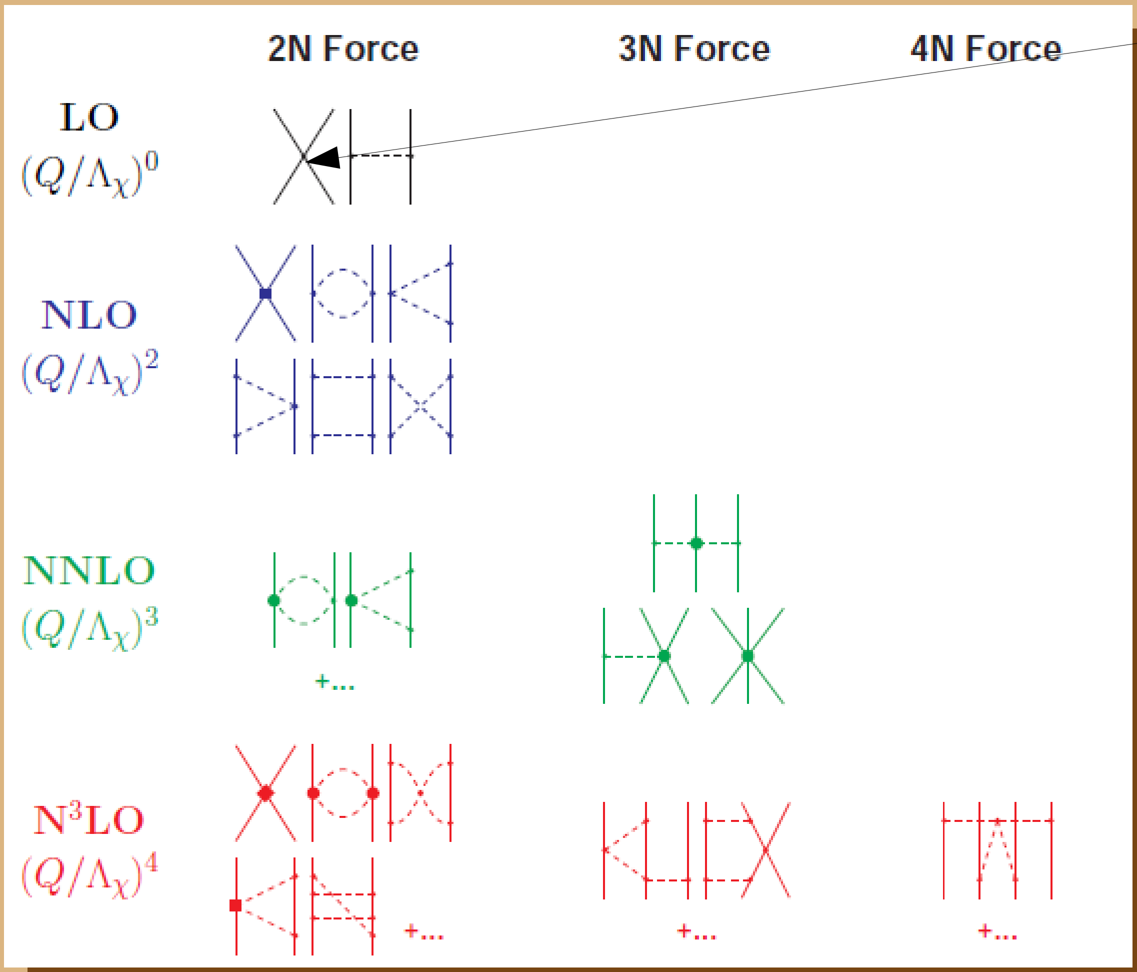

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Merely the standard choice.

Actually 4 terms in full set
consistent with the symmetries of QCD

$$V_{\rm ct}^{(0)} = C_1 + C_2\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + C_3\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + C_4\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Pick 2 and antisymmetrize

How to go beyond?

Combine power of Quantum Monte Carlo with consistency of chiral Effective Field Theory

Write down a local energy-independent NN potential

- Use local pion-exchange regulator $f_{\text{long}}(r) = 1 - e^{-(r/R_0)^4}$

cf. $f(p, p') = e^{-(p/\Lambda)^{2n}} e^{-(p'/\Lambda)^{2n}}$

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Write down a local energy-independent NN potential

- Use local pion-exchange regulator $f_{\rm long}(r) = 1 - e^{-(r/R_0)^4}$
- Pick 7 different contacts at NLO, just make sure that when antisymmetrized they lead to a set obeying the required symmetry principles

$$
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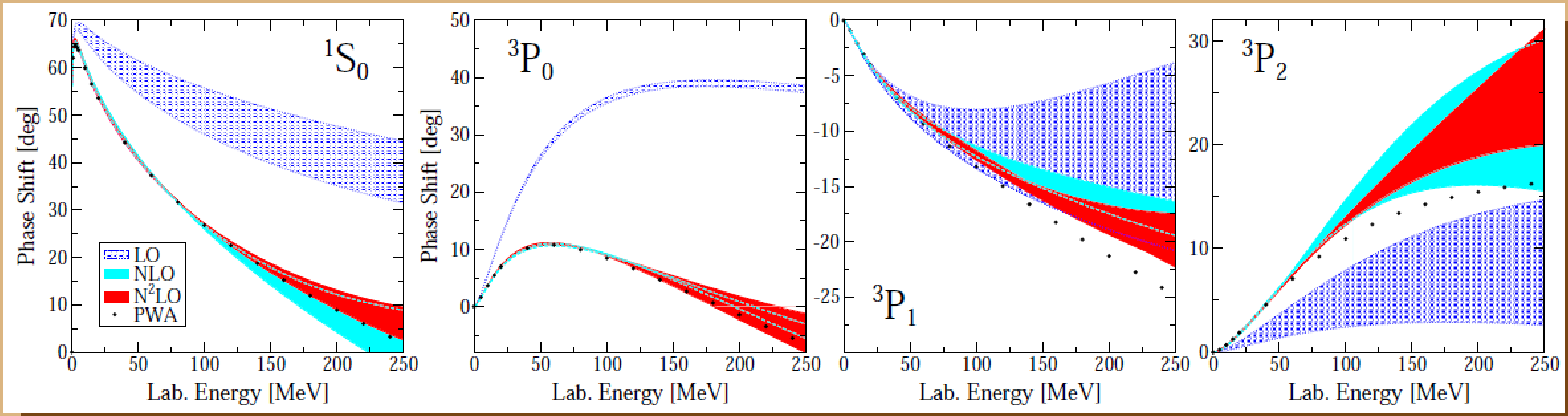
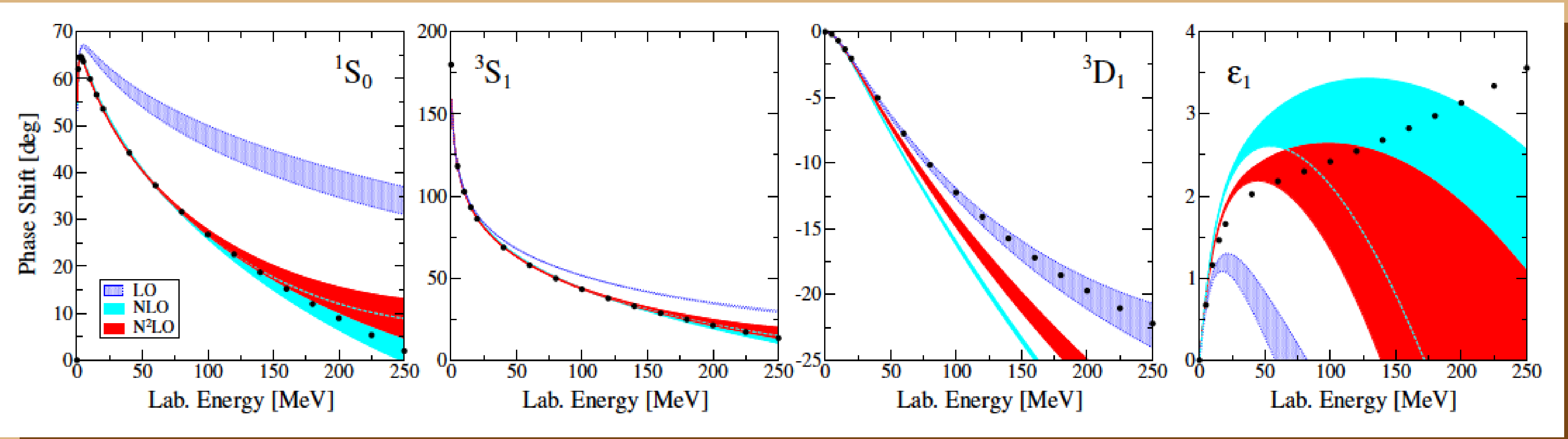
- Write down a local energy-independent NN potential
- Before doing many-body calculations, fit to NN phase shifts



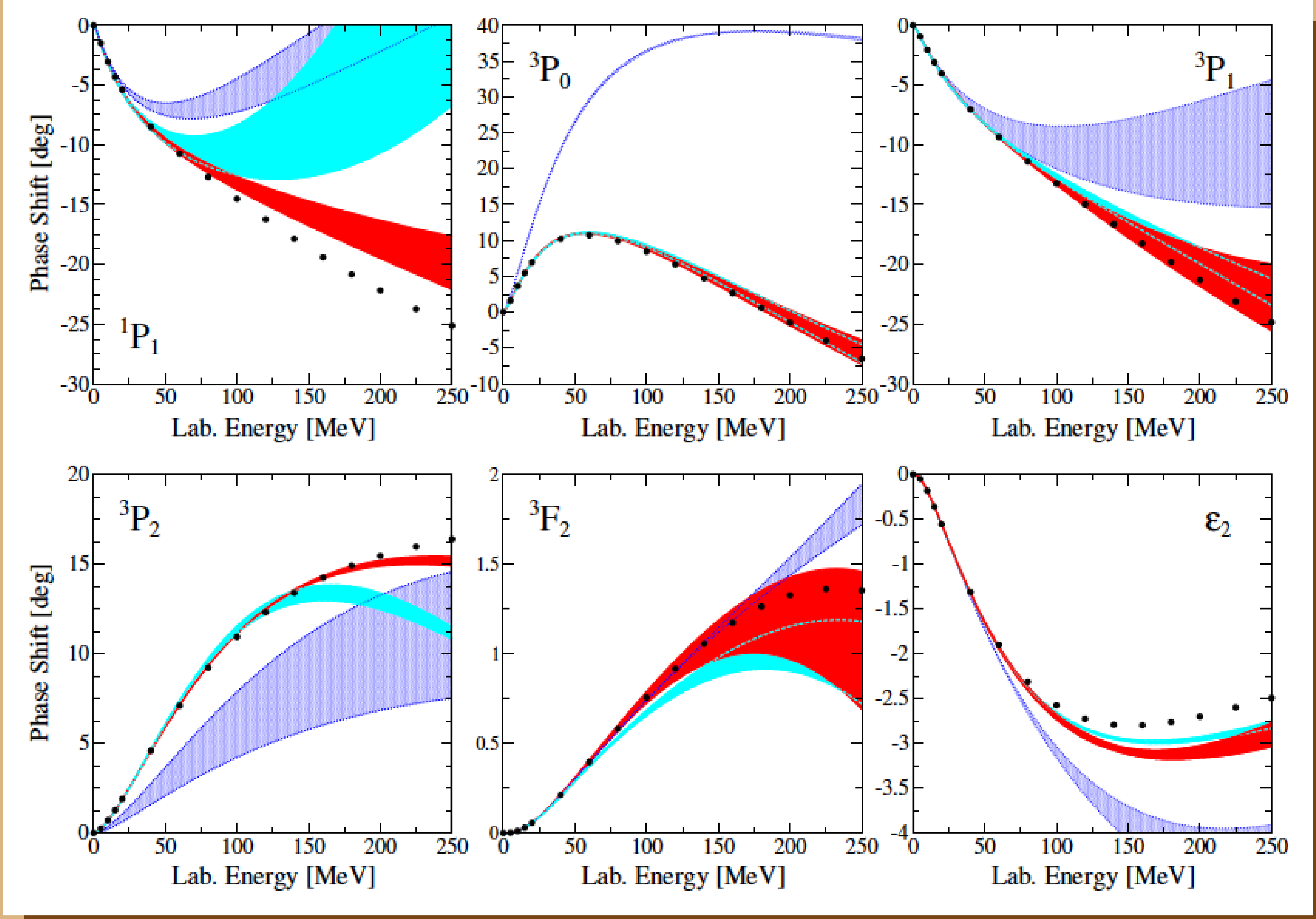
A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. **111**, 032501 (2013).

Updated phase shifts



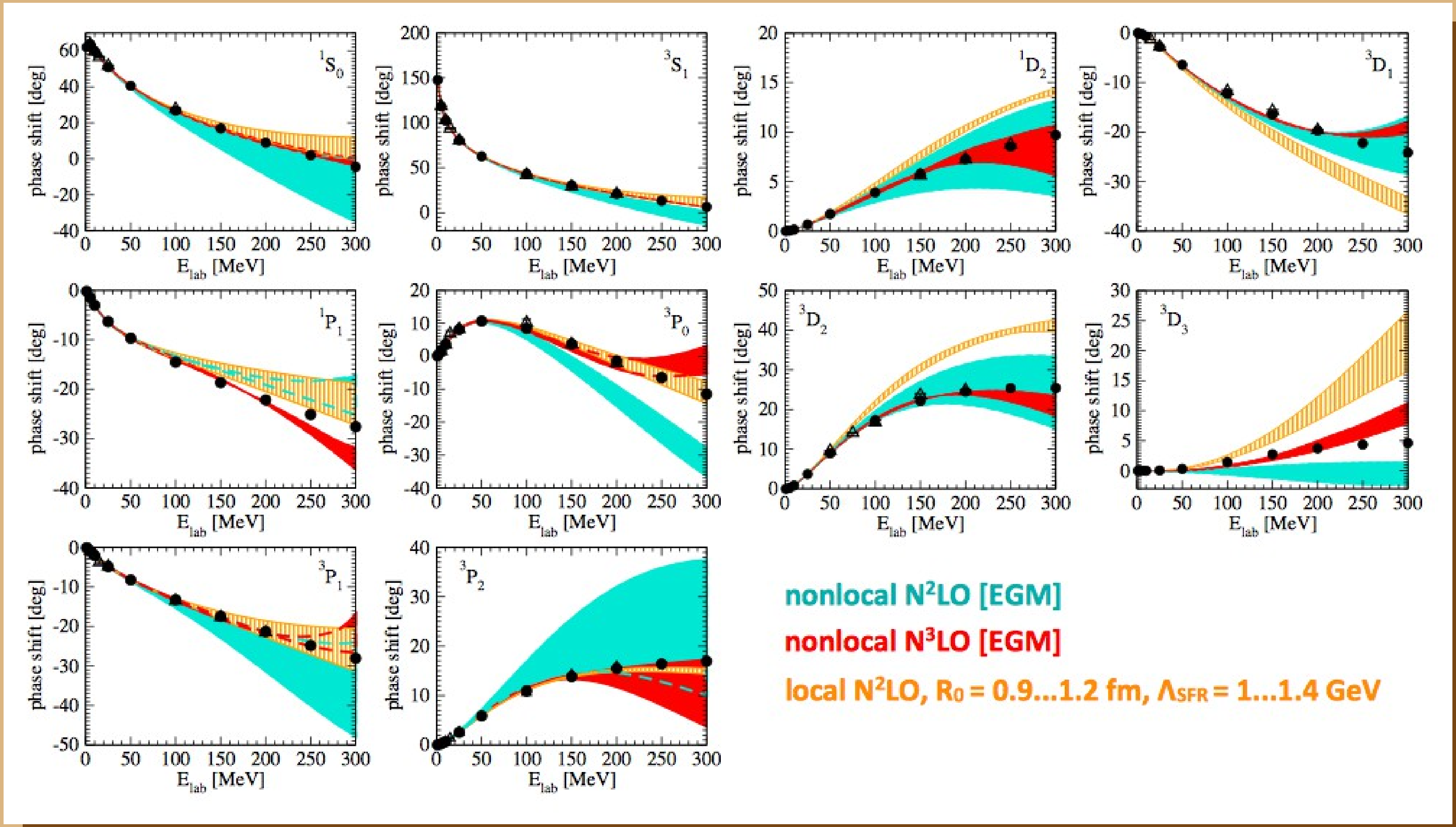
A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, arxiv:1406.0454

Updated phase shifts



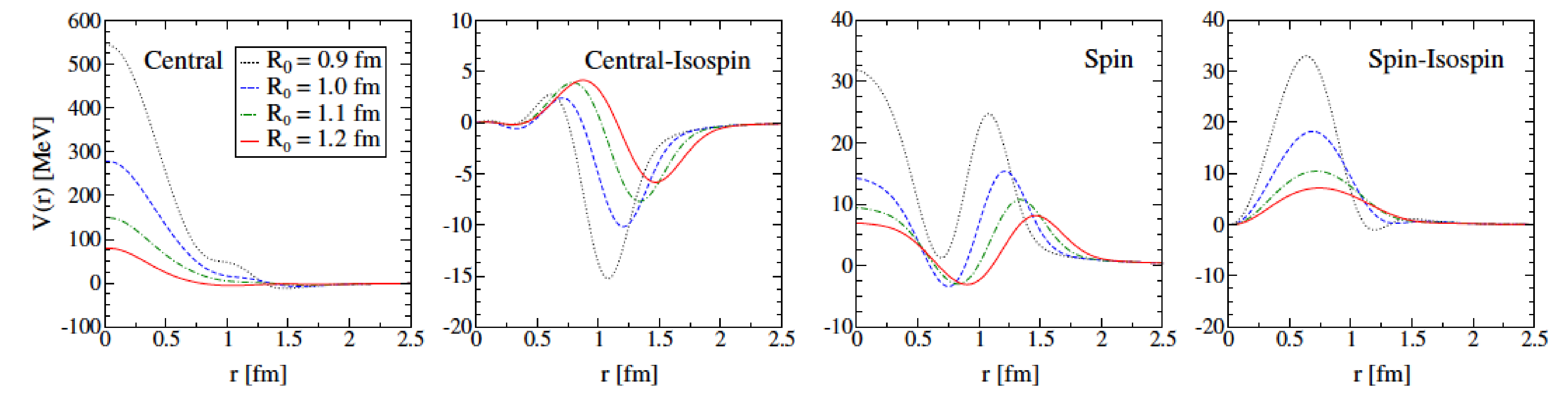
A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, arxiv:1406.0454

Compare to non-local EGM



Credit: Hermann Krebs

Since it's local, let's plot it (N^2LO)



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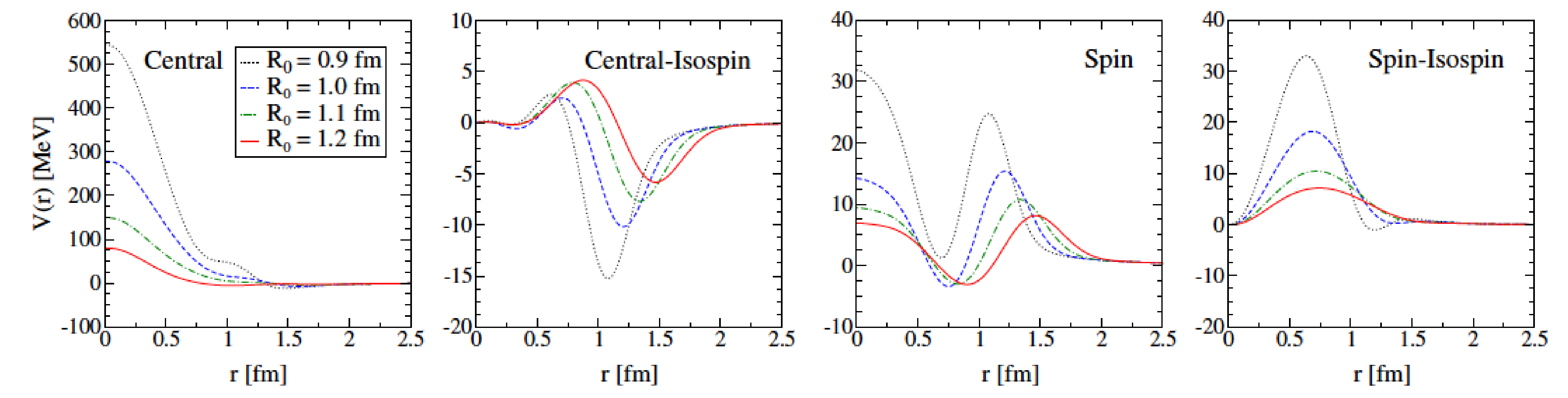
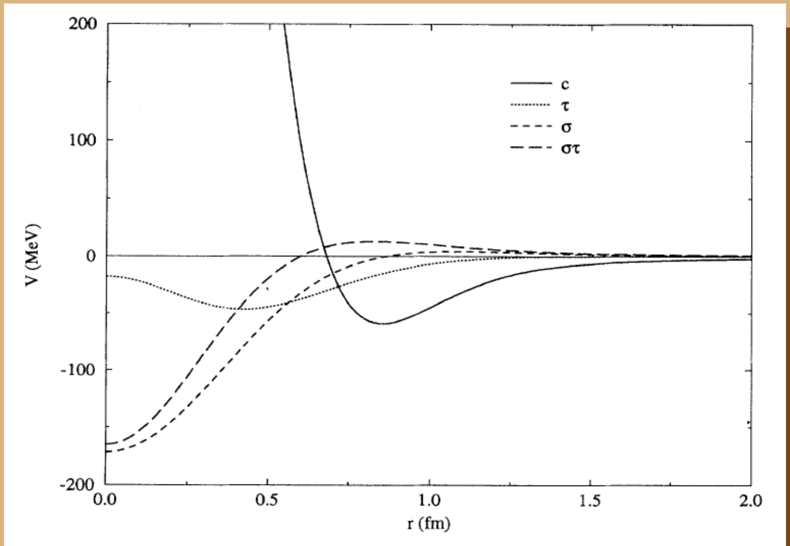


Compare with AV18



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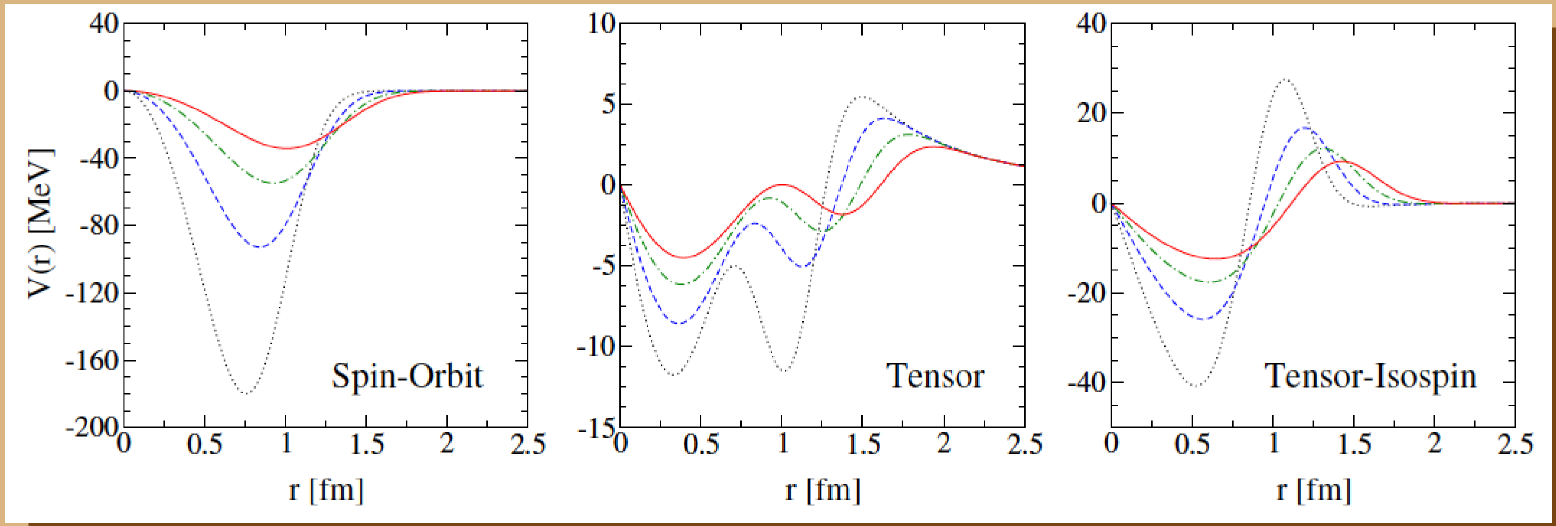

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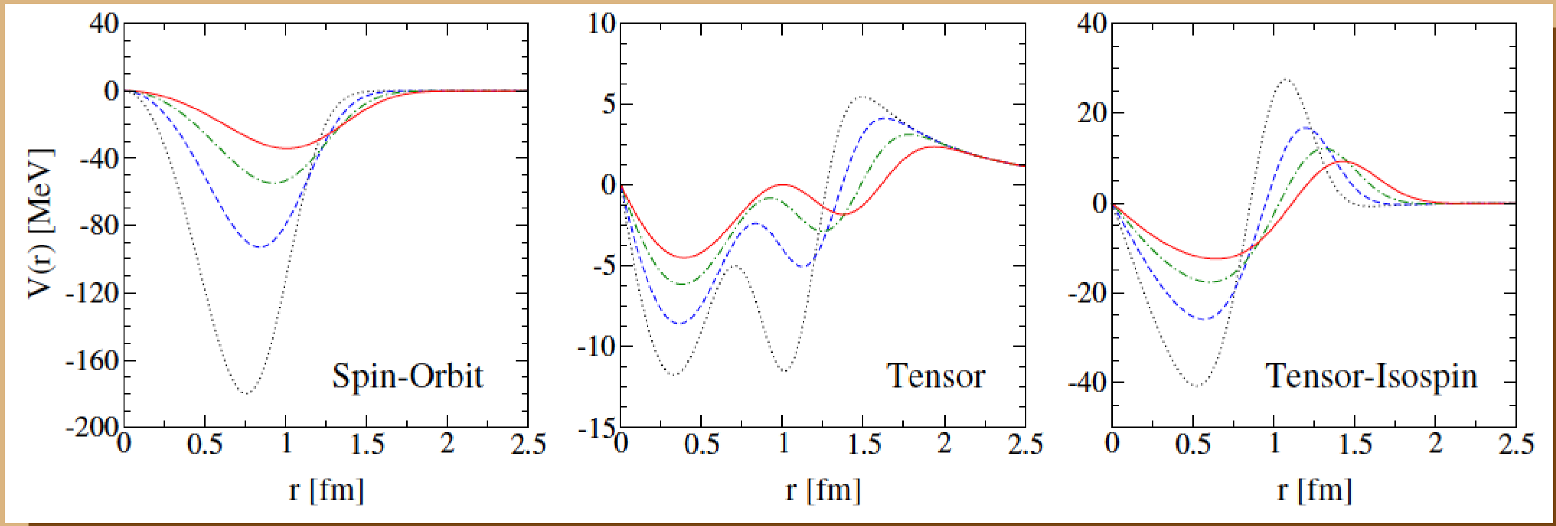

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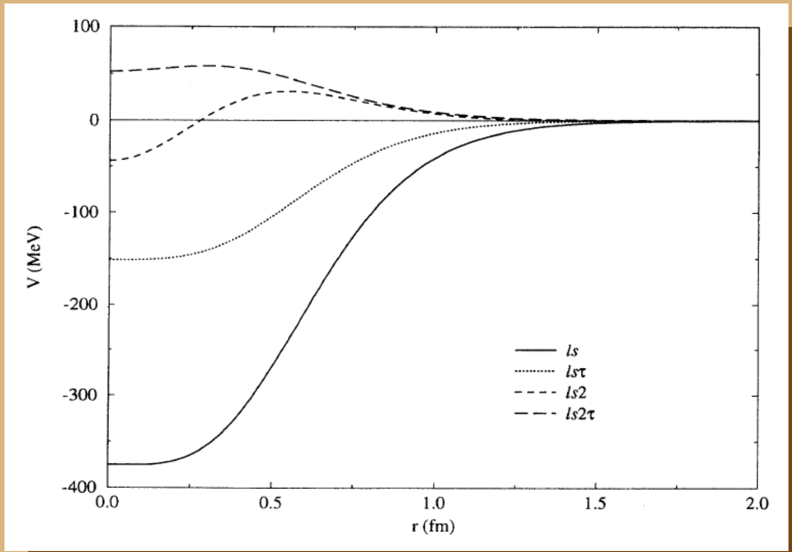
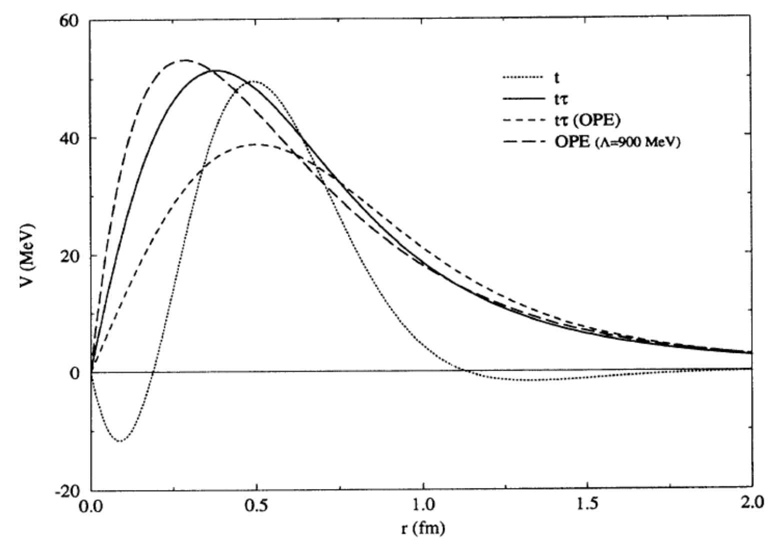
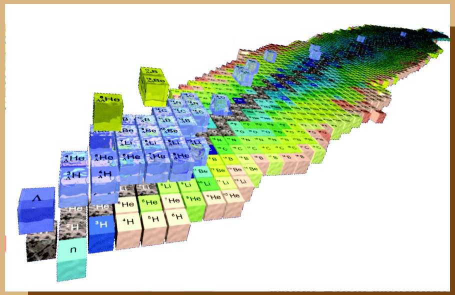




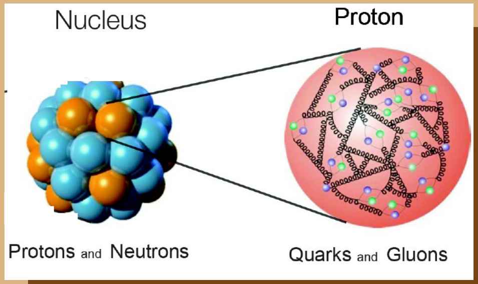
A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, arxiv:1406.0454

Outline



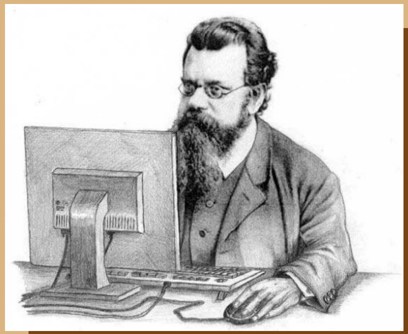
Many-nucleon problem

- Chart of nuclides
- Neutron stars
- Perturbative vs non-perturbative



Nuclear forces

- Chiral Effective Field Theory (EFT)
- Local chiral EFT

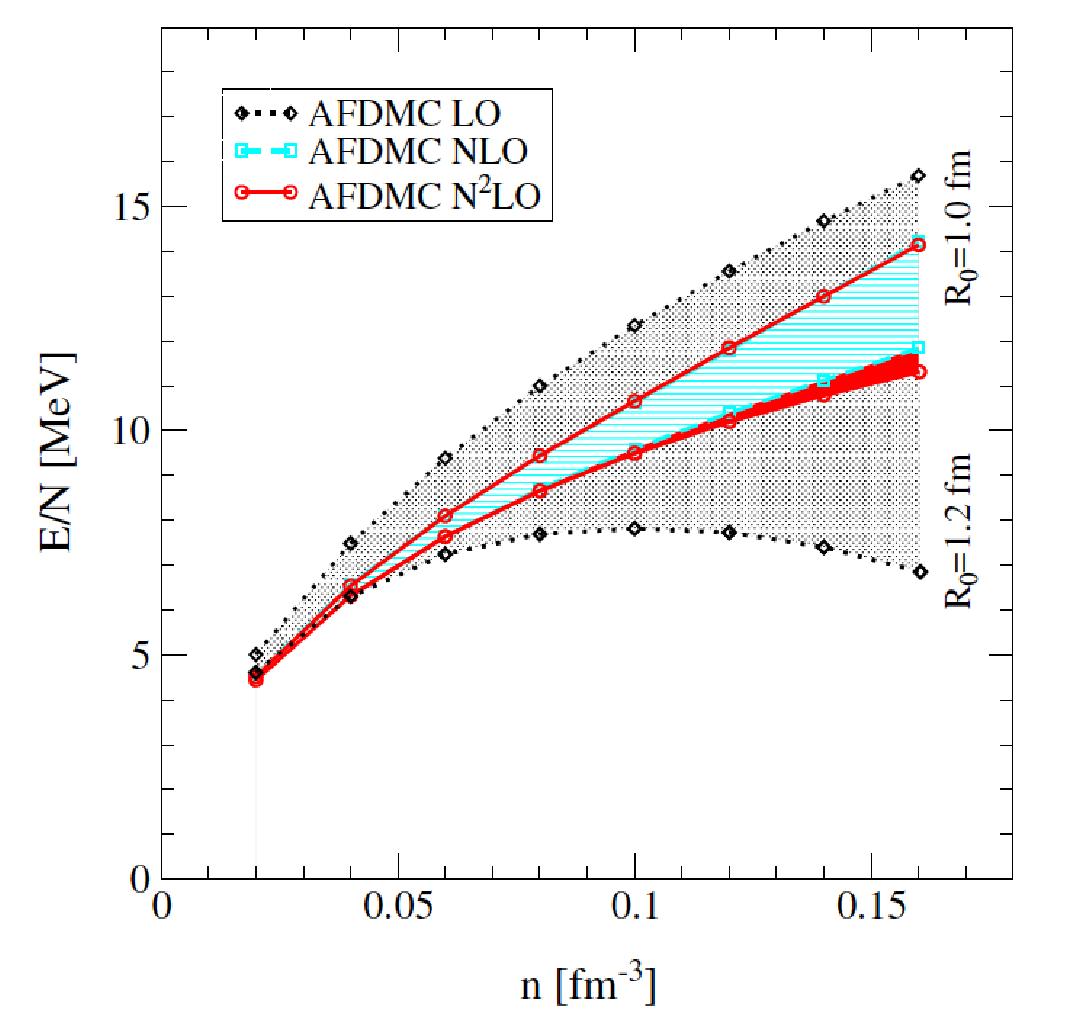


Credit: Bernhard Reischl

Few- and many-body results

- Pure neutron matter
- Light nuclei

Chiral EFT in QMC: neutron matter



- Use Auxiliary-Field Diffusion Monte Carlo to handle the full interaction
- First ever non-perturbative systematic error bands
- Band sizes to be expected
- Many-body forces will emerge systematically

NEUTRONS

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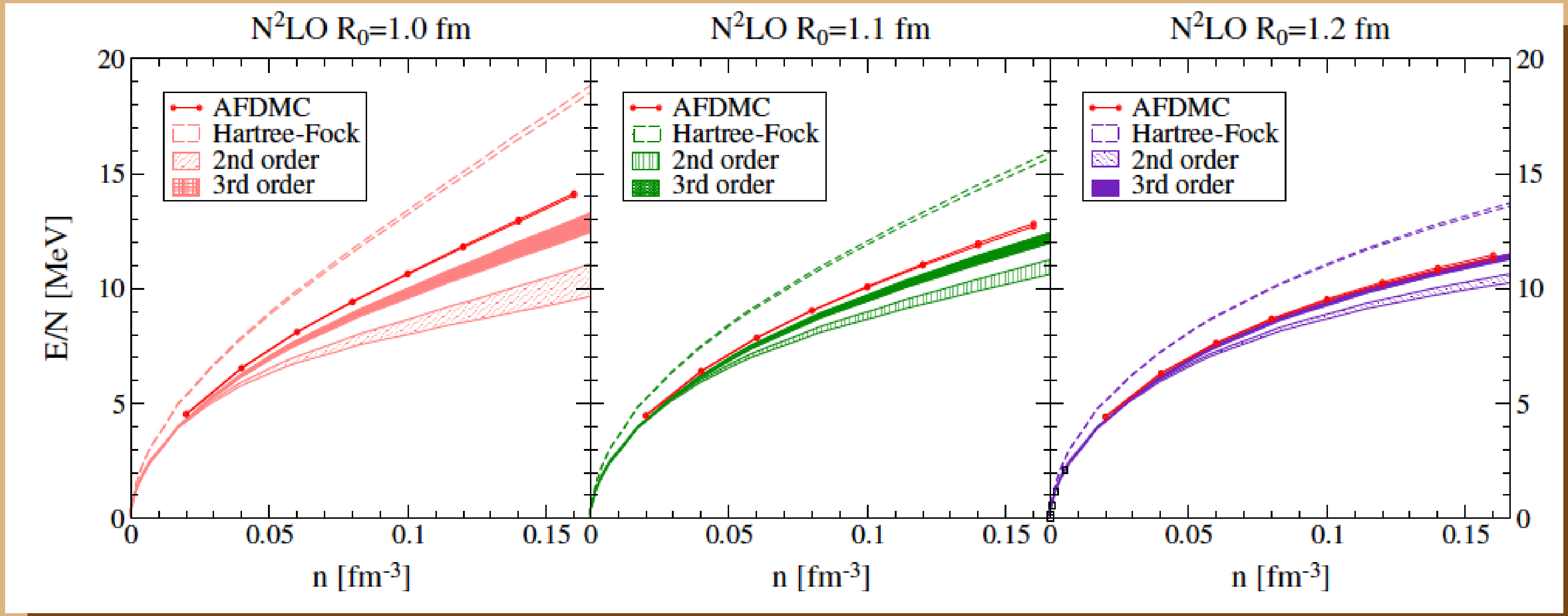
QMC vs MBPT



- MBPT bands come from diff. single-particle spectra
- Soft potential in excellent agreement with AFDMC

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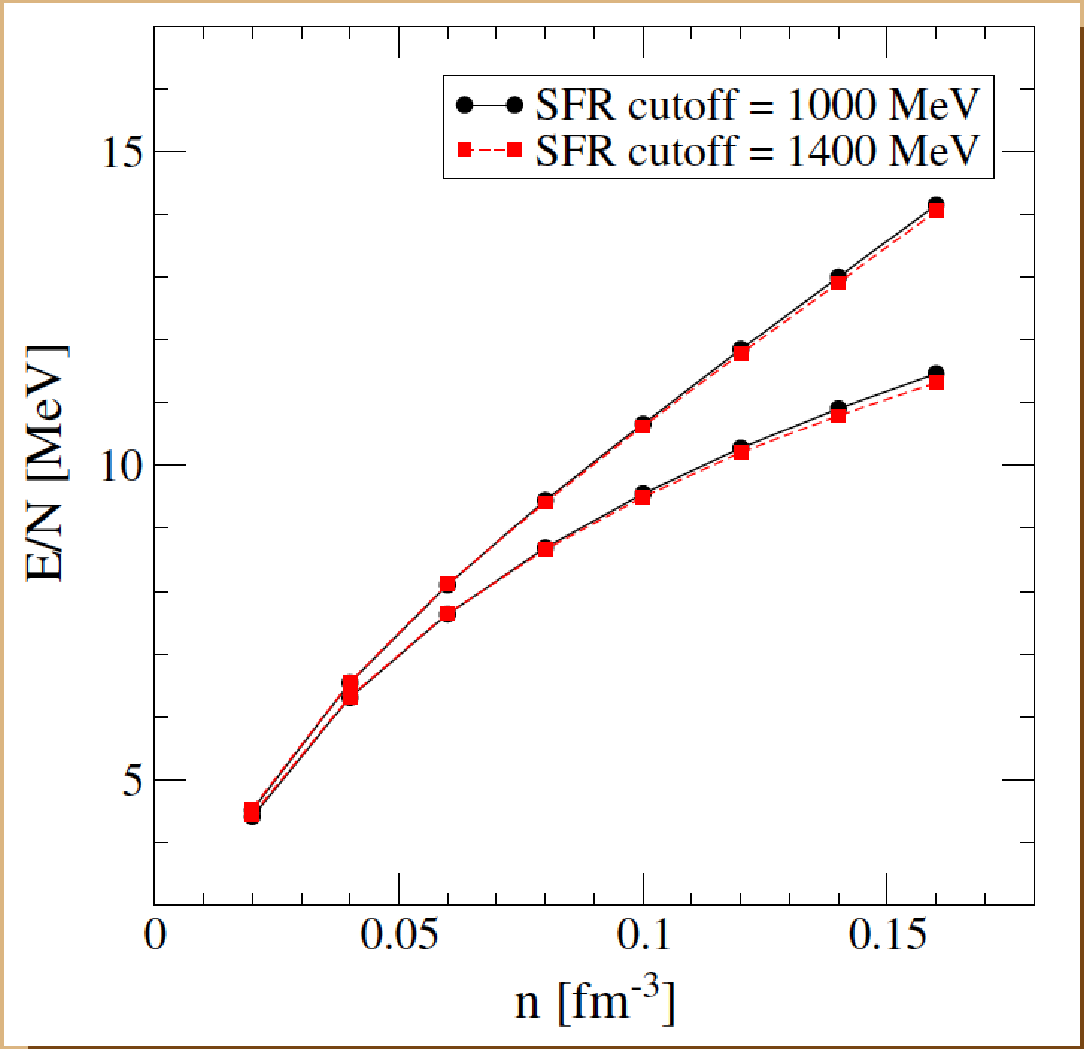
Varying the SFR cutoff



- SFR cutoff expected not to matter
- Spectral-function regularization cutoff (kept fixed before) now varied
- Clearly shows lack of dependence

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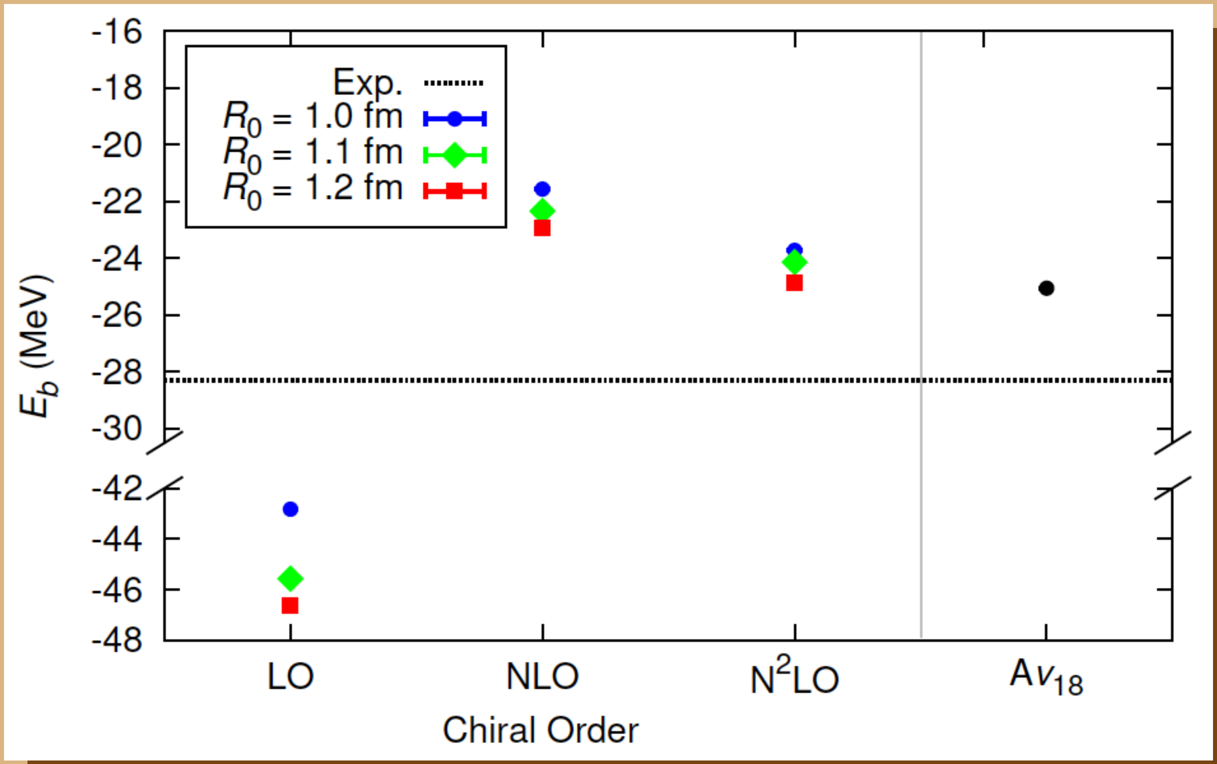
Nuclear GFMC for light nuclei



- Binding energy of ^{4}He
- Non-perturbative systematic error bands
- All results are strong force + Coulomb, no NNN

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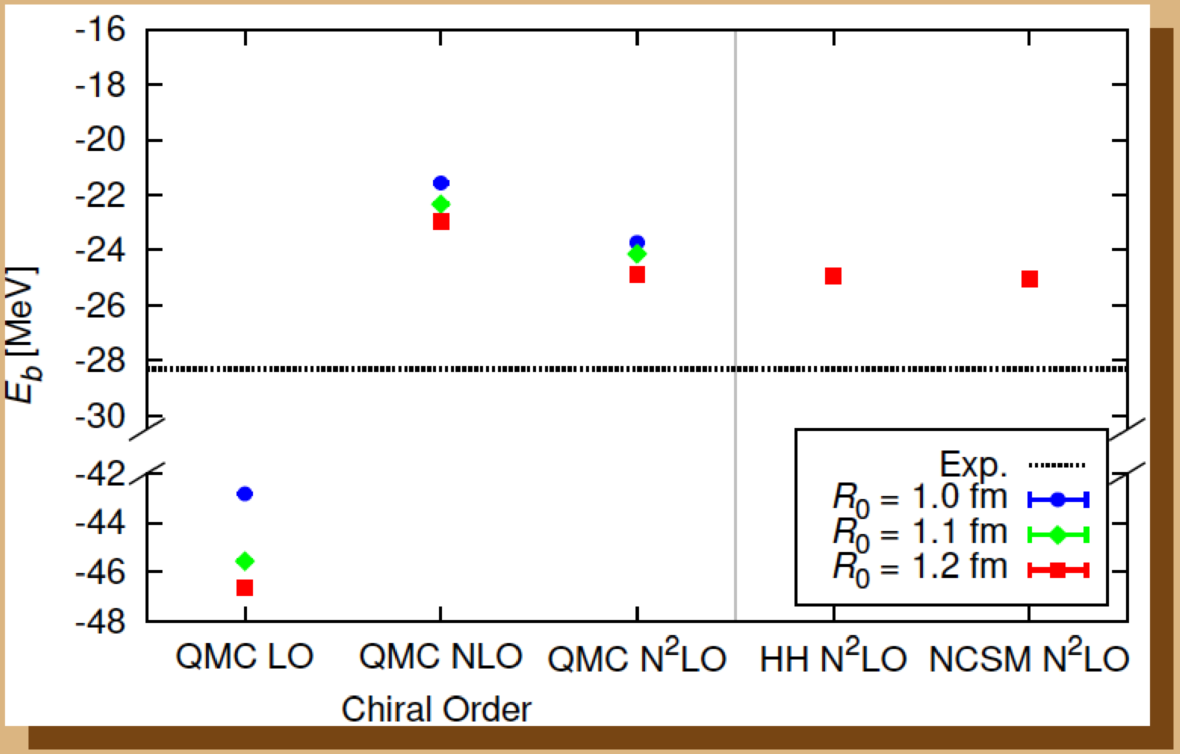
Local chiral EFT in other methods

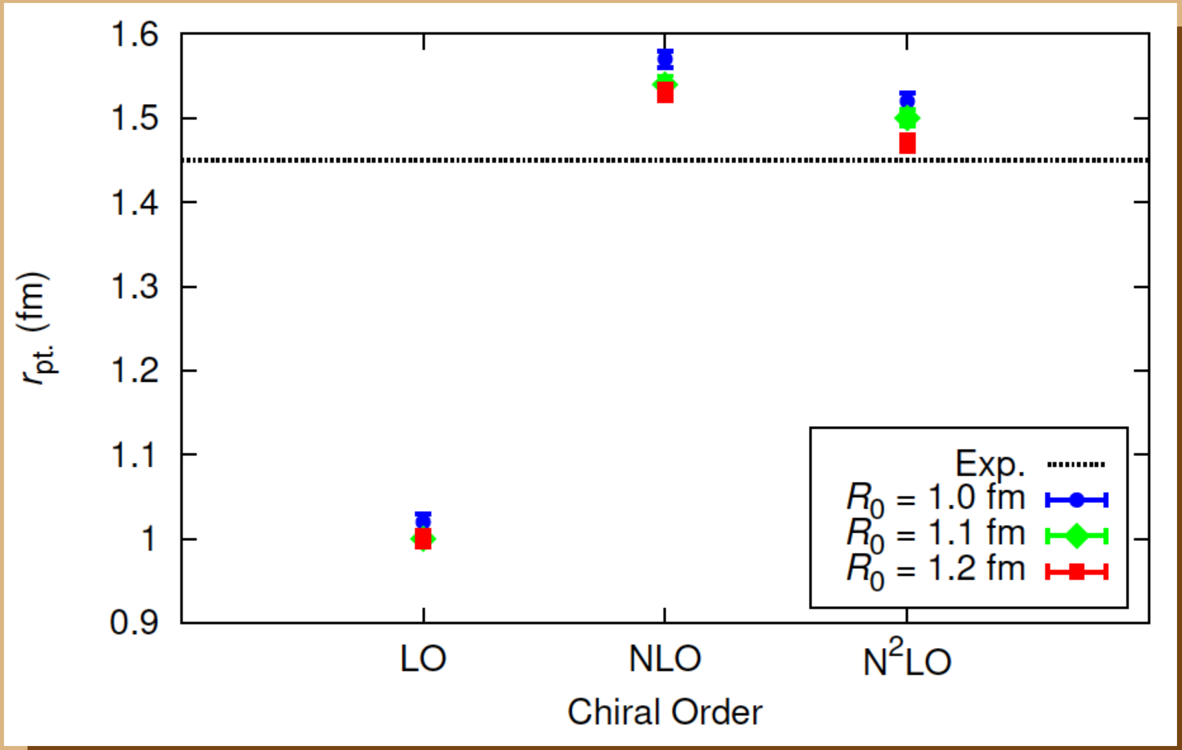


- Binding energy of ^{4}He
- Detailed benchmarking with hyperspherical harmonics (HH) and no-core shell model (NCSM)
- Probing perturbativeness

NUCLEONS

In collaboration with S. Bacca and P. Navrátil

Nuclear GFMC for light nuclei



- rms point radius of ^4He
- Non-perturbative systematic error bands
- All results are strong force + Coulomb, no NNN

NUCLEONS

J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, A. Gezerlis, A. Schwenk, arxiv:1406.2787

Conclusions

- Chiral EFT can now be used in continuum Quantum Monte Carlo methods
- Non-perturbative systematic error bands can be produced
- Back to TOV as soon as three-neutron forces have been incorporated

Conclusions

- Chiral EFT can now be used in continuum Quantum Monte Carlo methods
- Non-perturbative systematic error bands can be produced
- Back to TOV as soon as three-neutron forces have been incorporated
- “Jt is ywrite that every thing Hymself sheweth in the tastyng”

Acknowledgments

Funding

NSERC
CRSNG

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Collaborators

- Sonia Bacca (TRIUMF)
- Joe Carlson (LANL)
- Evgeny Epelbaum (Bochum)
- Stefano Gandolfi (LANL)
- Joel Lynn (LANL)
- Petr Navrátil (TRIUMF)
- Achim Schwenk (Darmstadt)
- Ingo Tews (Darmstadt)