The long arm of the lattice

randy.lewis@yorku.ca

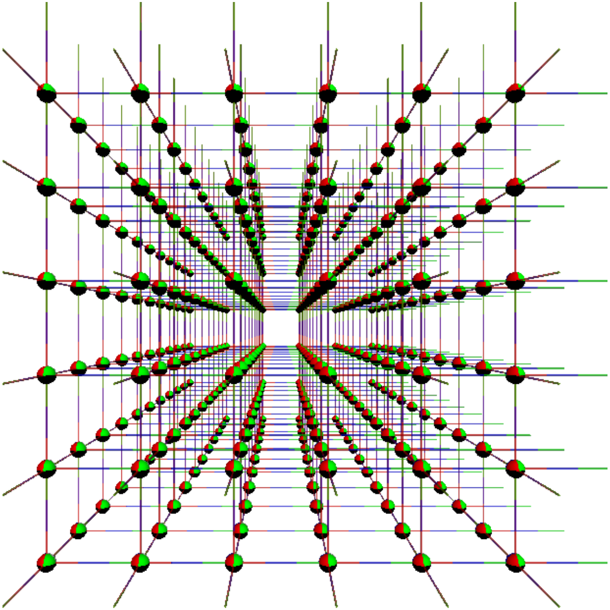
Lattice simulations are having a broad impact.
A few recent examples are presented here.

fundamental QCD parameters
Higgs decays at LHC and ILC
composite Higgs
CKM matrix
exotic hadrons
dark matter searches
quark gluon plasma
nuclei and hypernuclei

(None of my own publications are mentioned in this presentation.)

the lattice method

Quantum field theories involve
path integrals throughout spacetime.



On a spacetime lattice,
these integrals become finite sums,

$$\int d^4x \;\to\; a^4 \sum_{n_x} \sum_{n_y} \sum_{n_z} \sum_{n_t},$$

and $a \to 0$ is the continuum limit.

40 years of research by many people
has made a practical tool from this basic idea.
Let's survey some of the recent phenomenology.

fundamental QCD parameters

The quark masses m_q and the strong coupling α_s appear in the QCD Lagrangian.

How can you determine m_q when quarks are confined within hadrons?

Lattice studies of hadrons determine quark masses precisely.

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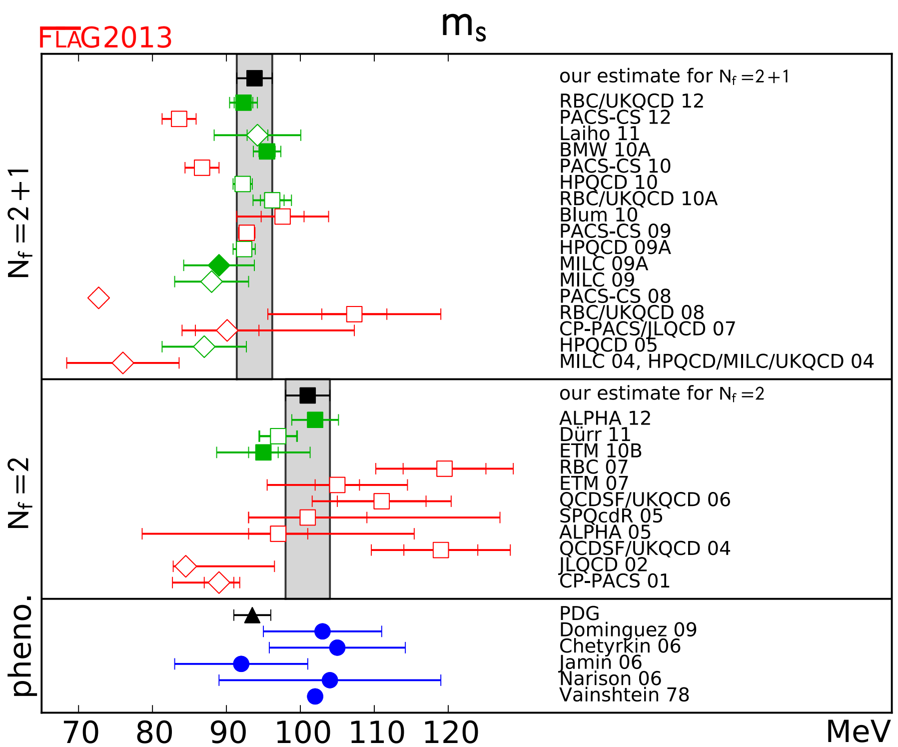

Higgs decays at LHC and ILC

The Higgs boson has been discovered.
Does it match the Standard Model exactly?

The High-Luminosity LHC and ILC will measure couplings precisely.
Standard Model calculations are not yet precise enough.

Lattice QCD will compute α_s, m_b and m_c to the required accuracy.

Expected Precision of Higgs Boson Partial Widths within the Standard Model

Lepage, Mackenzie and Peskin, 1404.0319

	$\delta m_b(10)$	$\delta\alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g
current errors [10]	0.70	0.63	0.61	0.77	0.89	0.78
+ PT	0.69	0.40	0.34	0.74	0.57	0.49
+ LS	0.30	0.53	0.53	0.38	0.74	0.65
+ LS^2	0.14	0.35	0.53	0.20	0.65	0.43
+ PT + LS	0.28	0.17	0.21	0.30	0.27	0.21
+ PT + LS^2	0.12	0.14	0.20	0.13	0.24	0.17
+ PT + LS^2 + ST	0.09	0.08	0.20	0.10	0.22	0.09
ILC goal				0.30	0.70	0.60

Table 1: Projected fractional errors, in percent, for the $\overline{\mathrm{MS}}$ QCD coupling and heavy quark masses under different scenarios for improved analyses. The improvements considered are: PT - addition of 4$^{\mathrm{th}}$ order QCD perturbation theory, LS, LS^2 - reduction of the lattice spacing to 0.03 fm and to 0.023 fm; ST - increasing the statistics of the simulation by a factor of 100. The last three columns convert the errors in input parameters into errors on Higgs couplings, taking account of correlations. The bottom line gives the target values of these errors suggested by the projections for the ILC measurement accuracies.

composite Higgs

Could the Higgs boson be a bound state of some new strong interaction?

QCD-like theories don't have a light scalar boson, but near-conformal theories can.

Lattice studies of SU(3) gauge theories with 12 fundamental fermions observe a light composite scalar particle.

Preliminary lattice studies of SU(3) gauge theories with one sextet fermion also provide hints of a Higgs impostor.

(Fodor, Holland, Kuti, Nogradi and Wong, 1401.2176)

Light composite scalar in twelve-flavor QCD on the lattice

Y. Aoki et al. Phys. Rev. Lett. **111**, 162001 (2013)

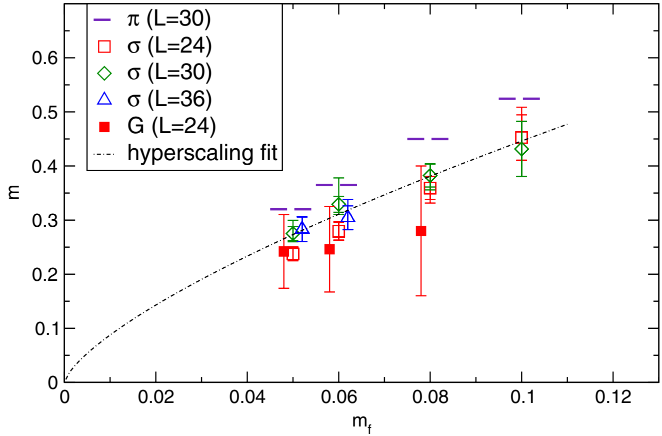

FIG. 4 (color online). The mass of the flavor-singlet scalar meson σ (see Table I) compared to the mass of the pseudoscalar π state and the mass m_G from gluonic operators. Errors are statistical, and systematics are added in quadrature. The hyperscaling curve is described in the text. The triangle and filled square symbols are slightly shifted for clarity.

CKM matrix

The Cabibbo-Kobayashi-Maskawa matrix is in the Standard Model Lagrangian.

It describes quark mixing and CP violation, and thus affects the matter-antimatter asymmetry of the universe.

If the CKM matrix is not unitary, then new physics exists beyond the Standard Model.

Lattice studies can determine most CKM matrix elements.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \sim \quad \begin{pmatrix} \pi \to \ell\nu & K \to \ell\nu(\pi) & B \to \pi\ell\nu \\ D \to \ell\nu(\pi) & D_{(s)} \to \ell\nu(K) & B \to D^{(*)}\ell\nu \\ B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s & - \end{pmatrix}$$

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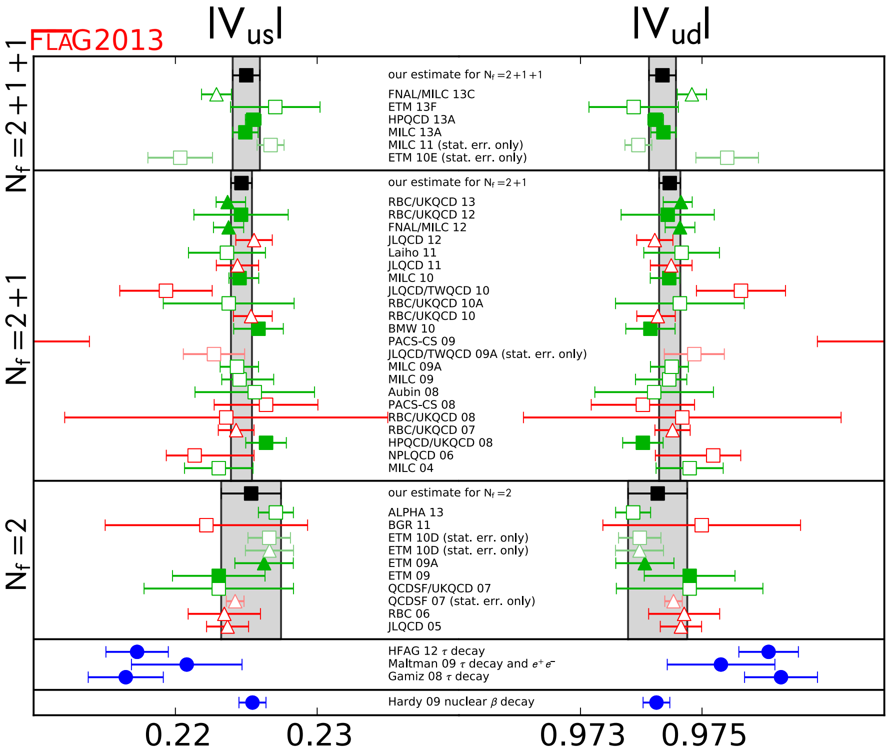


exotic hadrons

All familiar hadrons are quark-antiquark or quark-quark-quark.

Does QCD allow other color-singlet options as well?

Experiments in the past decade provide evidence for more than 20 exotic states.

Are they tetraquarks or meson molecules or hybrid mesons or something else?

Lattice calculations can study this extended particle spectrum.

Evidence for a charged charmonium-like Z_c^+ from QCD

Prelovsek, Lang, Leskovec and Mohler, 1405.7623

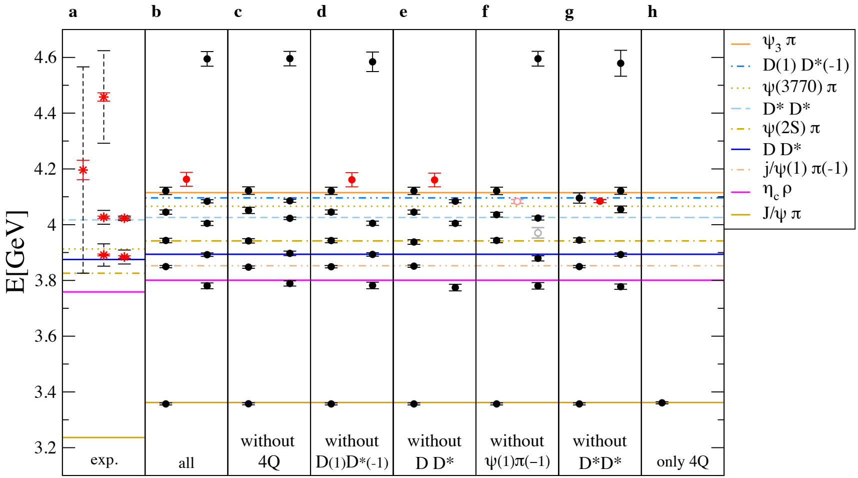


Figure 3: **The spectrum in the channel $\mathbf{I^G(J^{PC}) = 1^+(1^{+-})}$. a**,**b** same as in Fig. 1, where the lattice spectrum is based on the full 18×18 correlation matrix; **c** shows the lattice spectrum based on the 14×14 correlation matrix without diquark-antidiquark interpolating fields $\mathcal{O}^{4q}_{1-4}$; spectra **d-g** are based on truncated correlation matrices as described in the figure; spectrum **h** is based on $\mathcal{O}^{4q}_{1-4}$ only. The horizontal lines represent energies of the non-interacting two-particle states (2). Statistical errors on the lattice spectrum are shown.

Excited and exotic charmonium spectroscopy from lattice QCD

L. Liu et al. JHEP **07**, 126 (2012)

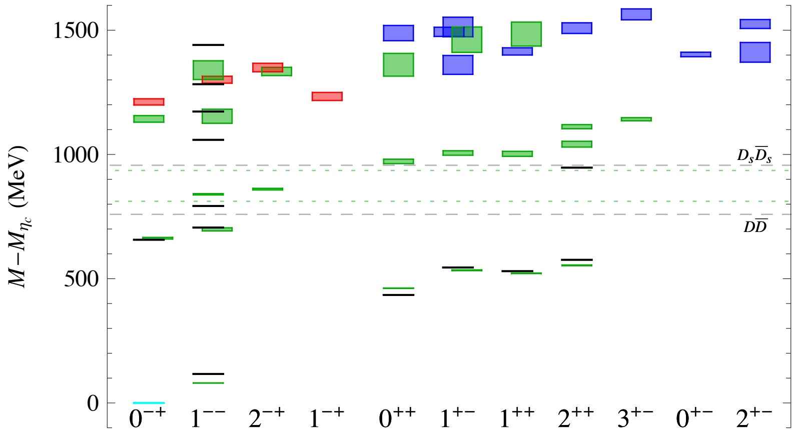


Figure 16. Charmonium spectrum up to around 4.5 GeV showing only J^{PC} channels in which we identify candidates for hybrid mesons. Red (dark blue) boxes are states suggested to be members of the lightest (first excited) hybrid supermultiplet as described in the text and green boxes are other states, all calculated on the 24^3 volume. As in Fig. 14, black lines are experimental values and the dashed lines indicate the lowest non-interacting $D\bar{D}$ and $D_s\bar{D}_s$ levels.

dark matter searches

Perhaps dark matter is a WIMP (weakly-interacting massive particle).

WIMP detection requires knowledge of WIMP-nucleon interactions.

The low-energy limit of a spin-independent interaction is scalar.

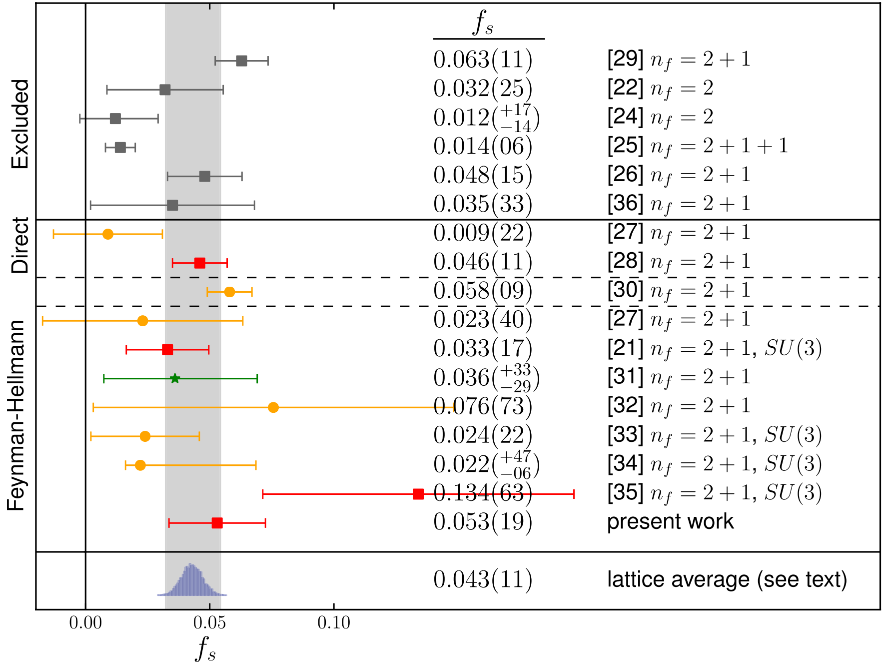
Lattice QCD can determine the necessary matrix elements,

$$f_{u,d} = \frac{(m_u + m_d)\langle N|\bar{u}u + \bar{d}d|N\rangle}{2m_N} \quad \text{and} \quad f_s = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N}\,.$$

Recent lattice results confirm that f_s is smaller than some previous estimates.

Scalar strange content of the nucleon from lattice QCD

Junnarkar and Walker-Loud, Phys. Rev. D87, 114510 (2013)



quark gluon plasma

Heavy ion collisions observe a quark-gluon plasma above the QCD transition temperature, $T_c = 154 \pm 9$ MeV.

Generalized "chemical potentials" are defined for baryon number B, electric charge Q and strangeness S.

Derivatives of ln(partition function) provide cumulants,

$$\chi_{ijk}^{BQS} = \frac{1}{V^3 T} \; \frac{\partial^i}{\partial(\mu_B/T)^i} \; \frac{\partial^j}{\partial(\mu_Q/T)^j} \; \frac{\partial^k}{\partial(\mu_S/T)^k} \ln Z$$

Lattice calculations show the phenomenology between the limits of
(a) a hadron resonance gas (low temperature)
(b) a free quark gas (high temperature)

The last word(s) on CPOD 2013

Karsch, Proc. of Science, CPOD2013 (2013) 046

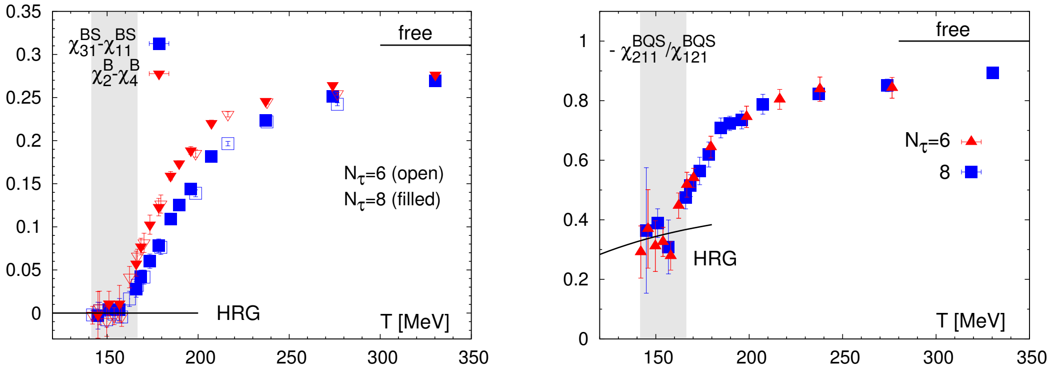


Figure 2: Left: The difference of 4^{th} and 2^{nd} order cumulants combined in such a way that they vanish in an uncorrelated hadron resonance gas (in Boltzmann approximation) [25]. **Right:** The ratio of two mixed cumulants that project onto the quantum numbers of strange quarks and yield identical results in a free strange quark gas.

[25] Strangeness at high temperatures: from hadrons to quarks,
Bazavov et al., Phys. Rev. Lett. 111, 082301 (2013).

nuclei and hypernuclei

In principle, nuclei are ultimately described by QCD.

These many-quark systems require large lattice volumes.

Binding energies $\ll \Lambda_{\mathrm{QCD}}$ are particularly difficult.

If lattice proves itself with known nuclear physics,
then it could be valuable for exotic nuclei and extreme environments.

Light Nuclei and Hypernuclei from Quantum Chromodynamics in the Limit of SU(3) Flavor Symmetry

Beane et al., Phys. Rev. D87, 034506 (2013)

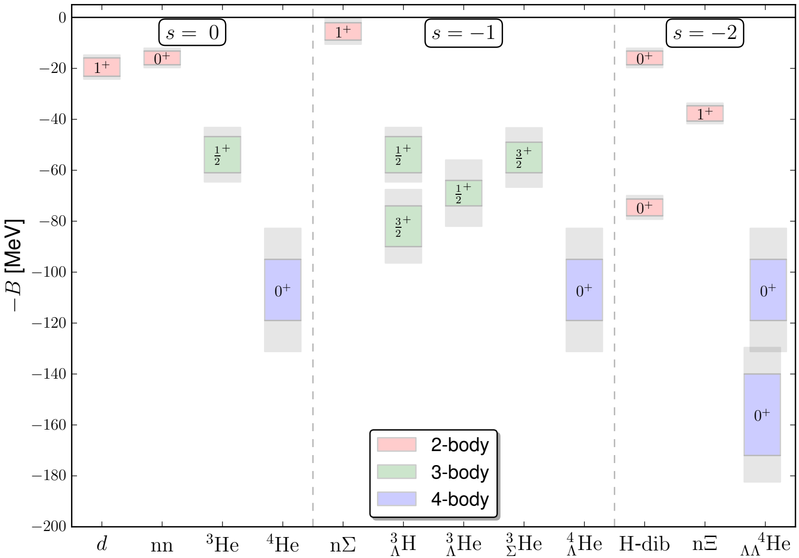


FIG. 19: A compilation of the nuclear energy levels, with spin and parity J^π, determined in this work.

concluding remarks

Lattice field theory is rigorous,
systematic,
increasingly precise,
broadly applicable within QCD and beyond.

fundamental QCD parameters, Higgs decays at LHC and ILC, composite Higgs, CKM matrix, exotic hadrons, dark matter searches, quark gluon plasma, nuclei and hypernuclei

This has been a *very truncated* glimpse of recent lattice activities.

Next week, more than 400 practitioners will attend the (annual)
32nd International Symposium on Lattice Field Theory.