Inside a black hole

detectors as topological probes

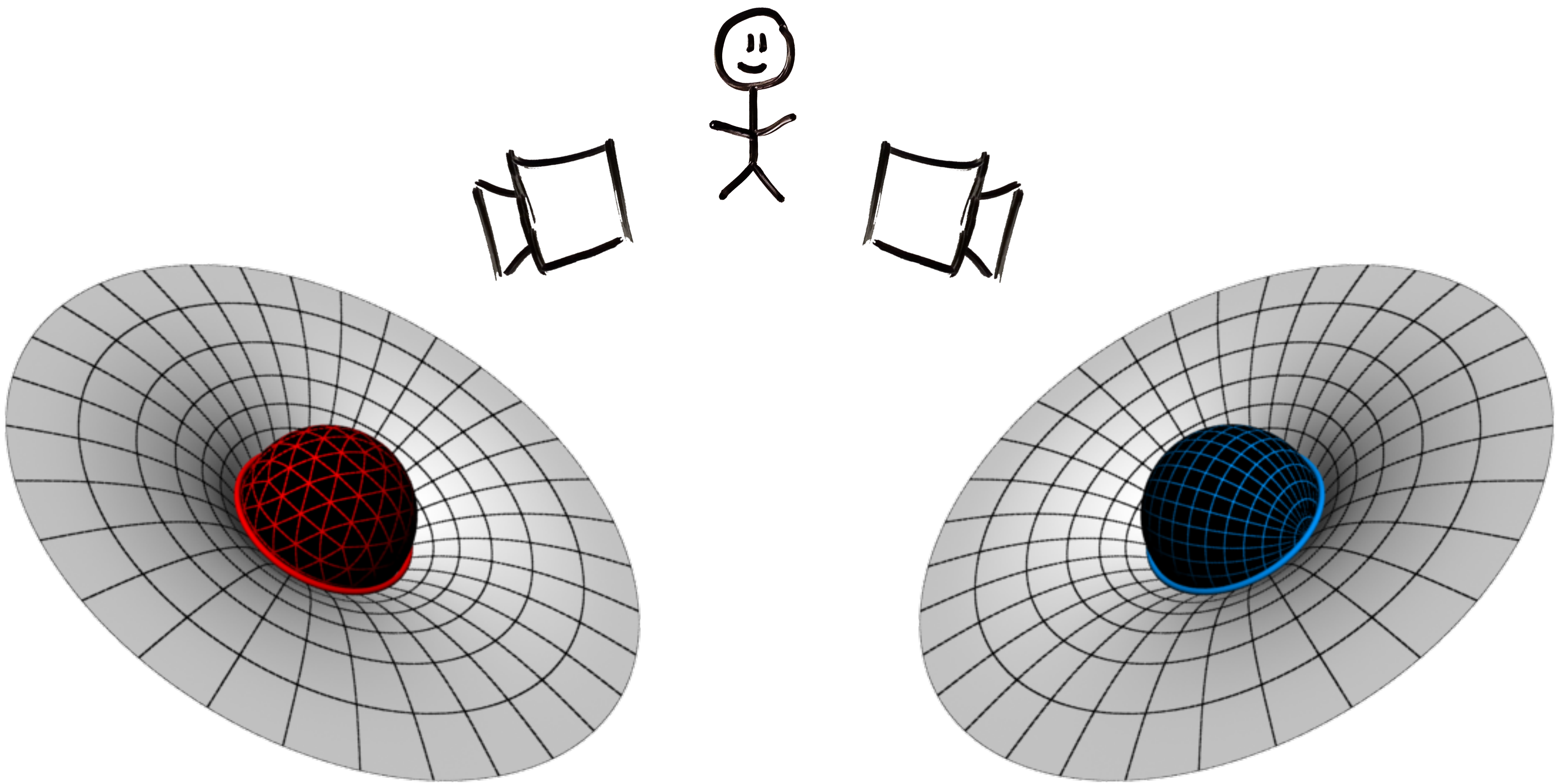
Alexander Smith and Robert Mann

A. R. H. Smith and R. B. Mann, Class. Quantum Grav. 31 (2014) 082001

“Black hole voyeurism” CQG+: http://cqgplus.com/2014/04/30/black-hole-voyeurism/#more-432

Looking inside a black hole

A. R. H. Smith and R. B. Mann, Class. Quantum Grav. 31 (2014) 082001



BTZ black hole

$\mathbb{RP}^2$ geon

The BTZ black hole

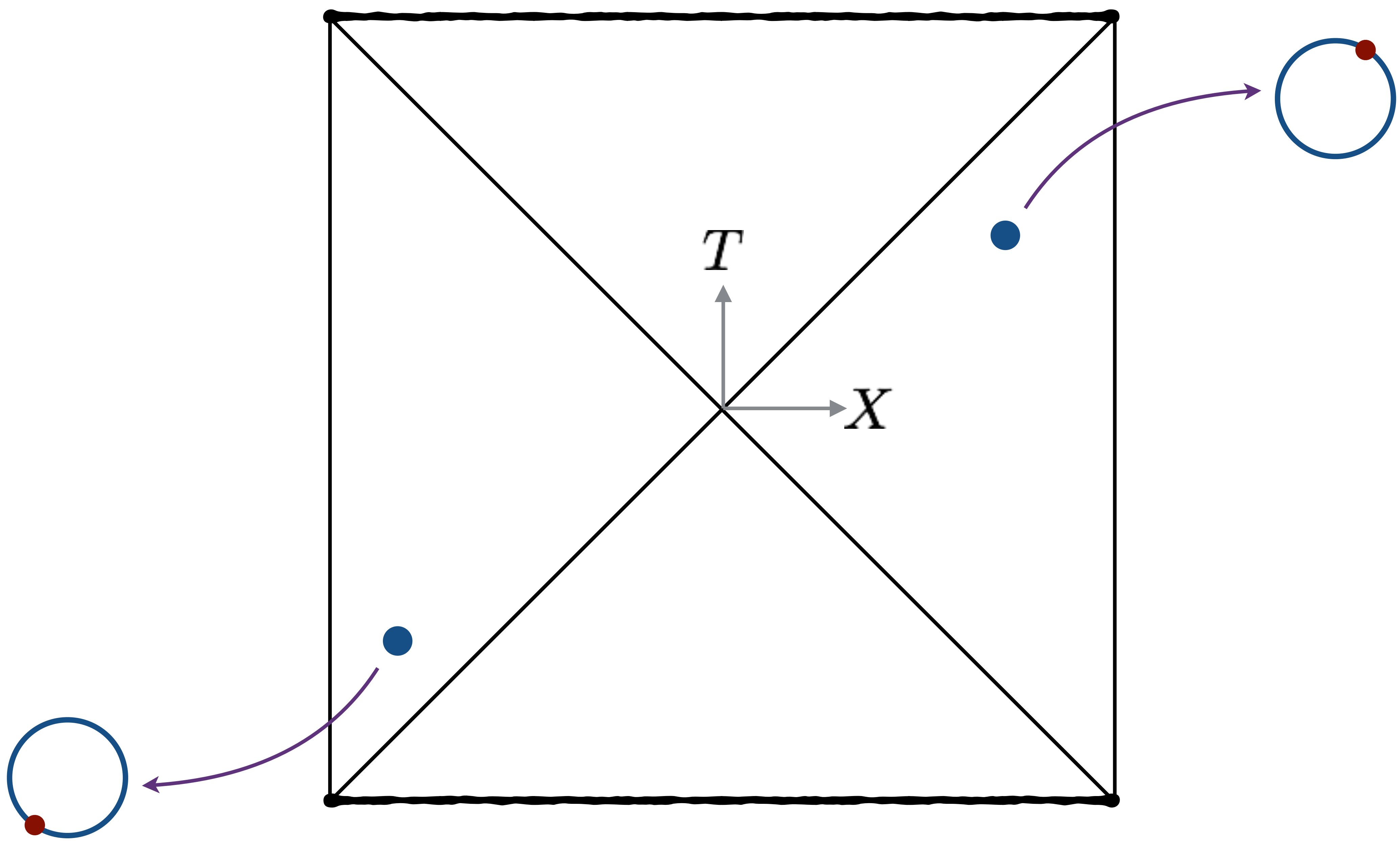
M. Banados, et. al., Phys. Rev. Let. 69 (1992) 1849

$$ds^2 = \left(-M + \frac{r^2}{\ell^2}\right) dt^2 + \left(-M + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\phi^2$$

$$= -\frac{l^2}{(1+UV)^2}\left[-4dU\,dV + M\left(1-UV\right)^2 d\phi^2\right]$$

The geon construction

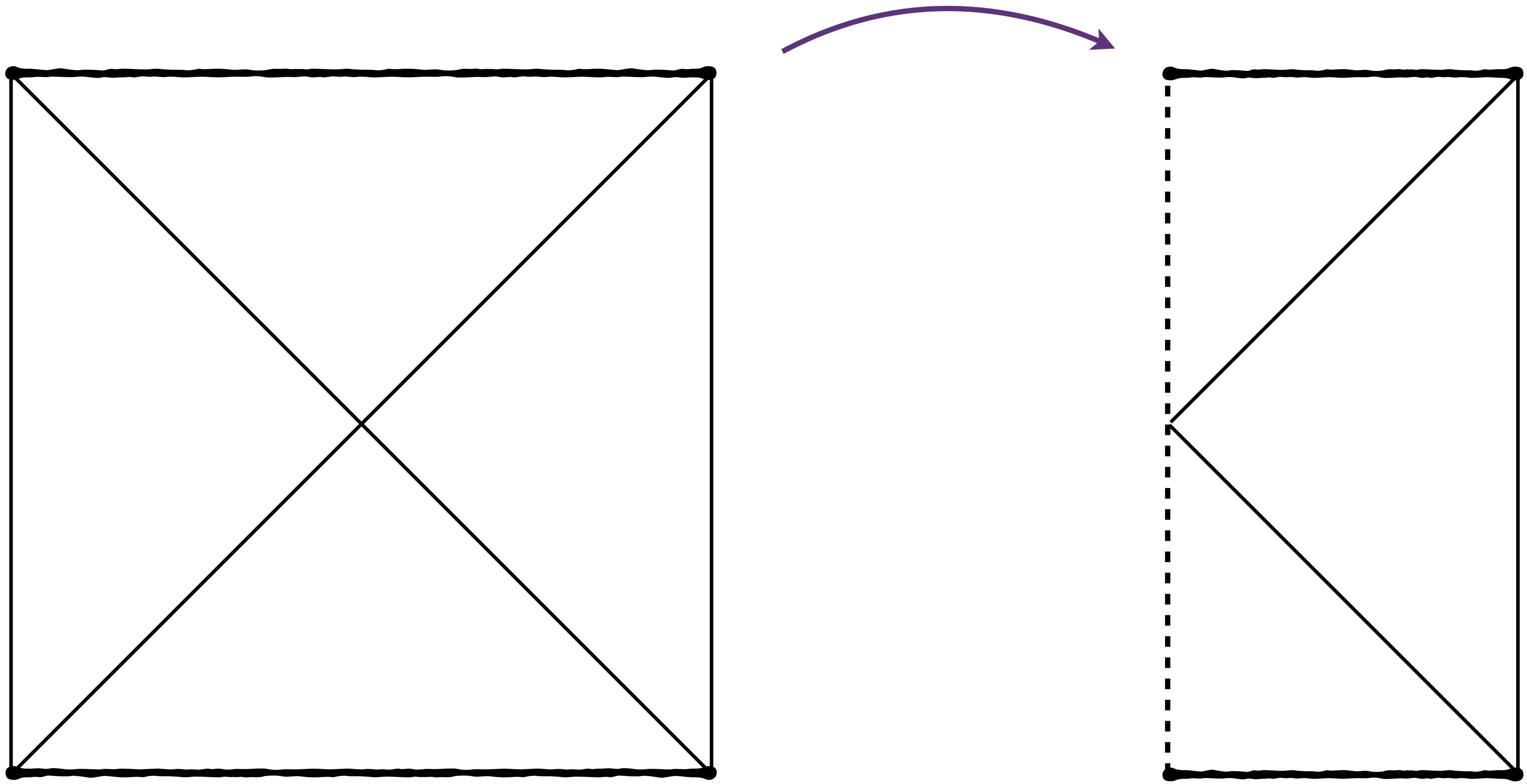
$$J : (T, X, \phi) \rightarrow (T, -X, P(\phi))$$



The ~~BTZ black hole~~ $\mathbb{R}\mathbf{P}^2$ geon

The geon construction

$$J:(T,X,\phi)\to(T,-X,P(\phi))$$



The BTZ black hole

The $\mathbb{RP}^{\mathbf{2}}$ geon

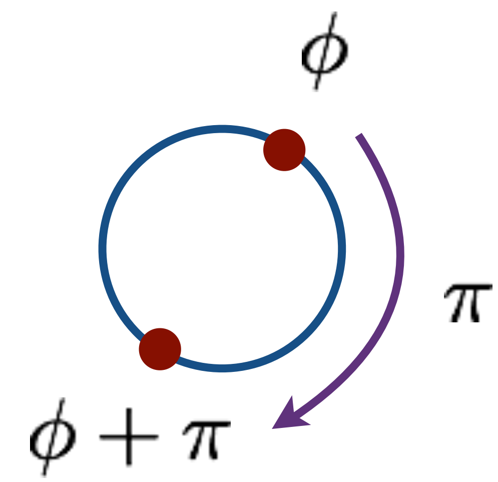
The $\mathbb{RP}^2$ geon

J. Louko, et. al., Phys. Rev. D 59 (1999) 066002

The geon map is

$$J : (T, X, \phi) \to (T, -X, P(\phi))$$

$$(T, X, \phi) \to (T, -X, \phi + \pi)$$



In light cone coordinates

$$U = T - X$$

$$V = T + X$$

the map becomes

$$J : (U, V, \phi) \to (V, U, \phi + \pi)$$

The $\mathbb{RP}^2$ geon

J. Louko, et. al., Phys. Rev. D 59 (1999) 066002

$$J : (U, V, \phi) \to (V, U, \phi + \pi)$$

Now we note that

$$\Gamma := \{I, J\} \simeq \mathbb{Z}_2$$

which means the geon can be seen as a quotient spacetime

$$\mathcal{M}_{\text{geon}} = \mathcal{M}_{\text{BTZ}}/\mathbb{Z}_2$$

Recall the BTZ metric

$$ds^2 = -\frac{l^2}{(1+UV)^2}\left[-4dU\,dV + M\,(1-UV)^2\,d\phi^2\right]$$

$\Longrightarrow$ The BTZ and geon metric are identical!

The Unruh-DeWitt detector

Unruh, W. G. (1976) Phys. Rev. D, 14, 870
DeWitt, B.S. (1979) in General Relativity, eds. S.W. Hawking and W. Israel

 $=$ Two level quantum system $\{|0\rangle_d, |E\rangle_d\}$

monopole moment

detectors trajectory

$$H_{\rm int} = c\chi(\tau)\mu(\tau)\phi(x(\tau))$$

coupling constant

switching function

scalar field

The Unruh-DeWitt detector

Unruh, W. G. (1976) Phys. Rev. D, 14, 870
DeWitt, B.S. (1979) in General Relativity, eds. S.W. Hawking and W. Israel

The probability of transition is given by

$$P(E) = c^2 \left| \langle 0_d | \, \mu(0) \, | E_d \rangle \right|^2 \mathcal{F}(E)$$

with being $\mathcal{F}(E)$ the response function

$$\mathcal{F}(E) = \int_{-\infty}^{\infty} d\tau' \chi(\tau') \int_{0}^{\infty} d\tau'' \chi(\tau'') e^{-iE(\tau' - \tau'')} G^{+}(x(\tau'), x(\tau''))$$

The Wightman function

$$G^{+}(x, x') := \langle 0 | \, \phi(x) \phi(x') \, | 0 \rangle$$

and $(\nabla^{\mu} \nabla_{\mu} + \cancel{m}) \, G^{+}(x, x') = 0$

$m = 0$

The Unruh-DeWitt detector

Unruh, W. G. (1976) Phys. Rev. D, 14, 870
DeWitt, B.S. (1979) in General Relativity, eds. S.W. Hawking and W. Israel

We define the instantaneous transition rate as

$$\dot{\mathcal{F}}(E) = \frac{1}{4} + 2\int_0^{\Delta\tau} ds \ \mathrm{Re}\ \left[e^{-iEs} G^+(\tau, \tau - s)\right]$$

If the detector was turned on in the asymptotic past

$$\dot{\mathcal{F}}(E) = \frac{1}{4} + 2\int_0^{\infty} ds \ \mathrm{Re}\ \left[e^{-iEs} G^+(\tau, \tau - s)\right]$$

Quantum field theory on the BTZ spacetime

Carlip S. (1995) arXiv:gr-qc/9506079

The BTZ spacetime can be see as the quotient space

$$\mathcal{M}_{\mathrm{BTZ}} = \mathcal{M}_{\mathrm{AdS}_3}/\mathbb{Z}$$

where the identification is

$$\mathbb{Z} \simeq (t, r, \phi) \sim (t, r, \phi + 2\pi) =: \Lambda$$

Thus we can make use of the method of images to obtain the BTZ Wightman functions

$$G^{+}_{\mathrm{BTZ}}(x, x') = \sum_{n} G^{+}_{\mathrm{AdS}_3}(x, \Lambda^n x') = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\sqrt{\Delta X_n^2(x, x')}}$$

Hartle-Hawking Vacuum

where

$$\Delta X_n^2(x, x') = -1 + \sqrt{\alpha(r)\alpha(r')}\cosh\left[\frac{r_h}{\ell}(\phi - \phi' - 2\pi n)\right] - \sqrt{(\alpha(r) - 1)(\alpha(r') - 1)}\cosh\left[\frac{r_h}{\ell^2}(t - t')\right]$$

$$\alpha(r) := r^2/r_h^2$$

Quantum field theory on the $\mathbb{RP}^2$ geon

Recall that

$$\mathcal{M}_{\text{geon}} = \mathcal{M}_{\text{BTZ}}/\mathbb{Z}_2$$

with $\Gamma := \{I, J\} \simeq \mathbb{Z}_2$ and $J : (U, V, \phi) \to (V, U, \phi + \pi)$

$$G^+_{\text{geon}}(x,\ x') = \sum_{m \in \{0,1\}} G^+_{\text{BTZ}}(x, J^m x')$$

$$= G^+_{\text{BTZ}}(x, x') + G^+_{\text{BTZ}}(x, Jx')$$

$$\Delta G^+(x, x')$$

$$= G^+_{\text{BTZ}}(x, x') + \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\sqrt{\Delta X_n^2(x, J(x'))}}$$

The transition rate of a detector in the BTZ spacetime

Let us consider a detector at a fixed distance:

$$t = \ell \frac{1}{\sqrt{\alpha - 1}} \tau; \qquad r = \text{constant}; \qquad \phi = 0$$

So the transition rate along this trajectory is

$$\dot{\mathcal{F}}_{\text{BTZ}}(E) = \frac{1}{4} + 2 \int_0^\infty ds \ \text{Re} \left[e^{-iEs} G^+_{\text{BTZ}}(x(\tau), x(\tau - s)) \right]$$

where

$$G^+_{\text{BTZ}}(x(\tau), x(\tau - s)) = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi \sqrt{\Delta X_n^2 (x(\tau), x(\tau - s))}}$$

and

$$\Delta X_n^2(x(\tau), x(\tau - s)) = -1 + \alpha(r) \cosh \left[\frac{r_h}{\ell} 2\pi n \right] - (\alpha(r) - 1) \cosh \left[\frac{r_h}{\ell^2} s \right]$$

The transition rate of a detector in the $\mathbb{RP}^2$ geon spacetime

Again, we consider a detector at a fixed distance:

$$t = \ell \frac{1}{\sqrt{\alpha - 1}} \tau; \qquad r = \text{constant}; \qquad \phi = 0$$

In the geon space time the transition rate is given by

$G^+_{\text{BTZ}} + \Delta G^+$

$$\dot{\mathcal{F}}_{\text{geon}}(E) = \frac{1}{4} + 2 \int_0^\infty ds \, \text{Re} \left[e^{-iEs} G^+_{\text{geon}}(x(\tau), x(\tau - s)) \right]$$

$$= \dot{\mathcal{F}}_{\text{BTZ}}(E) + \int_0^\infty ds \, \text{Re} \left[e^{-iEs} \, \Delta G^+(x(\tau), x(\tau - s) \right]$$

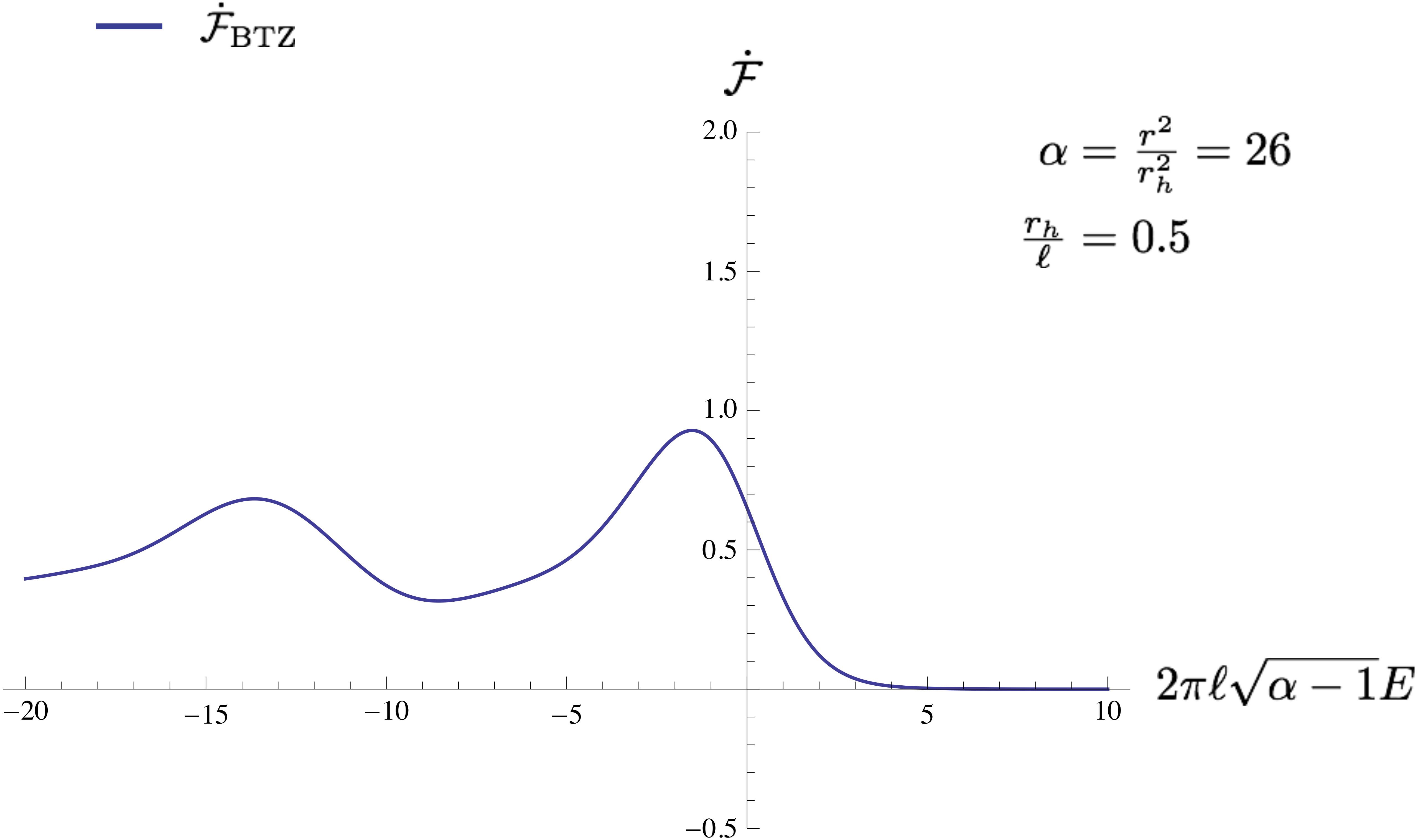
where

$$\Delta G^+ \left(x(\tau), x(\tau - s) \right) = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi \sqrt{\Delta X_n^2(x(\tau), Jx(\tau - s))}}$$

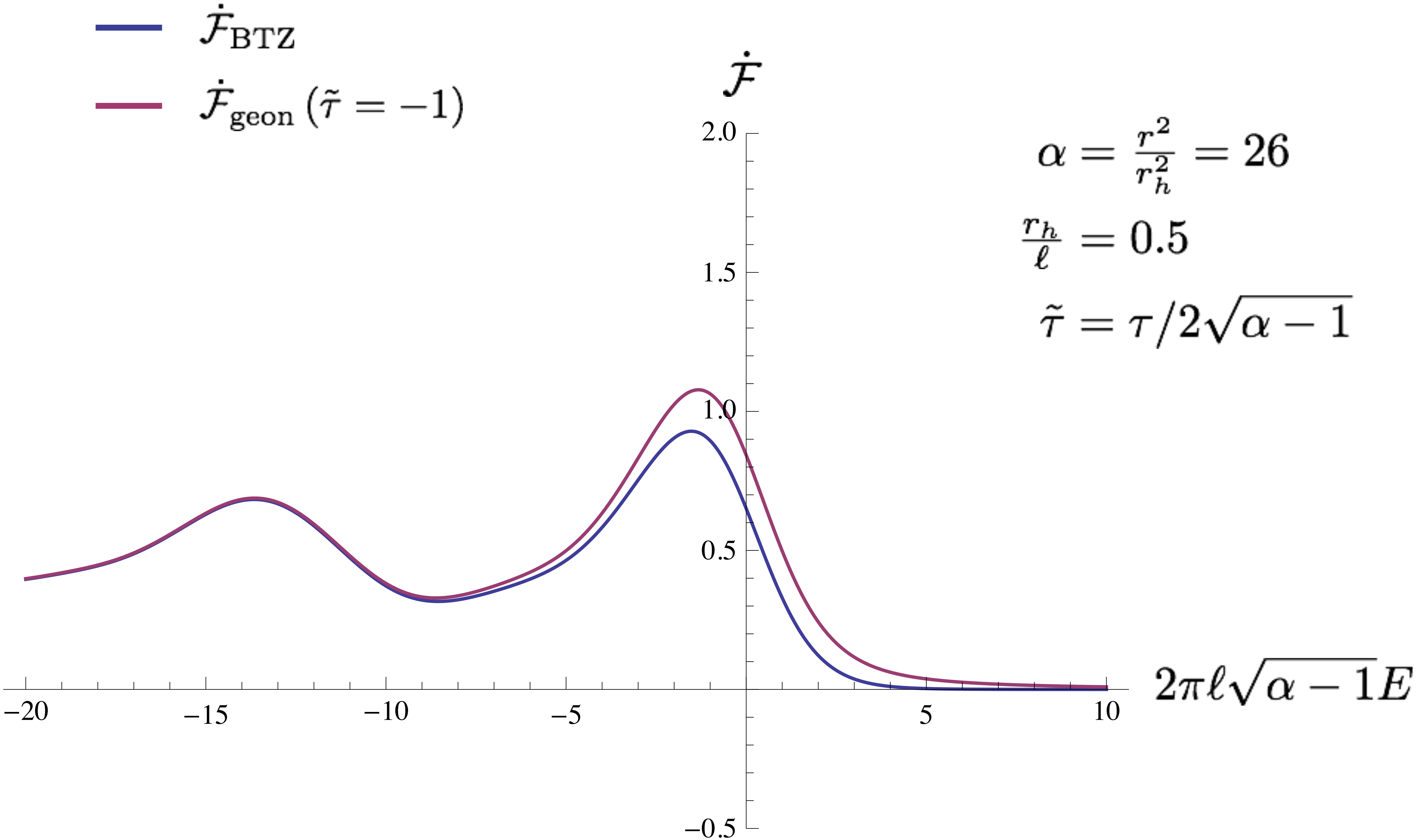
and

$$\Delta X_n^2(x(\tau), Jx(\tau - s)) = -1 + \alpha(r) \cosh \left[\frac{r_h}{\ell} 2\pi \left(n + \tfrac{1}{2} \right) \right] - (\alpha(r) - 1) \cosh \left[\frac{r_h}{\ell^2} (2\tau - s) \right]$$

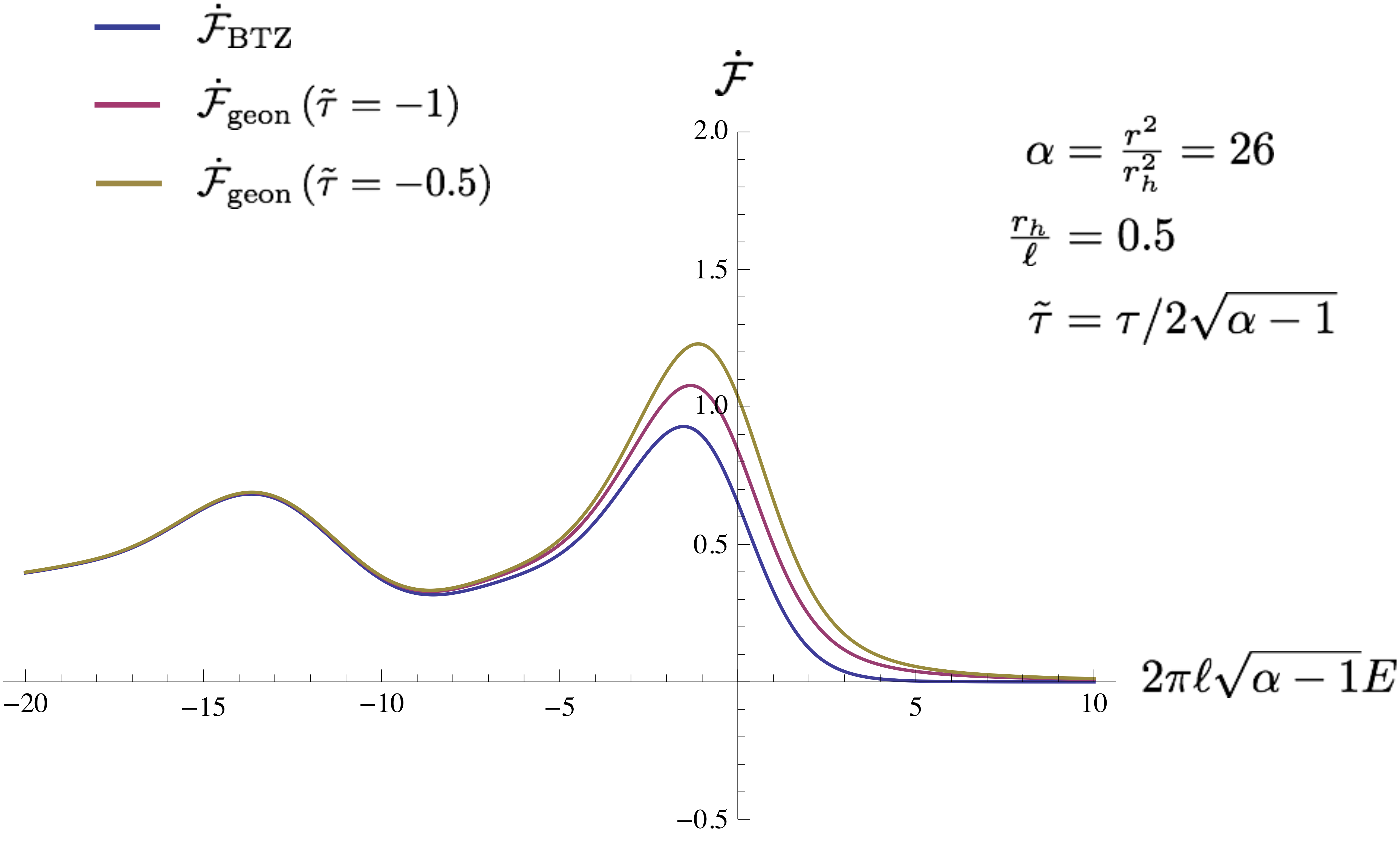
The transition rate as a function of energy gap



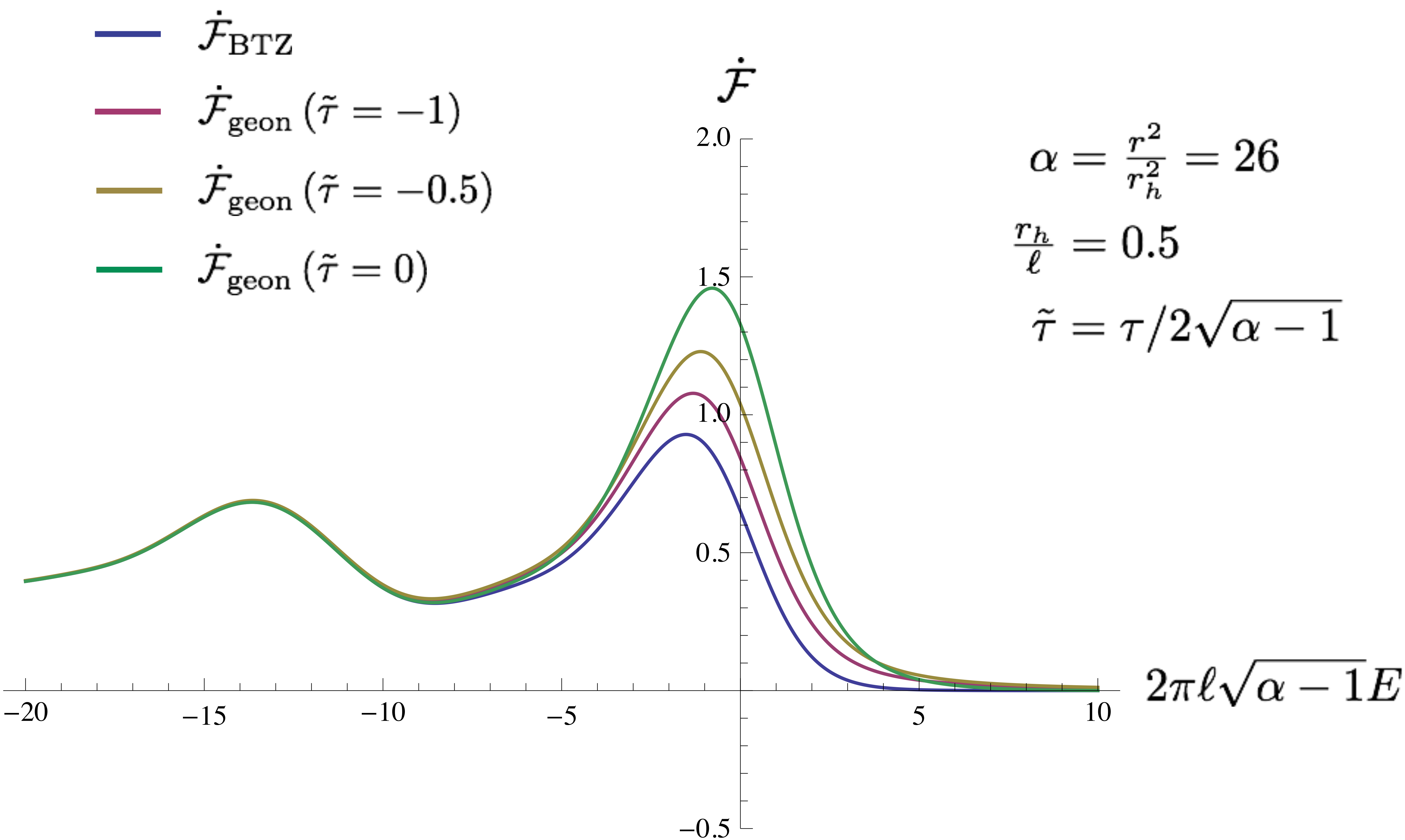
The transition rate as a function of energy gap



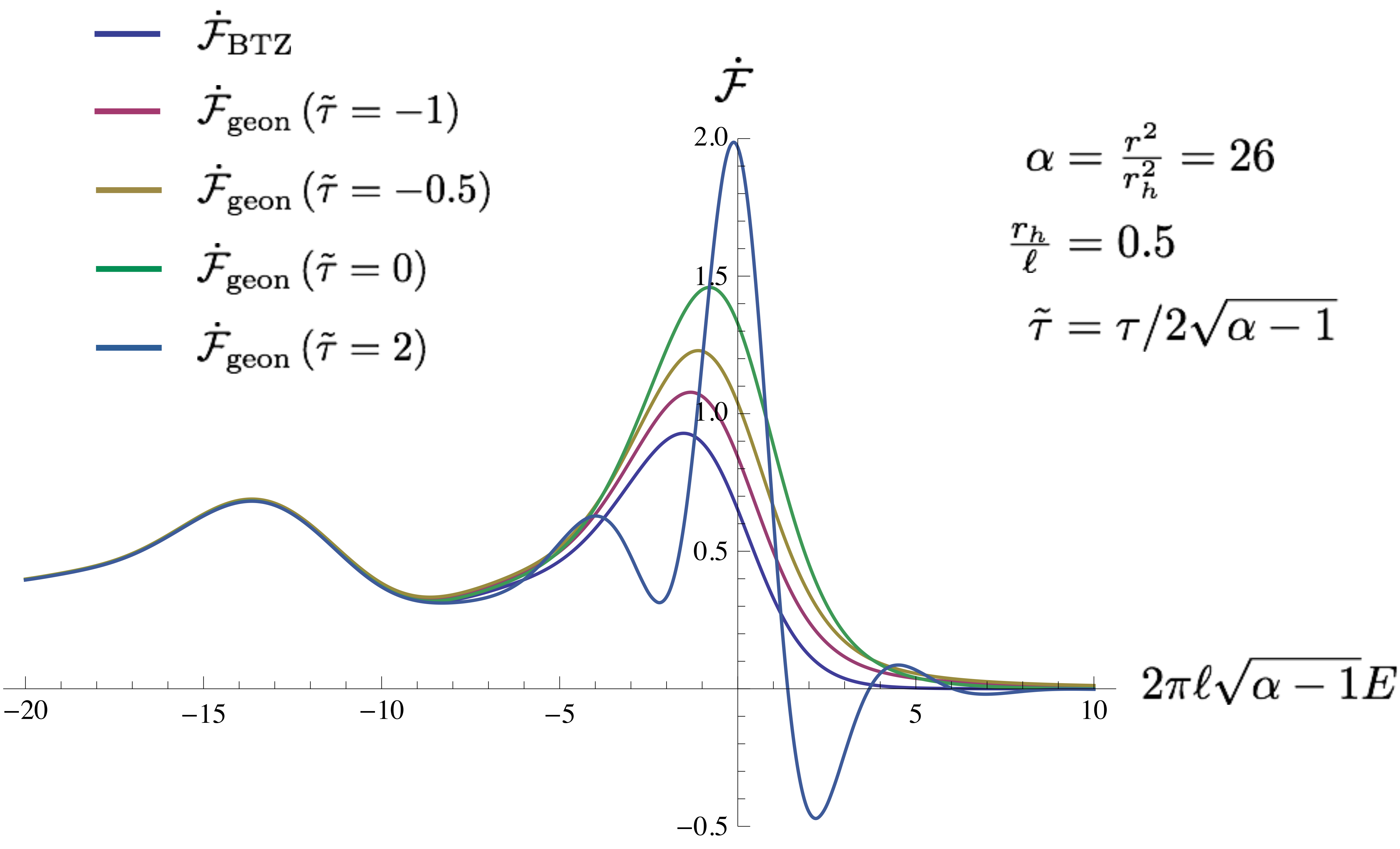
The transition rate as a function of energy gap



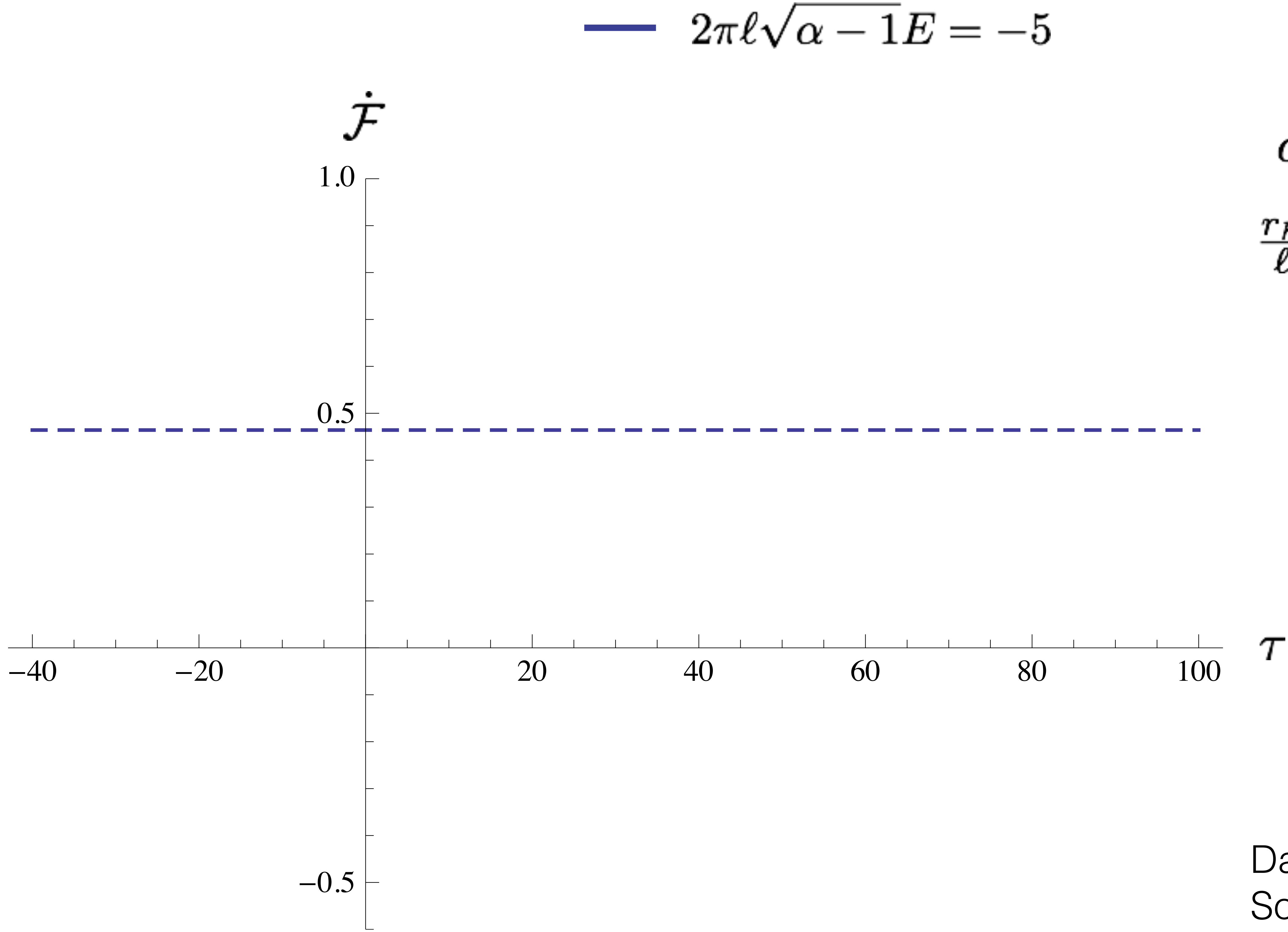
The transition rate as a function of energy gap



The transition rate as a function of energy gap



The transition rate as a function of energy gap

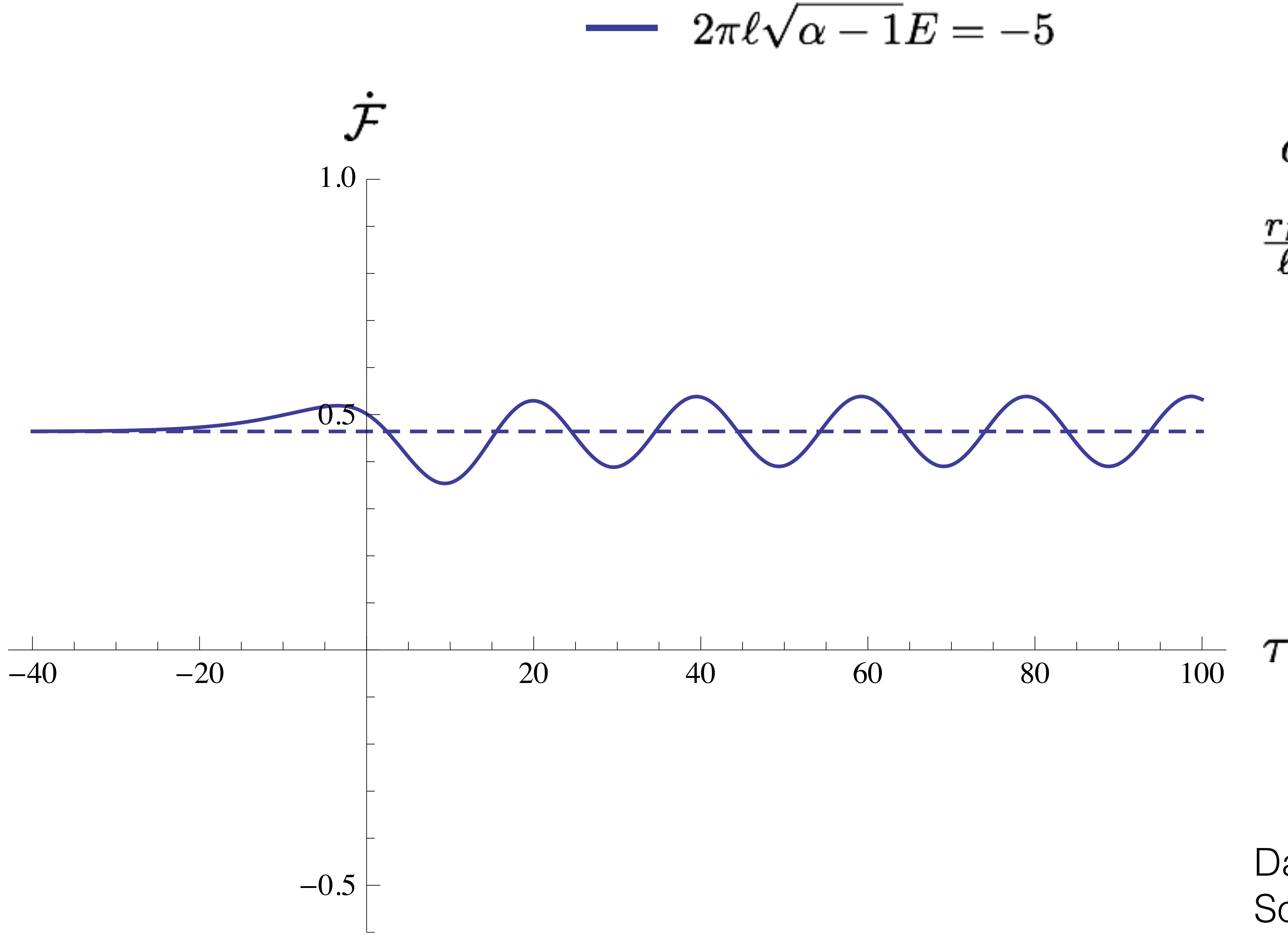


$\alpha = \frac{r^2}{r_h^2} = 26$

$\frac{r_h}{\ell} = 0.5$

Dashed = BTZ
Solid = geon

The transition rate as a function of proper time

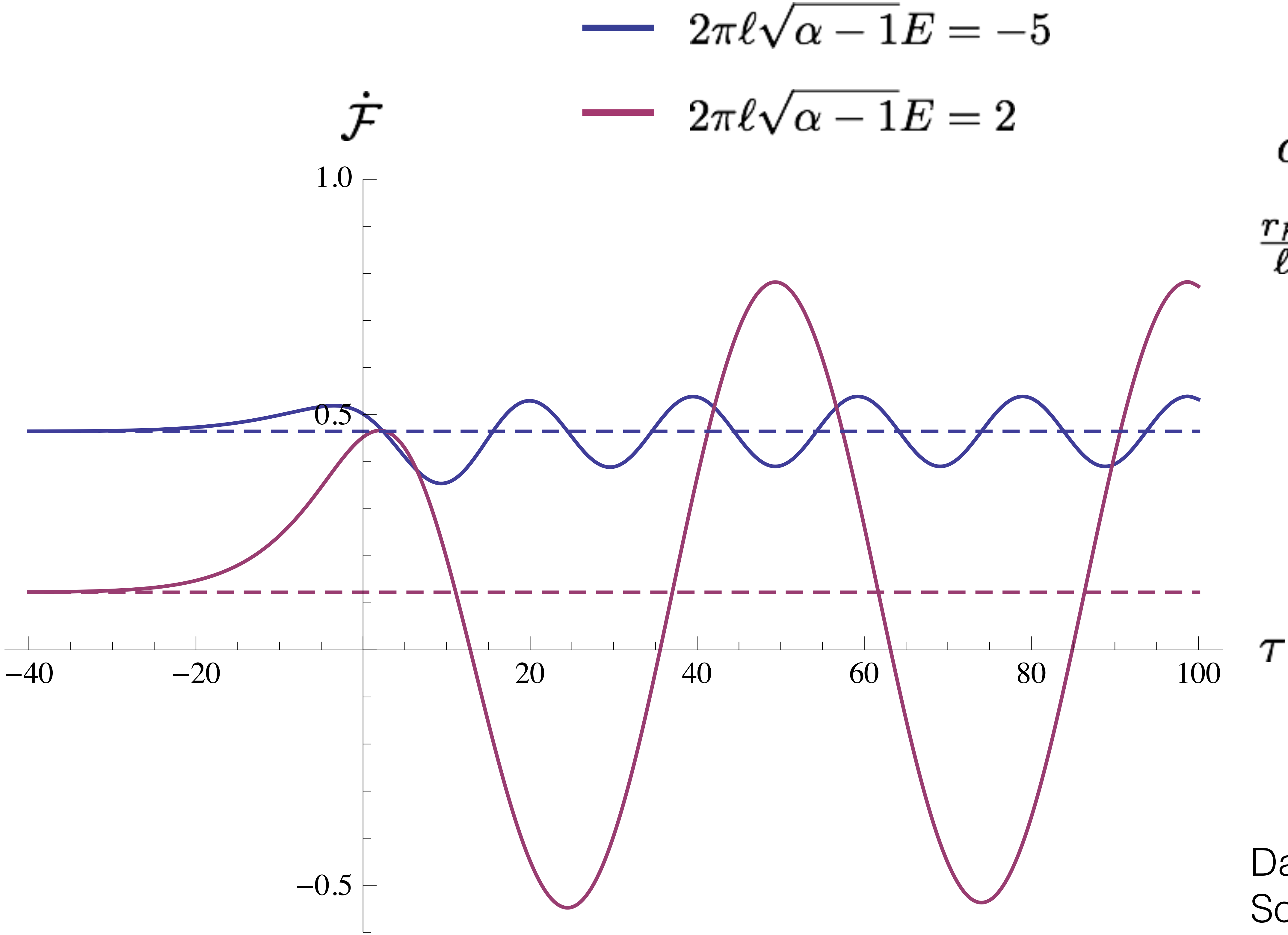


$$\alpha = \frac{r^2}{r_h^2} = 26$$

$$\frac{r_h}{\ell} = 0.5$$

Dashed = BTZ
Solid = geon

The transition rate as a function of proper time



$\alpha = \frac{r^2}{r_h^2} = 26$

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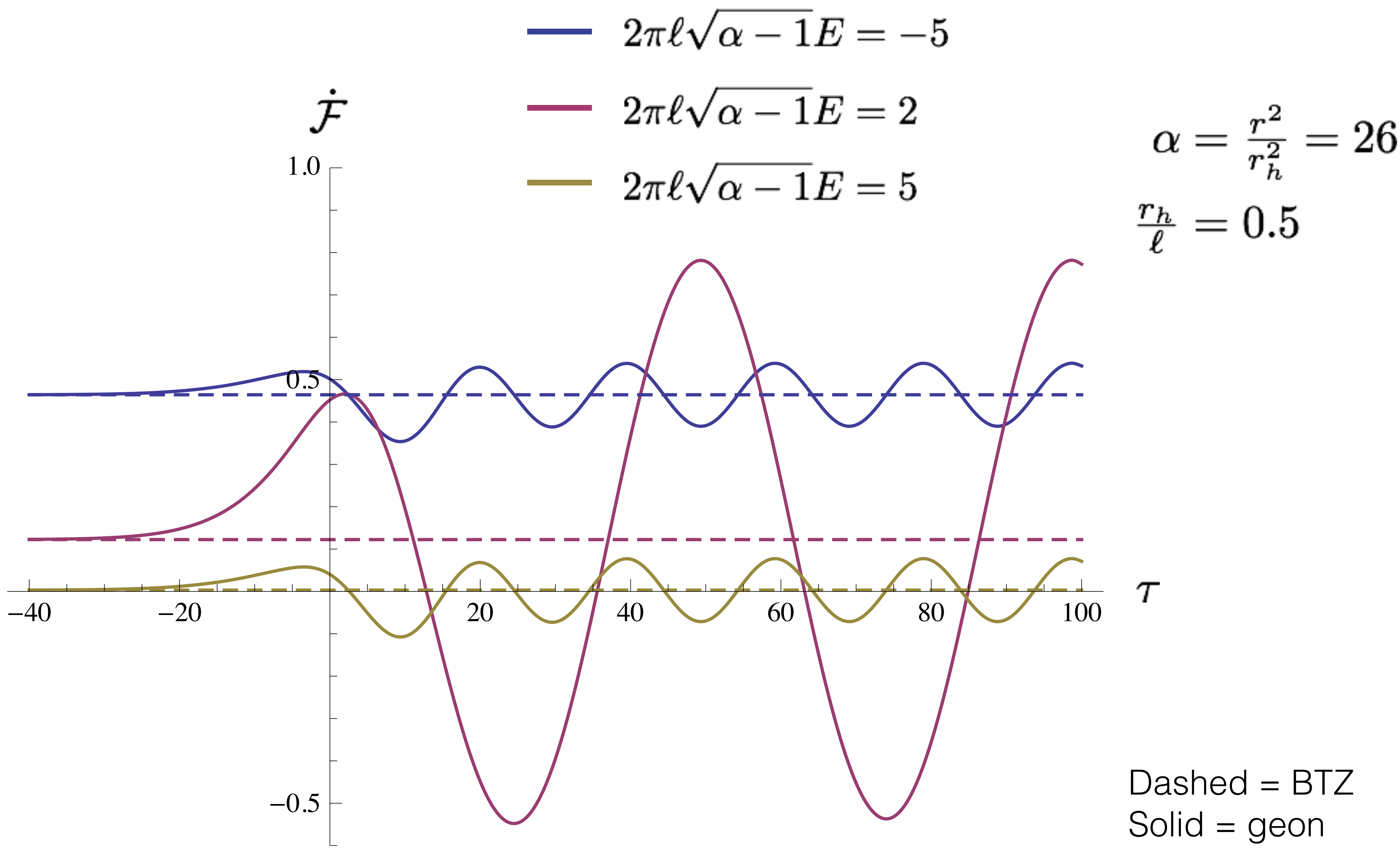
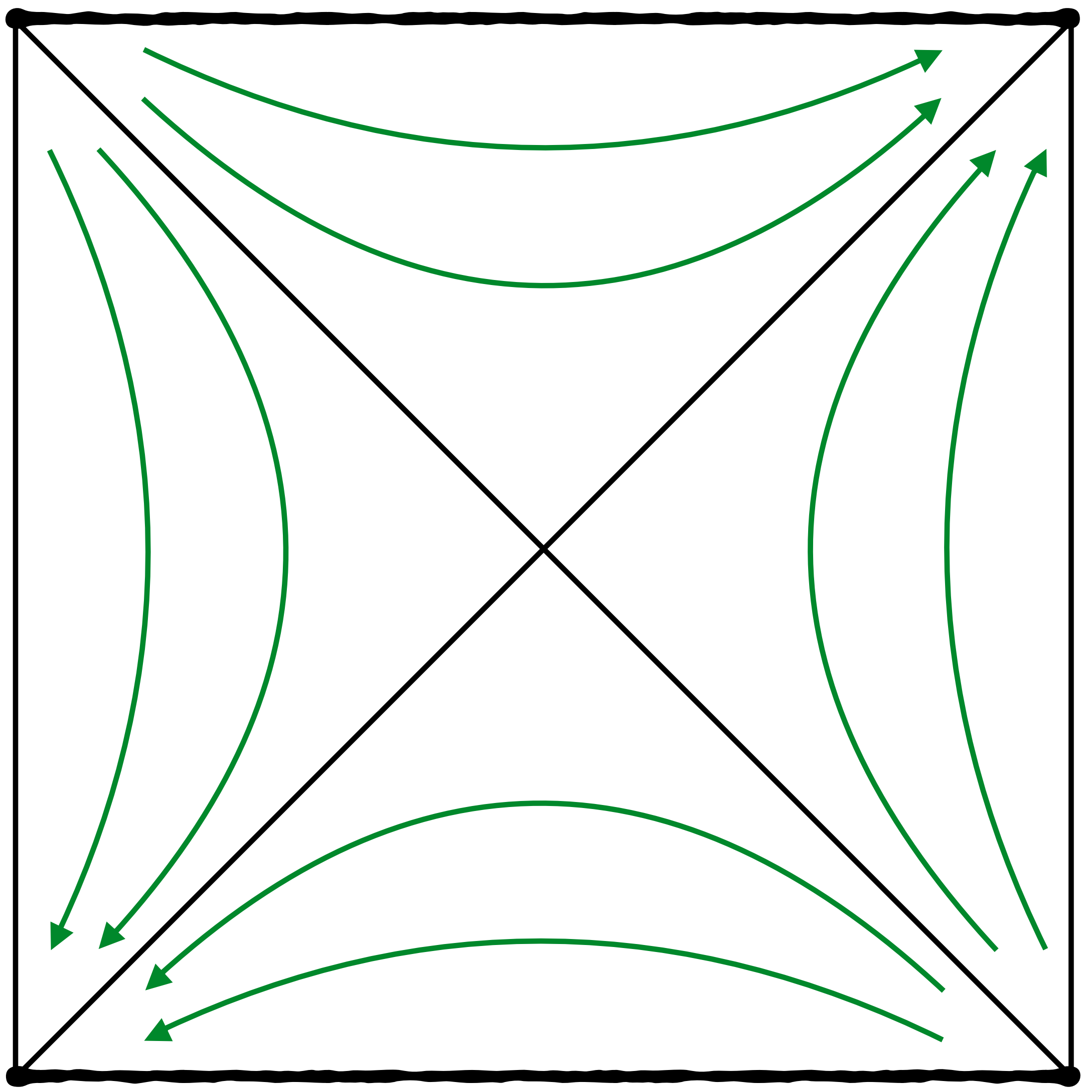
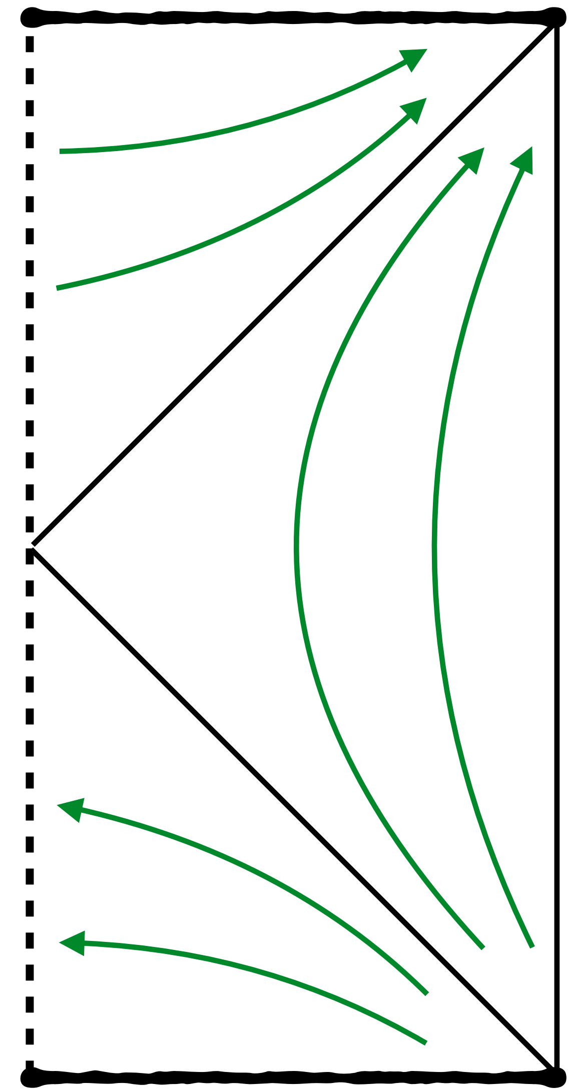
The transition rate as a function of proper time





The BTZ black hole



The $\mathbb{R}\mathbf{P}^2$ geon

Summary

- The response of a detector is different in the BTZ black hole and its associated geon
- Non-stationary effects are observed in the geon spacetime

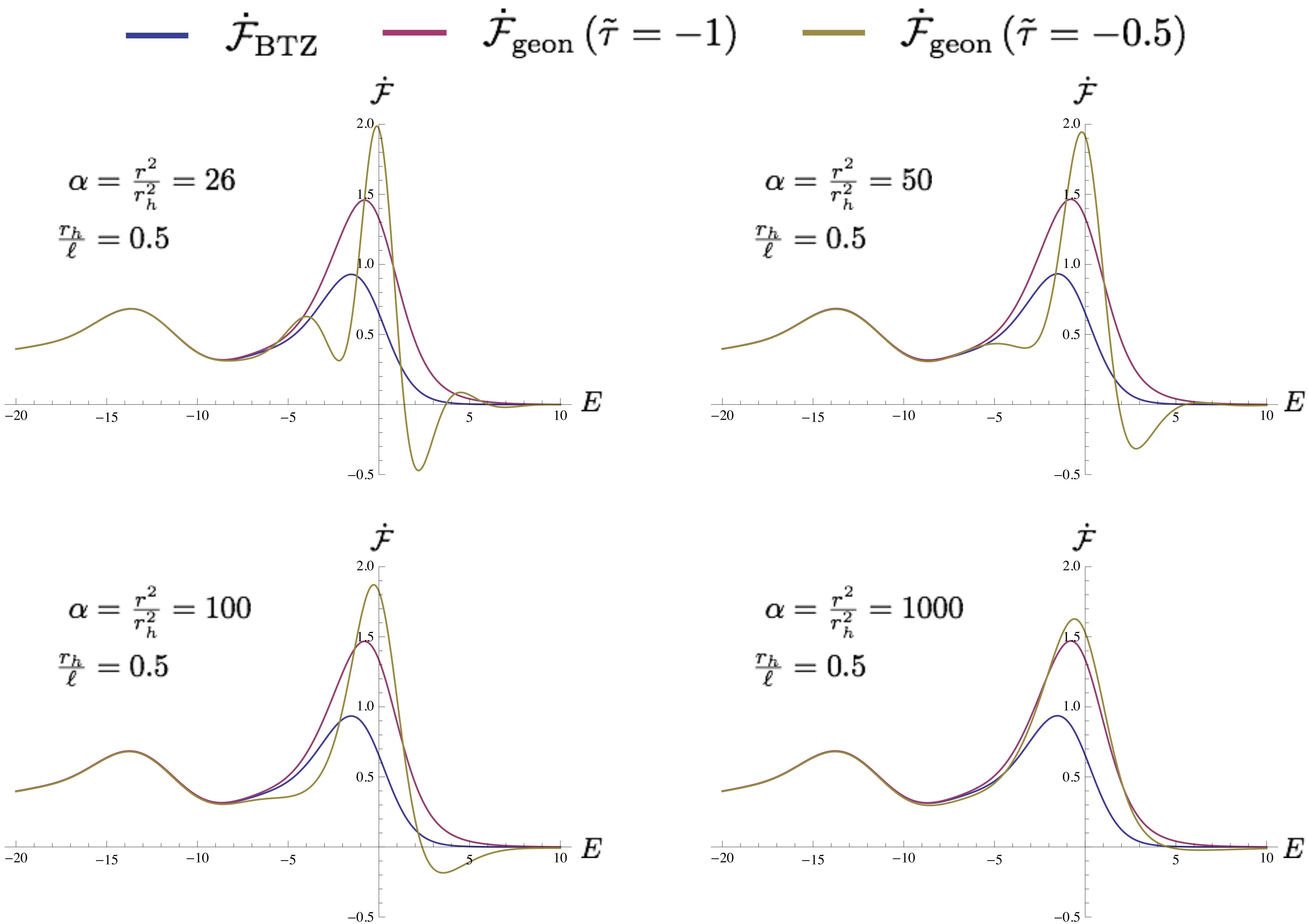
Future work

- Finite detector times
- Examine transition rates along different trajectories (radial infalling and circular)
- Impact for the topological censorship theorem
- More realistic spacetimes

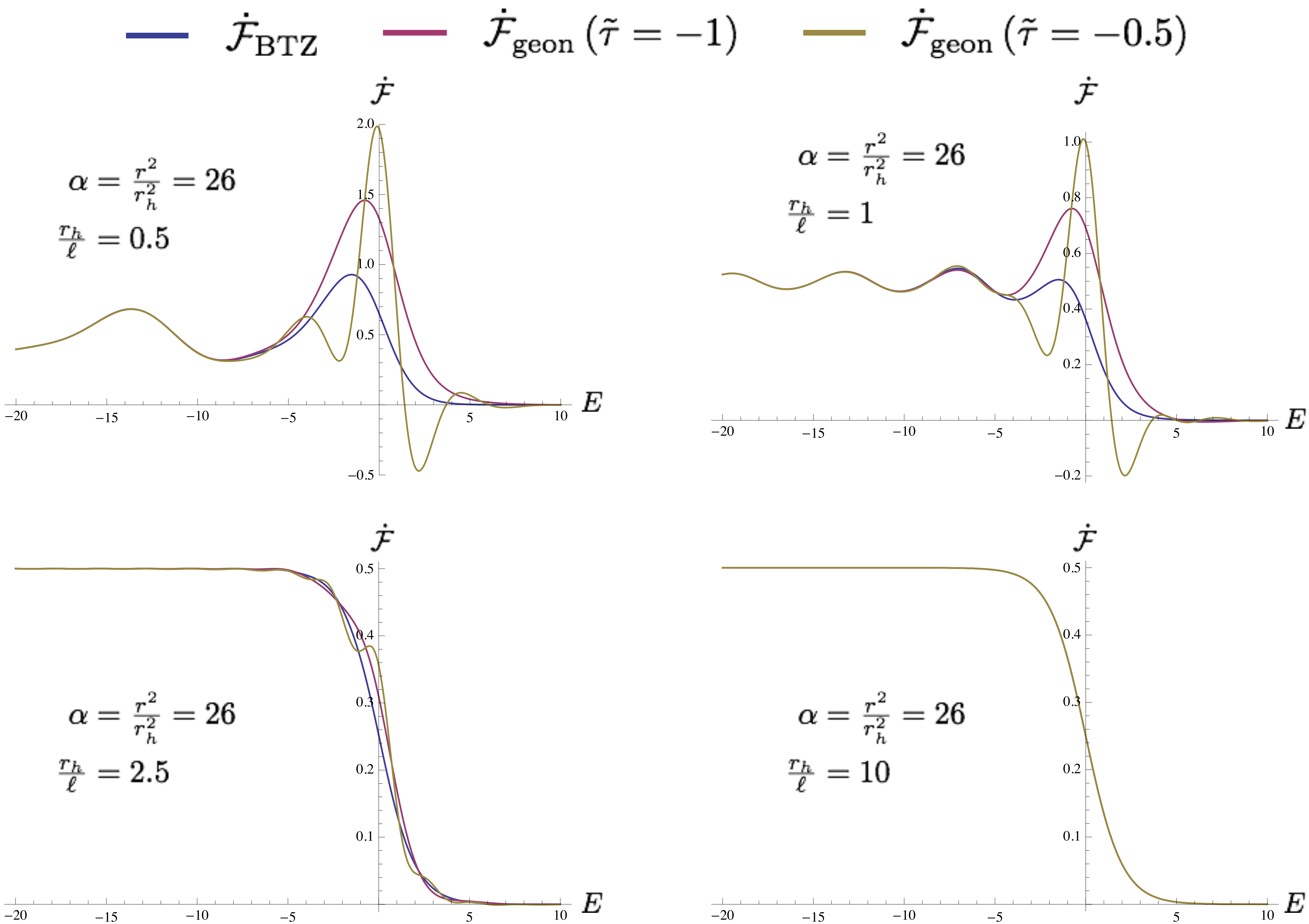
Thank you!

EXTRA SLIDES

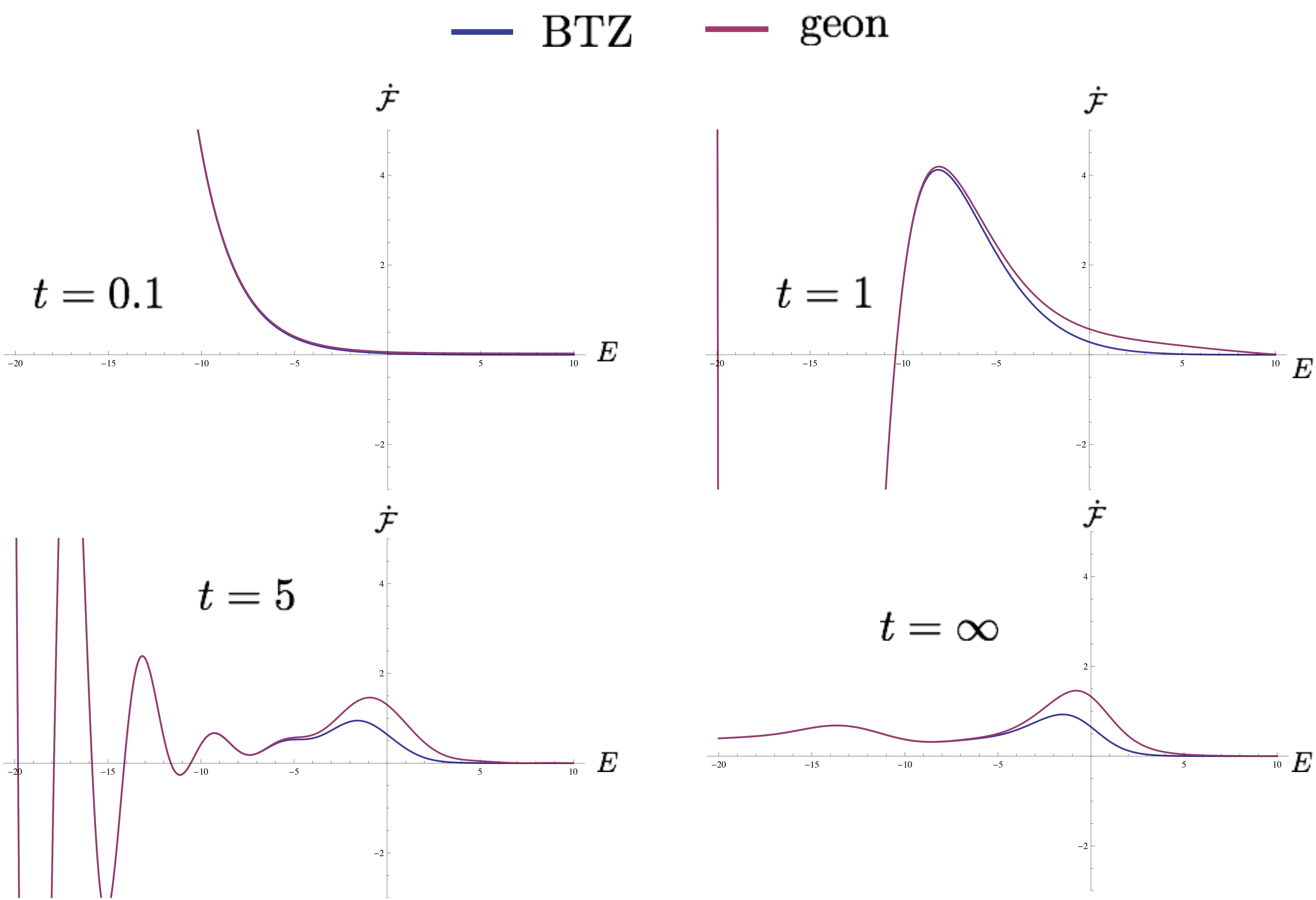
Moving the detector further away



Changing the size of the black hole



Changing the length of the interaction



Quantum field theory on the $\mathbb{RP}^2$ geon

$$G^+_{\mathrm{geon}}(x,\ x') = G^+_{\mathrm{BTZ}}(x, x') + \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\sqrt{\Delta X_n^2(x, J(x'))}}$$

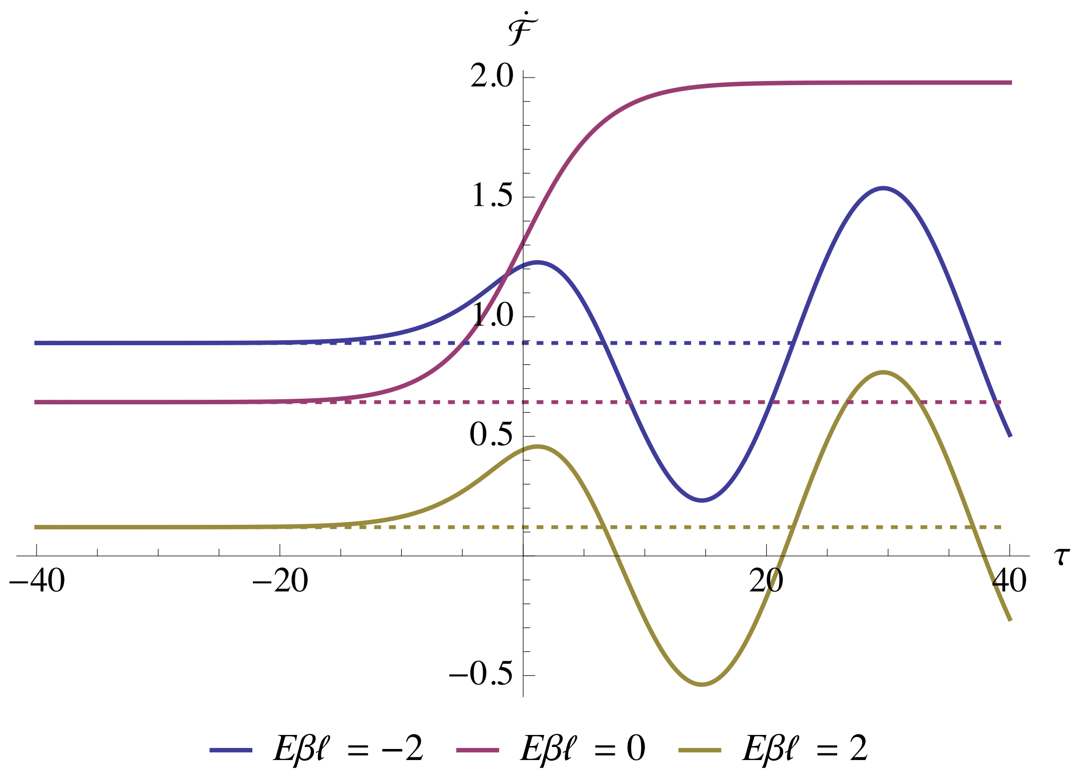
geon

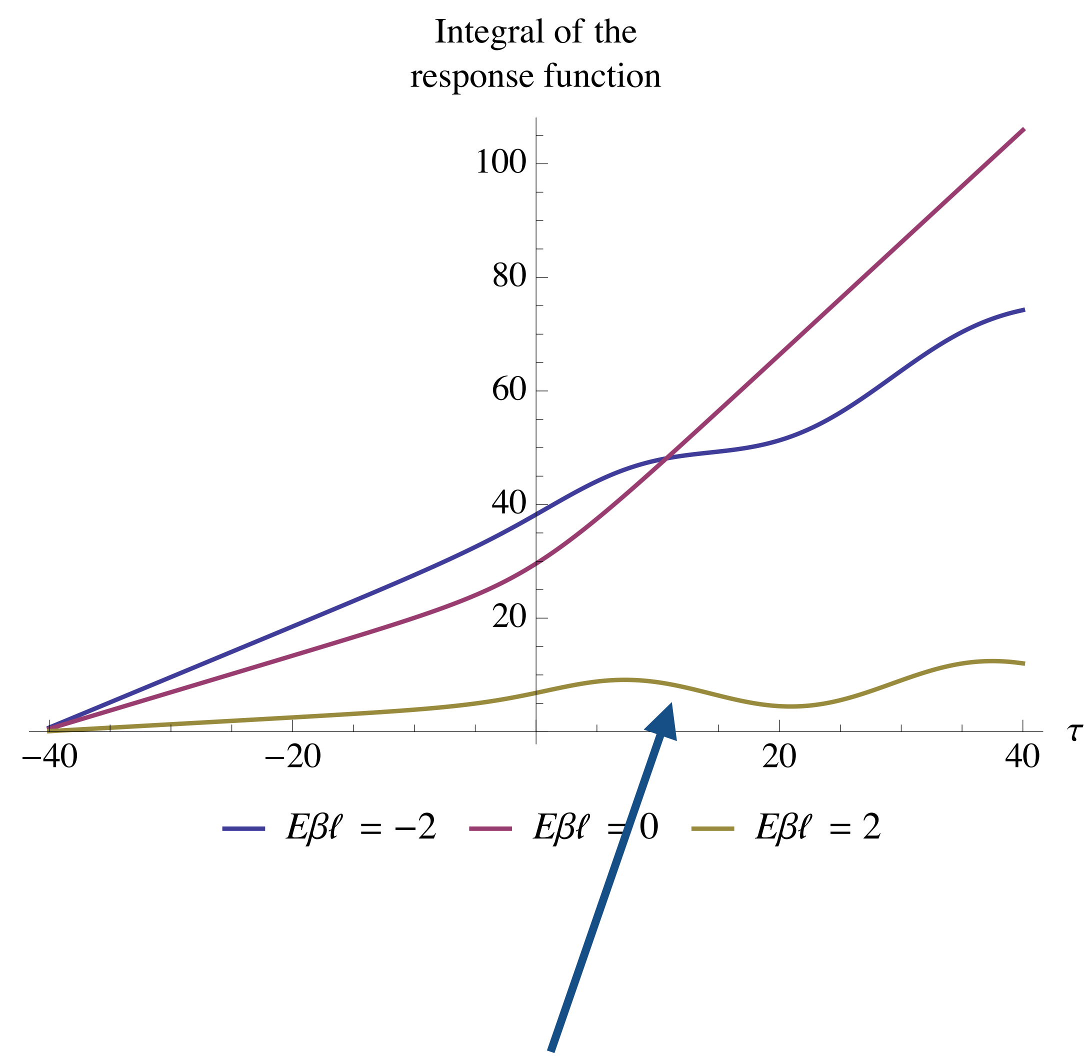
$$\Delta X_n^2(x, J(x')) = -1 + \sqrt{\alpha(r)\alpha(r')}\cosh\left[\frac{r_h}{\ell}\left(\phi - \phi' - \pi - 2\pi n\right)\right] - \sqrt{(\alpha(r)-1)\left(\alpha(r')-1\right)}\cosh\left[\frac{r_h}{\ell^2}\left(t + t'\right)\right]$$

geon

$$\Delta X_n^2(x, x') = -1 + \sqrt{\alpha(r)\alpha(r')}\cosh\left[\frac{r_h}{\ell}\left(\phi - \phi' - 2\pi n\right)\right] - \sqrt{(\alpha(r)-1)\left(\alpha(r')-1\right)}\cosh\left[\frac{r_h}{\ell^2}\left(t - t'\right)\right]$$

The integral of the response function





Stays positive!

$$\dot{\mathcal{F}}_{\tau_0,\tau}(E) = \lim_{\delta\tau \to 0} \frac{\mathcal{F}_{\tau+\delta\tau,\tau_0} - \mathcal{F}_{\tau,\tau_0}}{\delta\tau}$$

Quantum field theory on the BTZ spacetime

Carlip S. (1995) arXiv:gr-qc/9506079

AdS$_3$ can be defined as the submanifold

$$X_1^2 - T_1^2 + X_2^2 - T_2^2 = \ell^2$$

in $\mathbb{R}^4$ with the metric

$$ds^2 = -dT_1^2 - dT_2^2 + dX_1^2 + dX_2^2$$

Introduce coordinates

$$T_1 = \ell\sqrt{\alpha(r)}\cosh\left(\frac{r_h}{\ell}\phi\right)$$

$$X_1 = \ell\sqrt{\alpha(r)}\sinh\left(\frac{r_h}{\ell}\phi\right)$$

$$T_2 = \ell\sqrt{\alpha(r)-1}\cosh\left(\frac{r_h}{\ell^2}t\right)$$

$$X_2 = \ell\sqrt{\alpha(r)-1}\sinh\left(\frac{r_h}{\ell^2}t\right)$$

$\alpha(r) = r^2/r_h^2$

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\phi^2$$

where $f(r) = -M + \frac{r^2}{\ell^2}$

Quantum field theory on the BTZ spacetime

Lifschytz G. and Ortiz M. (1994) Phys. Rev. D 1929-1943

An AdS_3 Green's function

$$G^{+}_{\mathrm{AdS}_3}(x,\ x') = \frac{1}{4\pi\sqrt{\Delta X^2(x,x')}}$$

with transparent boundary conditions

where

$$\Delta X^2(x,x') := -(T_1 - T_1')^2 - (T_2 - T_2')^2 + (X_1 - X_1')^2 + (X_2 - X_2')^2$$

$$= -1 + \sqrt{\alpha(r)\alpha(r')}\cosh\left[\frac{r_h}{\ell}(\phi - \phi')\right] - \sqrt{(\alpha(r)-1)(\alpha(r')-1)}\cosh\left[\frac{r_h}{\ell^2}(t-t')\right]$$

So the BTZ Green's function is

$$G^{+}_{\mathrm{BTZ}}(x,\ x') = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\sqrt{\Delta X_n^2(x,x')}}$$

Hartle-Hawking Vacuum!!

$$\Delta X_n^2(x,x') = -1 + \sqrt{\alpha(r)\alpha(r')}\cosh\left[\frac{r_h}{\ell}(\phi - \phi' - 2\pi n)\right] - \sqrt{(\alpha(r)-1)(\alpha(r')-1)}\cosh\left[\frac{r_h}{\ell^2}(t-t')\right]$$

Quantum field theory on the $\mathbb{RP}^2$ geon

The action of $J : (U, V, \phi) \to (V, U, \phi + \pi)$

$$T_1 = \ell \left(\frac{1 - UV}{1 + UV} \right) \cosh \sqrt{M} \phi = \ell \sqrt{\alpha(r)} \cosh \left(\frac{r_h}{\ell} \phi \right)$$

$$X_1 = \ell \left(\frac{1 - UV}{1 + UV} \right) \sinh \sqrt{M} \phi = \ell \sqrt{\alpha(r)} \sinh \left(\frac{r_h}{\ell} \phi \right)$$

$$T_2 = \ell \left(\frac{V + U}{1 + UV} \right) = \ell \sqrt{\alpha(r) - 1} \cosh \left(\frac{r_h}{\ell^2} t \right)$$

$$X_2 = \ell \left(\frac{V - U}{1 + UV} \right) = \ell \sqrt{\alpha(r) - 1} \sinh \left(\frac{r_h}{\ell^2} t \right)$$

So

$$X_2 \to -X_2 \quad \Leftrightarrow \quad t \to -t$$

Quantum field theory on the $\mathbb{RP}^2$ geon

$$G^{+}_{\text{geon}}(x,\ x') = G^{+}_{\text{BTZ}}(x, x') + \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\sqrt{\Delta X_n^2(x, J(x'))}}$$

The transition rate in the geon spacetime

$$\dot{\mathcal{F}}_{\text{geon}}(E) = \dot{\mathcal{F}}_{\text{BTZ}}(E) + 2\int_0^{\infty} ds\ \text{Re}\ \left[e^{-iEs} G^{+}_{\text{BTZ}}(\tau, J(\tau - s))\right]$$

$$= \dot{\mathcal{F}}_{\text{BTZ}}(E) + \frac{1}{2\pi}\int_0^{\infty} ds\ \text{Re}\left[e^{-iEs}\sum_{n=-\infty}^{\infty} \frac{1}{4\pi\sqrt{\Delta X_n^2(\tau, J(\tau - s))}}\right]$$

with

$$\Delta X_n^2(\tau, J(\tau - s)) = -1 + \alpha(r)\cosh\left[\frac{r_h}{\ell}2\pi\left(n + \frac{1}{2}\right)\right] - (\alpha(r) - 1)\cosh\left[\frac{r_h}{\ell^2}\boxed{(2\tau}- s)\right]$$