$B \to \rho$ transition form factors in AdS/QCD model.

Mohammad Ahmady

Mount Allison University

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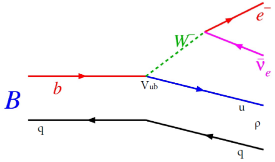
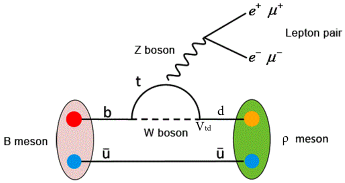
Motivation

Semileptonic $B \to \rho \ell \nu \Rightarrow V_{ub}$



Rare dileptonic $B \to \rho \ell \bar{\ell}$, $B \to \rho \gamma \Rightarrow V_{td}$ and possible new physics



Form factors

$B \to \rho$ transition form factors are defined as:

$$\begin{aligned}
\langle \rho(k,\varepsilon)|\bar{q}\gamma^\mu(1-\gamma^5)b|B(p)\rangle &= \frac{2iV(q^2)}{m_B+m_{K^*}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma - 2m_{K^*}A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu \\
&- (m_B+m_{K^*})A_1(q^2)\left(\varepsilon^{\mu *} - \frac{\varepsilon^*\cdot q q^\mu}{q^2}\right) \\
&+ A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_{K^*}}\left[(p+k)^\mu - \frac{m_B^2-m_{K^*}^2}{q^2}q^\mu\right]
\end{aligned}$$

$$\begin{aligned}
q_\nu\langle \rho(k,\varepsilon)|\bar{d}\sigma^{\mu\nu}(1-\gamma^5)b|B(p)\rangle &= 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho k_\sigma \\
&- iT_2(q^2)[(\varepsilon^*\cdot q)(p+k)_\mu - \varepsilon^*_\mu(m_B^2-m_{K^*}^2)] \\
&- iT_3(q^2)(\varepsilon^*\cdot q)\left[\frac{q^2}{m_B^2-m_{K^*}^2}(p+k)_\mu - q_\mu\right]
\end{aligned}$$

Semileptonic $B \to \rho\ell\nu$ decay

The transverse and longitudinal helicity amplitudes: for the decay $B \to \rho l \nu$

$$H_\pm(q^2) = (m_B + m_\rho) A_1(q^2) \mp \frac{\sqrt{\lambda(q^2)}}{m_B + m_\rho} V(q^2)$$

$$H_0(q^2) = \frac{1}{2m_\rho \sqrt{q^2}} \left\{ (m_B^2 - m_\rho^2 - t)(m_B + m_\rho) A_1(q^2) - \frac{\lambda(q^2)}{m_B + m_\rho} A_2(q^2) \right\}$$

$$\lambda(q^2) = (m_B^2 + m_\rho^2 - q^2)^2 - 4 m_B^2 m_\rho^2$$

Differential decay width:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \sqrt{\lambda(q^2)}\, q^2 \left(H_0^2(q^2) + H_+^2(q^2) + H_-^2(q^2) \right)$$

Rare dileptonic $B \to \rho\gamma$ decay

$B_{u,d} \to \rho + \gamma$ and $B_{u,d} \to K^* + \gamma$:

$$\frac{\Gamma(B_{u,d} \to \rho + \gamma)}{\Gamma(B_{u,d} \to K^* + \gamma)} = \frac{|V_{td}|^2}{|V_{ts}|^2} \frac{|T_1^{B\to\rho}(0)|^2}{|T_1^{B\to K^*}(0)|^2} \Phi_{u,d} \,,$$

$\Phi_{u,d}$ is a phase-space factor:

$$\Phi_{u,d} = \frac{(m_b^2 + m_d^2)}{(m_b^2 + m_s^2)} \frac{(m_{B_{u,d}}^2 - m_\rho^2)^3}{(m_{B_{u,d}}^2 - m_{K^*}^2)^3}$$

R. Sandapen, MA, PRD87,054013(2013)

Rare dileptonic $B \to \rho\mu^+\mu^-$ decay

$$\frac{d\mathcal{B}}{dq^2} = \tau_B \frac{G_F^2\alpha^2}{2^{11}\pi^5}\frac{|V_{tb}V_{td}^*|^2\sqrt{\lambda}v}{3m_B}((2m_\mu^2 + m_B^2 s)[16(|A|^2 + |C|^2)m_B^4\lambda + 2(|B_1|^2 + |D_1|^2)$$
$$\times\frac{\lambda + 12rs}{rs} + 2(|B_2|^2 + |D_2|^2)\frac{m_B^4\lambda^2}{rs} - 4[\Re e(B_1 B_2^*) + \Re e(D_1 D_2^*)]\frac{m_B^2\lambda}{rs}(1 - r - s)]$$
$$+6m_\mu^2[-16|C|^2 m_B^4\lambda + 4\Re e(D_1 D_3^*)\frac{m_B^2\lambda}{r} - 4\Re e(D_2 D_3^*)\frac{m_B^4(1-r)\lambda}{r} + 2|D_3|^2\frac{m_B^4 s\lambda}{r}$$
$$-4\Re e(D_1 D_2^*)\frac{m_B^2\lambda}{r} - 24|D_1|^2 + 2|D_2|^2\frac{m_B^4\lambda}{r}(2 + 2r - s)])$$

$\lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs$
$r = m_K^2/m_B^2$
$s = q^2/m_B^2$
$v = \sqrt{1 - 4m_\mu^2/q^2} \to$ muon velocity

Rare dileptonic $B^- \to \rho^- \mu^+ \mu^-$ decay (continue)

$$A = C_9^{eff} \left(\frac{V}{m_B + m_\rho} \right) + 4 C_7 \frac{m_b}{q^2} T_1$$

$$B_1 = C_9^{eff} (m_B + m_\rho) A_1 + 4 C_7 \frac{m_b}{q^2} (m_B^2 - m_\rho^2) T_2$$

$$B_2 = C_9^{eff} \left(\frac{A_2}{m_B + m_\rho} \right) + 4 C_7 \frac{m_b}{q^2} \left(T_2 + \frac{q^2}{m_B^2 - m_\rho^2} T_3 \right)$$

$$C = C_{10} \left(\frac{V}{m_B + m_\rho} \right)$$

$$D_1 = C_{10} (m_B + m_\rho) A_1$$

$$D_2 = C_{10} \left(\frac{A_2}{m_B + m_\rho} \right)$$

$$D_3 = -C_{10} \frac{2 m_\rho}{q^2} \left(\left(\frac{m_B + m_\rho}{2 m_\rho} \right) A_1 - \left(\frac{m_B - m_\rho}{2 m_\rho} \right) A_2 - A_0 \right)$$

Light cone sum rules

Z. Phys. C 63, 437–454 (1994)

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Exclusive radiative *B*-decays in the light-cone QCD sum rule approach

A. Ali[1,★], V.M. Braun [2,★★], H. Simma[3]

[1]Theory Division, CERN, CH-1211 Geneva 23, Switzerland
[2]Max-Planck-Institut für Physik-Werner-Heisenberg-Institut – D-80805, Munich, Germany
[3]Deutsches Elektronen Synchrotron DESY, D-22603 Hamburg, Germany

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Light cone sum rules: form factors obtained from distribution amplitudes

Consider the correlation function

$$i \int dx\, e^{iqx} \langle \rho(p,\lambda) | T\{\bar{\psi}(x)\sigma_{\mu\nu} q^{\nu} b(x) \bar{b}(0) i\gamma_5 \psi(0)\} | 0 \rangle$$

$$= i\epsilon_{\mu\nu\rho\sigma} e^{*(\lambda)\nu} q^{\rho} p^{\sigma} T((p+q)^2)$$

at $q^2 = 0,\ p^2 = m_\rho^2$

- $m_b^2 - (p+q)^2$ Euclidean of order several GeV^2
- ψ a generic notation for the field of the light quark

One can separate the contribution of the B-meson as the pole contribution to the invariant function $T((p+q)^2)$:

$$T((p+q)^2) = \frac{f_B m_B^2}{m_b + m_q} \frac{2T_1(0)}{m_B^2 - (p+q)^2} + \dots ,$$

Light cone sum rules: form factors obtained from distribution amplitudes (Continue)

b-quark has large virtuality $\sim m_b^2 - (p+q)^2 \Rightarrow$ one can use the perturbative expansion of the b-quark propagator.
The leading contribution is:

$$\int dx\, e^{iqx} \int \frac{dk}{(2\pi)^4}\, e^{-ikx} \frac{q_\nu}{m_b^2 - k^2} \langle \rho(p,\lambda) | T\{\bar{\psi}(x)\sigma_{\mu\nu}(m_b + \not{k}) i\gamma_5 \psi(0)\} |0\rangle .$$

Difference with traditional sum rules: write the non-local matrix element in the above equation in terms of the ρ meson's light cone distribution amplitudes.

Light cone distribution amplitudes

Light cone coordinates: $x^\mu = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp$ any combinations of x_1 and x_2.
At equal light-front time $x^+ = 0$ and in the light-front gauge $A^+ = 0$,

$$\langle 0|\bar{q}(0)\gamma^\mu q(x^-)|\rho(P,\lambda)\rangle = f_\rho M_\rho \frac{e_\lambda \cdot x}{P^+ x^-} P^\mu \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} \phi_\rho^{\parallel}(u,\mu)$$
$$+ f_\rho M_\rho \left(e_\lambda^\mu - P^\mu \frac{e_\lambda \cdot x}{P^+ x^-}\right) \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} g_\rho^{\perp(v)}(u,\mu)$$

$$\langle 0|\bar{q}(0)[\gamma^\mu, \gamma^\nu] q(x^-)|\rho(P,\lambda)\rangle = 2 f_\rho^\perp (e_\lambda^\mu P^\nu - e_\lambda^\nu P^\mu) \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} \phi_\rho^\perp(u,\mu)$$

$$\langle 0|\bar{q}(0)\gamma^\mu \gamma^5 q(x^-)|\rho(P,\lambda)\rangle = -\frac{1}{4}\epsilon^\mu_{\nu\rho\sigma} e_\lambda^\nu P^\rho x^\sigma f_\rho M_\rho \int_0^1 \mathrm{d}u\, e^{-iuP^+x^-} g_\rho^{\perp(a)}(u,\mu)$$

Vector meson's polarization vectors e_λ are chosen as

$$e_L = \left(\frac{P^+}{M_\rho}, -\frac{M_\rho}{P^+}, 0_\perp\right) \qquad \text{and} \qquad e_{T(\pm)} = \frac{1}{\sqrt{2}}(0,0,1,\pm i)$$

Light cone distribution amplitudes

$x^- \to 0$ limit: usual definitions of f_ρ and $f_\rho^\perp$ are recovered:

$$\langle 0|\bar{q}(0)\gamma^\mu q(0)|\rho(P,\lambda)\rangle = f_\rho M_\rho e_\lambda^\mu$$

$$\langle 0|\bar{q}(0)[\gamma^\mu,\gamma^\nu]q(0)|\rho(P,\lambda)\rangle = 2f_\rho^\perp(e_\lambda^\mu P^\nu - e_\lambda^\nu P^\mu)$$

Light cone sum rules: Form factors obtained from distribution amplitudes

$$T((p+q)^2) = \int_0^1 du \frac{1}{m_b^2 + \bar{u} u m_\rho^2 - u(p+q)^2} \left[m_b f_\rho^\perp \phi_\perp(u) + u m_\rho f_\rho g_\perp^{(v)}(u) + \frac{1}{4} m_\rho f_\rho g_\perp^{(a)} \right] + \frac{1}{4} \int_0^1 du \frac{m_b^2 - u^2 m_\rho^2}{(m_b^2 + \bar{u} u m_\rho^2 - u(p+q)^2)^2} m_\rho f_\rho g_\perp^{(a)}(u)$$

where $\bar{u} = 1 - u$. Introducing $s = m_b^2/u + \bar{u} m_\rho^2$, the above is similar to a dispersion integral $\sim 1/(s-(p+q)^2) \to \exp(-s/t) \Leftarrow$Borel Transform

Sum rules assumption: The contribution of the B-meson corresponds to intermediate states with masses smaller than a certain threshold $s < s_0 \Leftarrow$ Continuum threshold

$$\frac{f_B m_B^2}{m_b + m_q} 2 T_1(0) e^{-(m_B^2 - m_b^2)/t} = \\ = \int_0^1 du \frac{1}{u} \exp\left[-\frac{\bar{u}}{t} \left(\frac{m_b^2}{u} + m_\rho^2 \right) \right] \theta \left[s_0 - \frac{m_b^2}{u} - \bar{u} m_\rho^2 \right] \left\{ m_b f_\rho^\perp \phi_\perp(u) + u m_\rho f_\rho g_\perp^{(v)}(u) + \frac{m_b^2 - u^2 m_\rho^2 + ut}{4ut} m_\rho f_\rho g_\perp^{(a)}(u) \right\}$$

Light cone sum rules (continue)

For $q^2 \neq 0$

$$\frac{f_B m_B^2}{m_b + m_q} 2T_1(q^2) e^{-(m_B^2 - m_b^2)/t} =$$

$$= \int_0^1 du \frac{1}{u} \exp\left[-\frac{\bar{u}}{t}\left(\frac{m_b^2 - q^2}{u} + m_\rho^2\right)\right] \theta\left[s_0 - \frac{m_b^2 - \bar{u}q^2}{u} - \bar{u}m_\rho^2\right] \left\{ m_b f_\rho^\perp \phi_\perp(u) + u m_\rho f_\rho g_\perp^{(v)}(u) + \frac{m_b^2 + q^2 - u^2 m_\rho^2 + ut}{4ut} m_\rho f_\rho g_\perp^{(a)}(u) \right\}$$

- $t \sim 6 - 9\mathrm{GeV}^2$
- $s_0 \sim 33 - 35\mathrm{GeV}^2$

AdS/QCD holographic light front wavefunctions

G. F. de Teramond and S. J. Brodsky, PRL 102, 081601(2009).

The correspondence between string theory in five-dimensional antide Sitter (AdS)space and four-dimensional quantum chromodynamics (QCD) has enjoyed a number of successes.

The meson wavefunction in this model can be written as:

$$\phi(x,\zeta) = \mathcal{N}\frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\exp\left(-\frac{\kappa^2\zeta^2}{2}\right)\exp\left(-\frac{m_f^2}{2\kappa^2 x(1-x)}\right) \quad ,$$

$\mathcal{N}$ is fixed by normalization condition once spin wavefunction is included.

$\zeta = \sqrt{x(1-x)}r$

$x \Rightarrow$ the light-front longitudinal momentum fraction of the quark

$r \Rightarrow$ the quark-antiquark transverse separation

Light cone DAs in terms of LFWF

$$\phi_\rho^\parallel(z,\mu) = \frac{N_c}{\pi f_\rho M_\rho} \int \mathrm{d}r \; \mu J_1(\mu r)[M_\rho^2 z(1-z) + m_f^2 - \nabla_r^2] \frac{\phi_L(r,z)}{z(1-z)} \; ,$$

$$\phi_\rho^\perp(z,\mu) = \frac{N_c m_f}{\pi f_\rho^\perp} \int \mathrm{d}r \; \mu J_1(\mu r) \frac{\phi_T(r,z)}{z(1-z)} \; ,$$

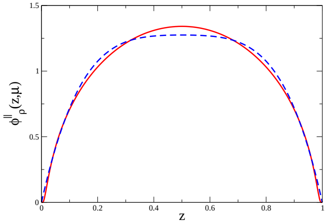
$$g_\rho^{\perp(v)}(z,\mu) = \frac{N_c}{2\pi f_\rho M_\rho} \int \mathrm{d}r \; \mu J_1(\mu r) \left[m_f^2 - (z^2 + (1-z)^2)\nabla_r^2\right] \frac{\phi_T(r,z)}{z^2(1-z)^2}$$

$$\frac{\mathrm{d}g_\rho^{\perp(a)}}{\mathrm{d}z}(z,\mu) = \frac{\sqrt{2} N_c}{\pi f_\rho M_\rho} \int \mathrm{d}r \; \mu J_1(\mu r)(1-2z)[m_f^2 - \nabla_r^2] \frac{\phi_T(r,z)}{z^2(1-z)^2} \; .$$

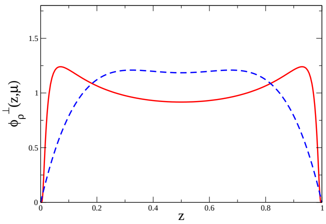
$$f_\rho = \frac{N_c}{m_V \pi} \int_0^1 \mathrm{d}z \; [z(1-z) m_V^2 + m_{\bar{q}} m_q - \nabla_r^2] \frac{\phi_L(r,z)}{z(1-z)} \bigg|_{r=0}$$

$$f_\rho^\perp(\mu) = \frac{N_c}{\pi} \int_0^1 \mathrm{d}z (m_q - z(m_q - m_{\bar{q}})) \int \mu J_1(\mu r) \frac{\phi_T(r,z)}{z(1-z)}$$

AdS/QCD DAs for ρ



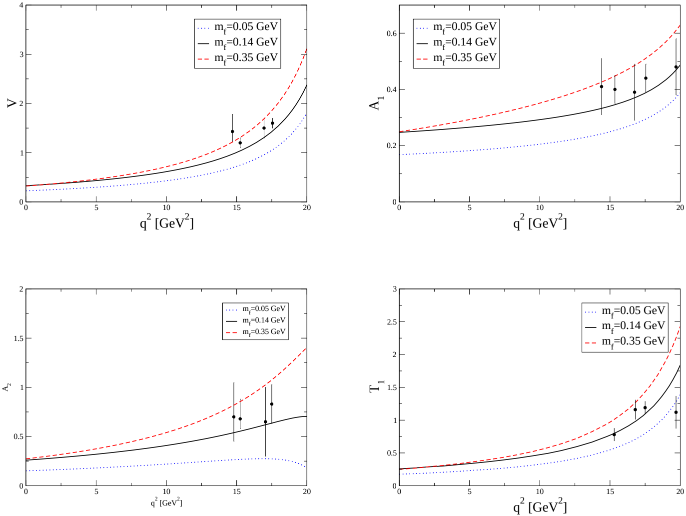
(a) Twist-2 DA for the longitudinally polarized ρ meson

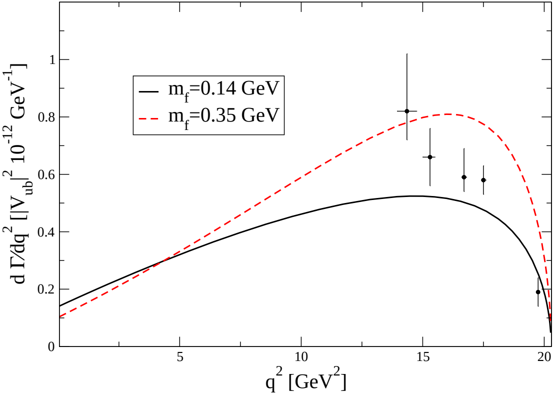


(b) Twist-2 DA for the transversely polarized ρ meson

Figure : Twist-2 DAs for the ρ meson. Solid Red: AdS/QCD DA at $\mu \sim 1$ GeV; Dashed Blue: Sum Rules DA at $\mu = 2$ GeV.

AdS/QCD prediction for $B \to \rho$ transition form factors





(a) Differential decay rate for the semileptonic $B \to \rho \ell \bar{\nu}$ decay.

Numerical predictions

Decay width	AdS/QCD	BB	FGM	ISGW2	Jaus	WSB	Melikhov
$\Gamma/\lvert V_{ub}\rvert^2$	$12.0, 15.9$	13.5 ± 4.0	5.4 ± 1.2	14.2	19.1	26	9.64

Table : Our predictions for the total decay width in units of ps^{-1} computed using quark masses $m_f = 0.14, 0.35$ GeV as compared to sum rules and quark models

Ratio	AdS/QCD	BB	FGM	ISGW2	Jaus	WSB	Melikhov
Γ_L/Γ_T	$0.59, 0.42$	0.52	0.5 ± 0.3	0.3	0.82	1.34	1.13

Table : Our predictions for the ratio of longitudinal to transverse decay widths using quark masses $m_f = 0.14, 0.35$ GeV compared to sum rules and quark models predictions

Numerical predictions

BaBar collaboration has measured partial decay widths in q^2 bins:PRD83, 032007 (2011)

$$\Delta\Gamma_{\text{low}} = \int_0^8 \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\mathrm{d}q^2 = (0.564 \pm 0.166) \times 10^{-4}$$

$$\Delta\Gamma_{\text{mid}} = \int_8^{16} \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\mathrm{d}q^2 = (0.912 \pm 0.147) \times 10^{-4}$$

$$\Delta\Gamma_{\text{high}} = \int_{16}^{20.3} \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}\mathrm{d}q^2 = (0.268 \pm 0.062) \times 10^{-4}$$

$$R_{\text{low}} = \frac{\Gamma_{\text{low}}}{\Gamma_{\text{mid}}} = 0.618 \pm 0.207$$

$$R_{\text{high}} = \frac{\Gamma_{\text{high}}}{\Gamma_{\text{mid}}} = 0.294 \pm 0.083$$

Our predictions for $m_f = 0.14, 0.35$:

$$R_{\text{low}} = 0.580, 0.424$$
$$R_{\text{high}} = 0.427, 0.503$$