Predicting the rare decay $B \to K^* \mu^+ \mu^-$ in light-front QCD

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$$B \to K^{*}\mu^{+}\mu^{-}$$

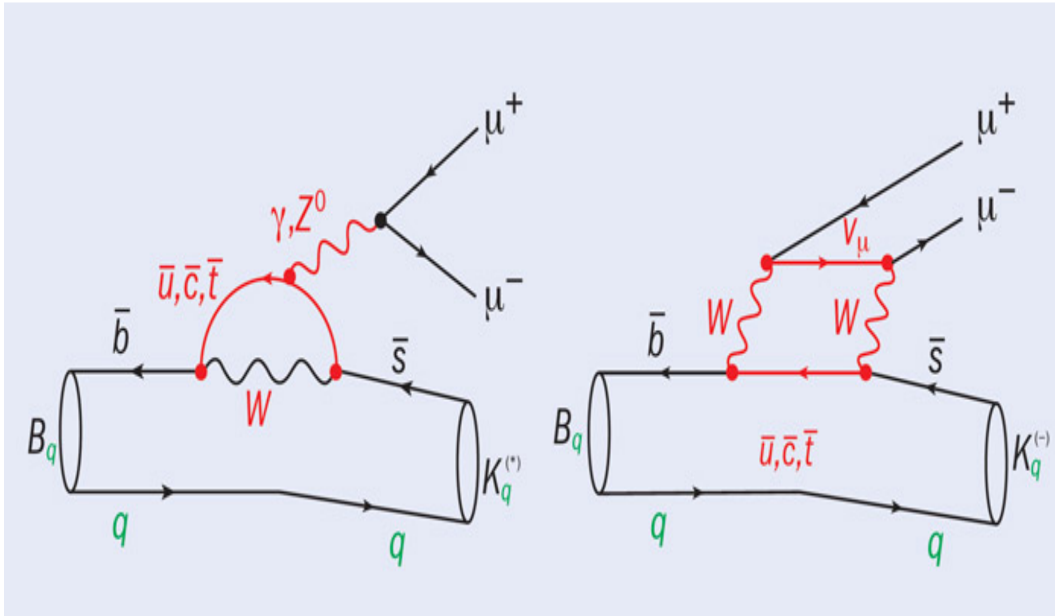


Figure from LHCb website

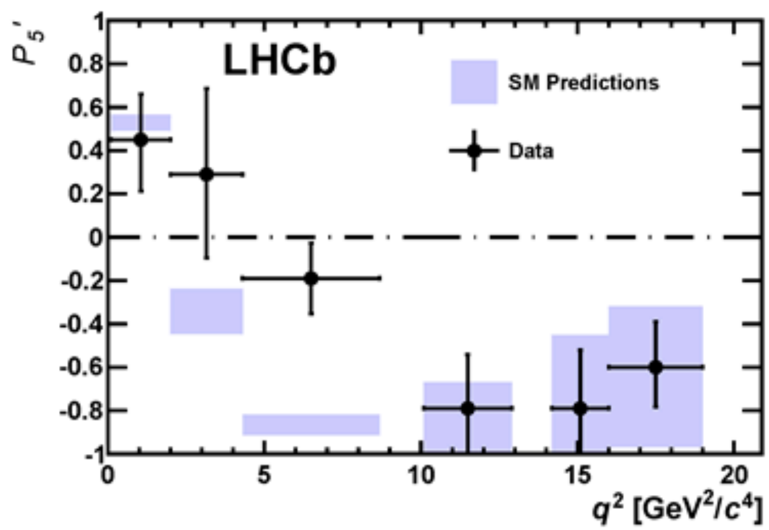
- Electroweak penguin and box diagram
- Loop suppressed in Standard Model
- Sensitive to New Physics

$$B \to K^* \mu^+ \mu^-$$

From LHCb website

9 August 2013: LHCb results hint at new physics?

The LHCb Collaboration has just published the results of a new analysis of the $B^0 \to K^{*0}\mu^+\mu^-$ decay, with $K^{*0} \to K^+\pi^-$. These results were presented three weeks ago at the European Physical Society Conference on High Energy Physics, EPSHEP, Stockholm, Sweden, and triggered very interesting discussions. The analysis of the $B^0 \to K^*\mu\mu$ decay is considered as a very promising channel to search for new physics effects, see the 14 June 2013 news for an introduction. A contribution from new physics particles could modify the angular distributions of the decay products. LHCb physicists have studied different variables related to these angular distributions as functions of the $\mu^+\mu^-$ invariant mass squared. In previously published results, no significant deviation from the Standard Model prediction has been found, see the 13 March 2012 news. In order to increase sensitivity to new physics effects LHCb physicists started to analyse additional observables (the so called P_i' observables) which are considered theoretically clean. This means that they are less sensitive than other observables to some theoretical parameters that are not precisely known (form-factors for experts). Four such observables, labelled P_4', P_5', P_6' and P_8', have been studied.



The image shows the distribution of the P_5' observable as a function of the $\mu^+\mu^-$ invariant mass squared q^2. The black data points are compared with the Standard Model prediction. A 3.7σ deviation of data above the prediction is observed for the third bin corresponding to q^2 between 4.3 and 8.68 GeV^2/c^4. Taking into account that this deviation is observed in one out of 24 bins investigated in this work (the so-called look-elsewhere effect), the significance of the deviation becomes 2.8σ.

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click the image for higher resolution

Observables largely free from form factor uncertainties

The differential branching ratio for $B \to K^* \mu^+ \mu^-$

T. Aliev, A. Ozpineci, M. Savci (1997)

$$\frac{d\mathcal{B}}{dq^2} = \tau_B \frac{G_F^2 \alpha^2}{2^{11}\pi^5} \frac{|V_{tb}V_{ts}^*|^2 \sqrt{\lambda} v}{3m_B} ((2m_\mu^2 + m_B^2 s)[16(|A|^2 + |C|^2) m_B^4 \lambda + 2(|B_1|^2 + |D_1|^2)$$
$$\times \frac{\lambda + 12rs}{rs} + 2(|B_2|^2 + |D_2|^2)\frac{m_B^4 \lambda^2}{rs} - 4[\Re e(B_1 B_2^*) + \Re e(D_1 D_2^*)]\frac{m_B^2 \lambda}{rs}(1 - r - s)]$$
$$+6m_\mu^2[-16|C|^2 m_B^4 \lambda + 4\Re e(D_1 D_3^*)\frac{m_B^2 \lambda}{r} - 4\Re e(D_2 D_3^*)\frac{m_B^4 (1-r)\lambda}{r} + 2|D_3|^2 \frac{m_B^4 s \lambda}{r}$$
$$-4\Re e(D_1 D_2^*)\frac{m_B^2 \lambda}{r} - 24|D_1|^2 + 2|D_2|^2 \frac{m_B^4 \lambda}{r}(2 + 2r - s)])$$

$$B_1 = C_9^{eff}(m_B + m_{K^*})A_1 + 4C_7 \frac{m_b}{q^2}(m_B^2 - m_{K^*}^2)T_2$$

$$C_9^{eff} = C_9 + Y(q^2)$$

Given in terms of form factors and Wilson coefficients

7 form factors

7 form factors required to compute Branching Ratio

$$\langle K^*(k,\varepsilon)|\bar{s}\gamma^\mu(1-\gamma^5)b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_{K^*}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma - 2m_{K^*}A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu$$
$$- (m_B+m_{K^*})A_1(q^2)\left(\varepsilon^{\mu *} - \frac{\varepsilon^*\cdot q q^\mu}{q^2}\right)$$
$$+ A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_{K^*}}\left[(p+k)^\mu - \frac{m_B^2 - m_{K^*}}{q^2}q^\mu\right]$$

and

$$q_\nu\langle K^*(k,\varepsilon)|\bar{s}\sigma^{\mu\nu}(1-\gamma^5)b|B(p)\rangle = 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho k_\sigma$$
$$- iT_2(q^2)[(\varepsilon^*\cdot q)(p+k)_\mu - \varepsilon^*_\mu(m_B^2 - m_{K^*}^2)]$$
$$- iT_3(q^2)(\varepsilon^*\cdot q)\left[\frac{q^2}{m_B^2 - m_{K^*}^2}(p+k)_\mu - q_\mu\right]$$

Light-cone sum rules for form factors

Valid for low to intermediate momentum transfer

T. Aliev, A. Ozpineci, M. Savci (1997)

$$V(q^2) = \left(\frac{m_B + m_{K^*}}{2}\right)\left(\frac{m_b}{f_B m_B^2}\right)\exp\left(\frac{m_B^2}{M^2}\right)\int_\delta^1 du \, \exp\left(-\frac{m_b^2 + p^2 u\bar{u} - q^2\bar{u}}{uM^2}\right) \times \left\{ m_b m_{K^*} \frac{f_{K^*} g_\perp^{(a)}(u)}{2u^2 M^2} + \frac{f_{K^*}^\perp \phi_\perp(u)}{u} \right\},$$

$$T_1(q^2) = \frac{1}{4}\left(\frac{m_b}{f_B m_B^2}\right)\exp\left(\frac{m_B^2}{M^2}\right)\int_\delta^1 \frac{du}{u}\exp\left(-\frac{m_b^2 + p^2 u\bar{u} - q^2\bar{u}}{uM^2}\right)\left\{ m_b f_{K^*}^\perp \phi_\perp(u) + f_{K^*} m_{K^*}\left[\Phi_\parallel(u) + u g_\perp^{(v)}(u) + \frac{g_\perp^{(a)}(u)}{4} + \frac{(m_b^2 + q^2 - p^2u^2) g_\perp^a(u)}{4uM^2}\right]\right\},$$

We compute the form factors using new AdS/QCD Distribution Amplitudes
Traditionally, these DAs are computed using additional Sum Rules

Distribution Amplitudes

Ball and Braun (1997)

DAs parametrize the OPE of vacuum-to-meson transition matrix elements of quark-antiquark non-local gauge invariant operators at light-like separations. At equal light-front time and in the light-front gauge:

$$\langle 0|\bar{q}(0)\gamma^\mu s(x^-)|K^*(P,\lambda)\rangle = f_{K^*}M_{K^*}\frac{e_\lambda\cdot x}{P^+x^-}P^\mu\int_0^1 \mathrm{d}u\, e^{-iuP^+x^-}\phi^{\parallel}_{K^*}(u,\mu) + f_{K^*}M_{K^*}\left(e^\mu_\lambda - P^\mu\frac{e_\lambda\cdot x}{P^+x^-}\right)\int_0^1 \mathrm{d}u\, e^{-iuP^+x^-}g^{\perp(v)}_{K^*}(u,\mu)$$

$$\langle 0|\bar{q}(0)[\gamma^\mu,\gamma^\nu]s(x^-)|K^*(P,\lambda)\rangle = 2f^{\perp}_{K^*}(e^\mu_\lambda P^\nu - e^\nu_\lambda P^\mu)\int_0^1 \mathrm{d}u\, e^{-iuP^+x^-}\phi^{\perp}_{K^*}(u,\mu)$$

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma^5 s(x^-)|K^*(P,\lambda)\rangle = -\frac{1}{4}\epsilon^\mu_{\nu\rho\sigma}e^\nu_\lambda P^\rho x^\sigma \tilde{f}_{K^*}M_{K^*}\int_0^1 \mathrm{d}u\, e^{-iuP^+x^-}g^{\perp(a)}_{K^*}(u,\mu)$$

A useful relation

Light-front wavefunction

$$P^+ \int \mathrm{d}x^- e^{ix^- zP^+} \langle 0|\bar{q}(0)\Gamma s(x^-)|K^*(P,\lambda)\rangle = \frac{N_c}{4\pi} \sum_{h,\bar{h}} \int^{|\mathbf{k}|<\mu} \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} S^{K^*,\lambda}_{h,\bar{h}}(z,\mathbf{k}) \phi^{\lambda}_{K^*}(z,\mathbf{k})$$

$$\times \left\{ \frac{\bar{v}_{\bar{h}}((1-z)P^+,-\mathbf{k})}{\sqrt{(1-z)}} \Gamma \frac{u_h(zP^+,\mathbf{k})}{\sqrt{z}} \right\} \quad ($$

- Light-front spinors
- Renormalization scale as ultraviolet cut-off on transverse momentum

Distribution Amplitudes are related to light-front wavefunctions

We can then deduce that

$$\phi^{\parallel}_{K^*}(z,\mu) = \frac{N_c}{\pi f_{K^*} M_{K^*}} \int \mathrm{d}r \mu J_1(\mu r)[M^2_{K^*} z(1-z) + m_f m_s - \nabla^2_r] \frac{\phi^L_{K^*}(r,z)}{z(1-z)}$$

$$\phi^{\perp}_{K^*}(z,\mu) = \frac{N_c}{\pi f^{\perp}_{K^*}} \int \mathrm{d}r \mu J_1(\mu r)[m_s - z(m_s - m_{\bar{q}})] \frac{\phi^T_{K^*}(r,z)}{z(1-z)}$$

$$g^{\perp(v)}_{K^*}(z,\mu) = \frac{N_c}{2\pi f_{K^*} M_{K^*}} \int \mathrm{d}r \mu J_1(\mu r) \left[(m_s - z(m_s - m_{\bar{q}}))^2 - (z^2 + (1-z)^2)\nabla^2_r\right] \frac{\phi^T_{K^*}(r,z)}{z^2(1-z)^2}$$

$$\frac{\mathrm{d}g^{\perp(a)}_{K^*}}{\mathrm{d}z}(z,\mu) = \frac{\sqrt{2}N_c}{\pi \tilde{f}_{K^*} M_{K^*}} \int \mathrm{d}r \mu J_1(\mu r)[(1-2z)(m_s^2 - \nabla^2_r) + z^2(m_s + m_{\bar{q}})(m_s - m_{\bar{q}})] \frac{\phi^T_{K^*}(r,z)}{z^2(1-z)^2}$$

4 DAs: Twist-2 (long & trans), Twist-3(vector & axial vector)

M. Ahmady & RS
PRD, 2013

Brodsky and de Teramond
PRL, 2009

Light-front Schroedinger equation

$$\phi(z,\zeta,\varphi) = f(z)\frac{\Phi(\zeta)}{\sqrt{2\pi\zeta}}\exp(iL\varphi)$$

$$\zeta = \sqrt{z(1-z)}r$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\Phi(\zeta) = M^2\Phi(\zeta)$$

A holographic light-front Schroedinger equation

Brodsky and de Teramond
PRL, 2009

- A relativistic QM equation for mesons in physical 4D spacetime which maps onto the classical wave equation for strings propagating in a modified 5D anti-de Sitter (AdS) space

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\Phi(\zeta)=M^2\Phi(\zeta)$$

- Transverse distance in 4D maps onto fifth dimension in 5D AdS

$$\zeta \leftrightarrow z_5$$

- Angular momentum in 4D maps onto 5D mass times curvature radius of anti-de Sitter space

$$(m_5 R)^2 \leftrightarrow L^2-(J-2)^2$$

- Interacting potential in 4D driven by the geometry in fifth dimension of AdS

$$U(z_5)=\frac{1}{2}\varphi''(z_5)+\frac{1}{4}\varphi'(z_5)^2+\frac{2J-3}{2z_5}\varphi'(z_5)$$

Unique potential

Phenomenological & theoretical arguments constrain the dilation profile to be quadratic

S. Brodsky, G. de Teramond & H. G. Dosch (2013)

$$\varphi \propto z_5^p$$

$$\Longrightarrow p = 2$$

$$\varphi = \kappa^4 z_5^2$$

- Only possibility consistent with chiral symmetry in the massless quark limit.
- Linear Regge trajectories

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)$$

Harmonic oscillator potential in QCD

Final form of the holographic AdS/QCD wavefunction

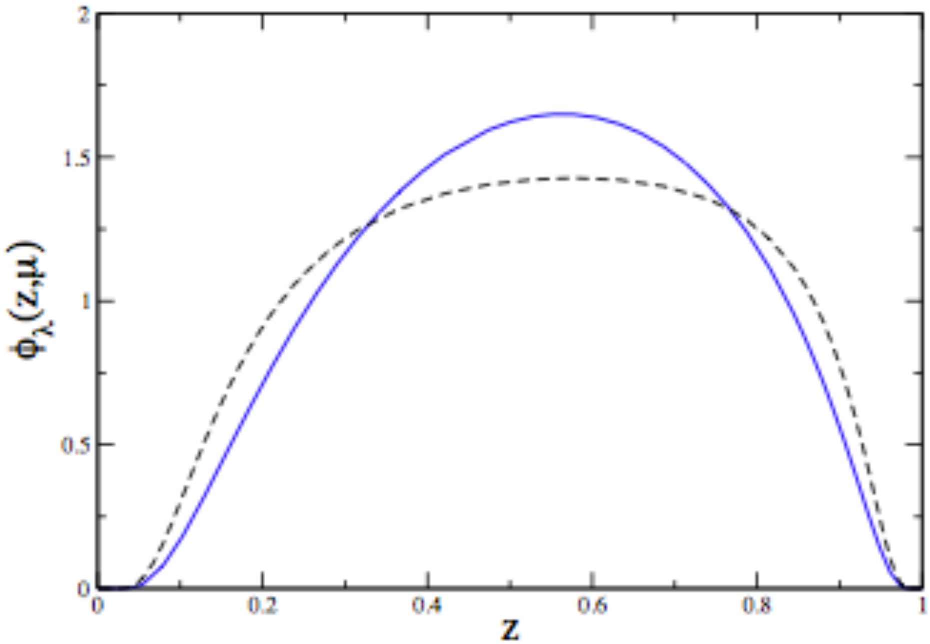
$$\phi^{\lambda}_{K^*}(z,\zeta) = \mathcal{N}_{\lambda}\frac{\kappa}{\sqrt{\pi}}\sqrt{z(1-z)}\exp\left(-\frac{\kappa^2\zeta^2}{2}\right)\exp\left\{-\left[\frac{m_s^2 - z(m_s^2 - m_{\bar{q}}^2)}{2\kappa^2 z(1-z)}\right]\right\}$$

$$\kappa = \frac{M_{K^*}}{\sqrt{2}} = 0.63\, GeV$$

$$f(z) = \sqrt{z(1-z)}$$

- Prefactor fixed by matching expression for pion EM form factor in AdS and physical spacetime
- Quarks masses introduced using Brodsky-de Teramond prescription
- We shall use constituent quark masses

Twist-2 DAs for K*



Solid blue: longitudinal twist-2 DA
Dashed black: transverse twist-2 DA

Fits for the form factors

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^4/m_B^4)}$$

F	$F(0)$	a	b
A_0	0.285	1.158	0.096
A_1	0.249	0.625	−0.119
A_2	0.235	1.438	0.554
V	0.277	1.642	0.600
T_1	0.255	1.557	0.499
T_2	0.251	0.665	−0.028
T_3	0.155	1.503	0.695

AdS/QCD LCSR fit

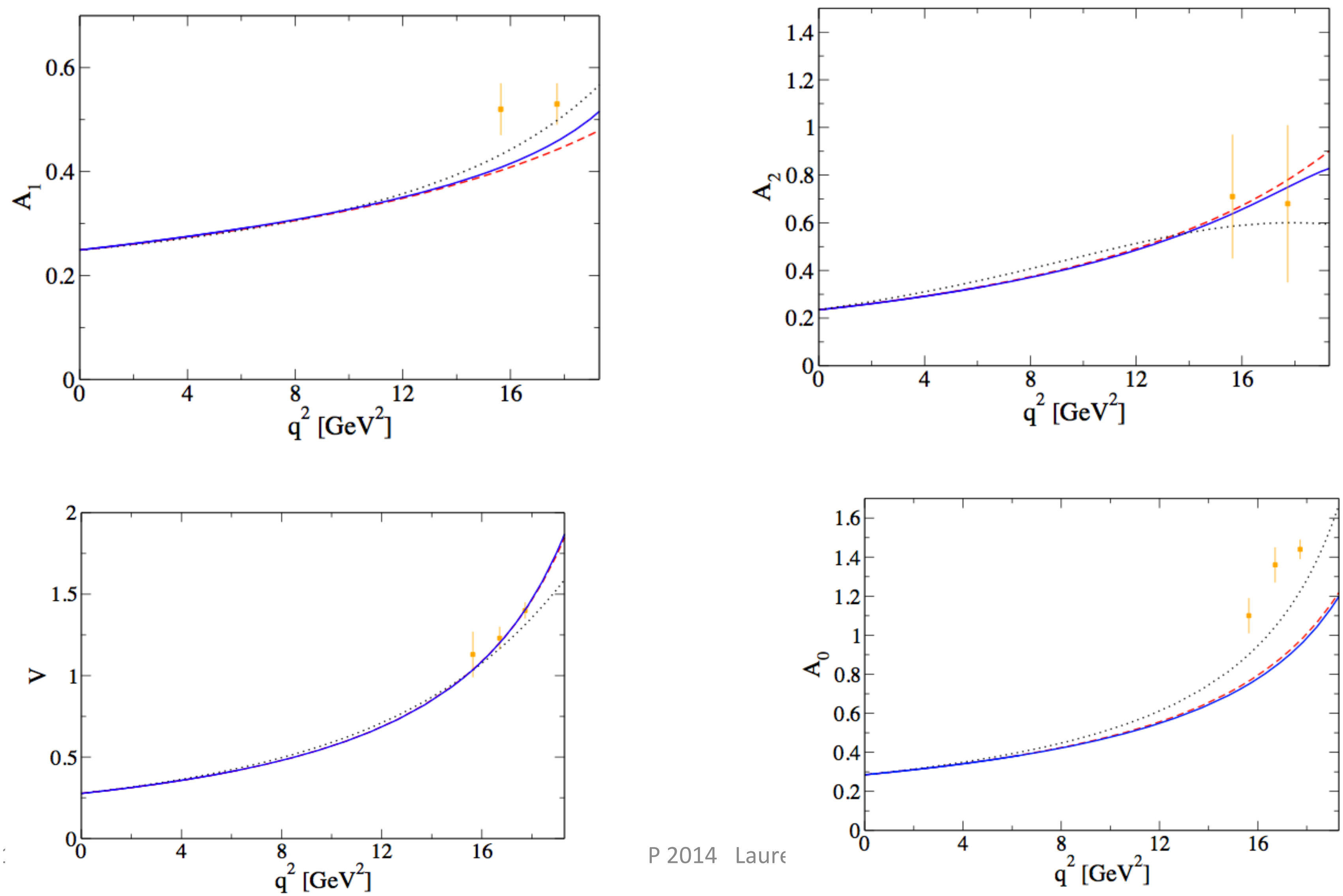
F	$F(0)$	a	b
A_0	0.285	1.314	0.160
A_1	0.249	0.537	−0.403
A_2	0.235	1.895	1.453
V	0.277	1.783	0.840
T_1	0.255	1.750	0.842
T_2	0.251	0.555	−0.379
T_3	0.155	1.208	−0.030

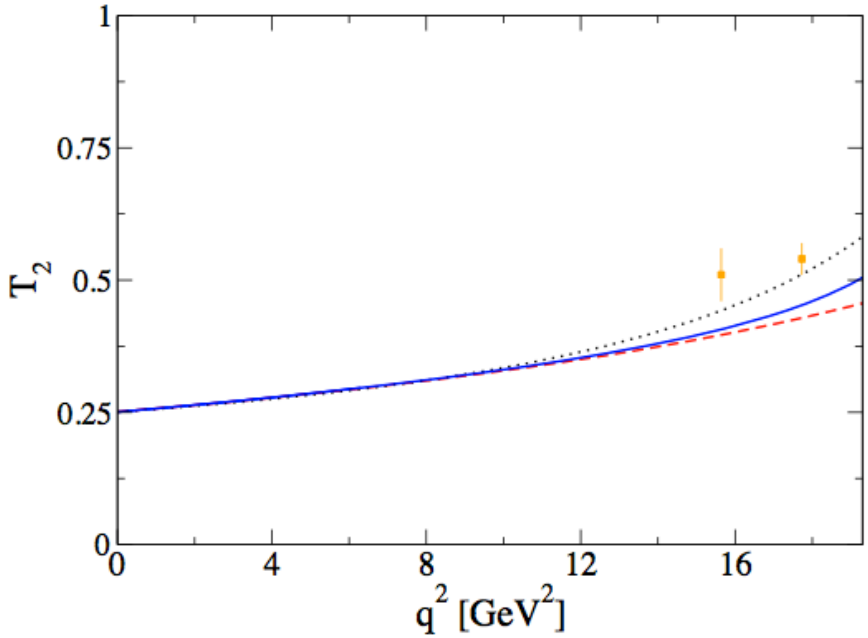
AdS/QCD LCSR + lattice fit

Form factors and lattice data

Blue: AdS/QCD
Red: AdS/QCD fit
Black: AdS/QCD + Lattice fit

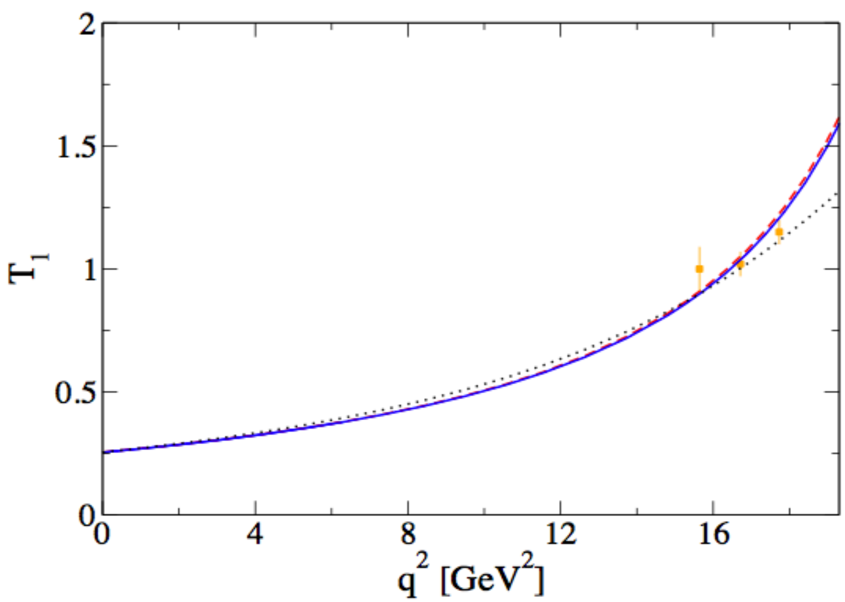
Lattice data from
R. R. Horgan et al. (2013)

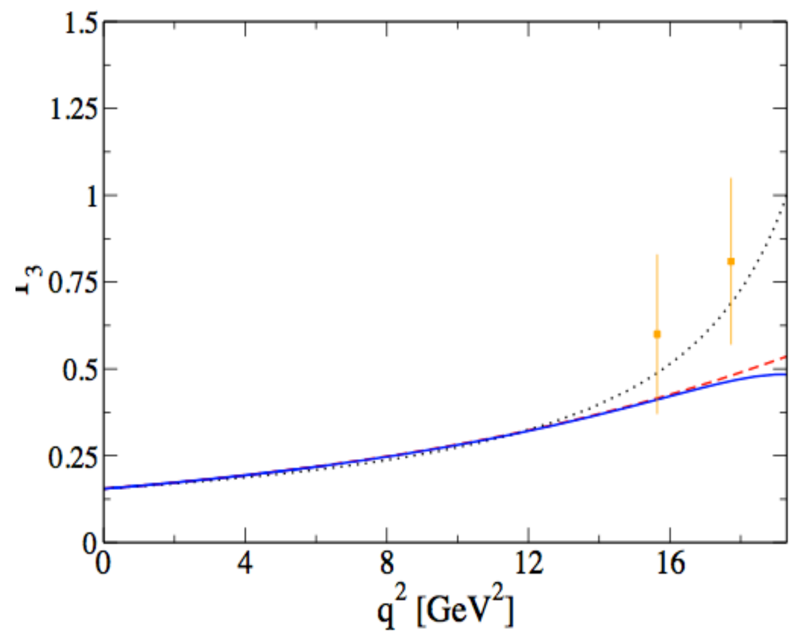




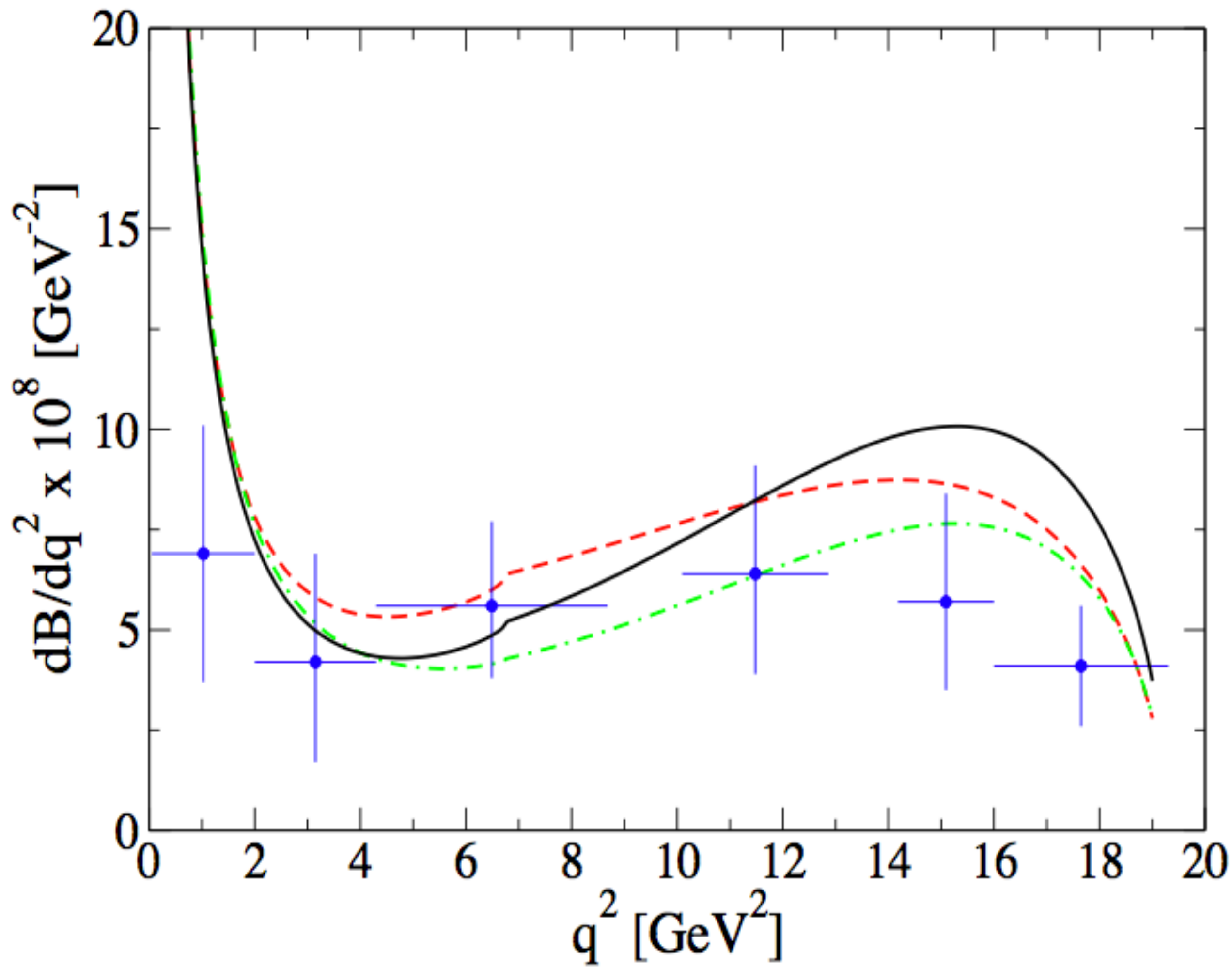
Form factors and lattice data

Blue: AdS/QCD
Red: AdS/QCD fit
Black: AdS/QCD + Lattice fit





Differential branching ratio



- Red: AdS/QCD
- Black: AdS/QCD + Lattice
- Green: AdS/QCD + Lattice + New Physics (reduced C_9)

- Data from LHCb

$$C_9^{NP} = -1.5$$

Conclusions and outlook

- We have computed the $B \to K^{*}$ transition form factors as well as the differential branching ratio for $B \to K^{*}\mu^{+}\mu^{-}$ using new holographic DAs for the vector meson

- Agreement with LHCb data except at large momentum transfer. Possible NP in C9 ?

- Now looking at other observables: isospin asymmetry, forward-backward asymmetry, …

Acknowledgements

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