# Andreev quantum dots in graphene-superconductor hybrids

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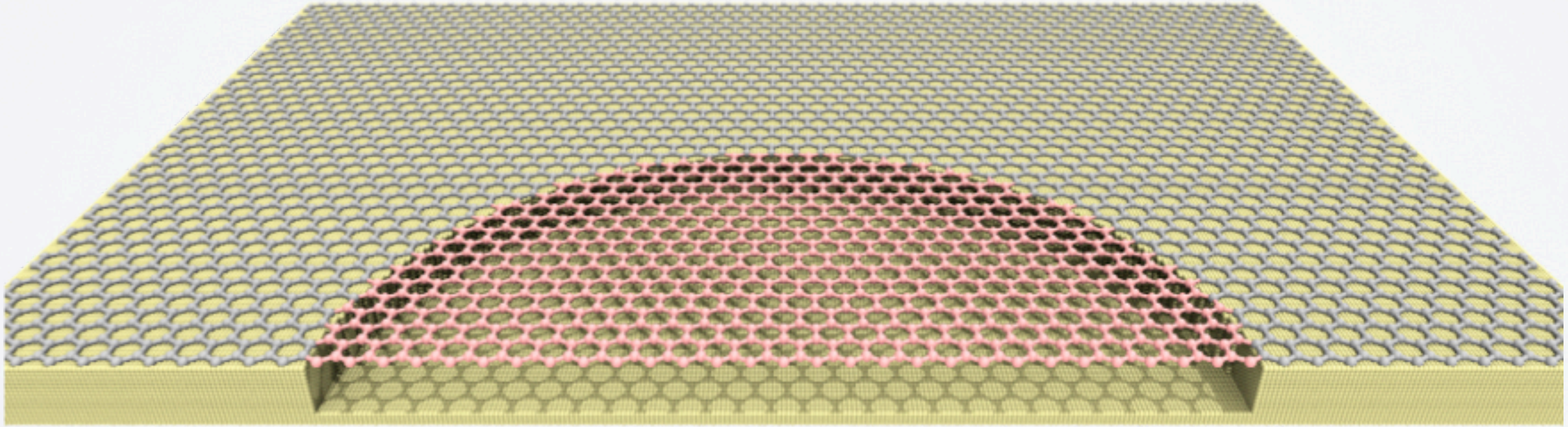
Universiteit Antwerpen, Belgium



# Outline:

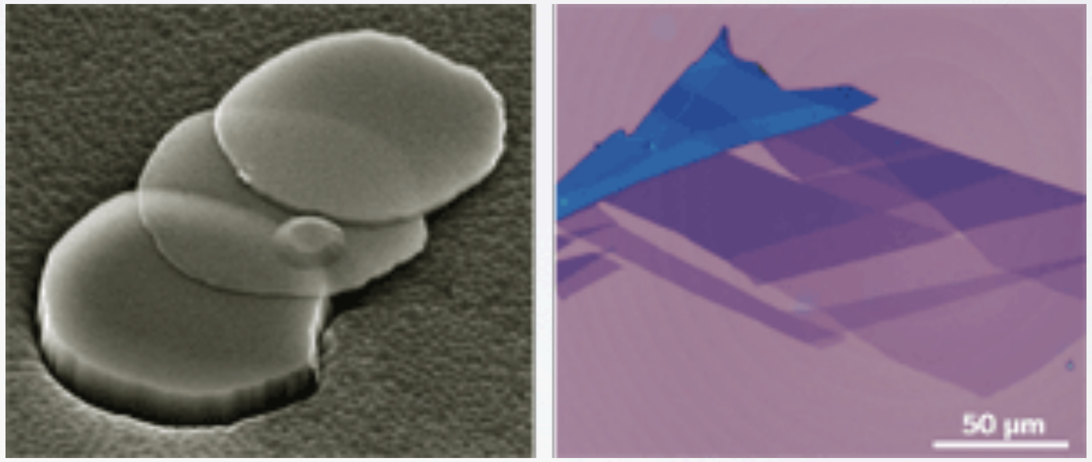
- Two dimensional materials (graphene and beyond)
- Quick intro to graphene physics
- Proximity effect and hybrid devices
- Robustness of Andreev quantum dots
- Manipulation of Andreev states

# Two dimensional materials: graphene

How to get them? Start from layered materials with weak van der Waals interlayer interaction and try to isolate monolayers

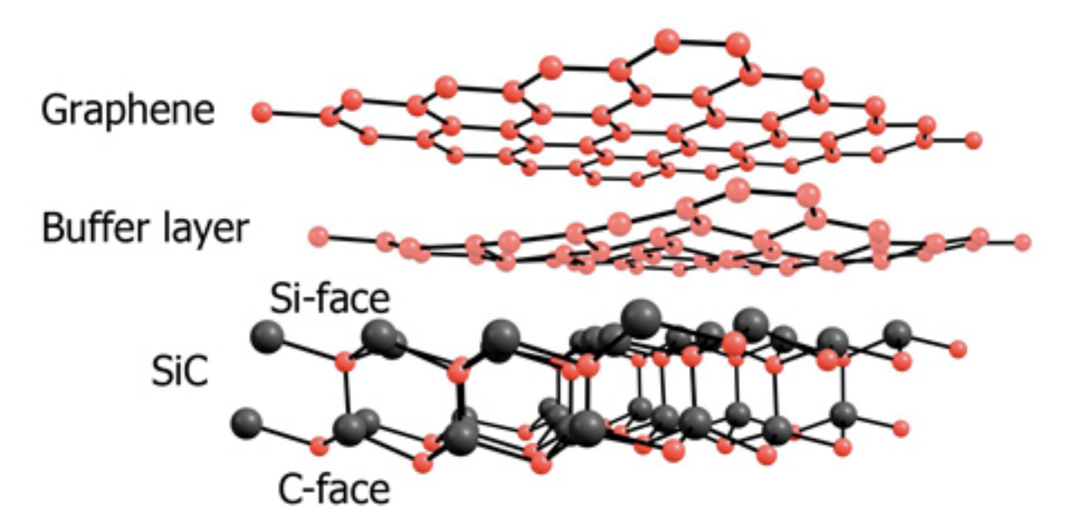
Graphite

“Scotch tape” method, exfoliation

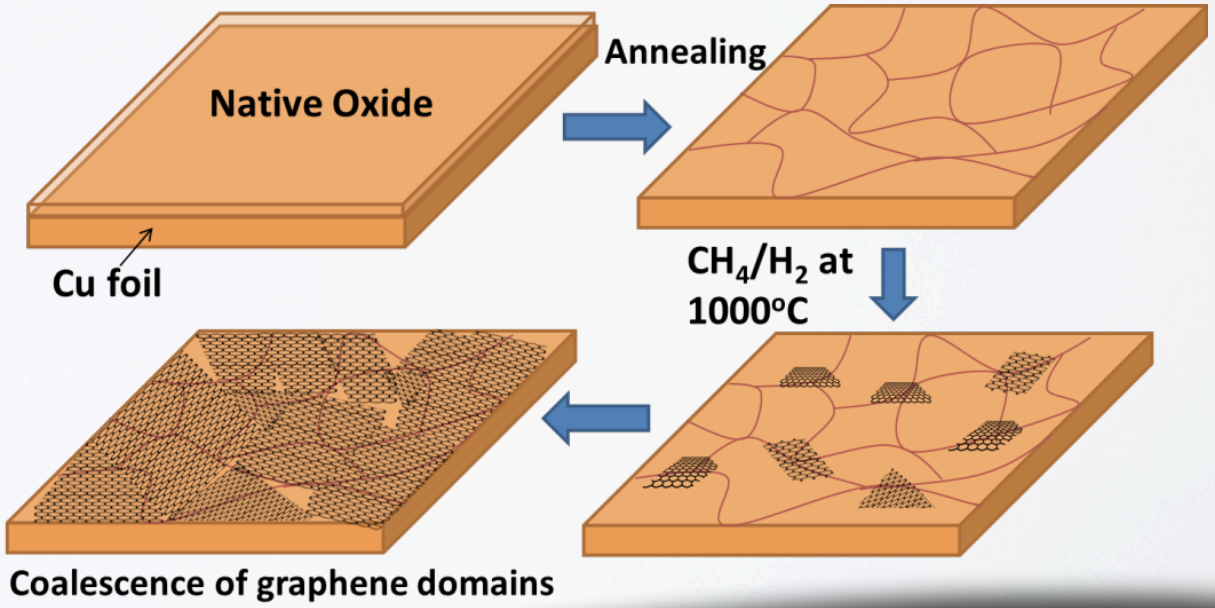


Geim and Novoselov, Nobel Prize 2010

Epitaxial growth



Chemical vapor deposition on Cu



# Two dimensional materials: graphene

How to get them? Start from layered materials with weak van der Waals interlayer interaction and try to isolate monolayers

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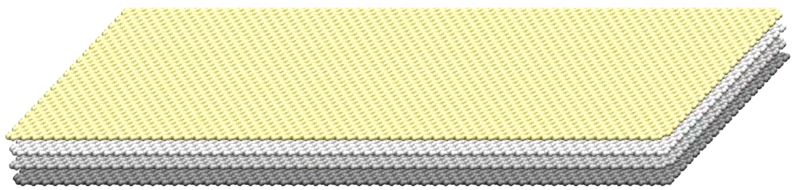

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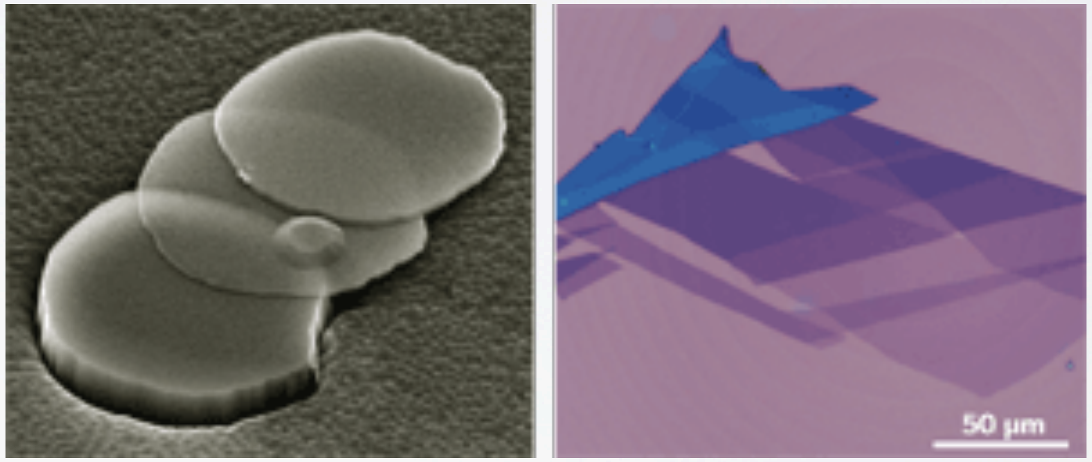
"Scotch tape" method, exfoliation



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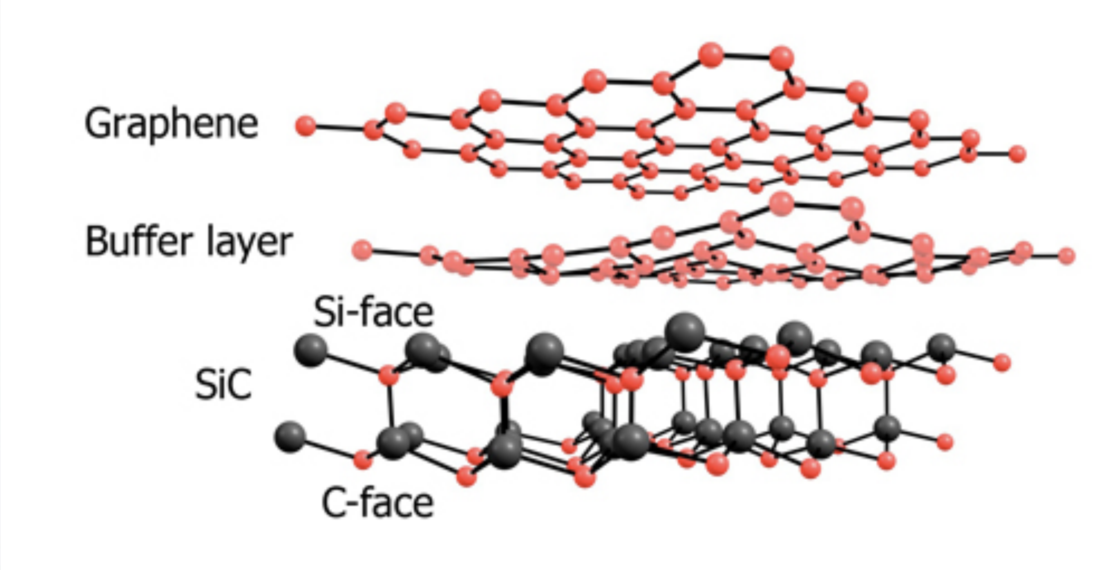

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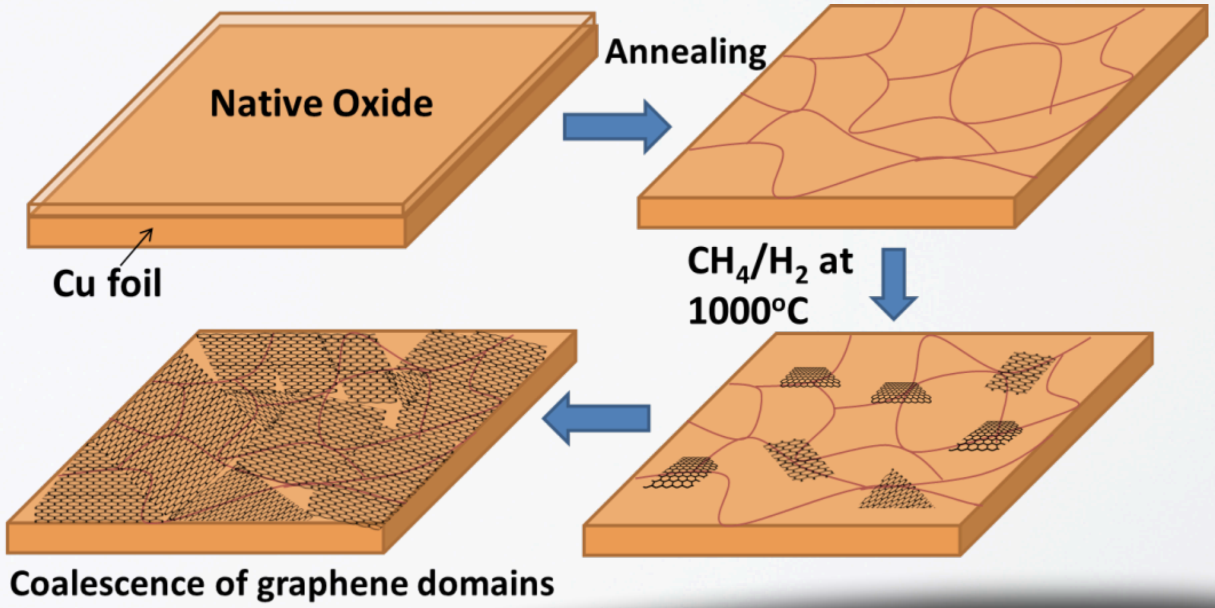
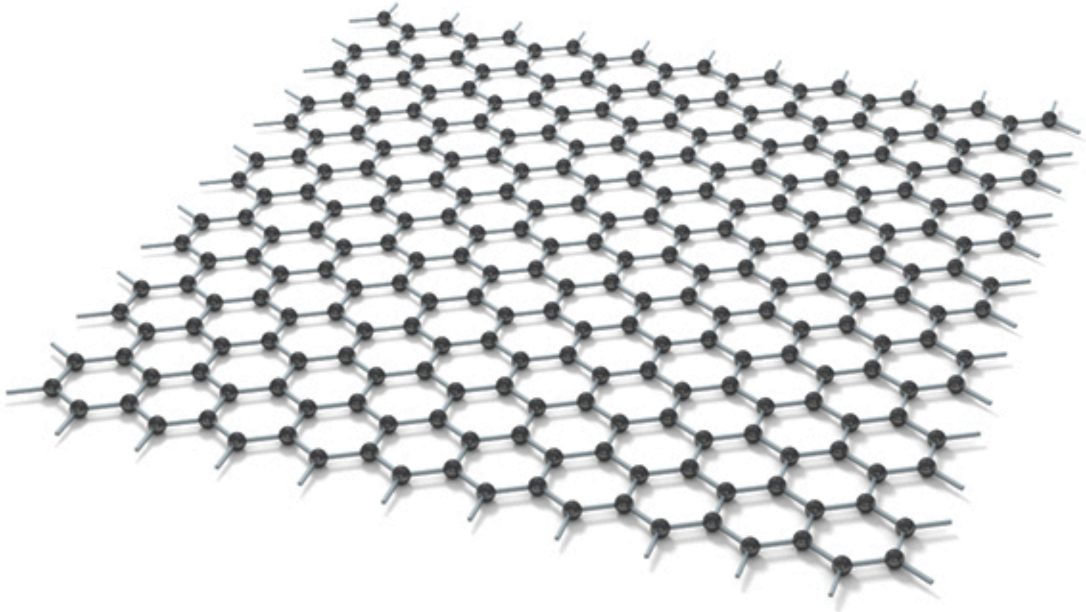
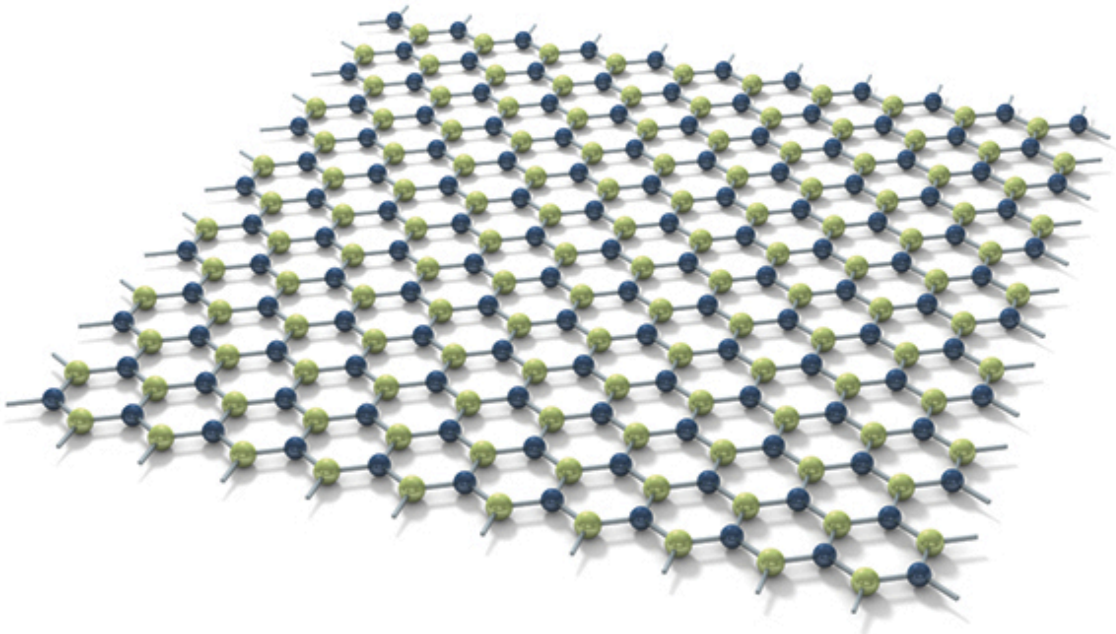


# Two dimensional materials: next generation

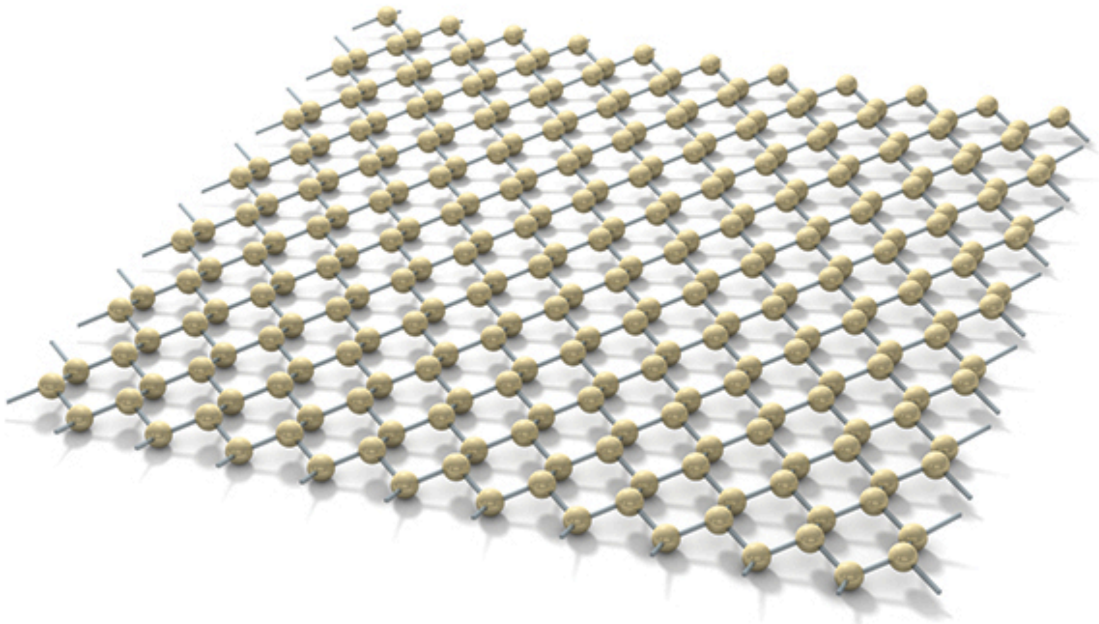
Graphene



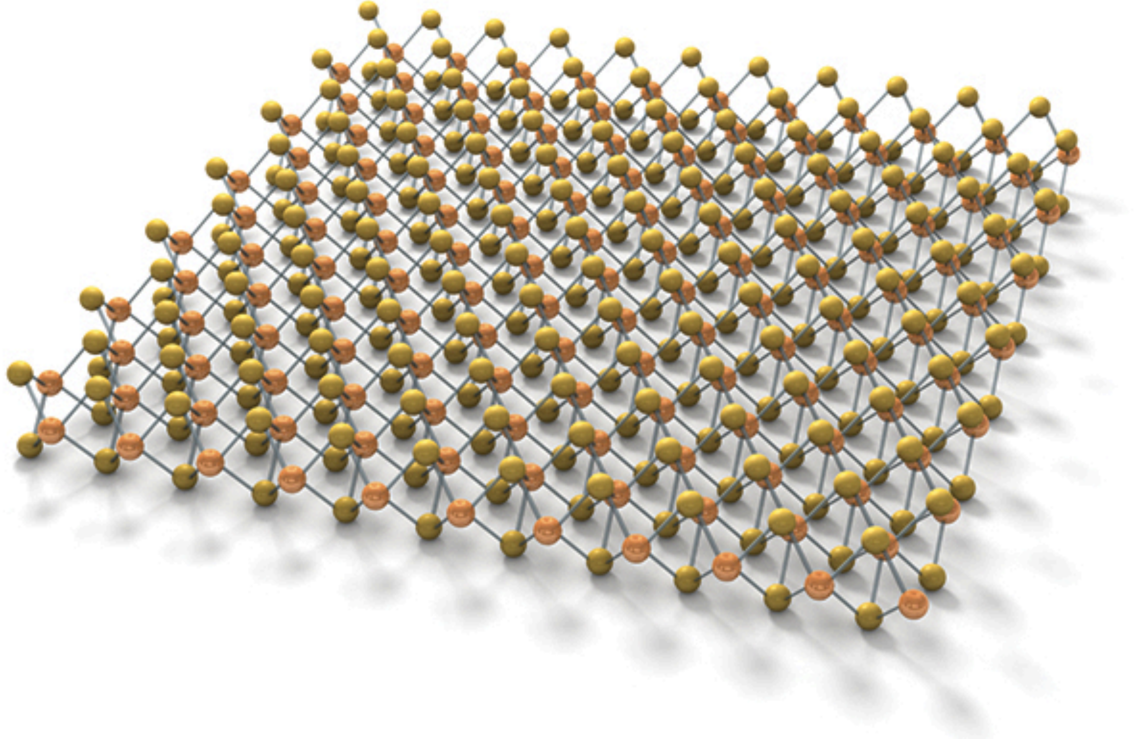
BN



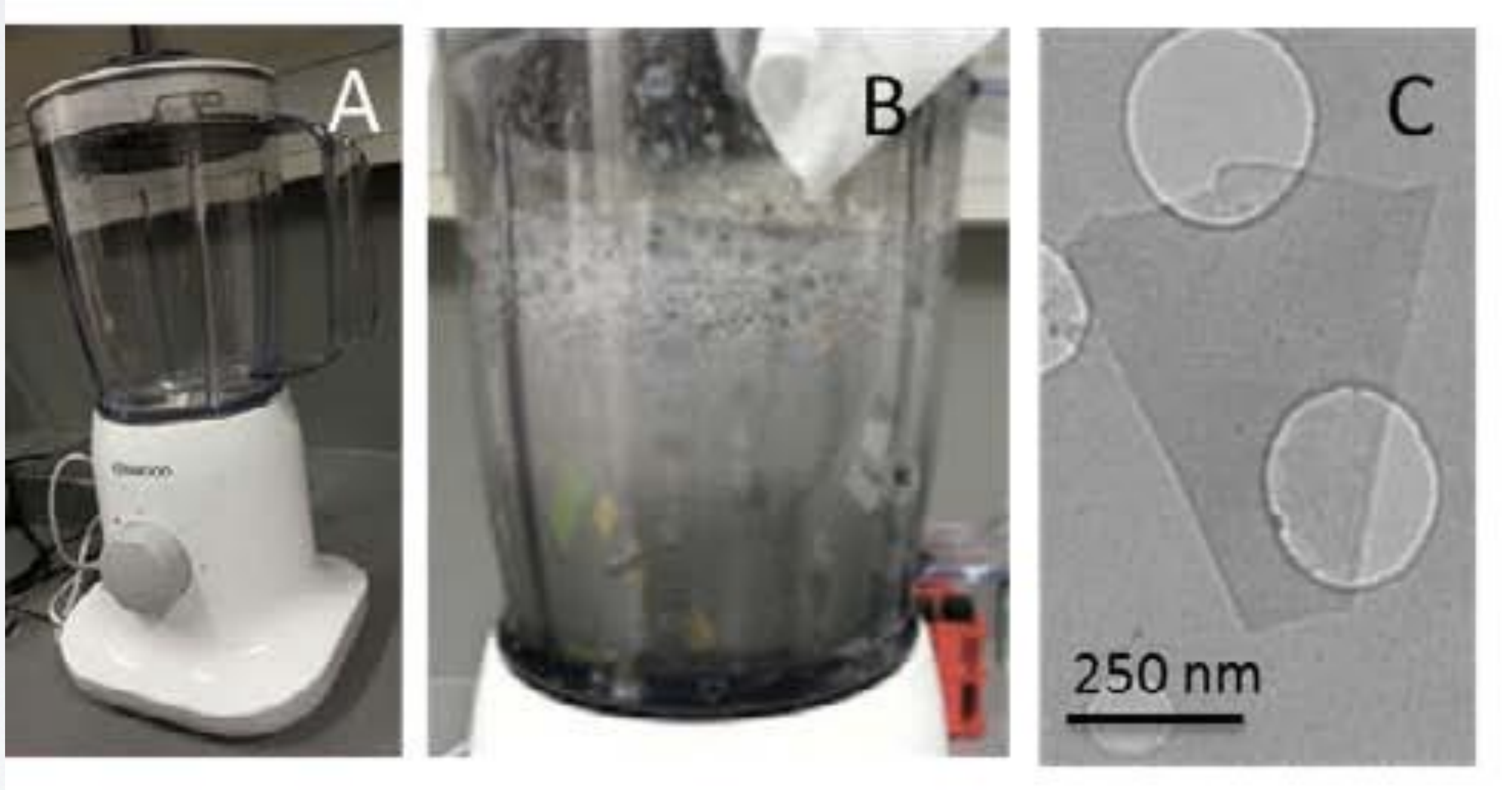
Silicene, Germanene



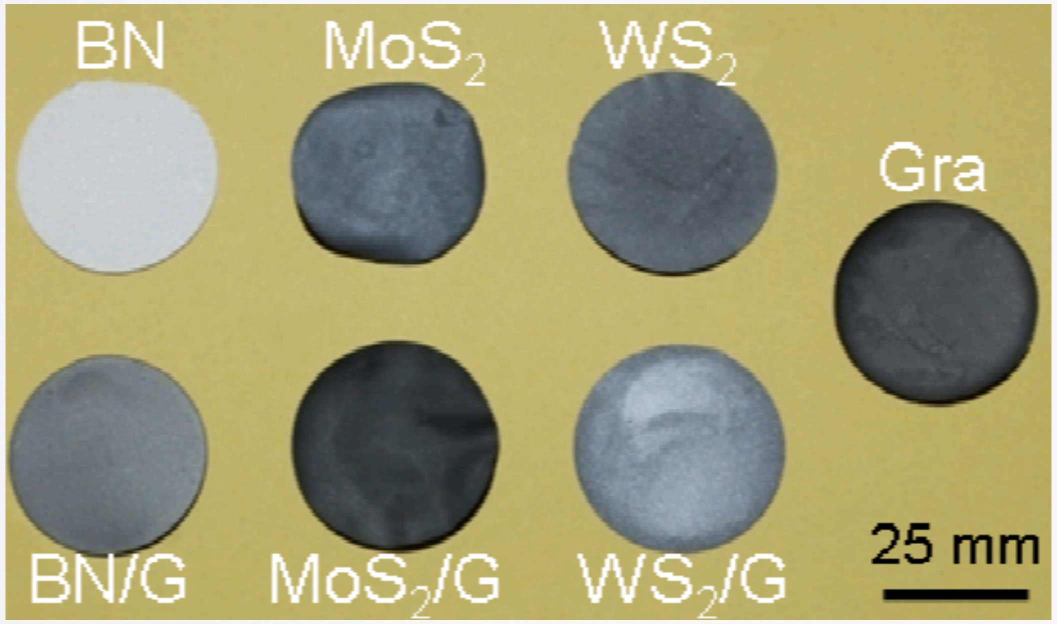
$MoS_2$, $NbSe_2$, $WS_2$



# Homemade graphene in the blender

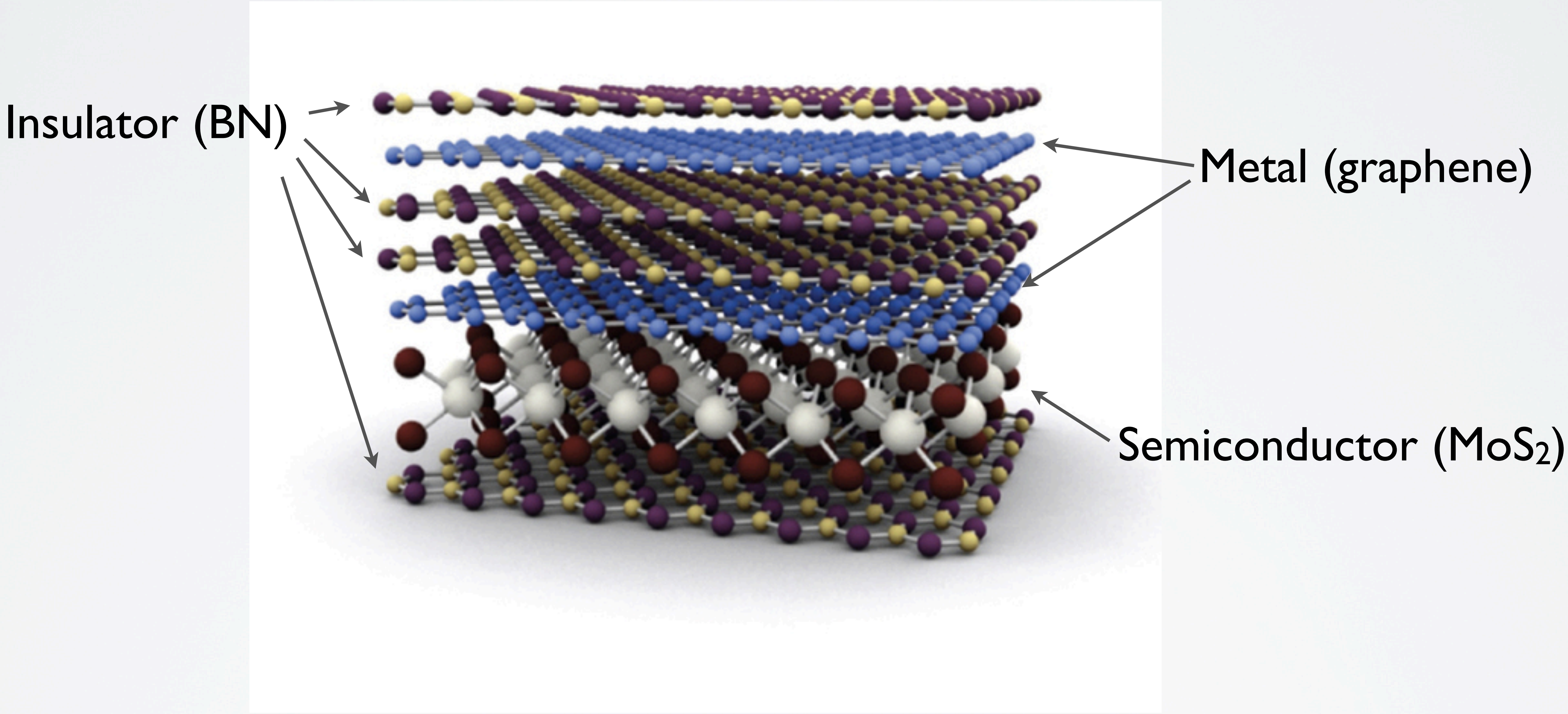


K.R. Paton et al, Nature Materials **13**, 624 (2014)



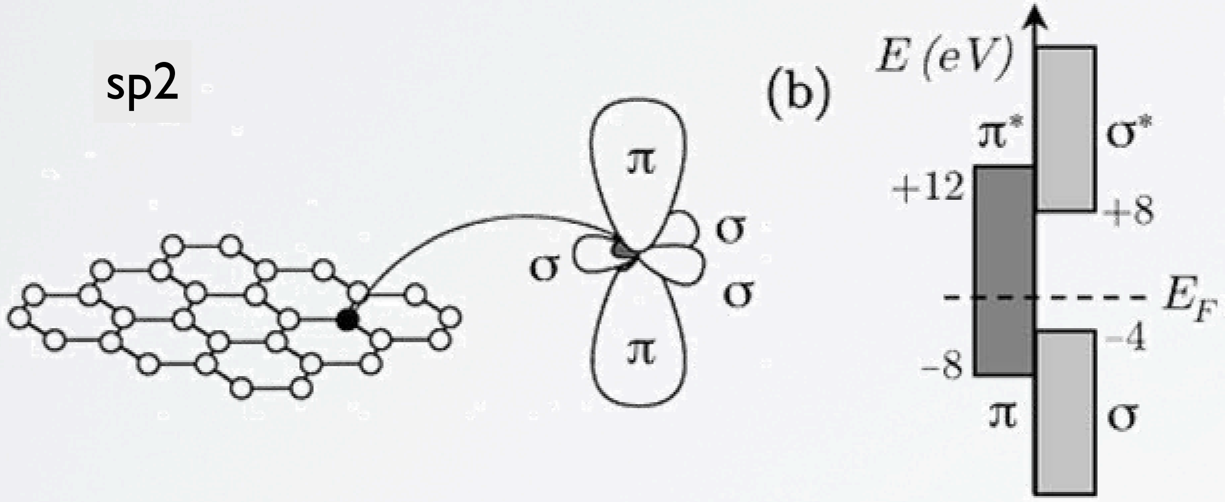
Jonathan N. Coleman et al , Science 331, 568 (2011)

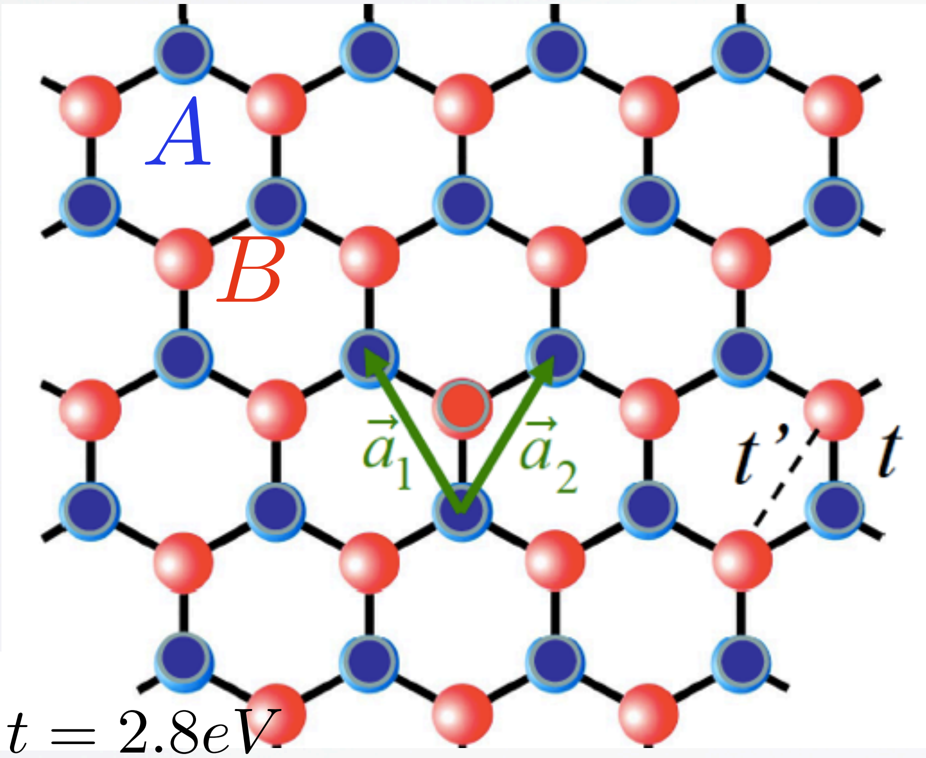
# Two dimensional hybrids (or on substrates)



- ✦ can be easily doped electrostatically
- ✦ Coulomb interactions are long range in graphene due to: vanishing DOS and 2D
- ✦ traverse the phase diagram by electrostatic doping: no induced disorder
- ✦ use screening from substrates to manipulate the phase diagram

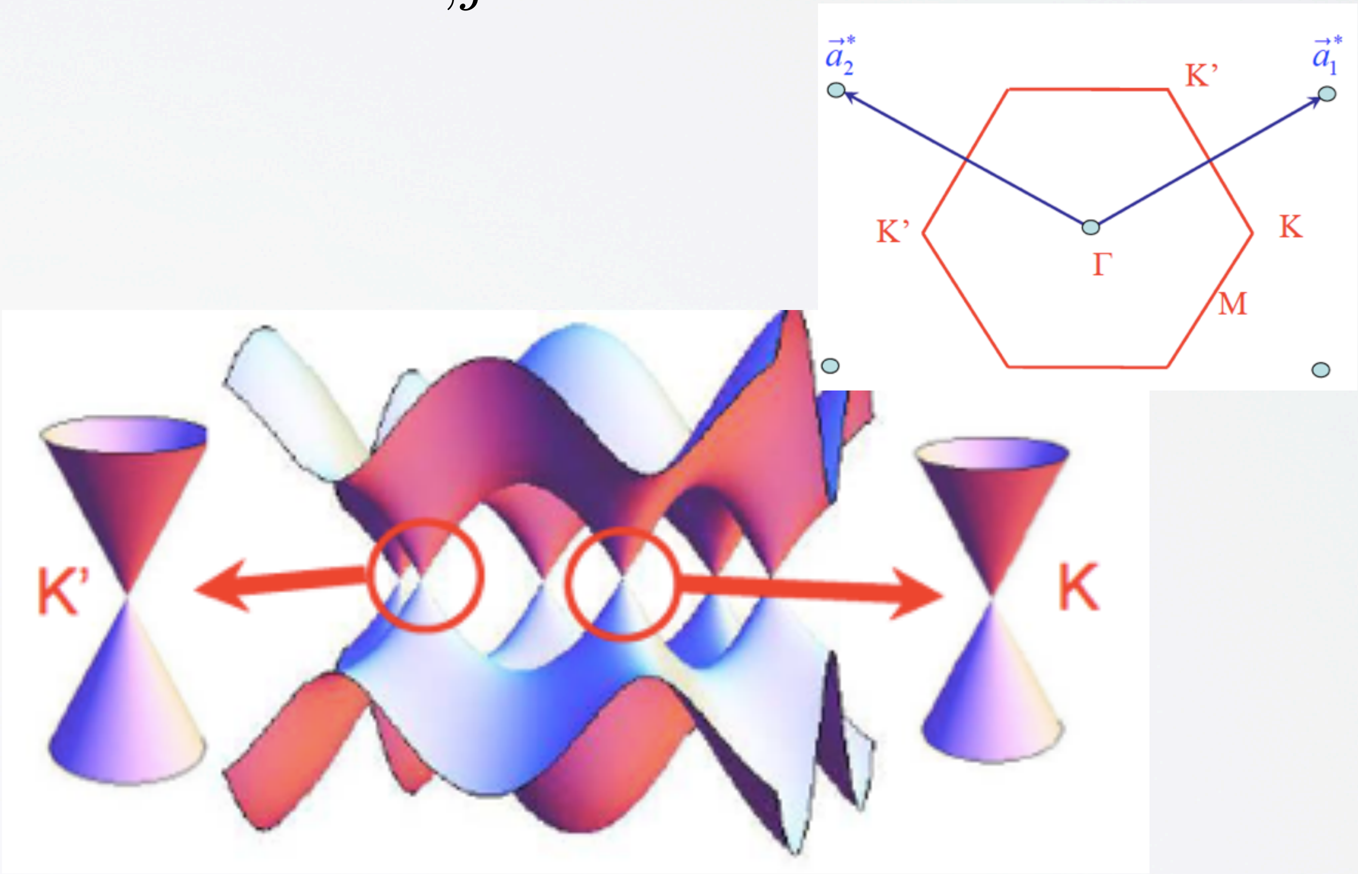
# Graphene: honeycomb lattice





- ✦ tight binding Hamiltonian for $p_z$ electrons

$$H_0 = \sum_{i,j} t_{ij}^{(ab)} (a_i^\dagger b_i + h.c)$$



- ✦ linear dispersion at the Fermi level

$$\boxed{E(\vec{q}) = \pm v_F |q|} \qquad v_F = \frac{3ta}{2} \approx \frac{c}{300}$$

# 2D continuum Dirac equation

Low energy Hamiltonian near K

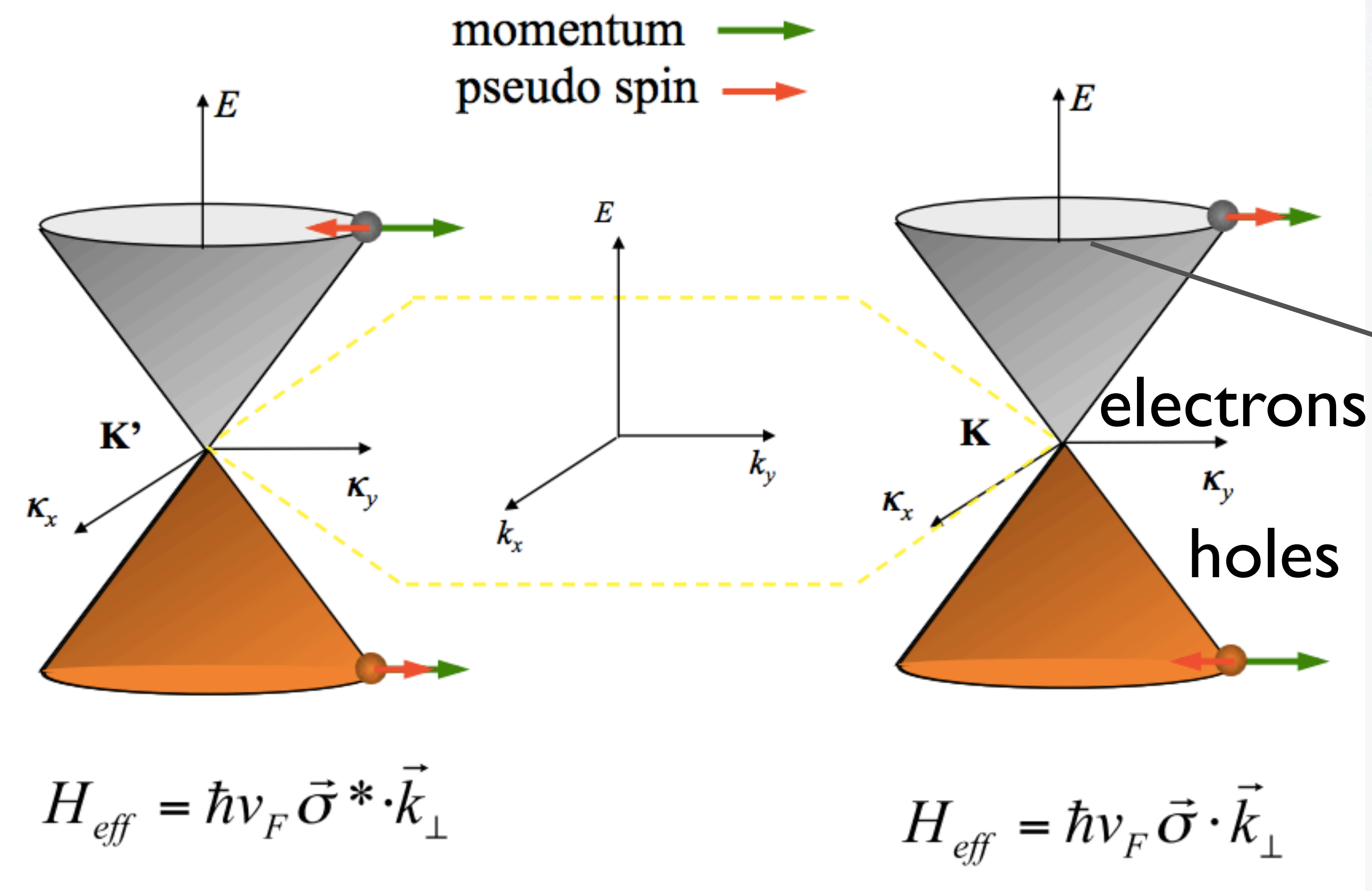
$$H_{eff} = \hbar v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix} = \hbar v_F \vec{\sigma}\cdot\vec{k}_\perp$$

**½ Spinor**

$$\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix} \begin{array}{l} p_z^{A}(r) \\ p_z^{B}(r) \end{array}$$

$$\left| k_\perp \right\rangle = e^{i\mathbf{k}\cdot\mathbf{r}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\theta_\kappa} \end{pmatrix}$$

$$\theta_k = \tan^{-1}(k_y / k_x)$$



$$H_{eff} = \hbar v_F \vec{\sigma}^{*}\cdot\vec{k}_\perp$$

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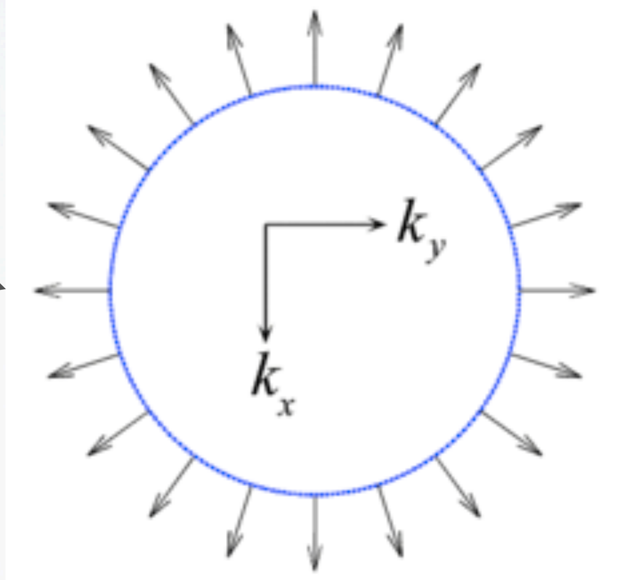


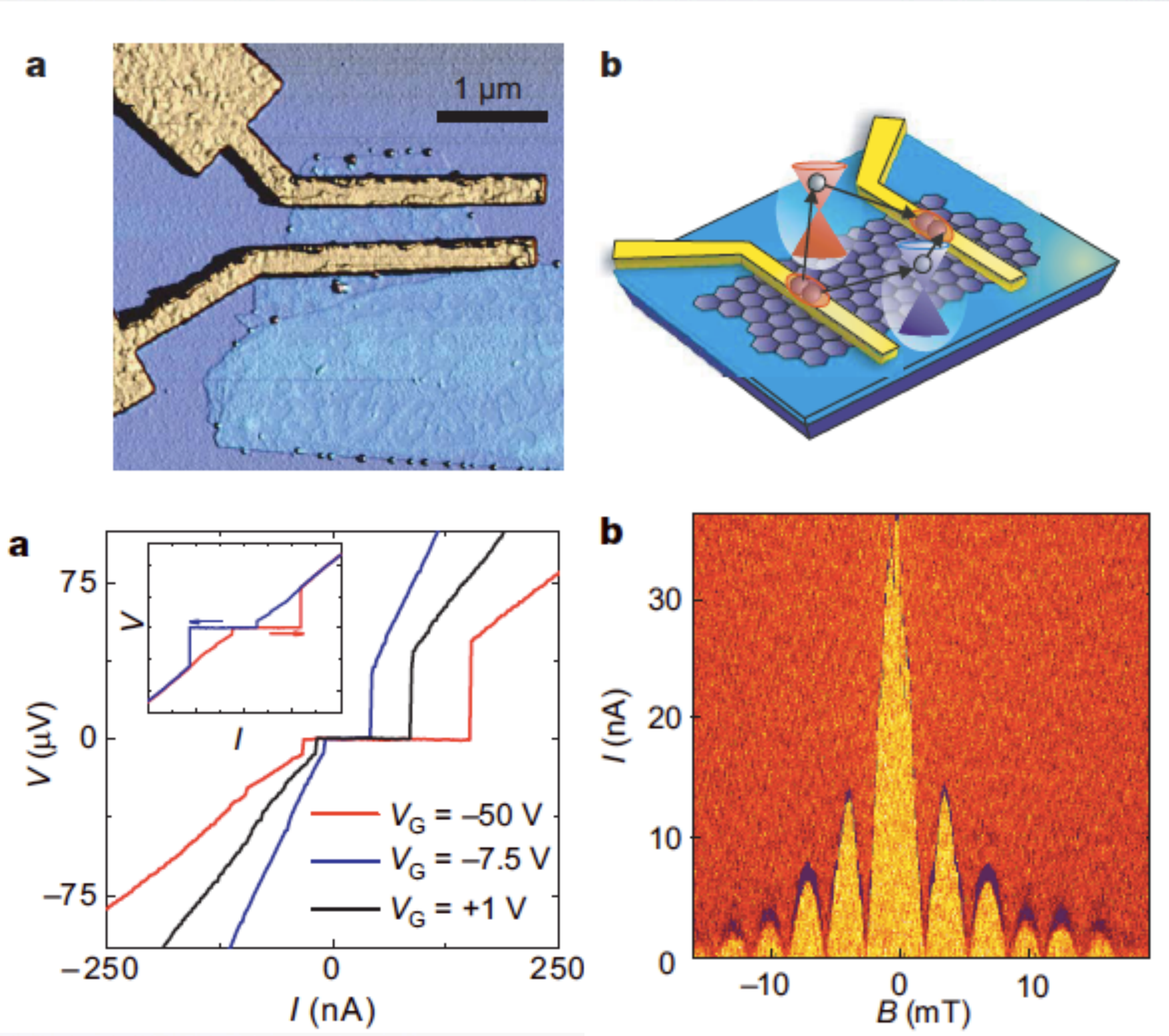
# Superconductivity in graphene (proximity effect)

- no intrinsic superconductivity due to the vanishing DOS at the Fermi level

- high doping from gates or metallic adatoms could allow SC

- Josephson current was predicted theoretically even when Fermi surface is tuned to be at the neutrality point, evanescent modes

  M. Titov et al., Phys Rev. B 74, 041401(R) (2006)

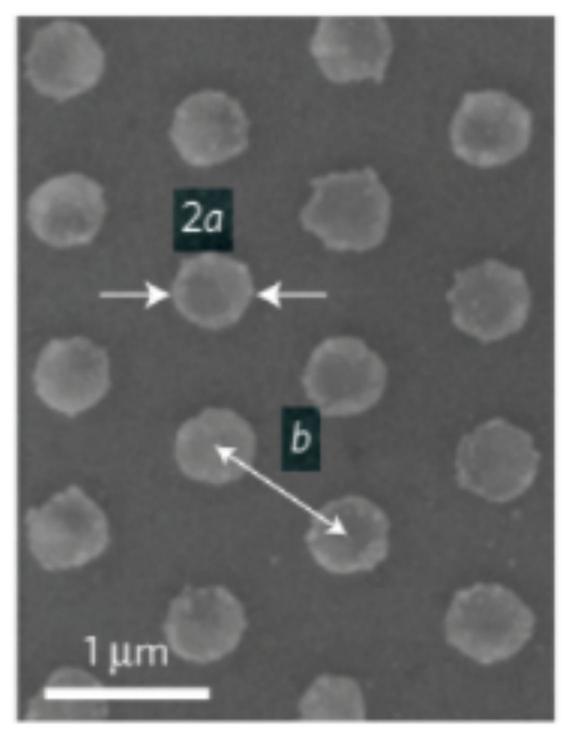


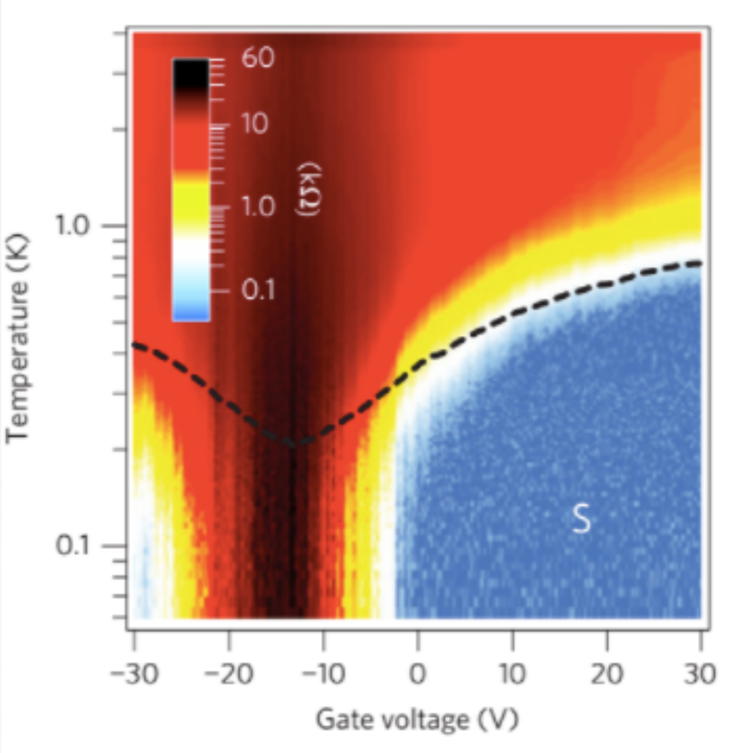
- graphene Josephson junction Al, Pb, NbSe$_2$, etc

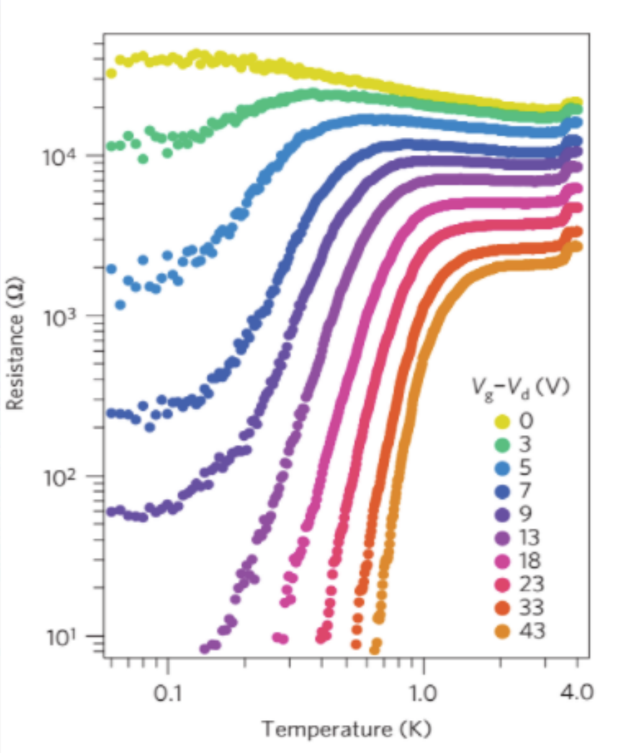
C. Ojeda-Aristizabal, M. Ferrier, S. Guéron, and H. Bouchiat, Physical Review B 79, (2009).
K. Komatsu, C. Li, S. Autier-Laurent, H. Bouchiat, and S. Guéron, Physical Review B 86, (2012).
I.V. Borzenets, U. C. Coskun, H. Mebrahtu, and G. Finkelstein, IEEE Transactions on Applied Superconductivity 22, 1800104 (2012).
D. Jeong, J.-H. Choi, G.-H. Lee, S. Jo, Y.-J. Doh, and H.-J. Lee, Physical Review B 83, (2011).
H. B. Heersche, P. Jarillo-Herrero, J. B. Oostinga, L. M. K. Vandersypen, and A. F. Morpurgo, Solid State Communications 143, 72 (2007).
H. Tomori, A. Kanda, H. Goto, S. Takana, Y. Ootuka, and K. Tsukagoshi, Physica C: Superconductivity 470, 1492 (2010).
H. B. Heersche, P. Jarillo-Herrero, J. B. Oostinga, L. M. K. Vandersypen, and A. F. Morpurgo, Nature 446, 56 (2007).

# Superconducting hybrid devices (gate tunable)

- ✦ quantum phase transition (metal to superconductor) in array of SC dots



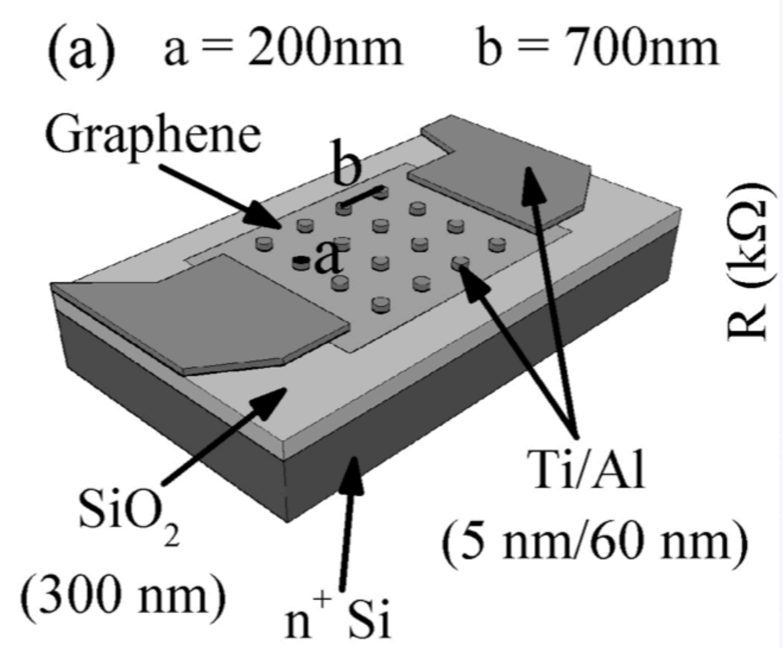


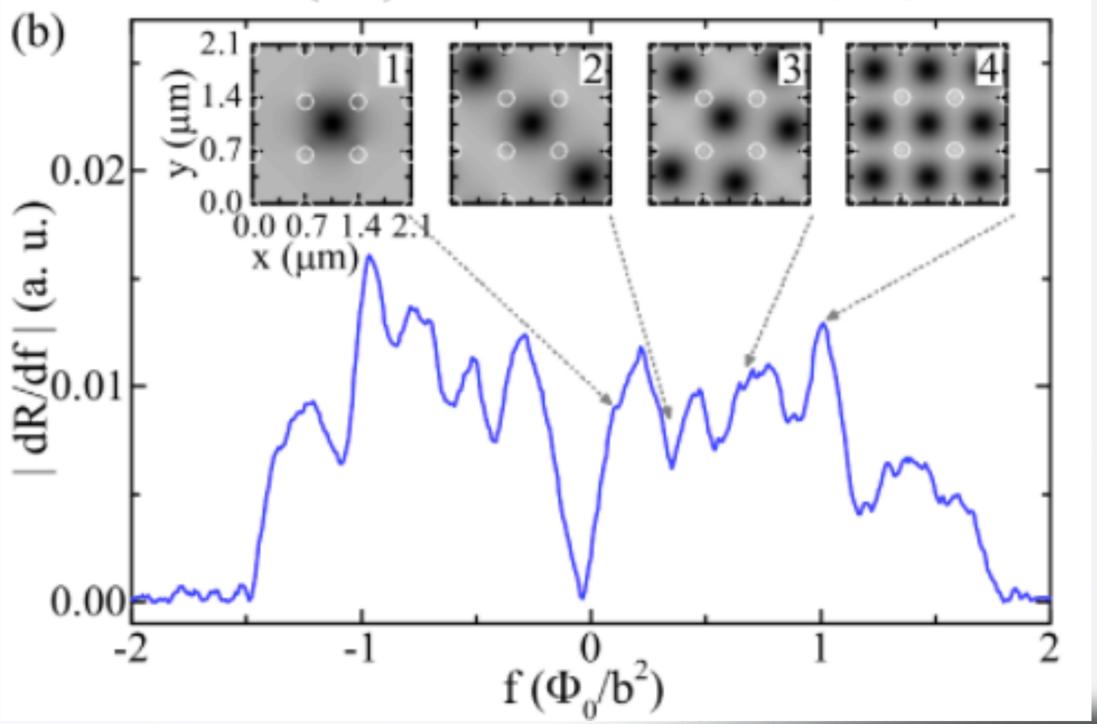


Z. Han, A. Allain, V. Bouchiat, Nat. Phys. 10, 380 (2014)

A. Allain, Z. Han, V. Bouchiat, Nat. Mater. 11, 590 (2012)

- ✦ Josephson junction array (signature of Josephson vortices)



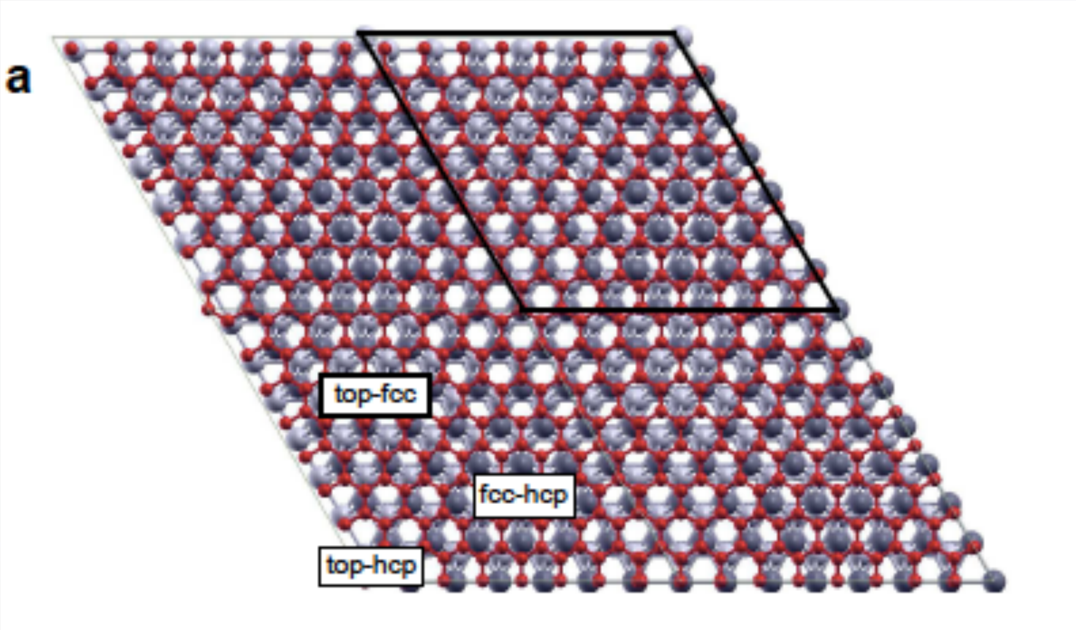


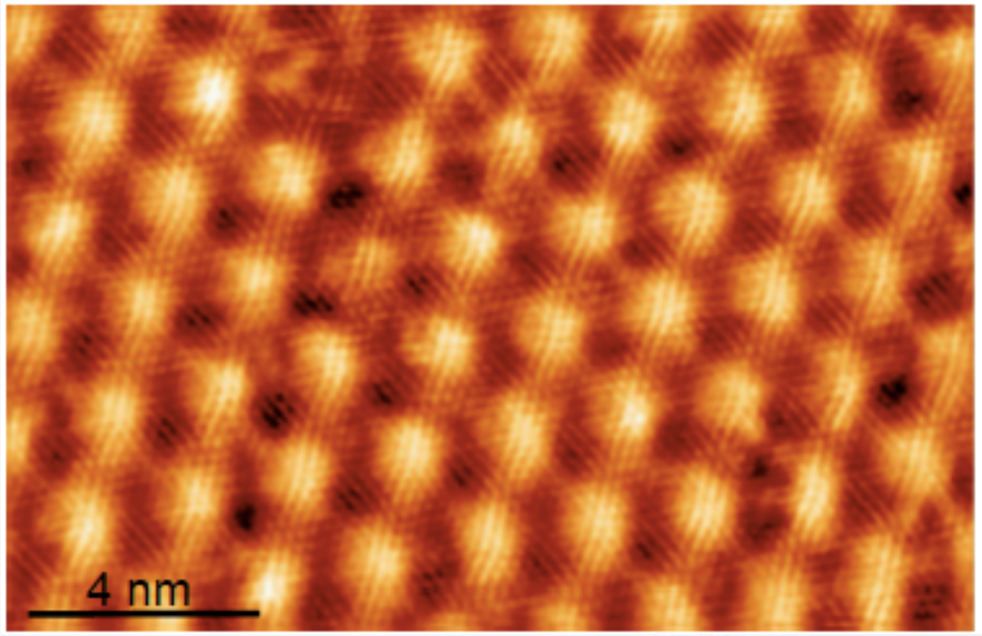
C. Richardson, L. Covaci, C. G. Smith. M.R. Connolly, tba

# Superconducting hybrid devices (Re substrate)

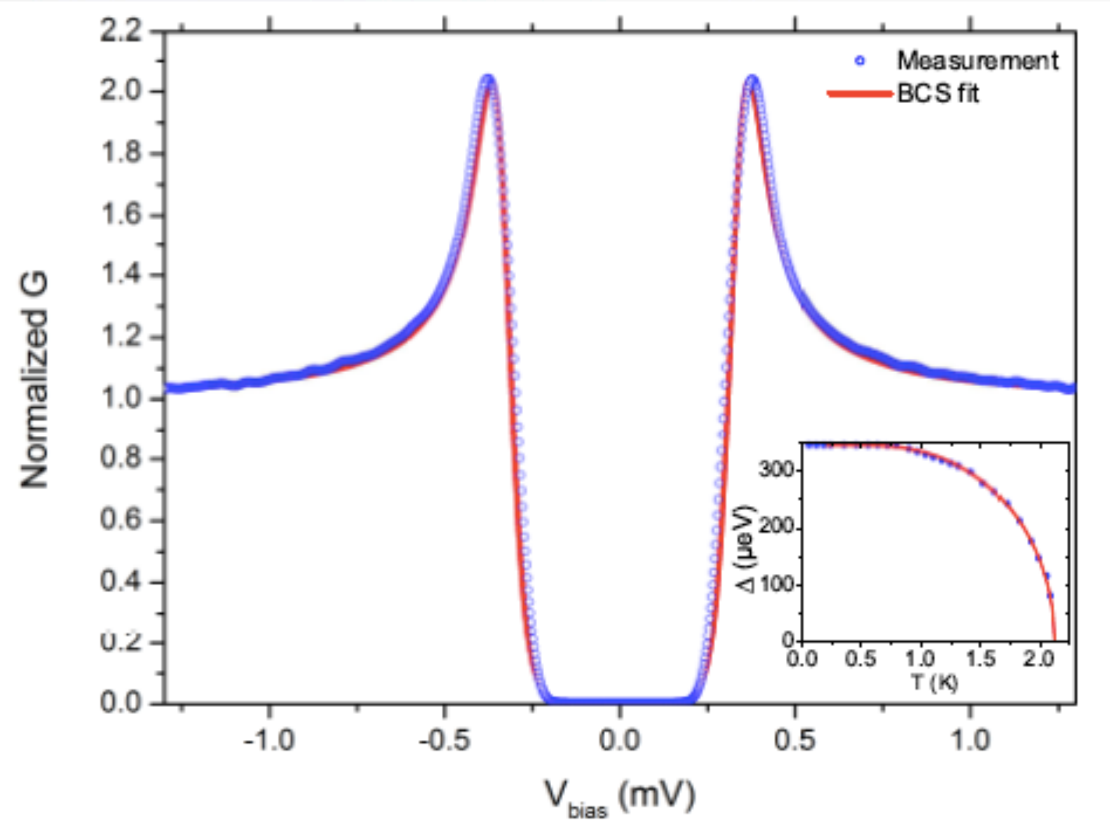
- CVD grown graphene on Re (0001), strongly coupled graphene/metal

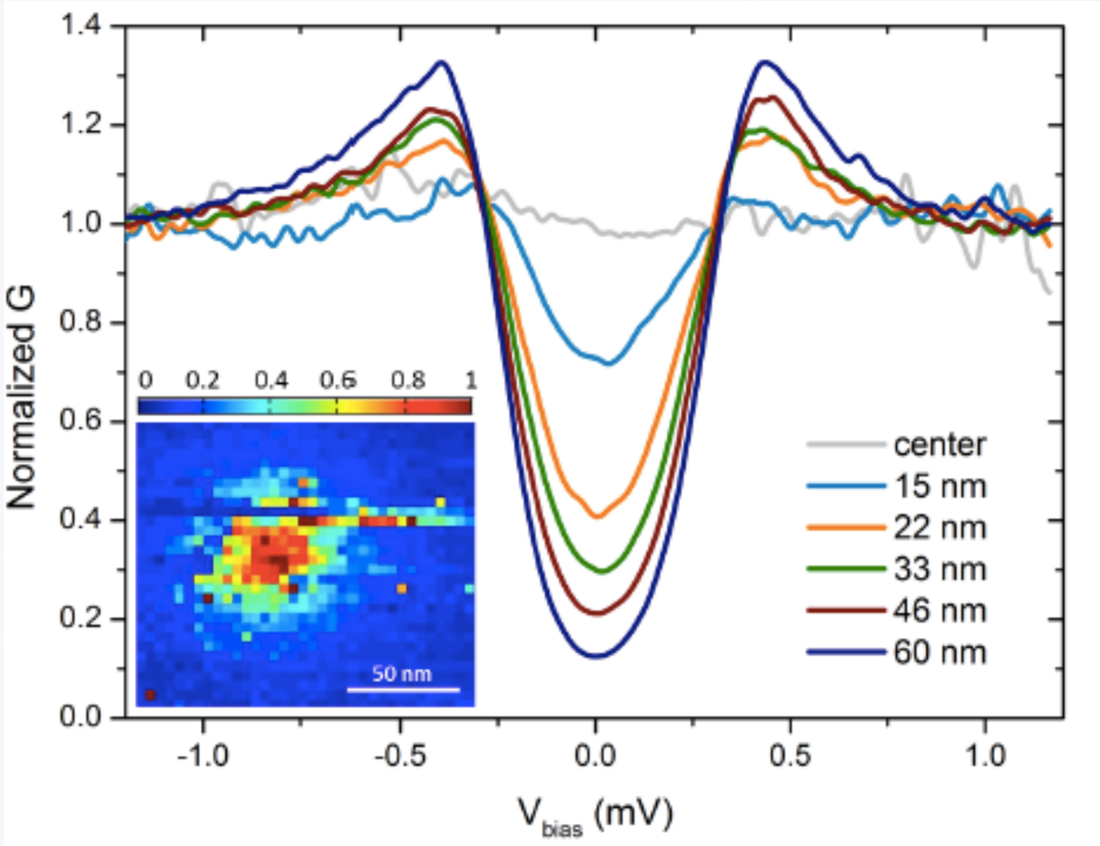
C. Tonnoir et al, Phys. Rev. Lett. 111, 246805 (2013)





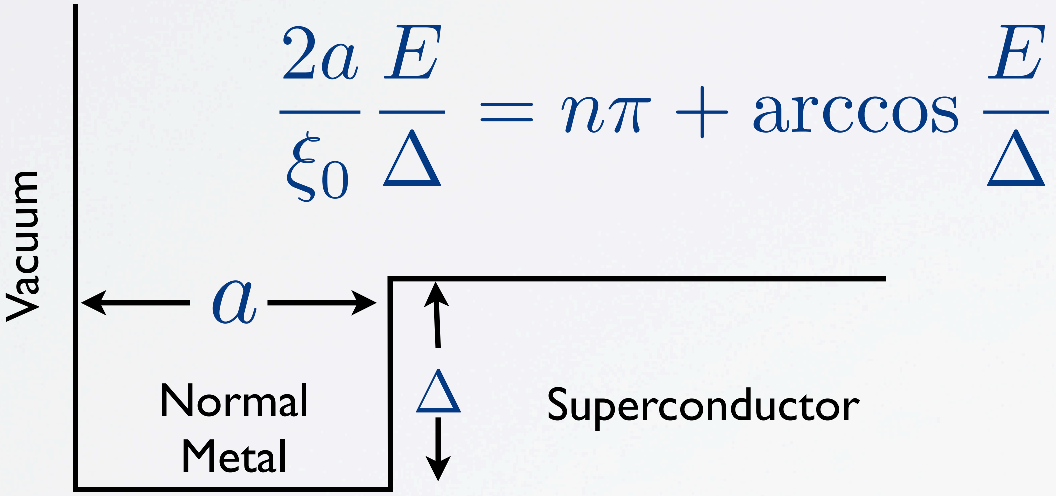
- Re becomes superconducting and a proximity gap is induced in graphene



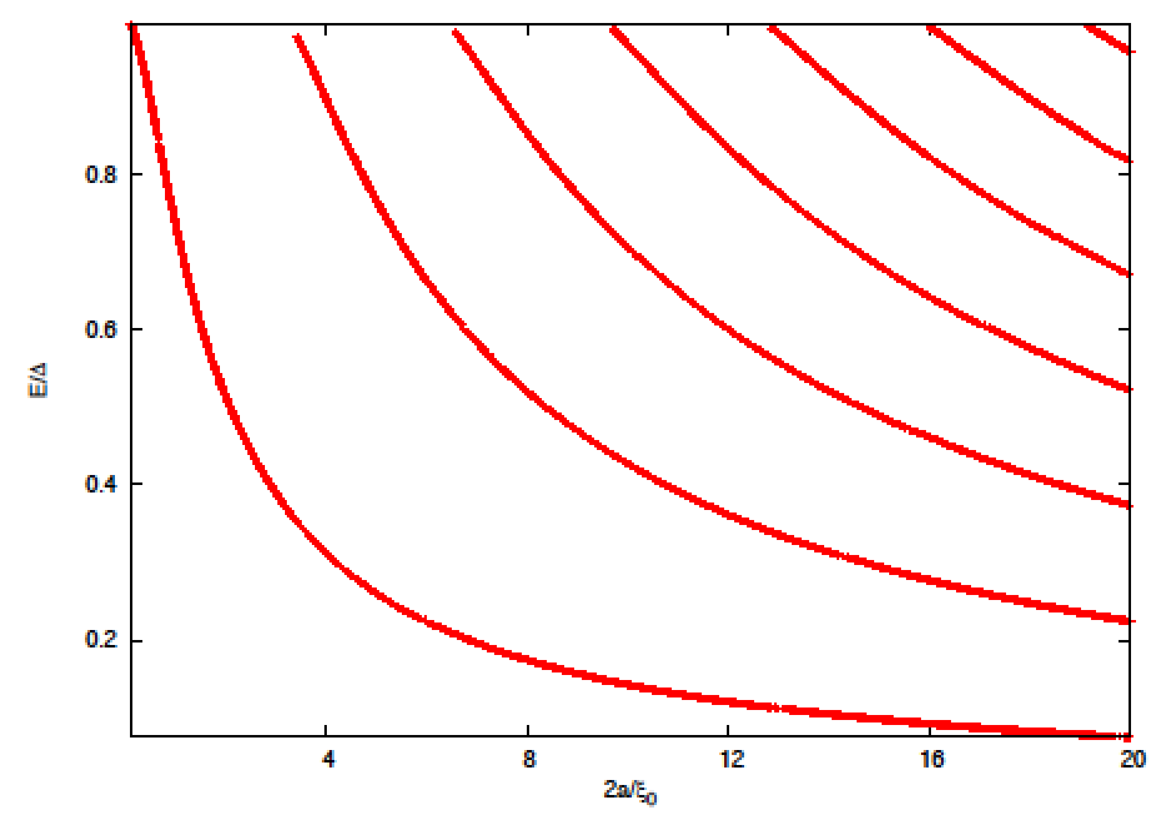


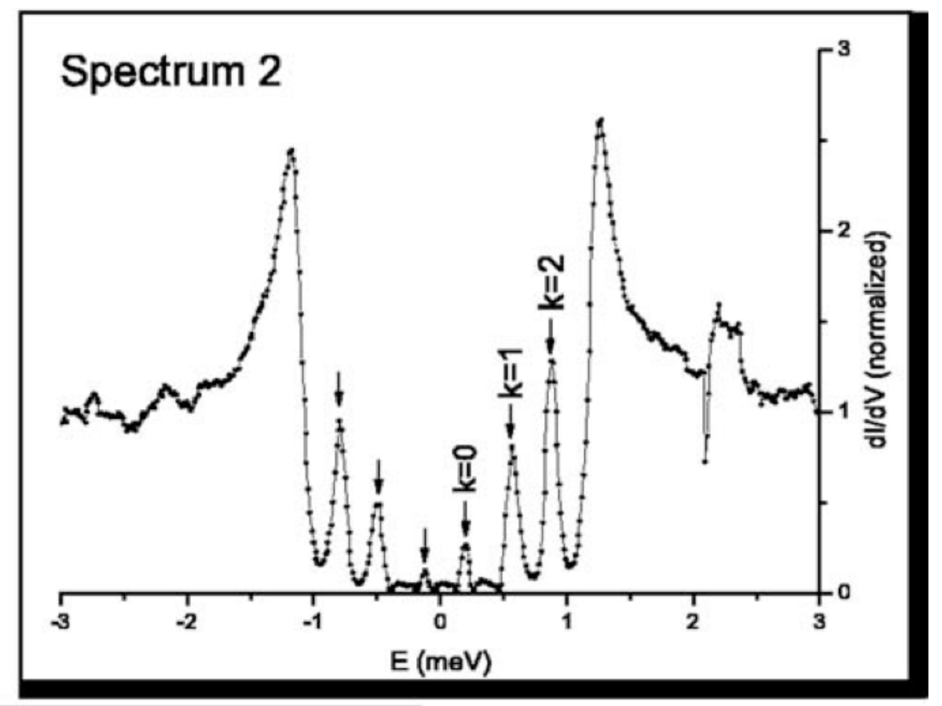
# Andreev states in 2D normal metal/SC

1D Normal metal - Superconductor

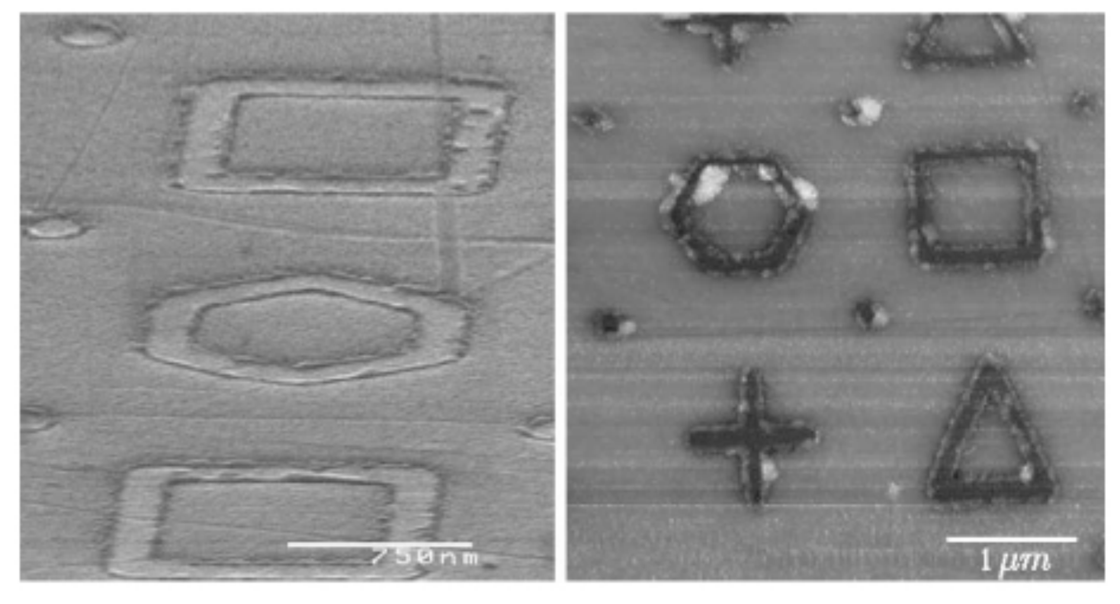


$$\frac{2a}{\xi_0}\frac{E}{\Delta} = n\pi + \arccos\frac{E}{\Delta}$$

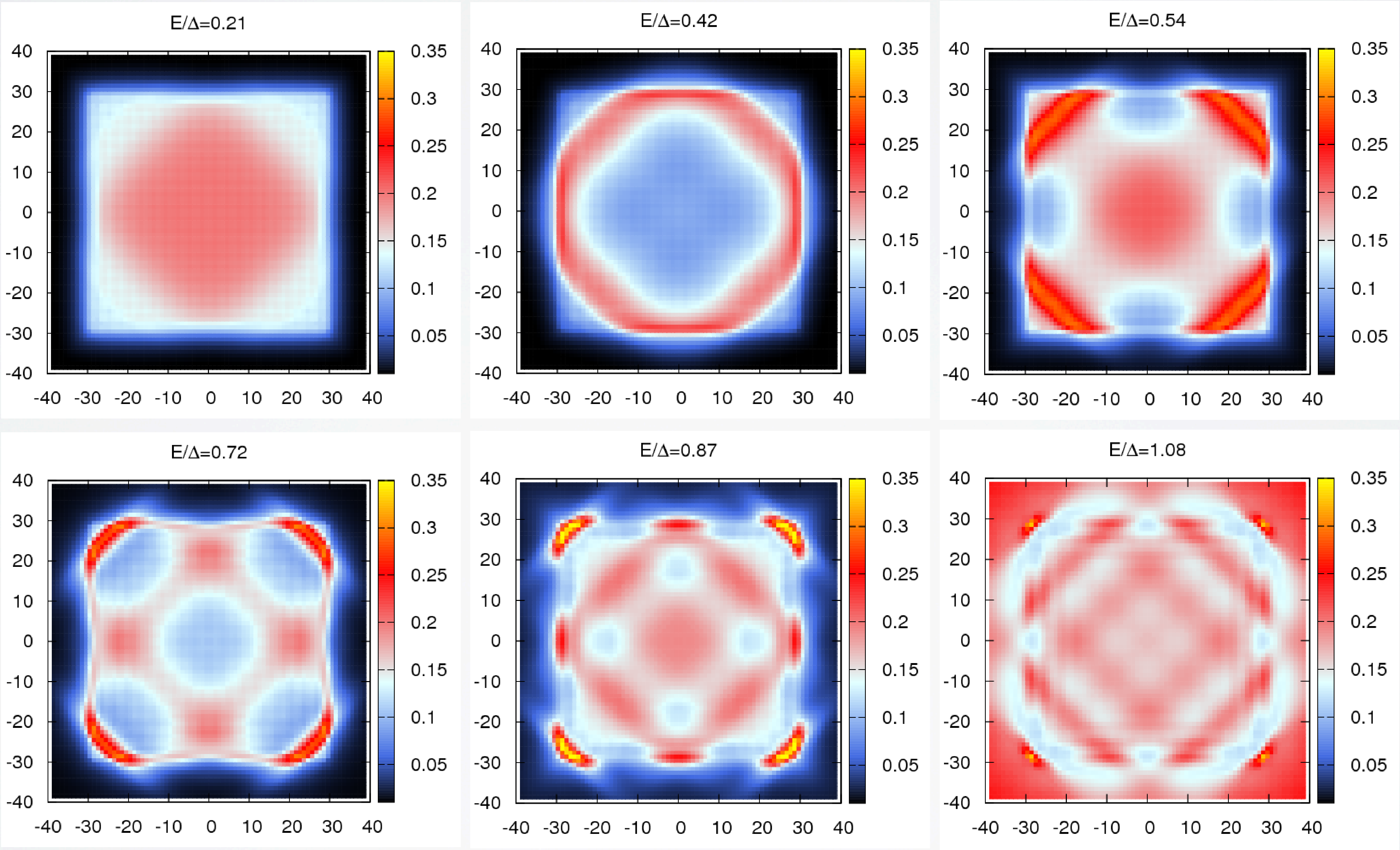




W. Escoffier et al., Phys. Rev. B 72, 140502(R) (2005)

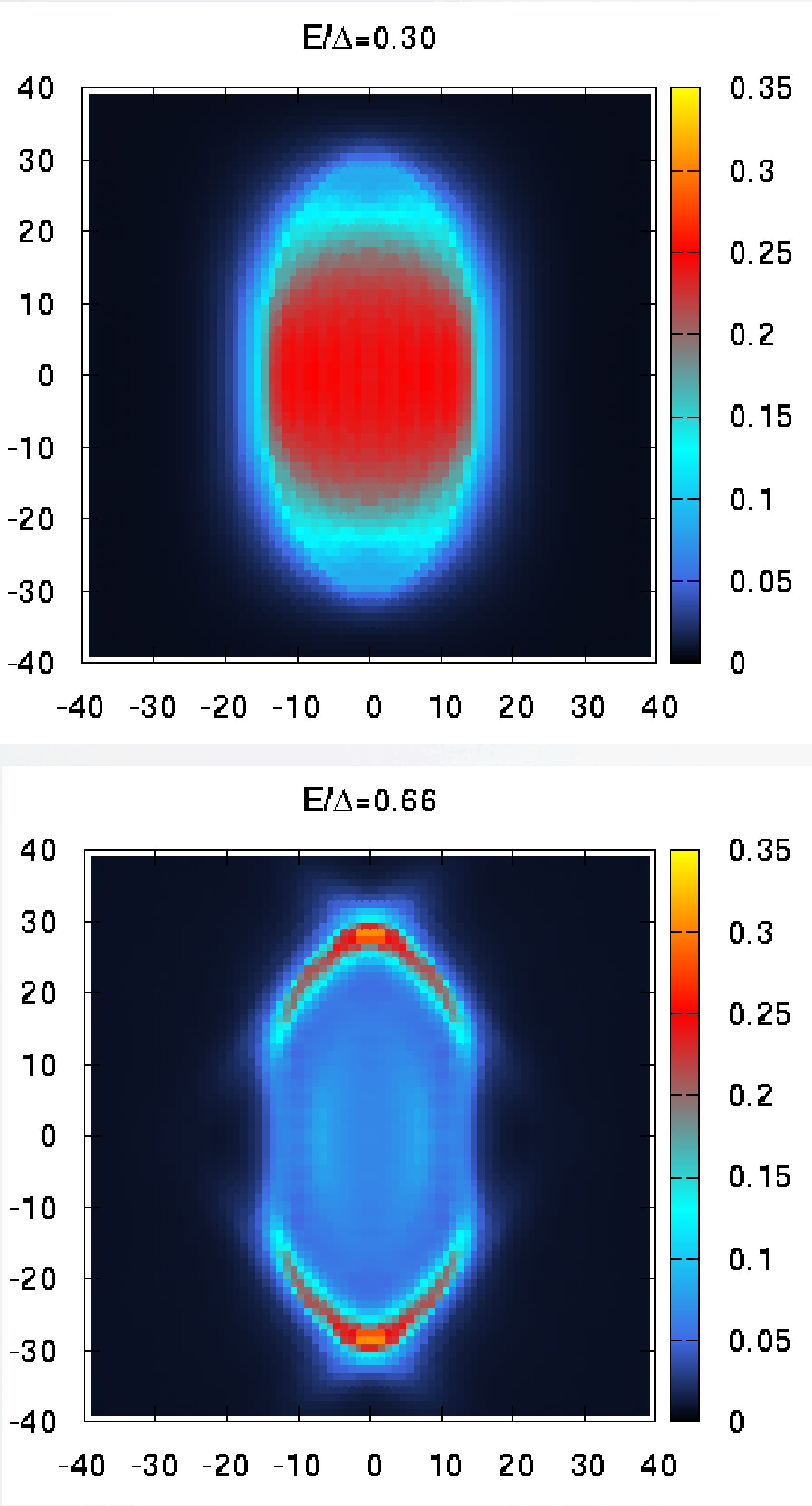


# Normal metal square in s-wave S: LDOS maps

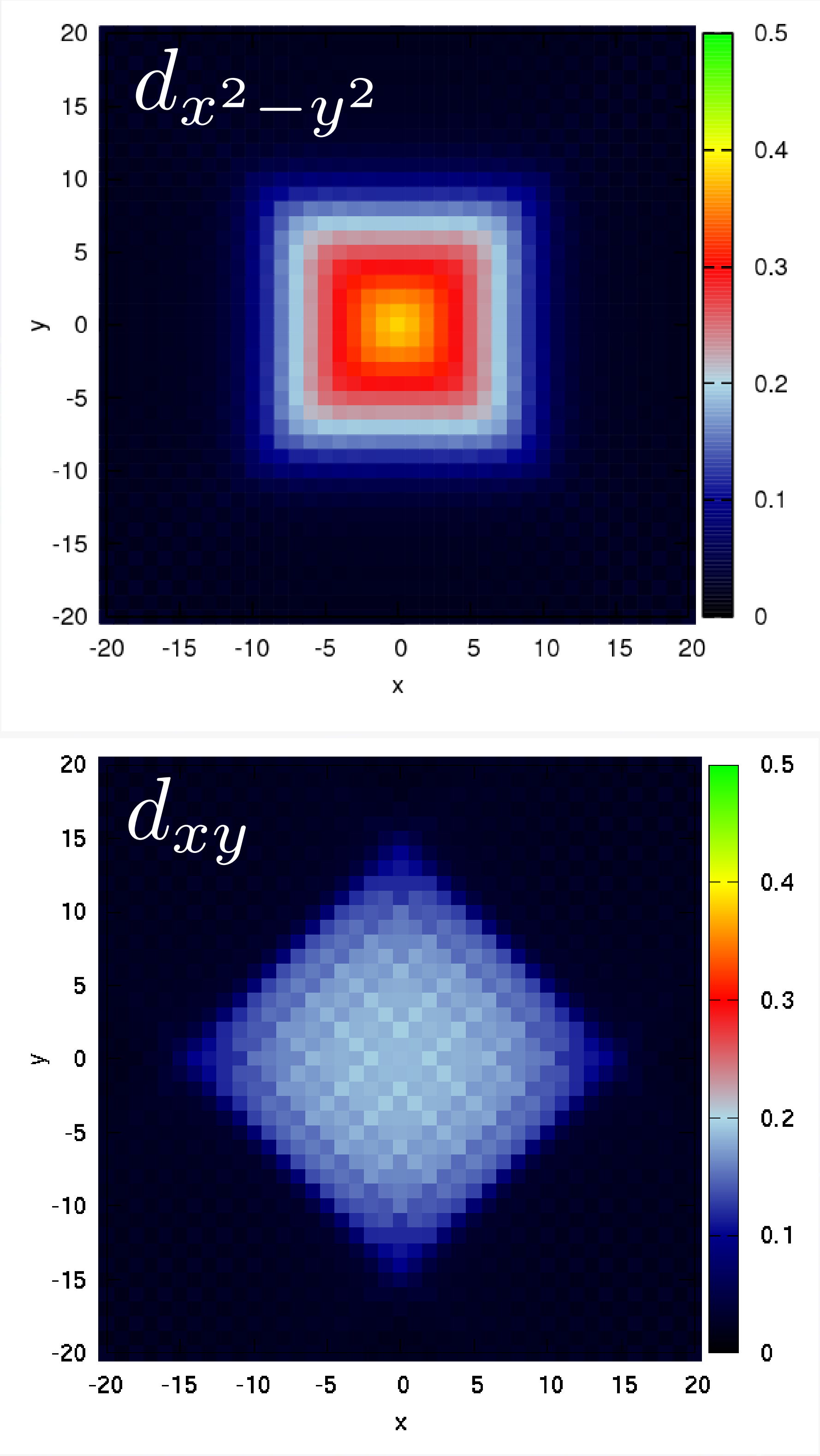


# Normal regions in SC: LDOS maps

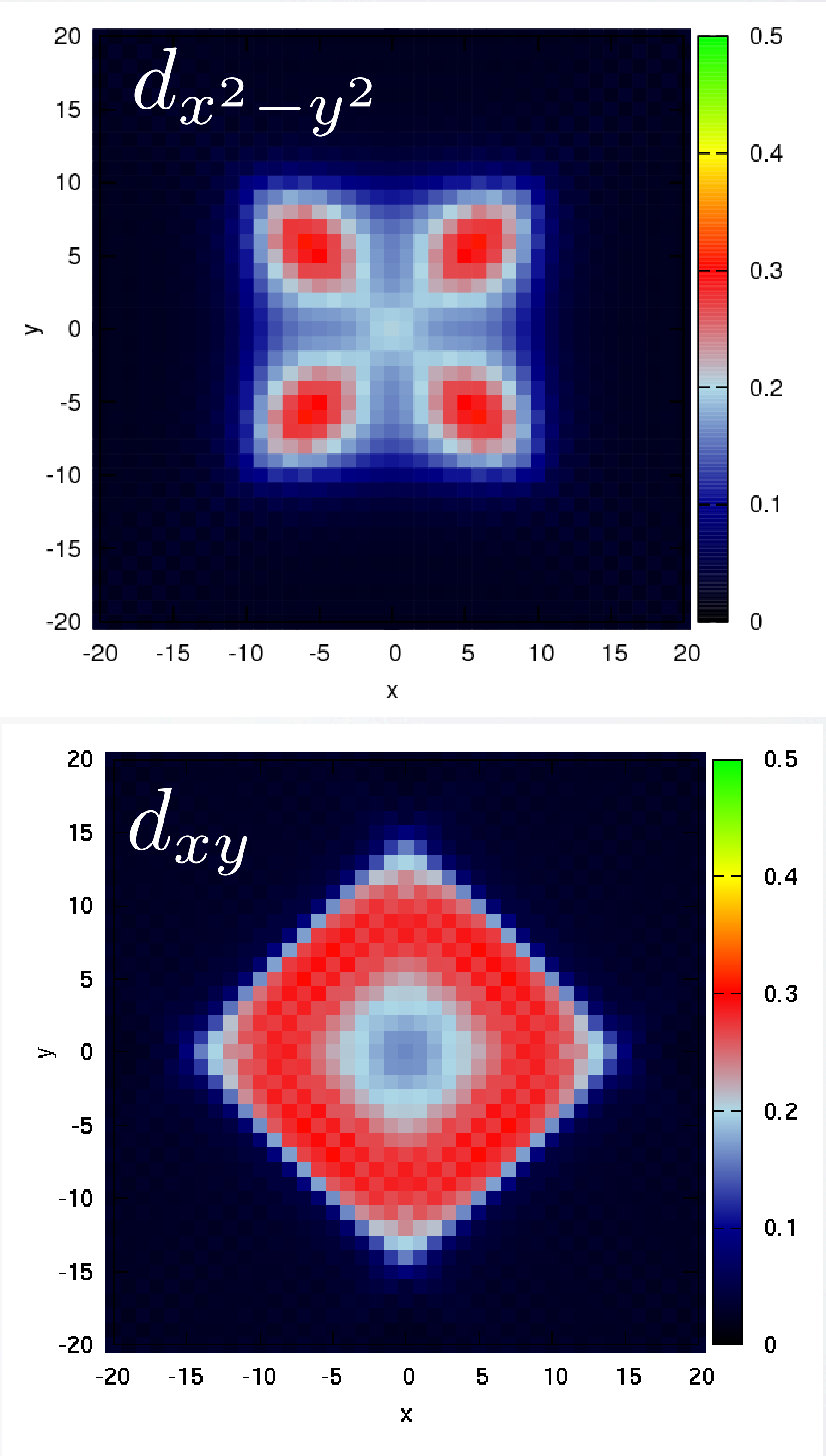
Ellipse(s-wave)



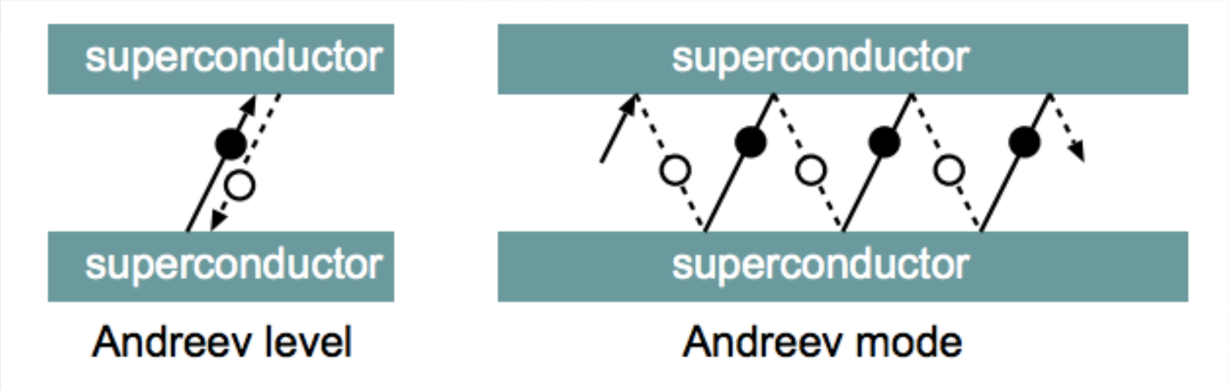
$E = 0$



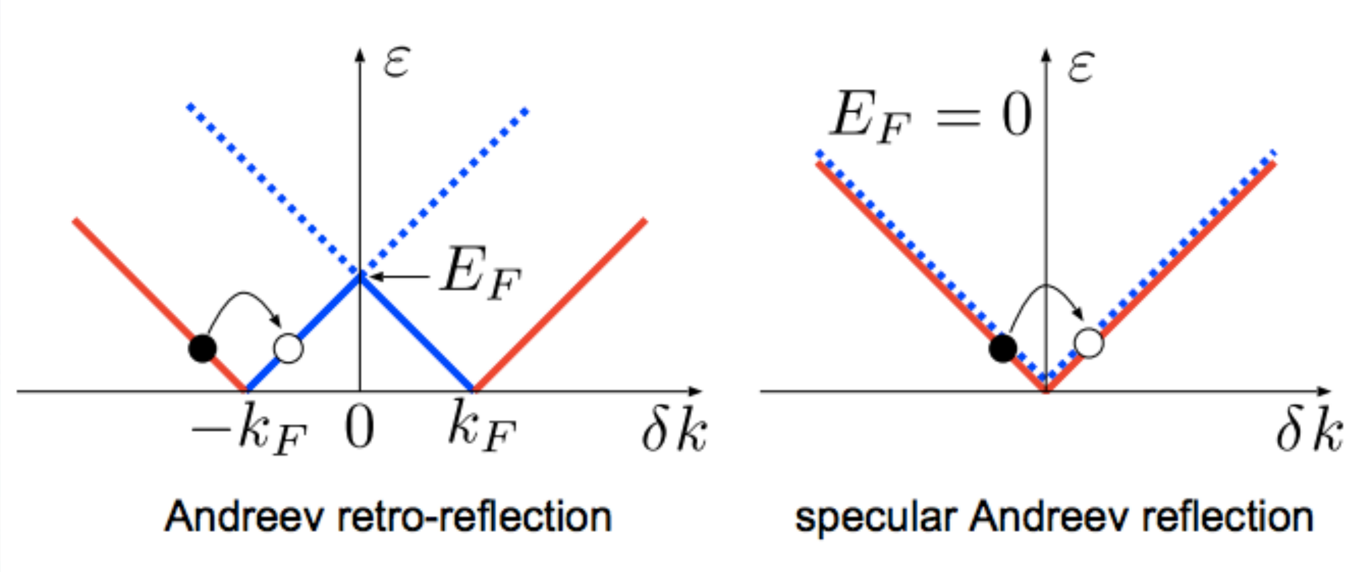
$E/\Delta = 0.3$



# Andreev reflection in graphene: SNS junctions



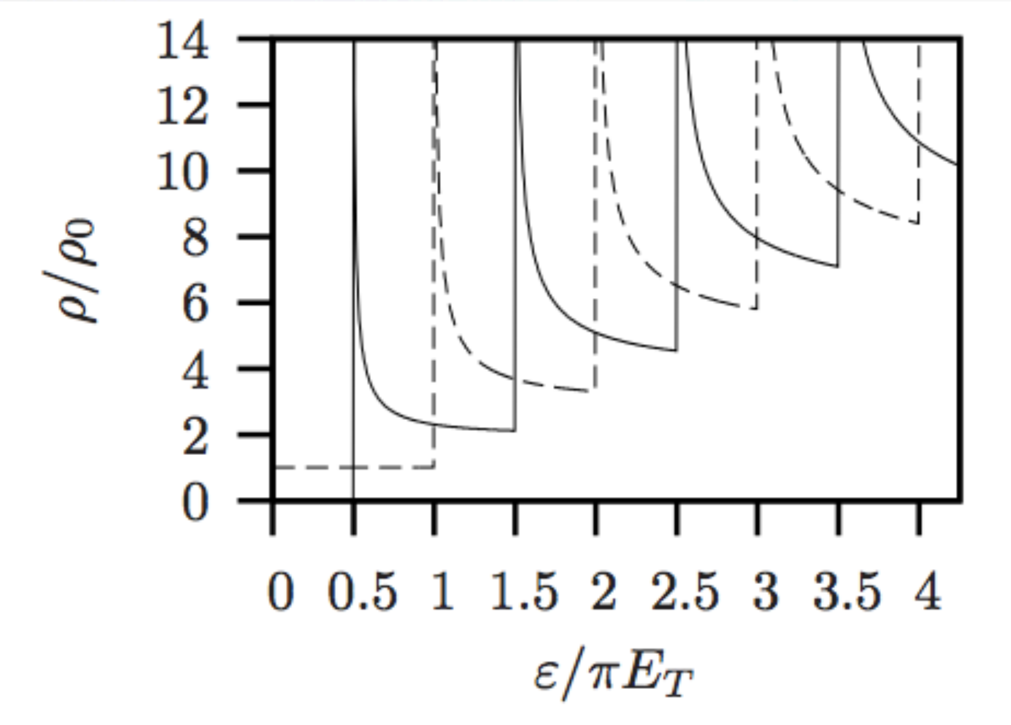
CWJ Beenaker, Rev.Mod.Phys. 80, 1337 (2008)

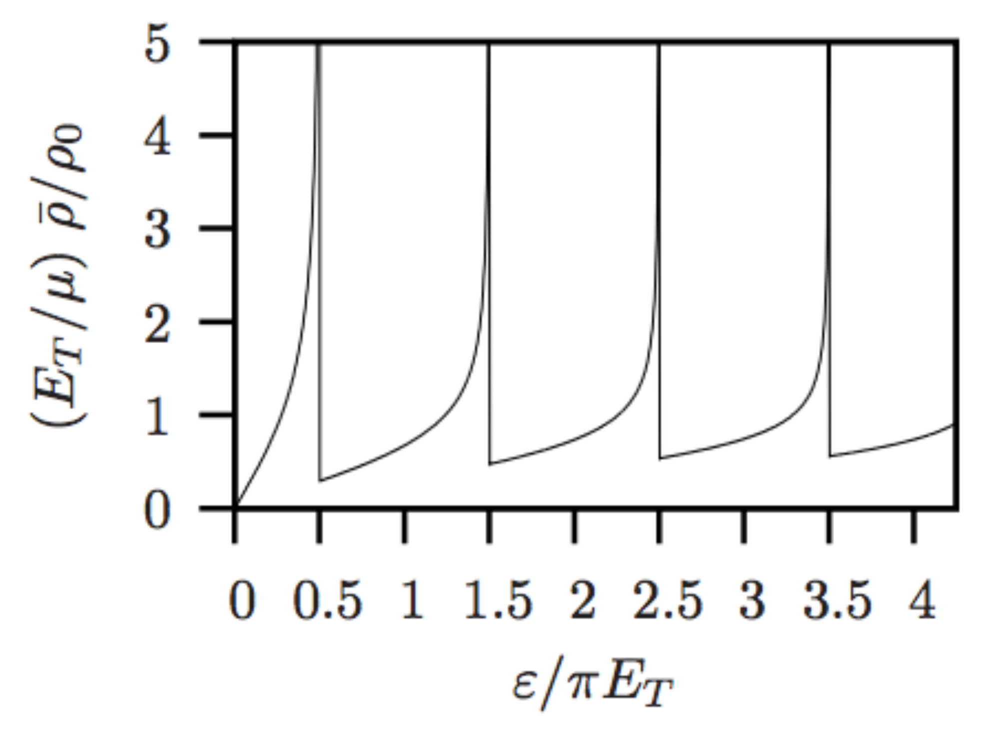


$\mu < E_T$

$$E_T = \hbar v_F / d$$

$\mu \gg E_T$



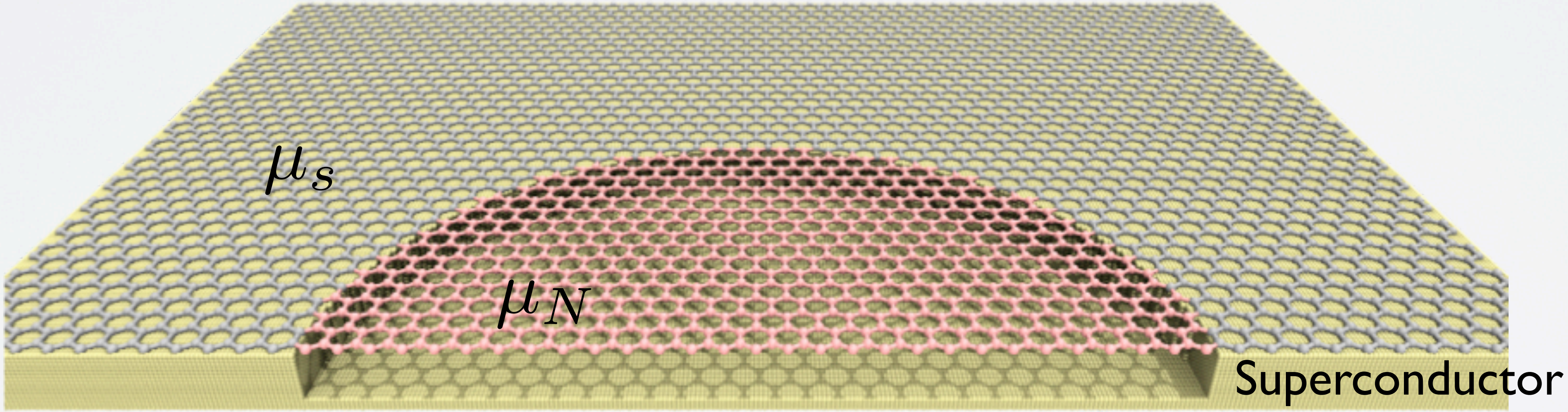


M. Titov, A. Ossipov, C. W. J. Beenakker, 75, 045417 (2007)

K. Halterman, O.T. Valls and M. Alidoust, Phys. Rev. B 84, 064509 (2011)

# Circular Andreev dot

- normal graphene circular region embedded in a superconducting one (due to proximity effect)



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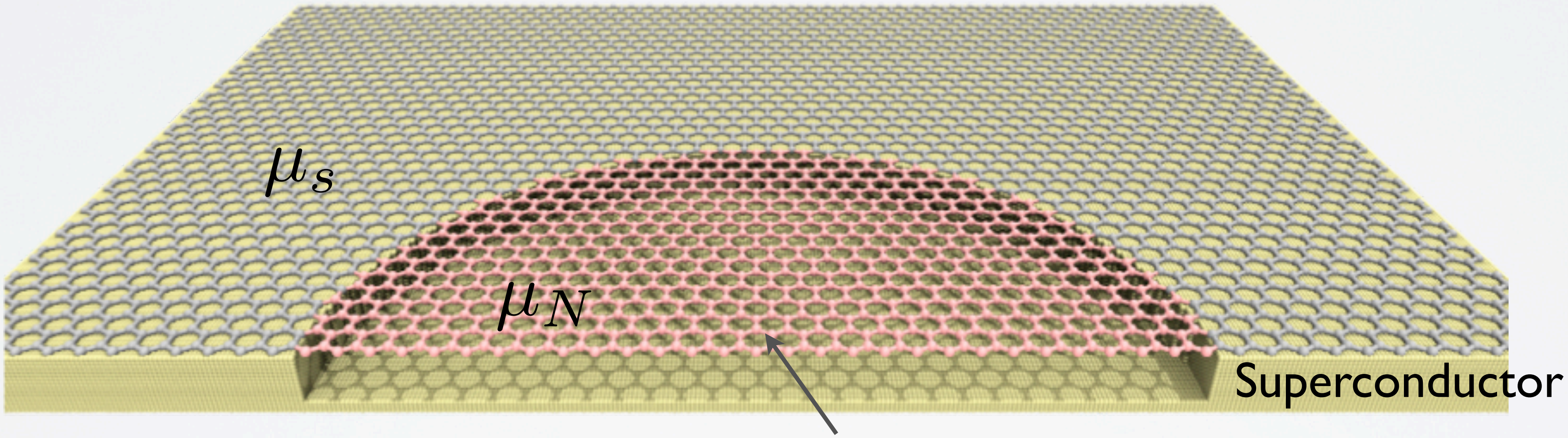
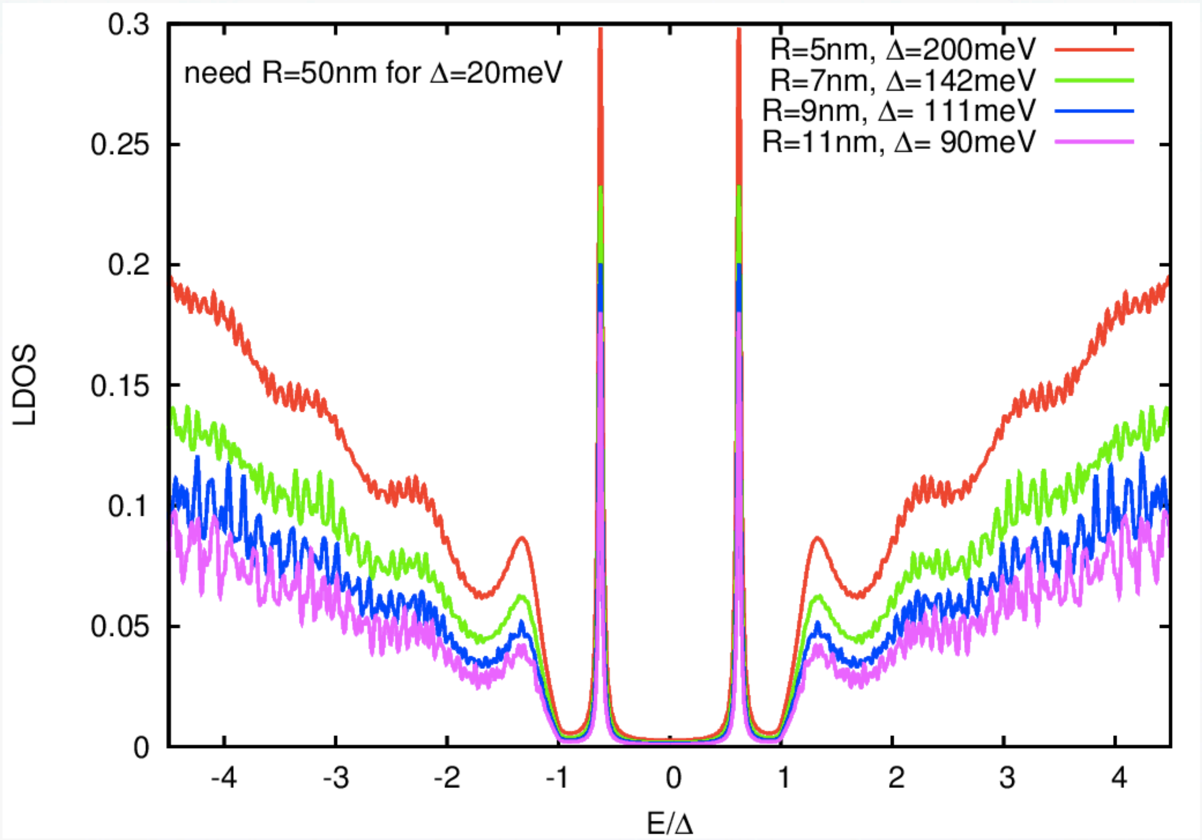




- keep $\xi/2R \sim (2R\Delta)^{-1}$ = const. –> the ABS are always at the same energy in units of $\Delta$

# Numerics: Green's functions approach, O(N) method

- expansion of the Green's function: $\hat{G}^{\alpha}_{ij}(t-t') = -\frac{i}{\hbar}\langle \mathcal{T} c_{i\alpha} c^{\dagger}_{j\alpha} \rangle$

- inhomogeneous strain ⟶ modify hopping amplitudes $\boxed{t_{ij} = t_0 e^{-3.37(\frac{|r_i - r_j|}{a} - 1)}}$

Chebyshev expansion of each spatial component (i,j): trivial parallelization

$$\boxed{G_{ij}(\omega) = \frac{-i}{\sqrt{1-\omega^2}}\left[\mu_0 + 2\sum_{n=1}^{\infty}\mu_n e^{-in\ arccos(\omega)}\right]}$$

$$\mu_n = \langle i \mid T_n(H) \, | j \rangle$$

$$T_n(x) = cos[n \ \ arcos(x)]$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}$$

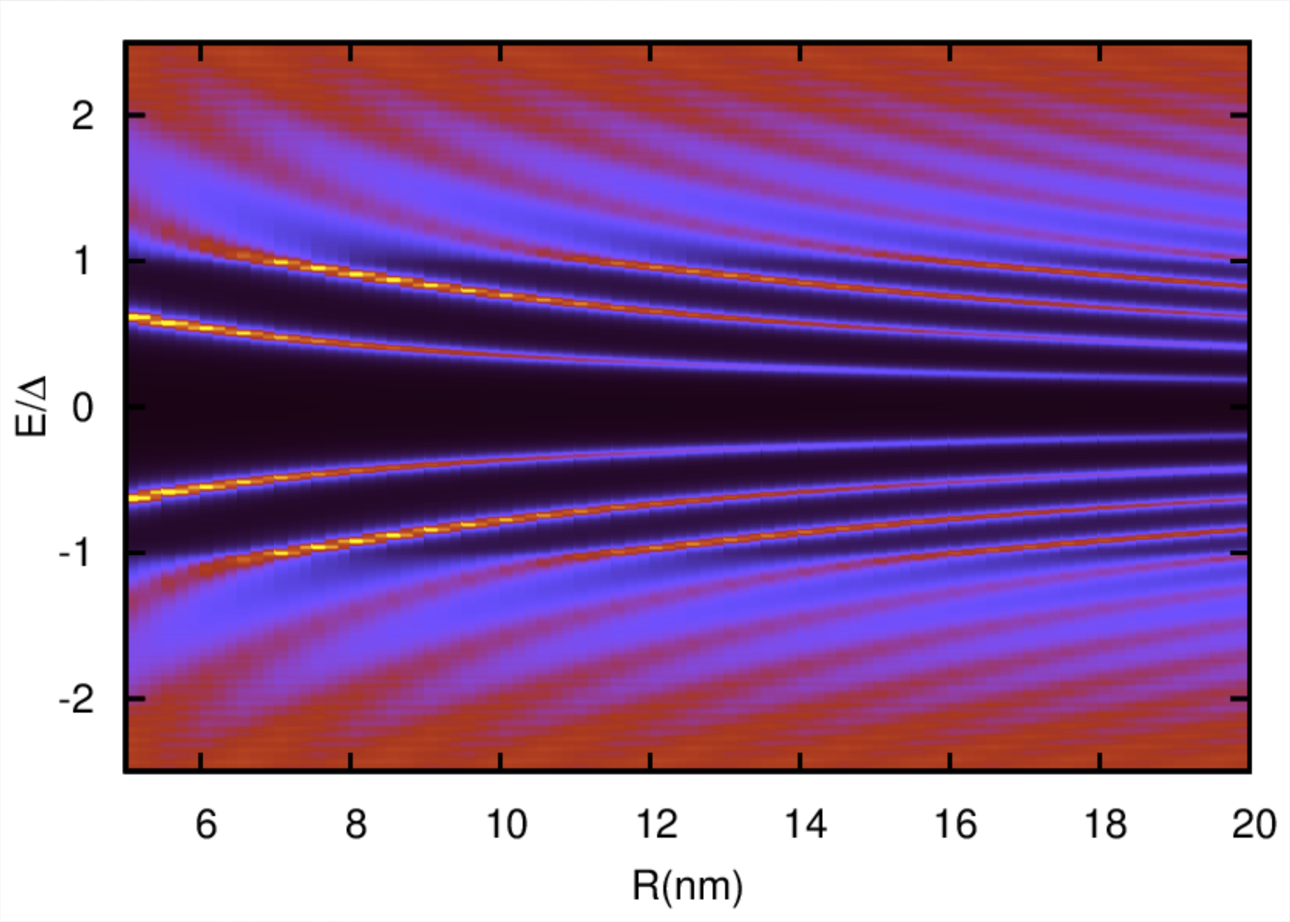
- Recursive procedure using only sparse matrix-vector multiplication : O(N)
- **Significant** speed improvement (x50) when using GPUs clusters

A. Weisse et al., Rev. Mod. Phys. 78, 275 (2006)

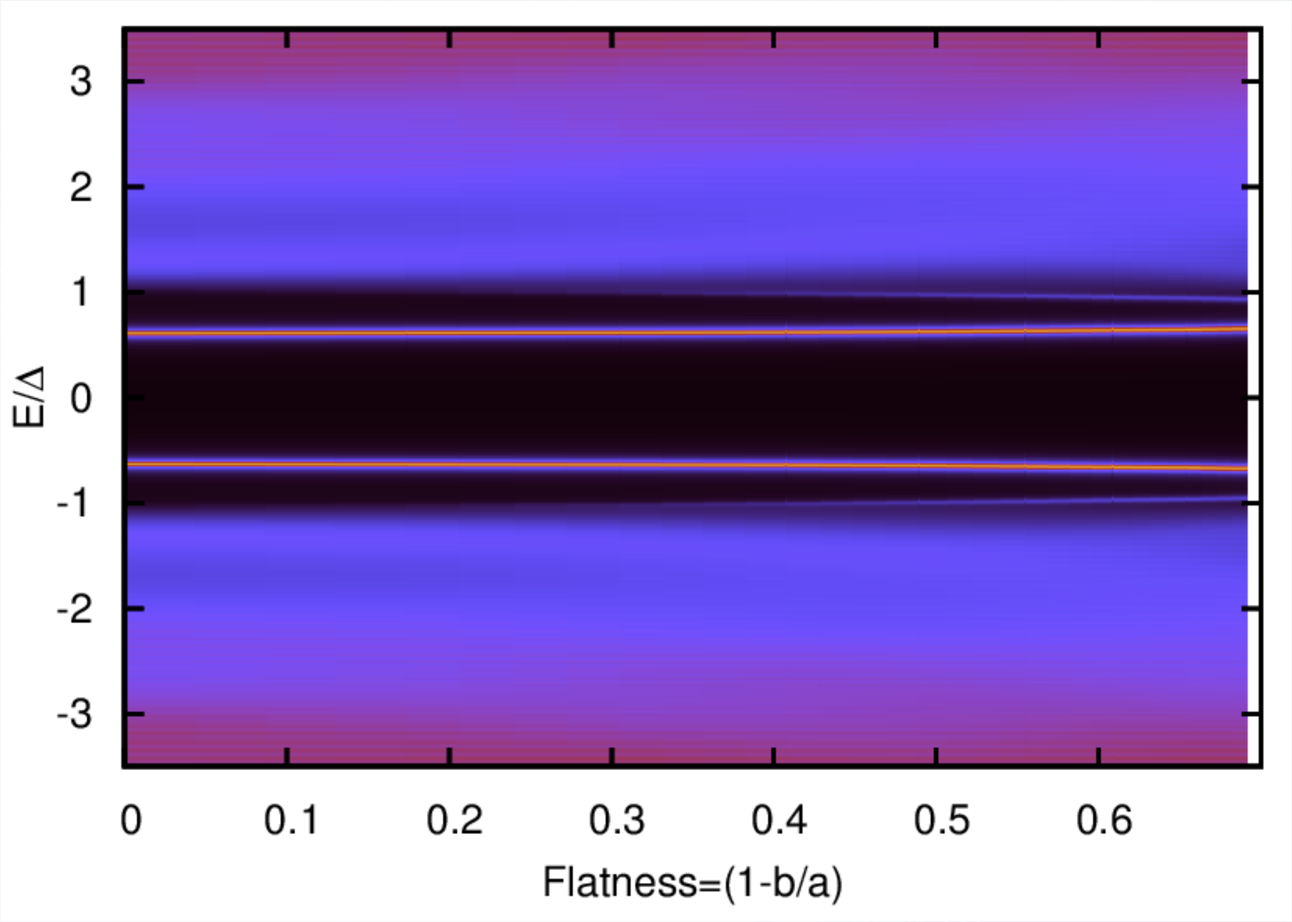
L. Covaci, F. Peeters and M. Berciu, Phys. Rev. Lett. 105, 167006 (2010)

# Circular Andreev dot: size dependence

Dependence on disc radius

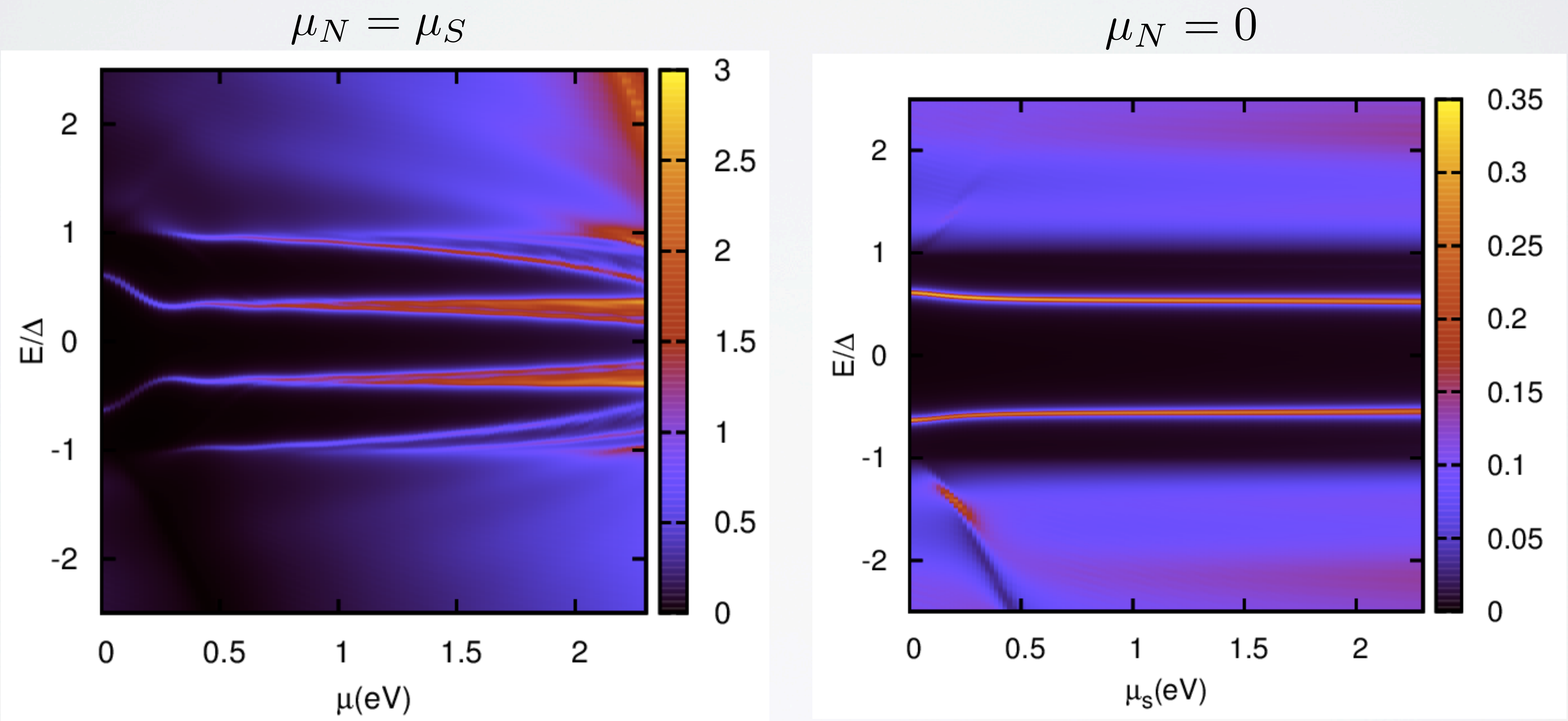


Dependence on flatness of ellipse



- ✦ multiple Andreev states enter the sub-gap region (visible also above the gap)
- ✦ the ellipsoidal dots still show well defined ABS (in contrast to regular NS system)
  - ✳ chaotic Andreev billiard is expected for the ellipse, but **not** observed here
  - ✳ quasiclassical billiard picture is not valid in this regime

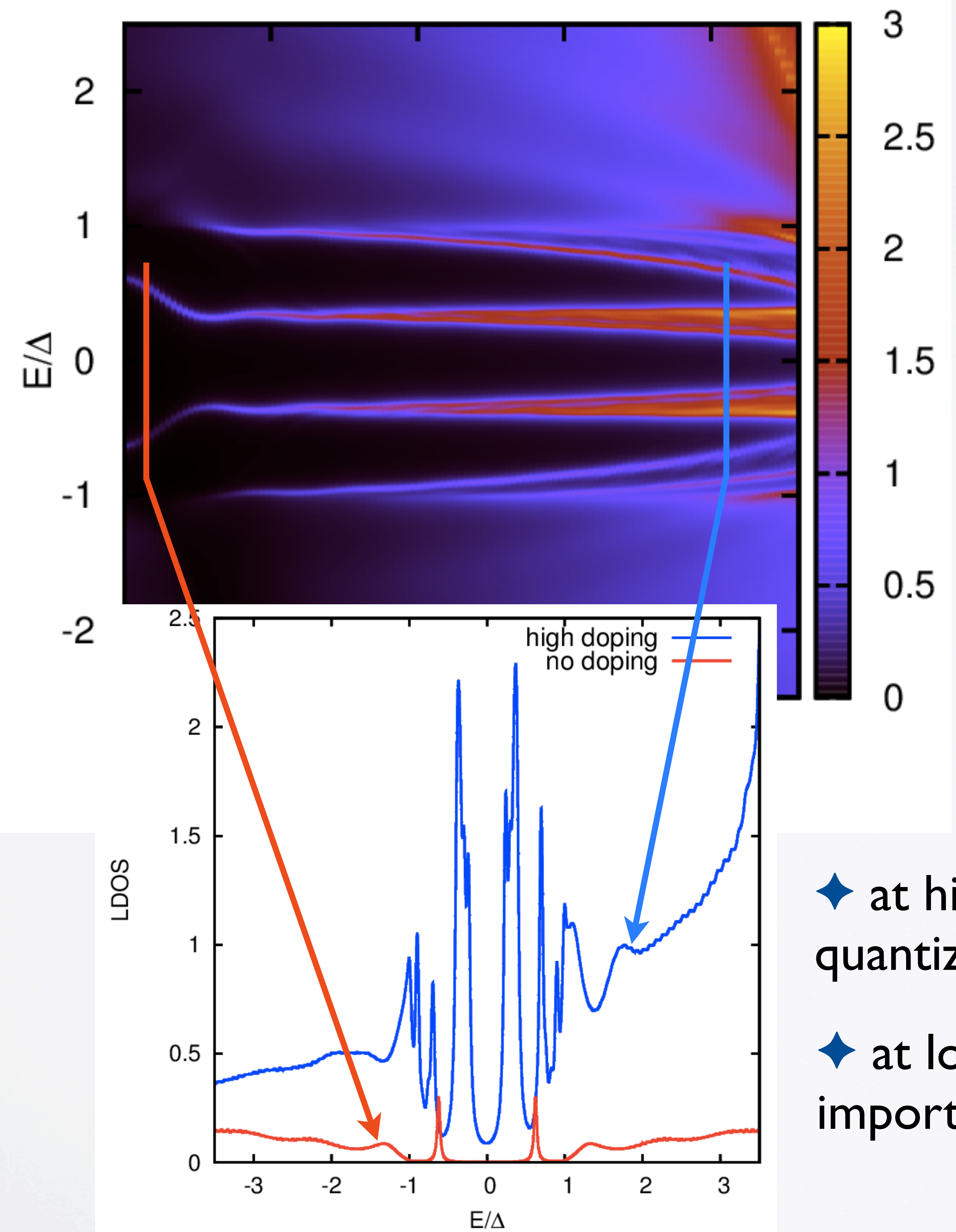
# Circular Andreev dot: doping dependence



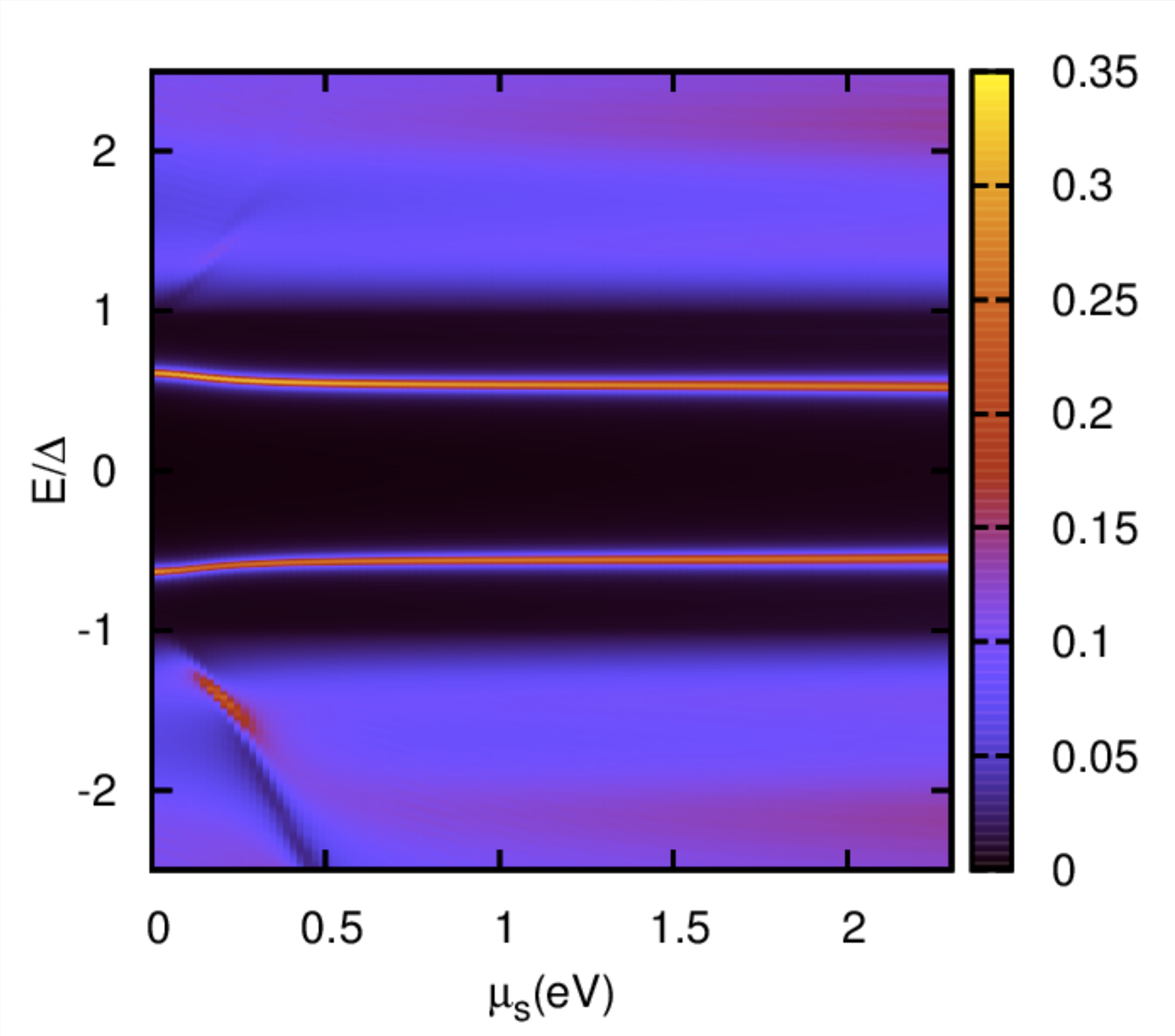
- ✦ at high-doping, the ABS LDOS does not show quantized level but continuum of states
- ✦ at low-doping, the type of scattering is not important for the quantization of the ABS energies

# Circular Andreev dot: doping dependence

$\mu_N = \mu_S$



$\mu_N = 0$



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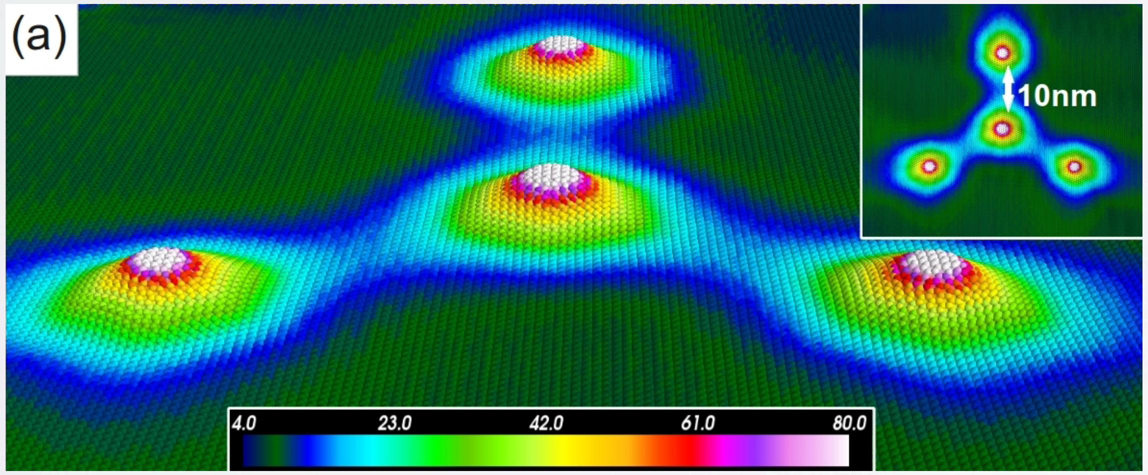
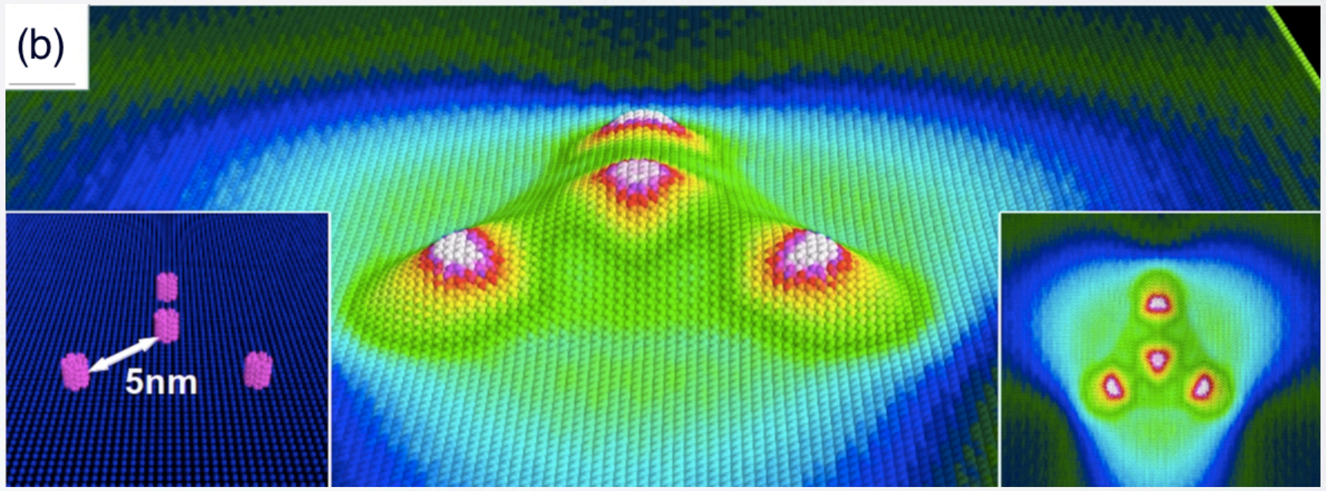
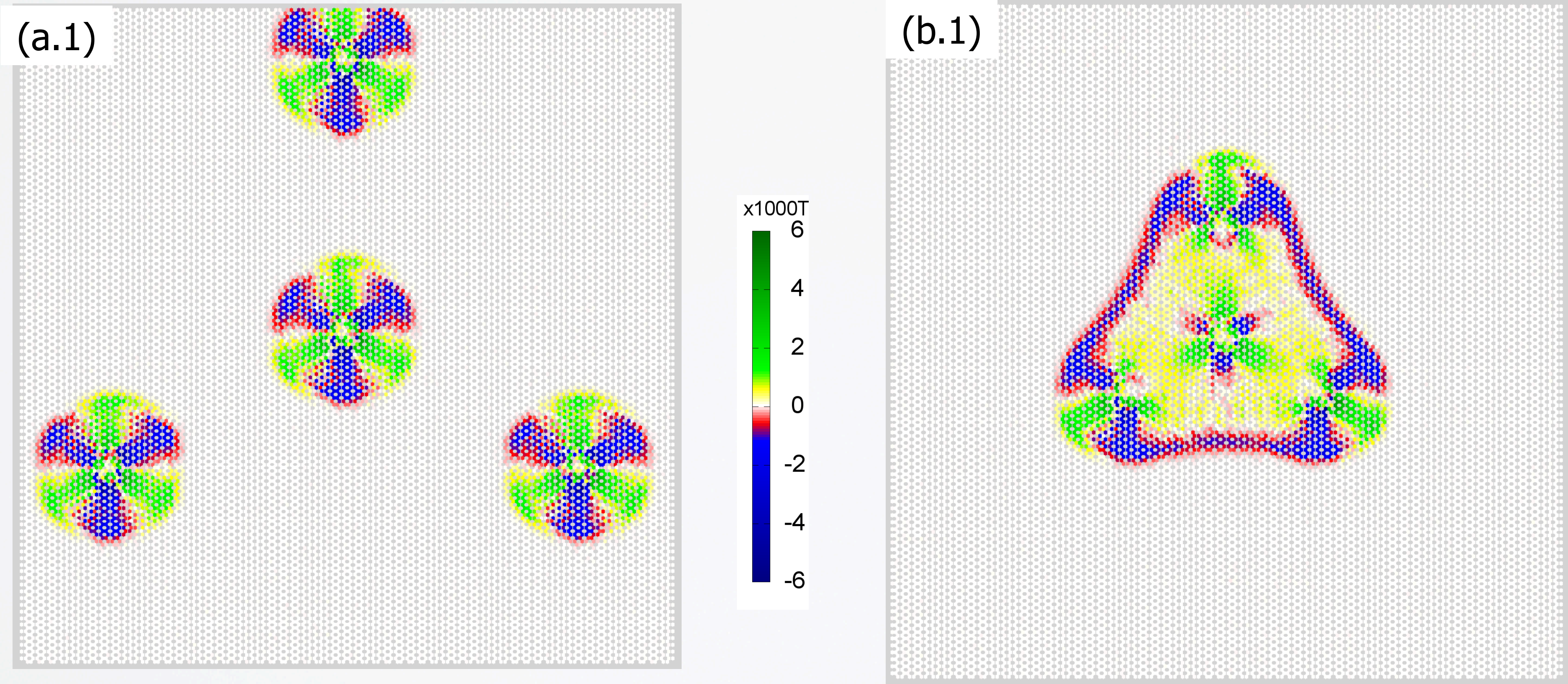
# Nano-engineering of strain configurations

## Molecular dynamics simulation of graphene sheet over substrate + pillars





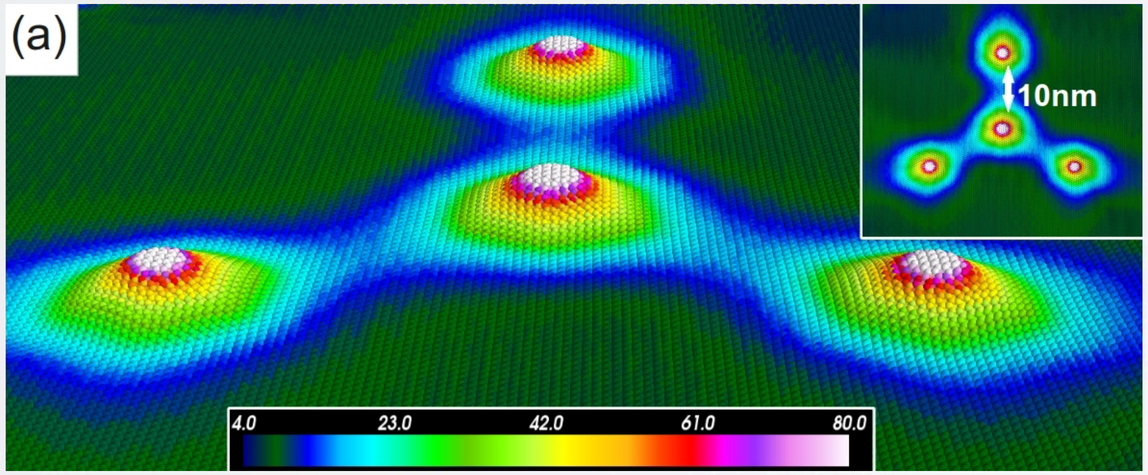
## Pseudo-magnetic field

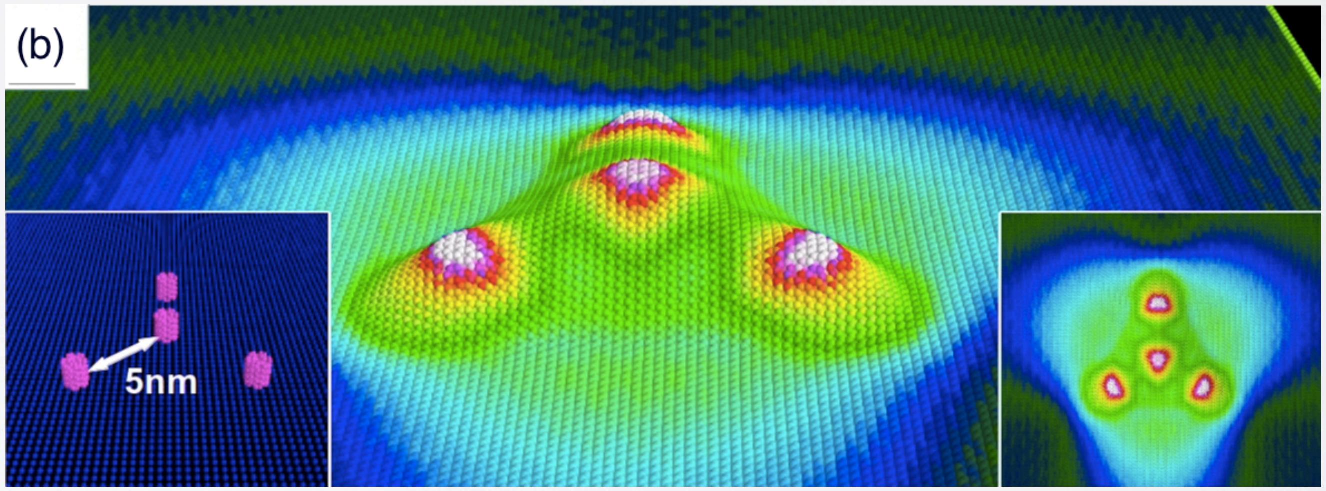


M. Neek-Amal, L. Covaci and F.M. Peeters, Phys. Rev. B 86, 041405(R) (2012)

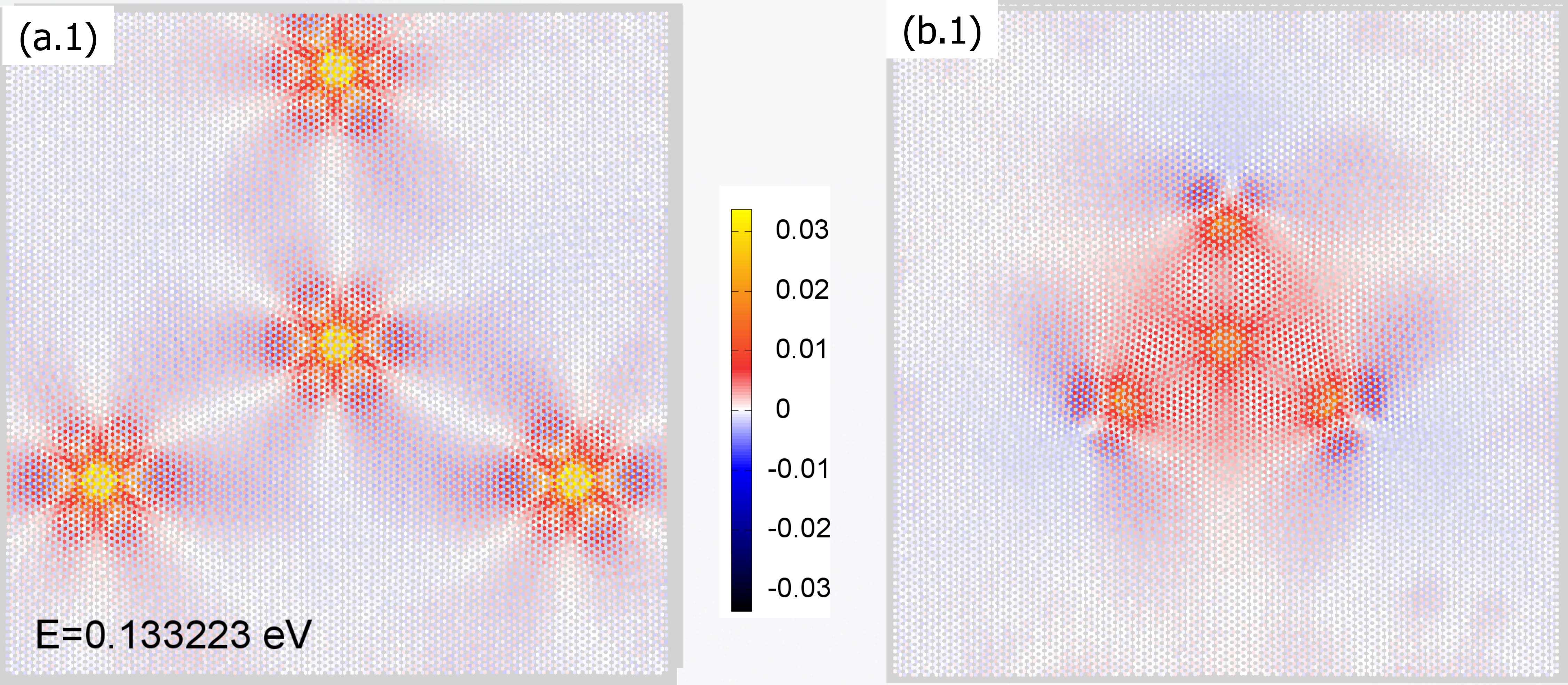
# Nano-engineering of strain configurations

Molecular dynamics simulation of graphene sheet over substrate + pillars



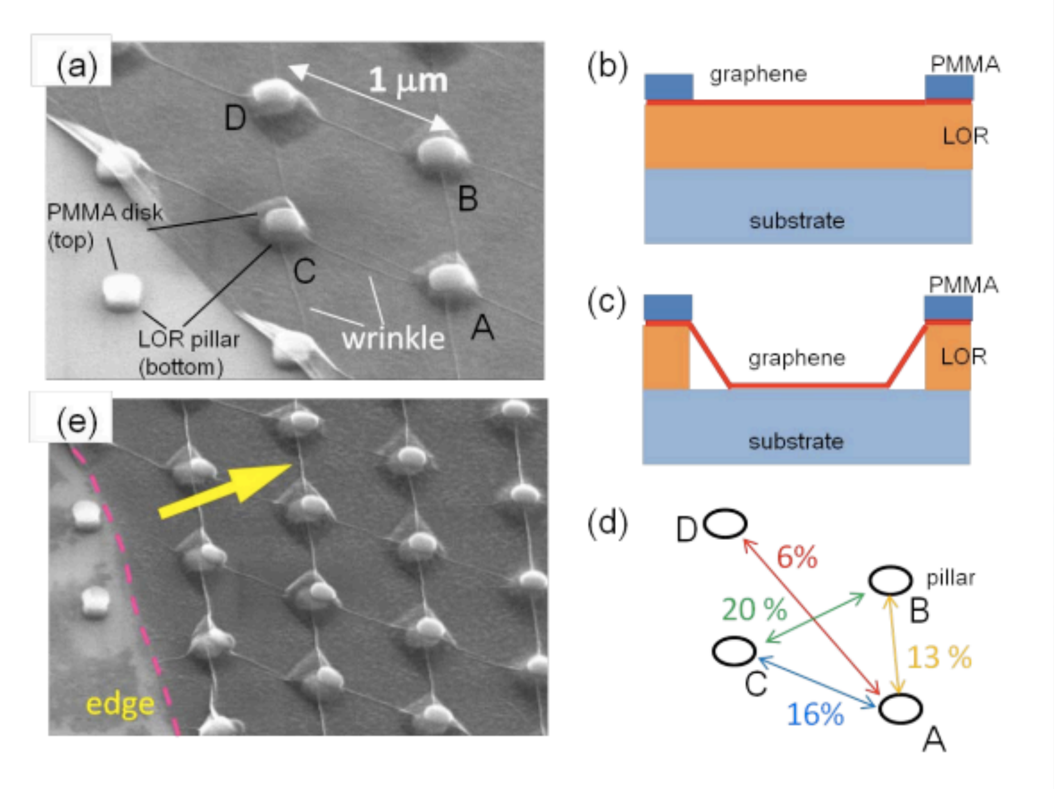


## LDOS

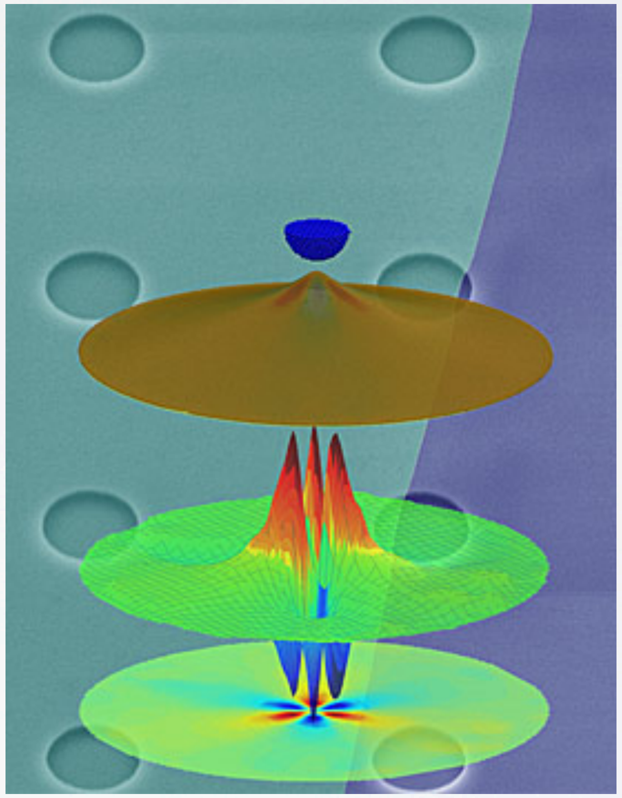


M. Neek-Amal, L. Covaci and F.M. Peeters, Phys. Rev. B 86, 041405(R) (2012)

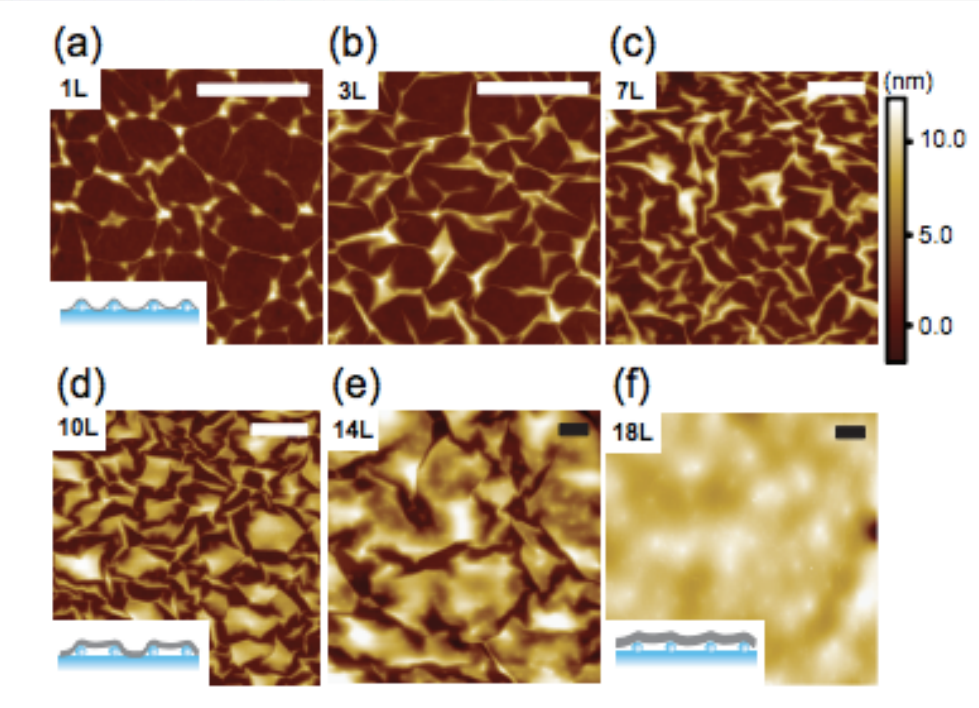
# Gaussian-like deformations (experimental)



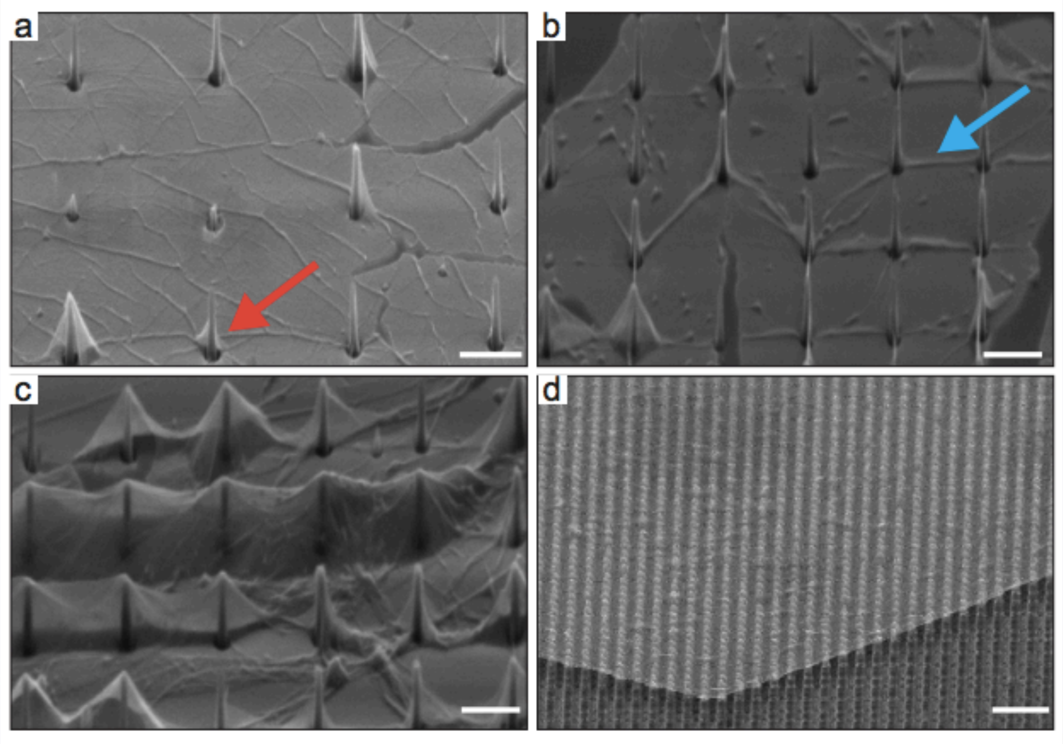
H. Tomori et al., Appl. Phys Exp. 4, 07510 (2012)



Klimov et al., Science 336, 1557 (2012)

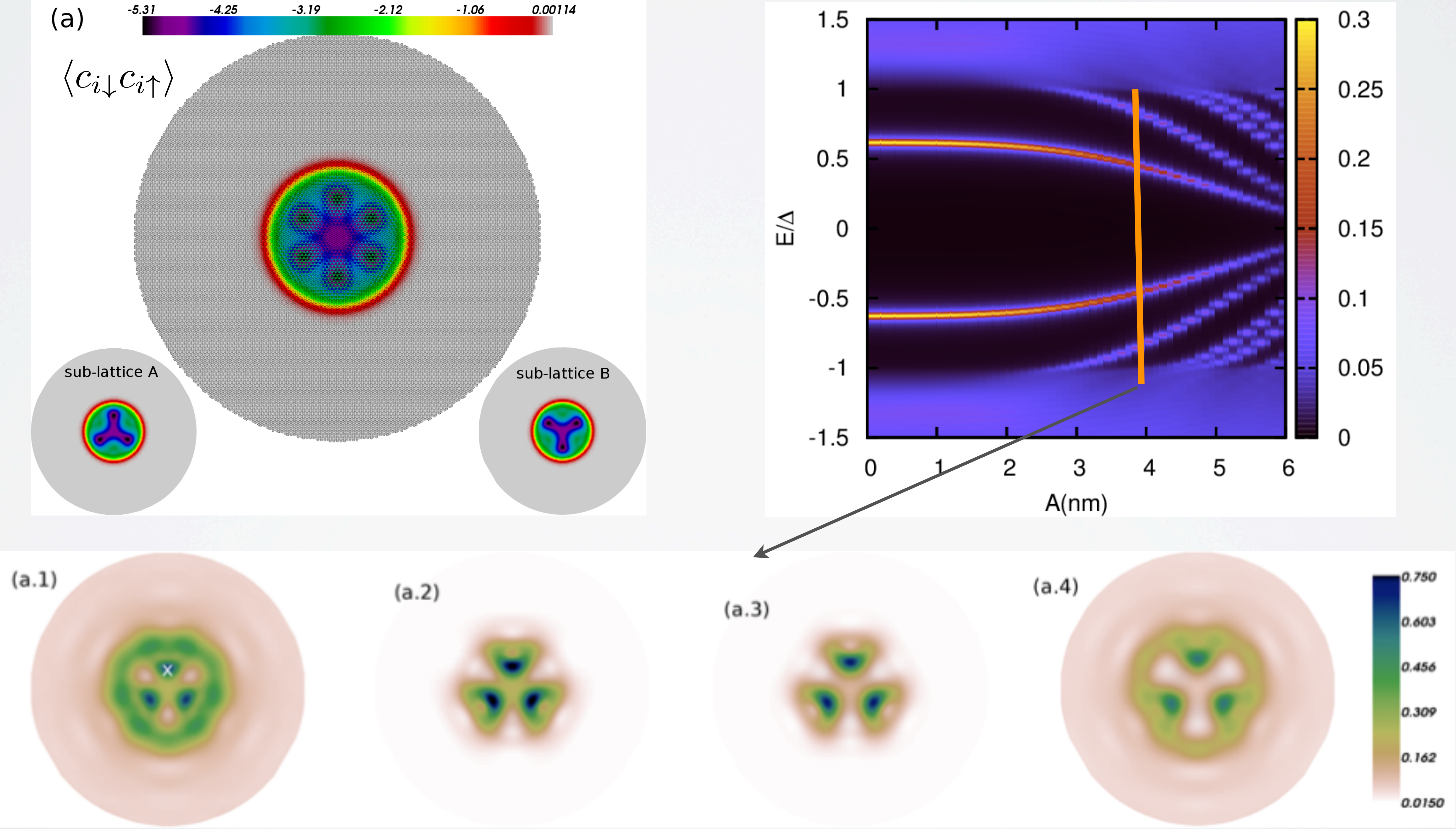


M. Yamamoto et al., Phys. Rev. X 2, 041018 (2012)



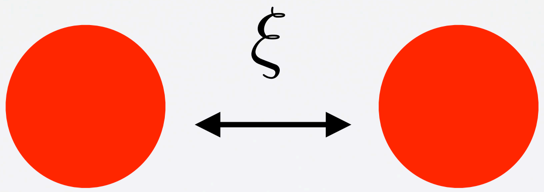
A.R. Plantey et al, arxiv:1404.5783

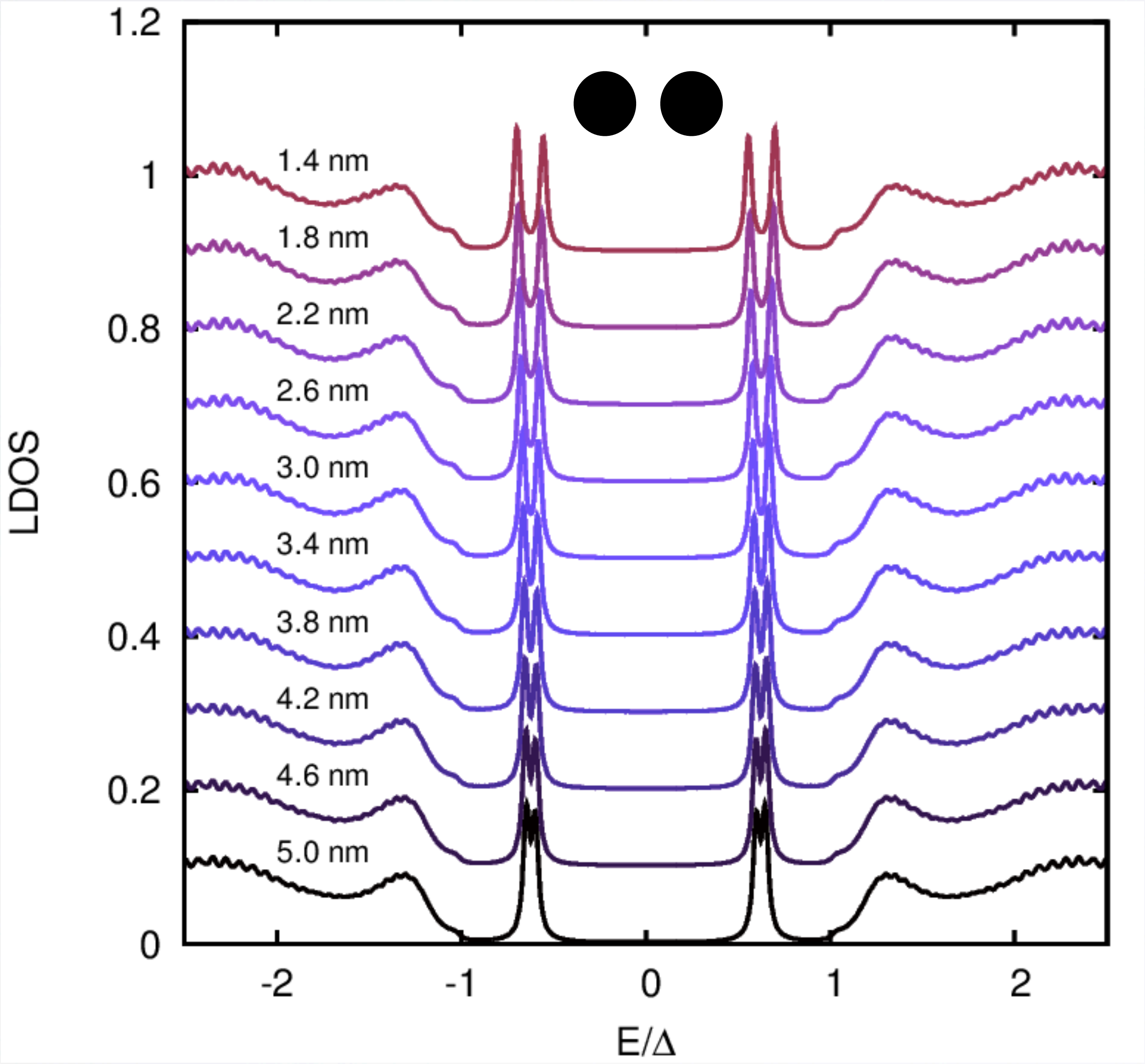
# How to manipulate ABS? 1. Gaussian deformation

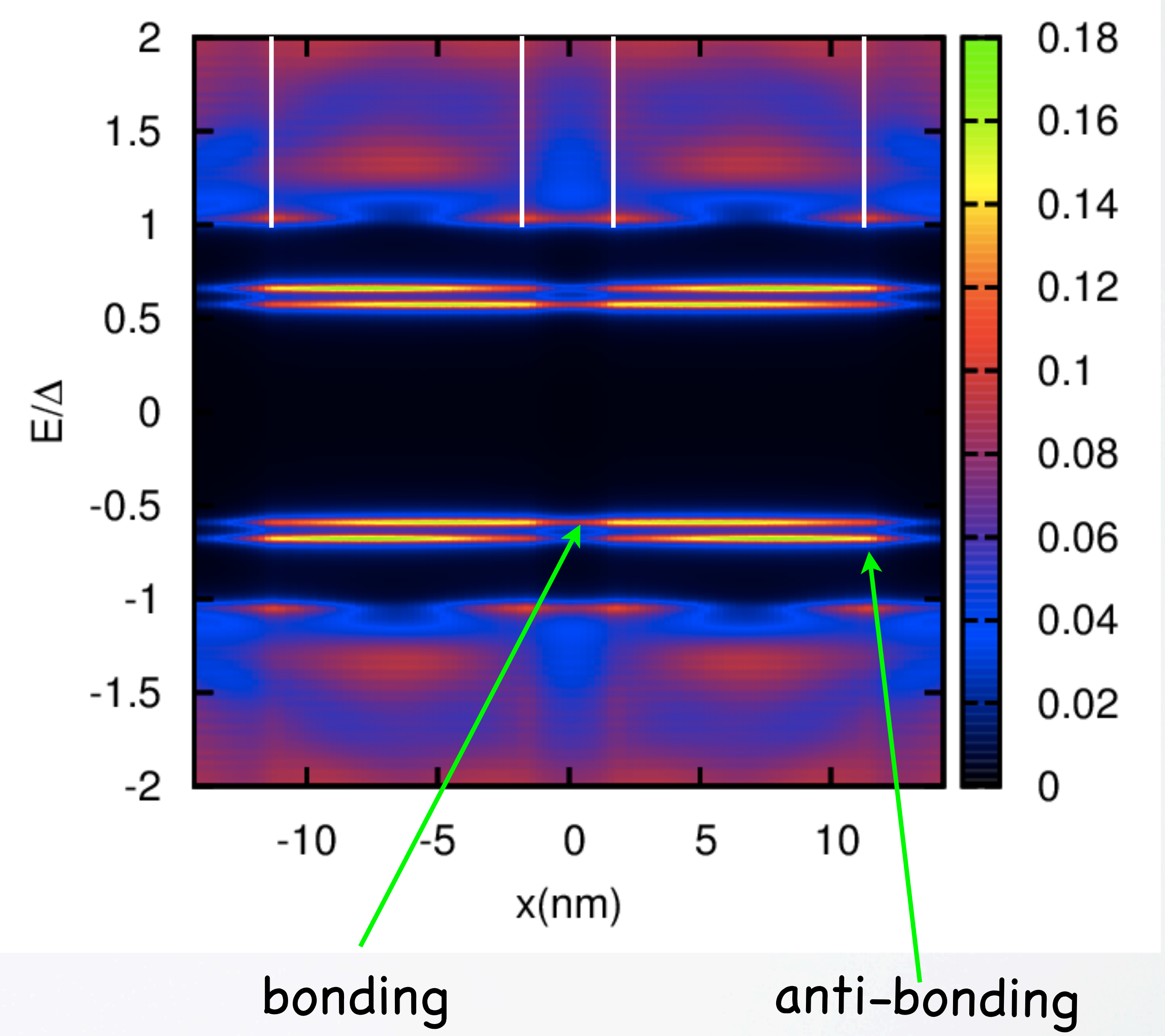


- ✦ the time between scatterings increases, therefore the energy gap is suppressed
- ✦ broken sub-lattice symmetry, modification of the wave-function

# How to manipulate ABS? 2. Inter-dot coupling







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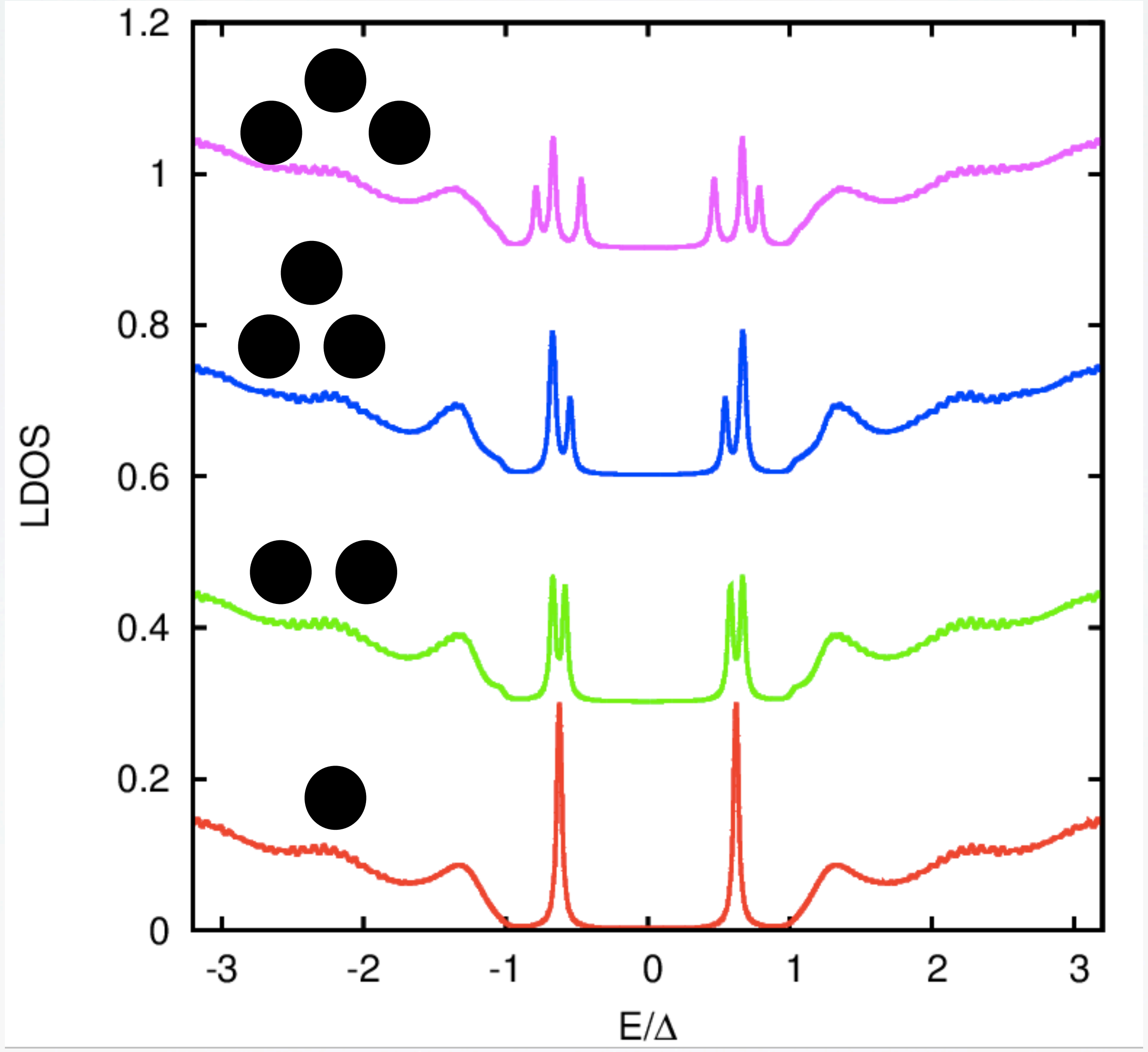
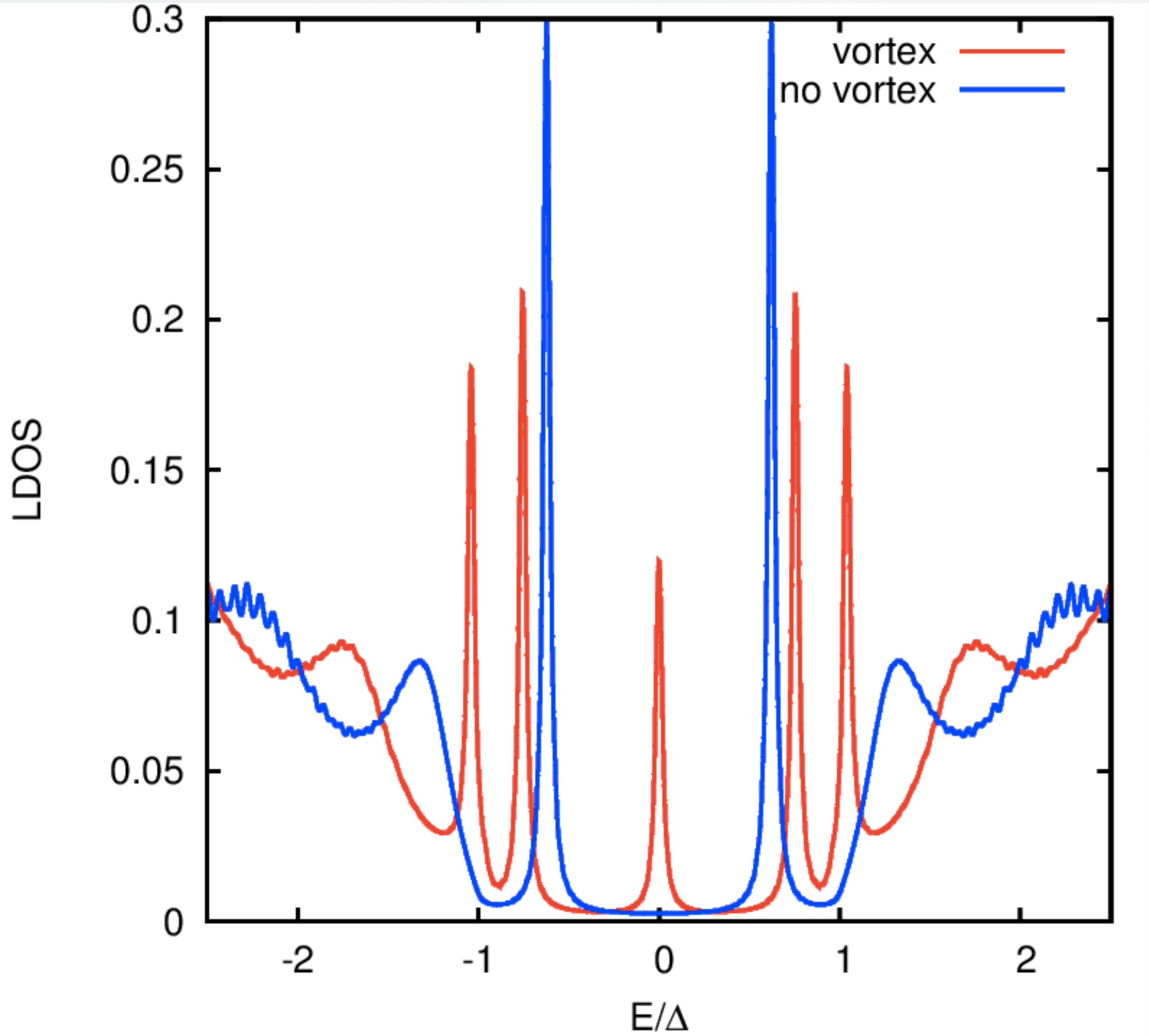
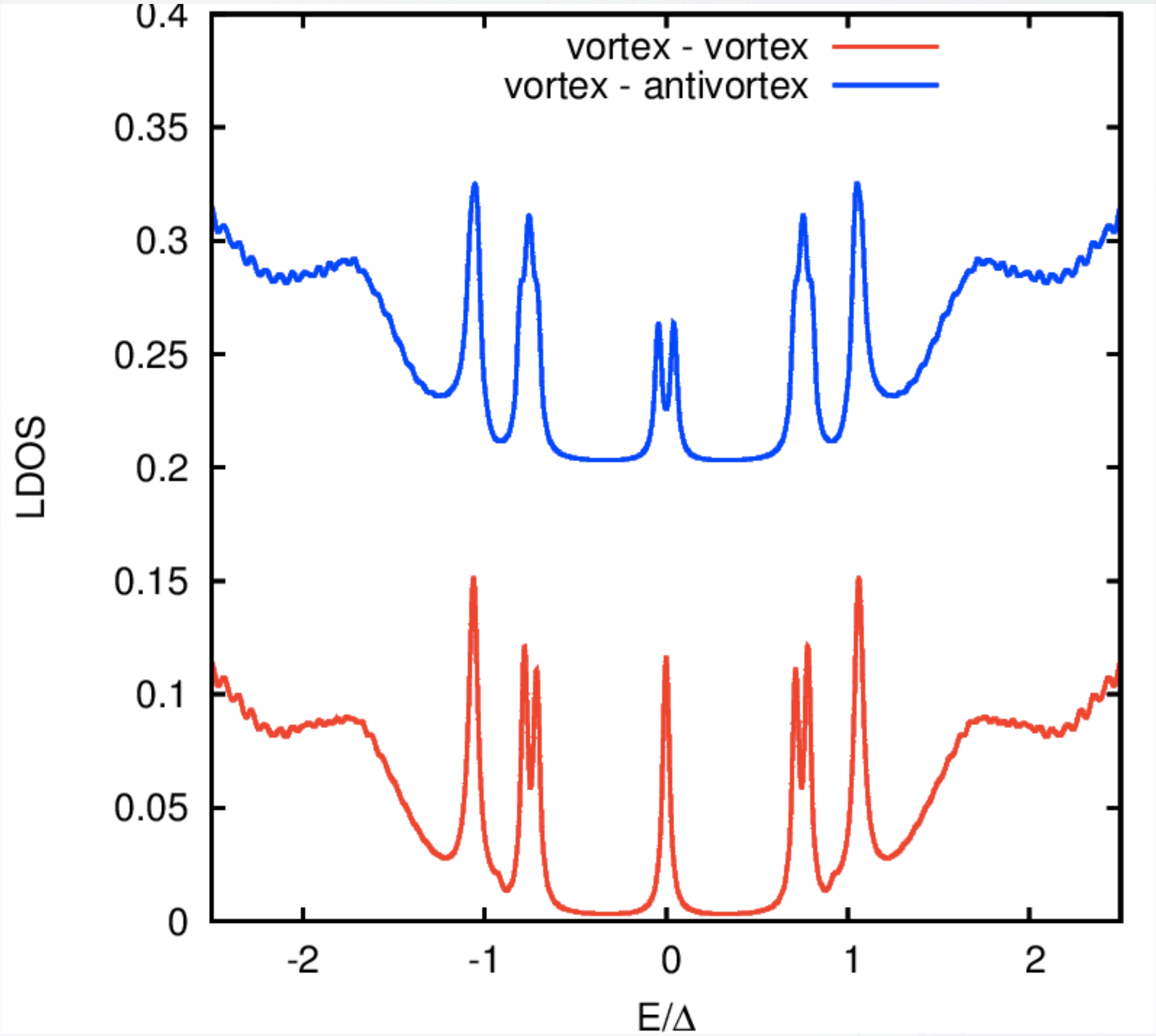


- ✦ multiple dots can be easily coupled through the superconducting contacts
- ✦ devise artificial atoms, artificial lattice or QD structures

# How to manipulate ABS? 3.Add vortices in the SC





I. M. Khaymovich et. al, Phys. Rev. B 79, 224506 (2009)

P. Ghaemi and F. Wilczek, Phys. Scr. T 146 (2012) 014019

D.L. Bergman and K. Le Hur, Phys Rev. B 79, 184520 (2009)

- ✦ zero energy modes in the vortex core (Majorana states) for all vorticities
- ✦ unfortunately there are even # of pairs of Majorana modes = fermions in each core
  - ✳ no non-Abelian statistics (add spin-orbit coupling to lift degeneracies)
- ✦ couple two dots with vortex-vortex or vortex-antivortex to achieve splitting of the zero energy modes

# Conclusions:

- SC is possible in graphene only by proximity effect (so far)
- at very low doping in the N region, Fermi wave length is comparable to R
- sub-gap quantized states appear in the dot (ABS)
- various ways to manipulate the ABS: strain, hybridization, vortices
- coupling of dots –> artificial atoms/band structure (two level systems)
- zero energy modes if a vortex is present in the hole

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**Thank you for your attention!**