Heavy-light diquark masses from QCD sum rules and constituent diquark models of XYZ states

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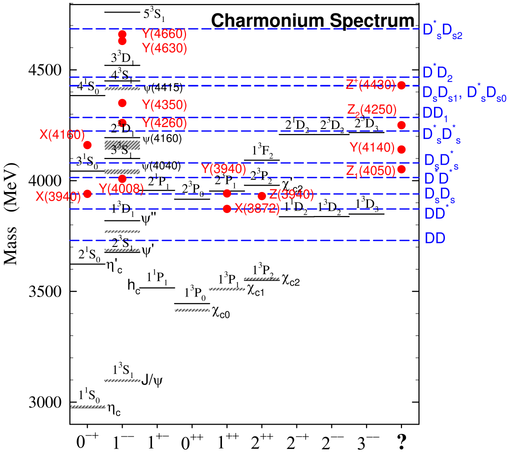
Outline

Heavy quarkonium-like states

- Heavy quarkonia:
  - Charmonium ($c\bar{c}$)
  - Bottomonium ($b\bar{b}$)
- Charmonium spectrum
  - predicted $c\bar{c}$ states
  - **unexpected** states!
  - Some in Bottomonium spectrum, too.
- Heavy quarkonium-like states ("XYZ's")
  - Difficult to interpret as $c\bar{c}$ or $b\bar{b}$.

(S. Godfrey, hep-ph/0910.3409)

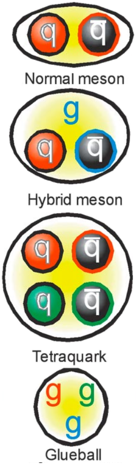


Exotic hadrons

- All $c\bar{c}$ states below 3.7 GeV have been discovered
  - Potential models reproduce masses of these very accurately.
  - The **XYZ's** were not predicted by potential models.
  - These only consider $c\bar{c}$ states.

- Quark model (assumed by potential models): hadronic spectrum just $q\bar{q}$ mesons, qqq baryons
  - QCD suggests hadrons other than $q\bar{q}$ and qqq

- Exotic hadrons: hadrons outside the constituent quark model
  - *What if the XYZ's are not $c\bar{c}$ states? Could they be exotics?*
  - **Use QCD sum rules (QSR) to predict properties of exotics containing heavy quarks.**



(physicsworld.com)

QCD Laplace sum rules (QSR)

Duality

Hadrons can be described via QCD or as resonances.

$$\underbrace{\underbrace{\Pi\left(Q^2\right)}_{\text{QCD correlation function}}}_{\text{calculate via OPE}} \quad \xleftrightarrow[\text{dispersion relation}]{} \quad \underbrace{\underbrace{\rho(t)}_{\text{hadronic spectral function}}}_{\text{Measure or model}}$$

Problem: unsuitable for studying hadrons in this form!

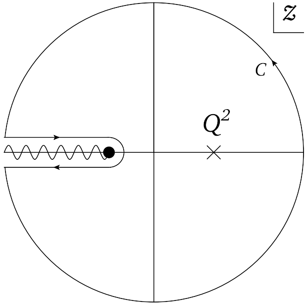
- Large contributions from excited states
- Unknown subtraction constants
- Field-theoretic divergences

QCD Laplace sum rules (QSR)

- $\Pi\left(Q^2\right)$ satisfies Schwarz reflection: $\Pi\left(Q^2\right) = \overline{\Pi}\left(\,\overline{Q^2}\,\right)$.
  $\rightarrow$ **It satisfies a dispersion relation, such as**

$$\Pi\left(Q^2\right) = \Pi(0) + Q^2\Pi'(0) + \frac{Q^4}{\pi}\int_{t_0}^{\infty} dt\, \frac{\mathrm{Im}\Pi(t)}{t^2\left(t+Q^2\right)}$$

- $\mathrm{Im}\Pi(t)$ is related to hadronic spectral function $\rho^{\mathrm{had}}(t)$
  $\rightarrow$ **something that can be measured/modeled**



QCD Laplace sum rules (QSR)

- Parametrize $\rho^{\text{had}}(t)$ in terms of resonance and continuum parts:

$$\rho^{\text{had}}(t) = \rho^{\text{res}}(t) + \theta(t - s_0)\,\text{Im}\Pi(t)$$

- Taking the **Borel (inverse Laplace) transform** of the dispersion relation yields

$$\boxed{\mathcal{R}_k^{\text{QCD}}(\tau, s_0) = \frac{1}{\pi}\int_{t_0}^{\infty} t^k e^{-t\tau} \rho^{\text{res}}(t)dt}$$

$$\mathcal{R}_k^{\text{QCD}}(\tau, s_0) = \frac{\hat{B}}{\tau}\left[\left(-Q^2\right)^k \Pi\left(Q^2\right)\right] - \frac{1}{\pi}\int_{s_0}^{\infty} t^k e^{-t\tau}\text{Im}\Pi\left(te^{-i\pi}\right)dt$$

- Excited states suppressed, subtraction constants removed

QCD Laplace sum rules (QSR)

- $\Pi(q)$ is calculated within the **Operator Product Expansion**

$$\Pi(q) = \sum_n C_n(q) \langle O_n \rangle \;\rightarrow\; C_n(q) \text{ expanded in } \alpha$$

$$\langle O_n \rangle \in \left\{ I \,,\, m_q \langle \bar{q} q \rangle \,,\, \langle \alpha\, G^2 \rangle \,,\, \langle \bar{q} \sigma G q \rangle \,,\, \alpha \langle \bar{q} q \rangle^2 \,,\, \ldots \right\}$$

- Resonance model in terms of **hadron mass M**

$$\rho^{\mathrm{res}}(t) = \pi f^2 \delta\left(t - M^2\right) \quad \rightarrow \quad \boxed{M = \sqrt{\frac{\mathcal{R}_1^{\mathrm{QCD}}(\tau\,,s_0)}{\mathcal{R}_0^{\mathrm{QCD}}(\tau\,,s_0)}}}$$

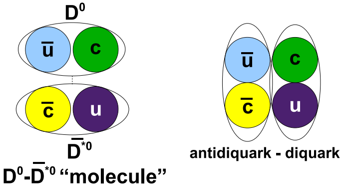
- **Stability of sum rule:** M has minimal τ dependence

Heavy quarkonium-like four-quark states

- Many XYZ's have been interpreted as four-quark states.

- $X(3872)$, for example:
  - D-meson molecule (*cf.* Deuteron), or
  - Tetraquark (diquark-antidiquark bound state).

(M. Nielsen *et al.*, PR 497 (2010) 41)



- **Diquarks**
  - two-quark (qq) clusters inside hadrons
  - Sometimes useful to think of as hadronic constituents
  - Spin-0 (scalar) or Spin-1 (vector)
  - Scalar and vector heavy-light (Qq) diquark masses should be degenerate

Constituent diquark models of tetraquarks

- Tetraquarks can be studied using constituent diquark models:
  - M_{Qq} from fits to $X(3872)$, $Y_b(10890)$
  - 0^+ and 1^+ diquarks assumed degenerate
  - $M_{cq} = 1.93\,\mathrm{GeV}$ (Maiani *et al.*, PRD71 (2005) 014028).
  - $M_{bq} = 5.20\,\mathrm{GeV}$ (Ali *et al.*, PRD85 (2012) 054011).

- Charged partners:
  - $Z_c^{\pm}(3900)$ discovered by BES-III (PRL 110 (2013) 252001), confirmed by Belle (PRL 110 (2013) 252002), CLEO (PLB 727 (2013) 366).
  - Predicted by Maiani *et al.* on basis on constituent diquark model
  - Other charged heavy quarkonium-like states found recently.

- QSR studies of heavy-light diquarks: calculate heavy-light constituent diquark mass.
  - **QCD-based test of constituent diquark models.**

QSR studies of four-quark states

- QSR studies of four-quark states among XYZ's.
  - Review: Nielsen, Navarra, Lee, Phys. Rept. 497 (2010) 41.
  - Most use four quark currents and are leading-order calculations.

- Molecular and tetraquark currents mix under Fierz transformations.
  - **QSR calculations that use four quark currents cannot distinguish between the molecular and tetraquark scenarios.**

- Fierz transformation ambiguity can be addressed by using diquark currents (Zhang, Huang, Steele, PRD76 (2007) 036004).
  - QSR determination of constituent diquark mass.
  - Diquarks are relevant for tetraquarks, not molecules.
  - Study pure tetraquark states via QSR.

NLO QSR studies of four-quark states

- Composite operator renormalization needed for NLO QSR calculations.
  - Composite operators can mix with lower dimensional operators with the same quantum numbers.
  - (Ex) Four-quark currents mix with other operators.
  - **Operator mixing is a significant technical barrier to NLO QSR calculations.**
- The diquark current is protected from operator mixing:
  - No lower dimensional operators with the same quantum numbers.
  - Renormalizes multiplicatively.
- Four-quark current vs. diquark current approach to studying four-quark states in QSR:
  - Diquark operator does not mix with other composite operators,
  - **QSR analyses of diquarks can be extended to higher orders more easily.**

Diquark current renormalization

- We have calculated the scalar $(J^P = 0^+)$ diquark renormalization factor to two-loop order:
  - **Kleiv, Steele, JPG38 (2011) 025001.**

- Exploited the relationship between diquark and meson currents.
  - Easy to extend to $J^P = 0^-\,, 1^\pm$ diquarks at one-loop level.

- The $J^P = 0^\pm\,, 1^\pm$ diquark operator renormalization factors are needed in NLO QSR studies of diquarks.
  - **Used in NLO calculation of heavy-light diquark mass.**

QSR studies of diquarks

- Original QSR studies of light (qq) diquarks:
  - Dosch, Jamin, Stech ZPC 42 (1989) 167; Jamin, Neubert PLB 238 (1990) 387
- Updated by Zhang, Huang, Steele, PRD76 (2007) 036004
  - Constituent light diquark mass $\to$ light tetraquarks
- Heavy-light (Qq) diquarks :
  - Wang, EPJC71 (2011) 1524
    $\to$ LO calculation of constituent diquark mass
  - **Kleiv, Steele, Zhang, Blokland, PRD 87 (2013) 125018**
    $\to$ NLO calculation of constituent diquark mass, including negative parity diquarks

Diquark Correlation Function

$$\Pi(q) = i \int d^4x\, e^{iq\cdot x} \langle 0 | T\, [\, J_\alpha(x)\, S_{\alpha\omega}[x\,,0]\, J^\dagger_\omega(0)\,]\, | 0 \rangle$$

$$J_\alpha = \epsilon_{\alpha\beta\gamma} Q^T_\beta C \mathcal{O} Q_\gamma \quad \langle 0 | J_\alpha | D_\beta \rangle = g_D \delta_{\alpha\beta}$$

$$S_{\alpha\omega}[x\,,0] = P \exp\left[ig \frac{\lambda_{\alpha\omega}}{2} \int_0^x dz^\mu A^a_\mu(z) \right]$$

$$= \delta_{\alpha\omega} + ig \frac{\lambda^a_{\alpha\omega}}{2} \int_0^1 d\xi A^a_\mu(\xi x)\, x^\mu$$

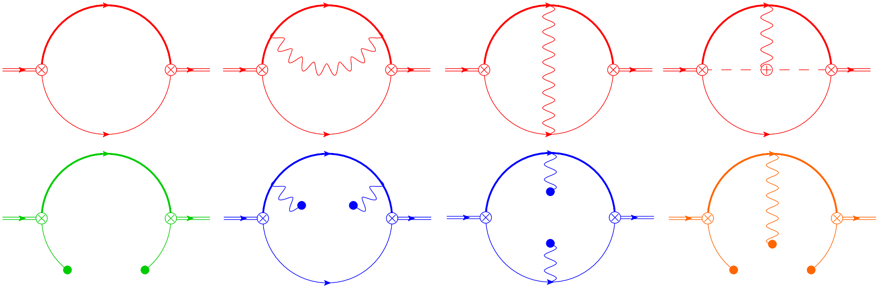
$$+ \mathcal{O}\left(g^2\right)$$

Current and J^{P}

$\mathcal{O}$	J^P
γ_5	0^+
I	0^-
γ_μ	$1^+\,, \ldots$
$\gamma_\mu\gamma_5$	$1^-\,, \ldots$

Diquark Correlation Function

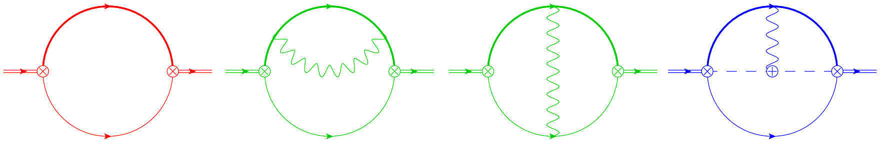
$$\Pi(q) = \mathbf{C^{I}}(\mathbf{q}) + \mathbf{C^{qq}}(\mathbf{q})\langle\bar{q}q\rangle + \mathbf{C^{GG}}(\mathbf{q})\langle\alpha G^2\rangle + \mathbf{C^{qGq}}(\mathbf{q})\langle\bar{q}\sigma G q\rangle + \mathbf{C^{qqqq}}(\mathbf{q})\langle\bar{q}q\rangle^2 + \ldots$$



Diquark Correlation Function

- Gauge invariance
  - **Schwinger string** exactly cancels gauge depedence of **perturbative** contributions
  - Condensates calculated in background gauge: $x^\mu A^a_\mu = 0$

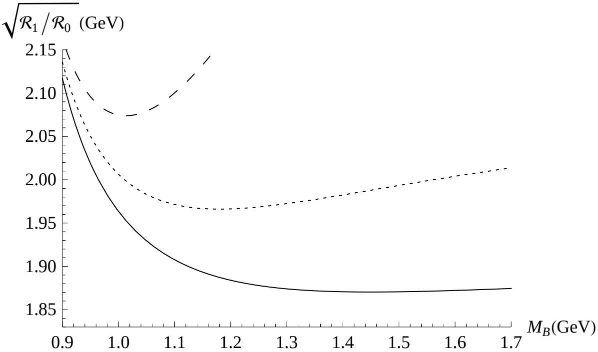
- Renormalization
  - Use known diquark current renormalization factor
  - Renormalization-induced contributions from **LO perturbative contribution**



Heavy-Light diquark mass predictions

J^P	M_{cq} (GeV)
0^+	1.86 ± 0.05
1^+	1.87 ± 0.10

J^P	M_{bq} (GeV)
0^+	5.08 ± 0.04
1^+	5.08 ± 0.04



- Scalar/vector masses are degenerate within uncertainty
- Negative parity sum rules are unstable $\rightarrow$ unable to make mass predictions (in agreement with QSR results for light diquarks)

Implications for the XYZ states

1. QSR determined heavy-light diquark masses are consistent with constituent models.
   $\rightarrow$ **QCD support for constituent diquark models of tetraquarks.**

2. Strengthens tetraquark interpretation of $X(3872)$, $Y_b(10890)$.

3. Support for tetraquark interpretation of charged heavy quarkonium-like states

Summary

- QSR predictions for heavy-light diquarks
  - QCD-based test of constituent diquark models
  - QSR support for tetraquark interpretation of $X(3872)$, $Y_b(10890)$ and charged heavy quarkonium-like states
  - Will help to determine the identities of the XYZ states

- Exciting time for hadron spectroscopy:
  - Many XYZ states discovered, perhaps more to come?
  - New experiments: Belle-II, BES-III, Glue-X, LHCb, $\overline{\text{P}}$ANDA.
  - *Lots of data to come!*

(belle2.kek.jp) (bes3.ihep.ac.cn) (halld.org) (cern.ch) (panda.gsi.de)

Acknowledgements

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- Preprints and links to published papers are available at
  - `http://arxiv.org/a/kleiv_r_1`

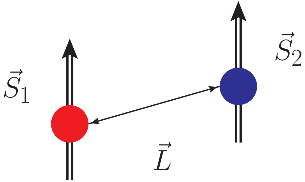
Thanks!

Backup Slides

J^{PC} quantum numbers

- Hadrons are classified by their J^{PC}:
  - $J = L + S$ is the angular momentum
  - P is parity ($P = +$ or $-$)
  - C is charge conjugation ($C = +$ or $-$)

- Conventional quarkonia have $P = (-)^{L+1}$ and $C = (-)^{L+S}$
  - **Possible (non-exotic)** J^{PC}: 0^{-+}, 1^{--}, 1^{+-}, 0^{++}, 1^{++}, 2^{++} ...
  - **Impossible (exotic)** J^{PC}: 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, ...



- *The observation of a state with exotic J^{PC} is a "smoking gun" for the existence of exotic hadrons.*

2012 RPP - Developments in Heavy Quarkonium Spectroscopy

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	$1^{++}/2^{-+}$	$B \to K\,(\pi^+\pi^- J/\psi)$	Belle [36,37] (12.8), BABAR [38] (8.6)	2003	OK
				$p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$	CDF [39–41] (np), D0 [42] (5.2)		
				$B \to K\,(\omega J/\psi)$	Belle [43] (4.3), BABAR [23] (4.0)		
				$B \to K\,(D^{*0}\overline{D}^0)$	Belle [44,45] (6.4), BABAR [46] (4.9)		
				$B \to K\,(\gamma J/\psi)$	Belle [47] (4.0), BABAR [48,49] (3.6)		
				$B \to K\,(\gamma\psi(2S))$	BABAR [49] (3.5), Belle [47] (0.4)		
				$pp \to (\pi^+\pi^- J/\psi) + ...$	LHCb [50] (np)		
$X(3915)$	3917.4 ± 2.7	$28^{+10}_{-\ 9}$	$0/2^{?+}$	$B \to K\,(\omega J/\psi)$	Belle [51] (8.1), BABAR [52] (19)	2004	OK
				$e^+e^- \to e^+e^-\,(\omega J/\psi)$	Belle [53] (7.7), BABAR [23] (np)		

Note: Here Y(3940) is listed as X(3915).

Sum rule window

- We need to balance perturbative and non-perturbative effects.
  - If perturbative effects are too large, continuum contributions dominate.
  - If non-perturbative effects are too large, we can have large errors in our mass prediction.
- An acceptable range of τ values can be determined using

$$f_{\text{cont}}\left(\tau, s_0\right) = \frac{\mathcal{L}_1^{\text{QCD}}\left(\tau, s_0\right) / \mathcal{L}_0^{\text{QCD}}\left(\tau, s_0\right)}{\mathcal{L}_1^{\text{QCD}}\left(\tau, \infty\right) / \mathcal{L}_0^{\text{QCD}}\left(\tau, \infty\right)} \geq 0.7$$

$$f_{\text{pow}}\left(\tau, s_0\right) = \left| \frac{\mathcal{L}_1^{\text{QCD}}\left(\tau, s_0\right) / \mathcal{L}_0^{\text{QCD}}\left(\tau, s_0\right)}{\mathcal{L}_1^{\text{pert}}\left(\tau, s_0\right) / \mathcal{L}_0^{\text{pert}}\left(\tau, s_0\right)} - 1 \right| \leq 0.15$$

- The mass function $M^2 = \mathcal{L}_1^{\text{QCD}}\left(\tau\, , s_0\right) / \mathcal{L}_0^{\text{QCD}}\left(\tau\, , s_0\right)$ must stabilize (have a critical point) within the sum-rule window.

- Optimal s_0 found though best fit of M^2 to a constant.