Effects of Time Ordering in Parametric-Down Conversion

Nicolás Quesada and J. E. Sipe

McLennan Physical Laboratories, University of Toronto

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Outline

1. Parametric Down-Conversion and the Magnus expansion
2. Entanglement and Time Ordering in SPDC
3. Summary

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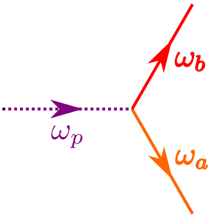
Spontaneous Parametric Down Conversion



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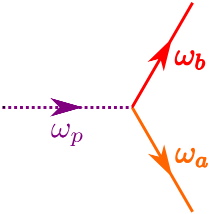

In a 1-D structure $\hat{H}^{\mathrm{PDC}}(t) =$

$$\varepsilon \int d\omega_a d\omega_b d\omega_p \Phi e^{i\Delta t} \hat{c}(\omega_p) \hat{a}^\dagger(\omega_a) \hat{b}^\dagger(\omega_b) + \text{h.c.}$$

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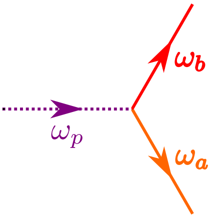

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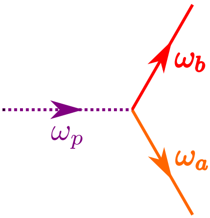

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- H^{PDC} is a Quadratic Bosonic Hamiltonian.
- Two mode squeezing generator.
- $[H^{\text{PDC}}(t), H^{\text{PDC}}(t')] \neq 0$

The phase matching function Φ

$$\Phi(\Delta k L/2) = \int_{-L/2}^{L/2} \frac{dz}{L} e^{iz(k_a + k_b - k_p)}$$

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$$\begin{aligned}\Phi(\Delta k L/2) &= \int_{-L/2}^{L/2} \frac{dz}{L} e^{iz(k_a+k_b-k_p)} = \mathrm{sinc}(\Delta k L/2) \\ \Delta k &= k_a(\omega_a) + k_b(\omega_b) - k_p(\omega_p)\end{aligned}$$

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For each k we linearize as follows:

$$k_x(\omega_x) \approx k_x(\bar{\omega}_x) + \left.\frac{dk_x}{d\omega_x}\right|_{\omega_x=\bar{\omega}_x} (\omega_x - \bar{\omega}_x) = k_x(\bar{\omega}_x) + \frac{\delta\omega_x}{v_x}$$

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The central frequencies $\bar{\omega}_x$ are such that:

$$\begin{aligned}\bar{\omega}_a + \bar{\omega}_b &= \bar{\omega}_p \text{ Energy conservation} \\ k_a(\bar{\omega}_a) + k_b(\bar{\omega}_b) &= k_p(\bar{\omega}_p) \text{ Momentum conservation}\end{aligned}$$

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Time ordering

Dyson

$$\hat{\mathcal{U}}(t,t_0) = \mathbb{I} - i\int_{t_0}^{t} dt_1 \hat{H}(t_1) + (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \hat{H}(t_1)\hat{H}(t_2) +$$

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$$\hat{\mathcal{U}}(t, t_0) = \exp\left(-i \int_{t_0}^{t} dt' \hat{H}(t') + \text{extra stuff}\right)$$

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Magnus is unitary to any order, Dyson is not.

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[S. Blanes, *et. al.* (2009) "The Magnus expansion and some of its applications". Phys. Rep. 470 (5-6): 151238.]

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$$\Omega_1 = -i\int dt'\hat{H}(t') = -2\pi i\varepsilon\times$$
$$\int d\omega_a d\omega_b J_1(\omega_a,\omega_b)$$
$$\times\ \hat{a}^\dagger(\omega_a)\hat{b}^\dagger(\omega_b) + \text{h.c.}$$
$$J_1 = \alpha(\omega_a+\omega_b)\times$$
$$\Phi(\omega_a,\omega_b,\omega_a+\omega_b)$$

2^{nd} Order Magnus Terms

$$\frac{(-i)^2}{2}\int dt_1\int^{t_1} dt_2\ \left[\hat{H}(t_1),\hat{H}(t_2)\right]$$
$$\left[a^\dagger(\omega_p)\,b^\dagger(\omega_q) + \text{h.c.},\right.$$
$$\left. a^\dagger(\omega_r)\,b^\dagger(\omega_s) + \text{h.c.}\right] =$$
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$$+\,\delta(\omega_p-\omega_r)b^\dagger(\omega_s)\,b(\omega_q) - \text{h.c.}$$

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The Magnus Expansion in SPDC

$$|\psi\rangle = e^{-2\pi i \int d\omega_1 d\omega_2 \left\{ (J(\omega_1,\omega_2)\hat{a}^\dagger(\omega_1)\hat{b}^\dagger(\omega_2) + \text{h.c.}) \right\}} \times e^{-2\pi i \int d\omega_1 d\omega_2 \left\{ G(\omega_1,\omega_2)\hat{a}^\dagger(\omega_1)\hat{a}(\omega_2) + H(\omega_1,\omega_2)\hat{b}^\dagger(\omega_1)\hat{b}(\omega_2) \right\}} |\text{vac}\rangle$$

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Magnus expansion is a perturbation theory "inside the exponential".

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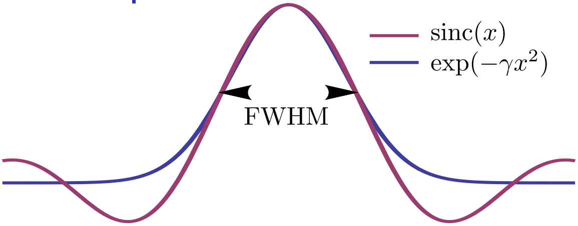
$$\approx e^{-\gamma(\Delta k L/2)^2}$$
$$\gamma \approx 0.193$$



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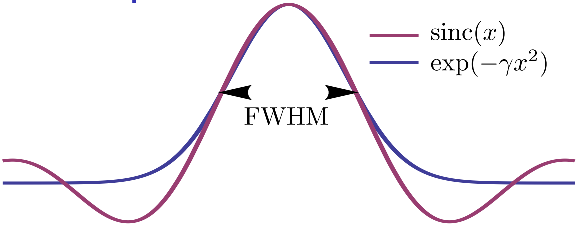

$$\Phi(\omega_a, \omega_b, \omega_p) = \exp\left(-\left(\frac{\sqrt{\gamma}L}{2v_a}\delta\omega_a + \frac{\sqrt{\gamma}L}{2v_b}\delta\omega_b - \frac{\sqrt{\gamma}L}{2v_p}\delta\omega_p\right)^2\right)$$

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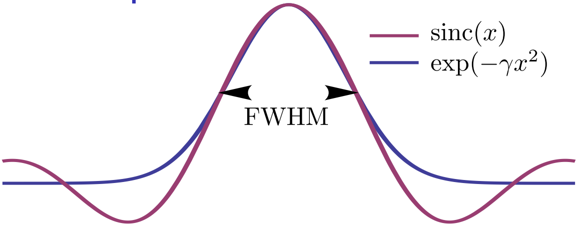

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$$= \exp\left(-(s_a\delta\omega_a + s_b\delta\omega_b - s_p\delta\omega_p)^2\right), \quad \left\{s_x = \frac{\sqrt{\gamma}L}{2v_x}\right\}$$

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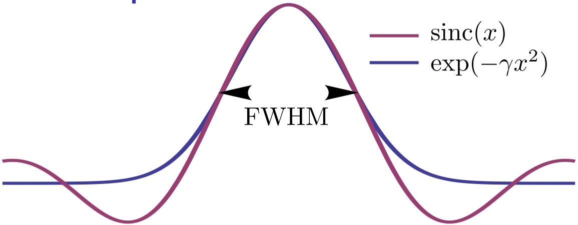

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The Hamiltonian is ($\Delta = \omega_a + \omega_b - \omega_p$)

$$\varepsilon\int d\omega_a d\omega_b d\omega_p \Phi e^{i\Delta t}\alpha(\omega_p)\hat{a}^\dagger(\omega_a)\hat{b}^\dagger(\omega_b) + \text{h.c.}$$

First order Magnus term

$$
\begin{aligned}
J_1(\omega_a, \omega_b) &= \alpha(\omega_a + \omega_b)\Phi(\omega_a, \omega_b, \omega_a + \omega_b) = \frac{\varepsilon \mathcal{T}}{\sqrt{\pi}} \exp\left(-\mathbf{u}\mathbf{N}\mathbf{u}^T\right) \\
\mathbf{N} &= \begin{pmatrix} \mu_a^2 & \mu^2 \\ \mu^2 & \mu_b^2 \end{pmatrix}, \quad \mathbf{u} = (\delta\omega_a, \delta\omega_b) \\
\eta_{a/b} &= s_p - s_{a/b}, \quad \mu^2 = \tau^2 + \eta_a \eta_b, \quad \mu_{a/b}^2 = \tau^2 + \eta_{a/b}^2.
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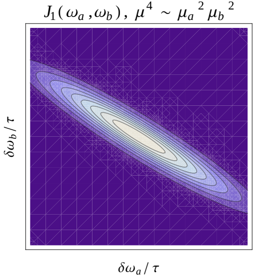

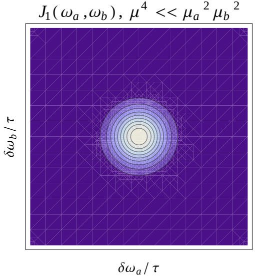

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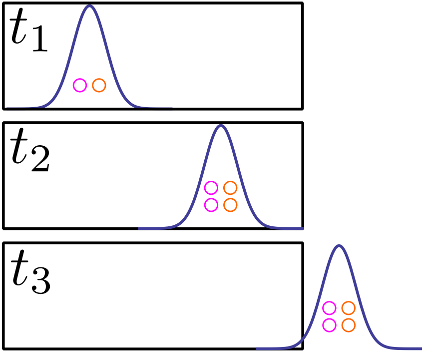

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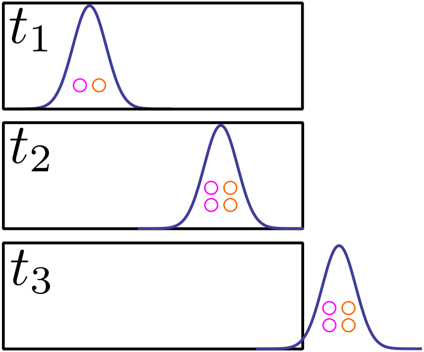

Entangled photons

$\det(\mathbf{N}) \sim 0$

$v_p\tau/|v_x - v_p| \ll L/v_x$

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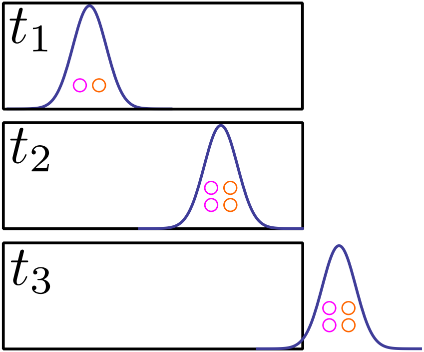

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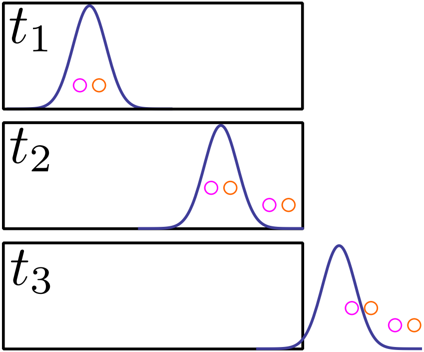

Magnus correction

$$\underbrace{J(\omega_a,\omega_b)}_{JSA} = \underbrace{\varepsilon J_1(\omega_a,\omega_b)}_{\text{Exact w.o. time order}} + \underbrace{\varepsilon^3(J_3(\omega_a,\omega_b) - iK_3(\omega_a,\omega_b))}_{\text{time order correction}} + \ldots$$

For an arbitrary Φ and α

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For the gaussian model

$$r \equiv \frac{\max_{\omega_a,\omega_b} |\varepsilon^3 J_3(\omega_a,\omega_b)|}{\max_{\omega_a,\omega_b} |\varepsilon J_1(\omega_a,\omega_b)|} < \underbrace{\frac{13\pi^2\varepsilon^2\tau^2}{6\sqrt{4(\eta_a-\eta_b)^2\tau^2+3(\eta_a\eta_b+\tau^2)^2}}}_{\equiv r_{\max}}$$

Time ordering and entanglement

if $(\det(\mathbf{N}) \sim 0)$

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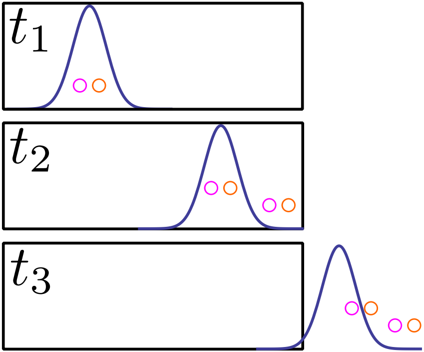

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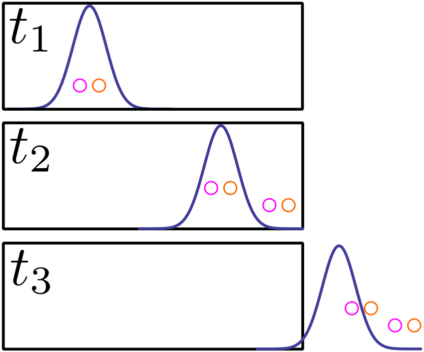

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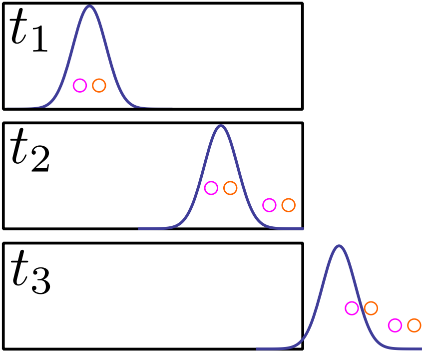

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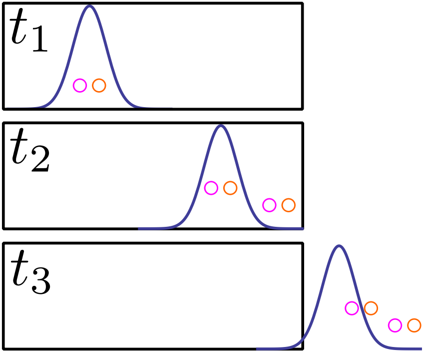

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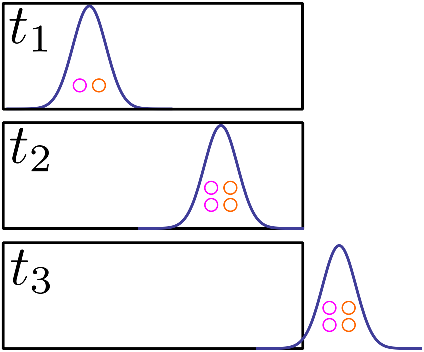

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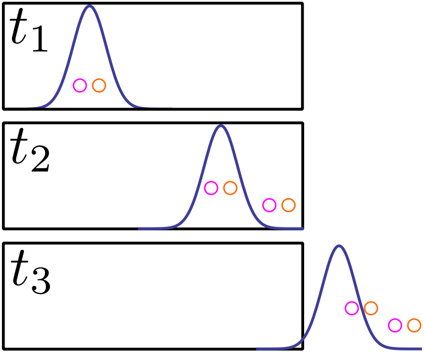

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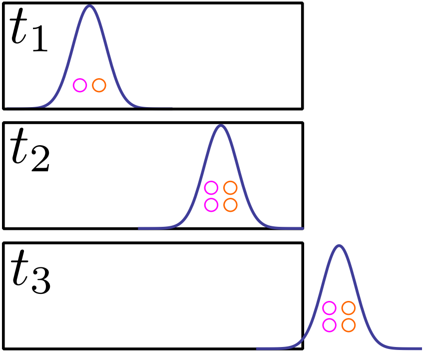

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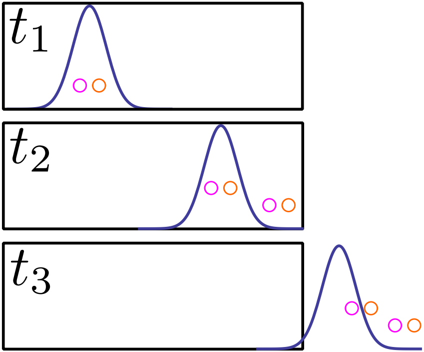

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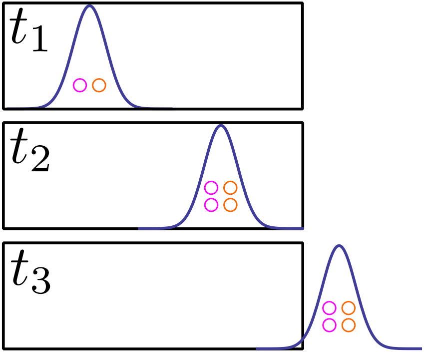

Time ordering more relevant for unentangled pair generation.

[consistent with numerics from A. Christ *et. al.* (2013) "Theory of quantum frequency conversion and type-II parametric down-conversion in the high-gain regime" New J. Phys. 15 053038]

Outline

1. Parametric Down-Conversion and the Magnus expansion
2. Entanglement and Time Ordering in SPDC
3. Summary

Take home Message

On the Magnus expansion

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