Tunneling and Domain Walls

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arXiv:14xx.xxxx

Quantum particles can go in regions forbidden classically (negative kinetic energy), leading to interesting phenomena such as:

- barrier penetration
- decay phenomena (eg nuclei)
- bubble nucleation
- vacuum decay

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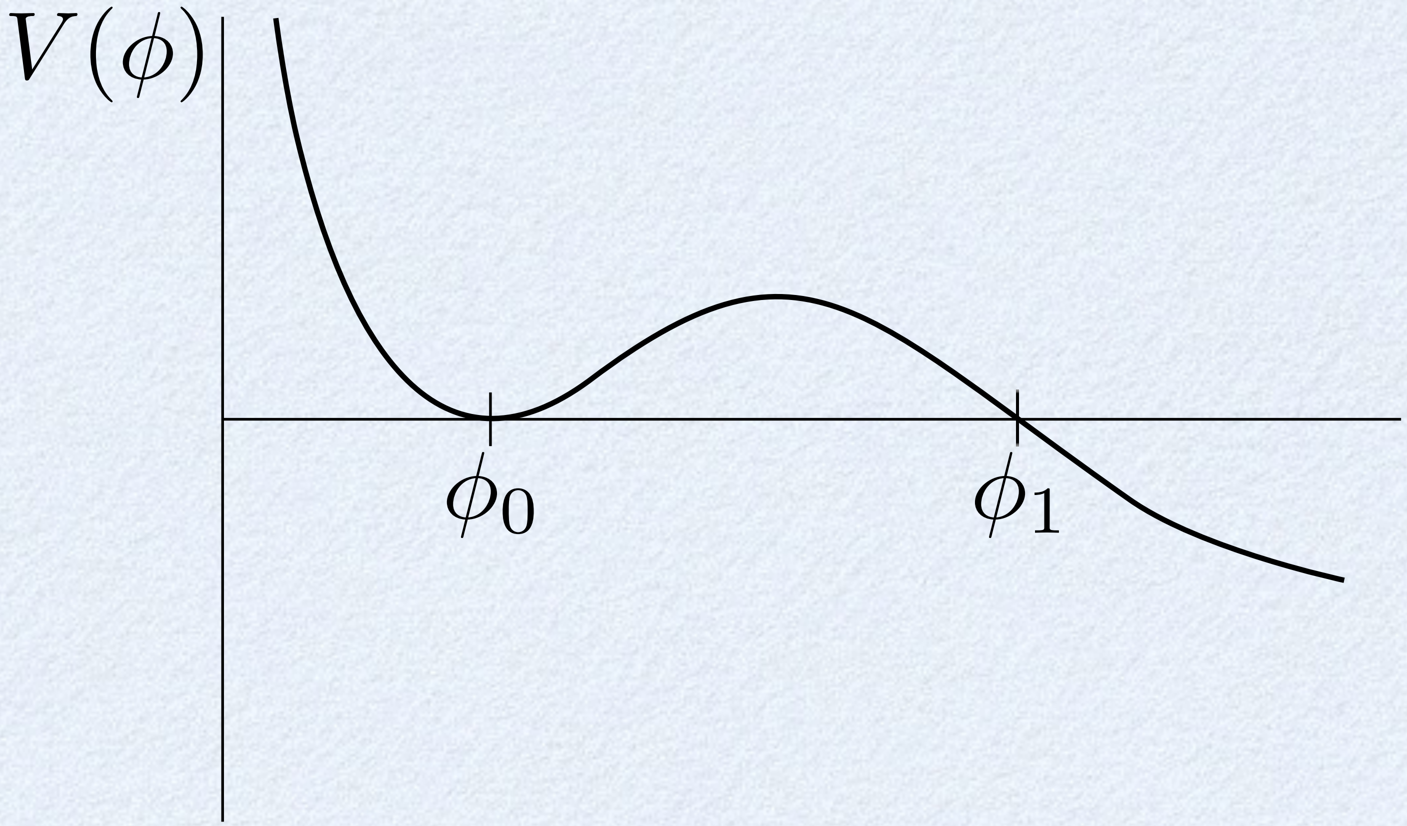
OUTLINE

1. Quick review of vacuum decay
2. Case of interest : symmetric true vacuum, symmetry-breaking false vacuum
3. R*eally quick* review of previous work: vortices and vacuum decay
4. Domain walls : existence; first steps towards studying vacuum decay
5. Conclusions and outlook

1. Quick Review of Vacuum Decay



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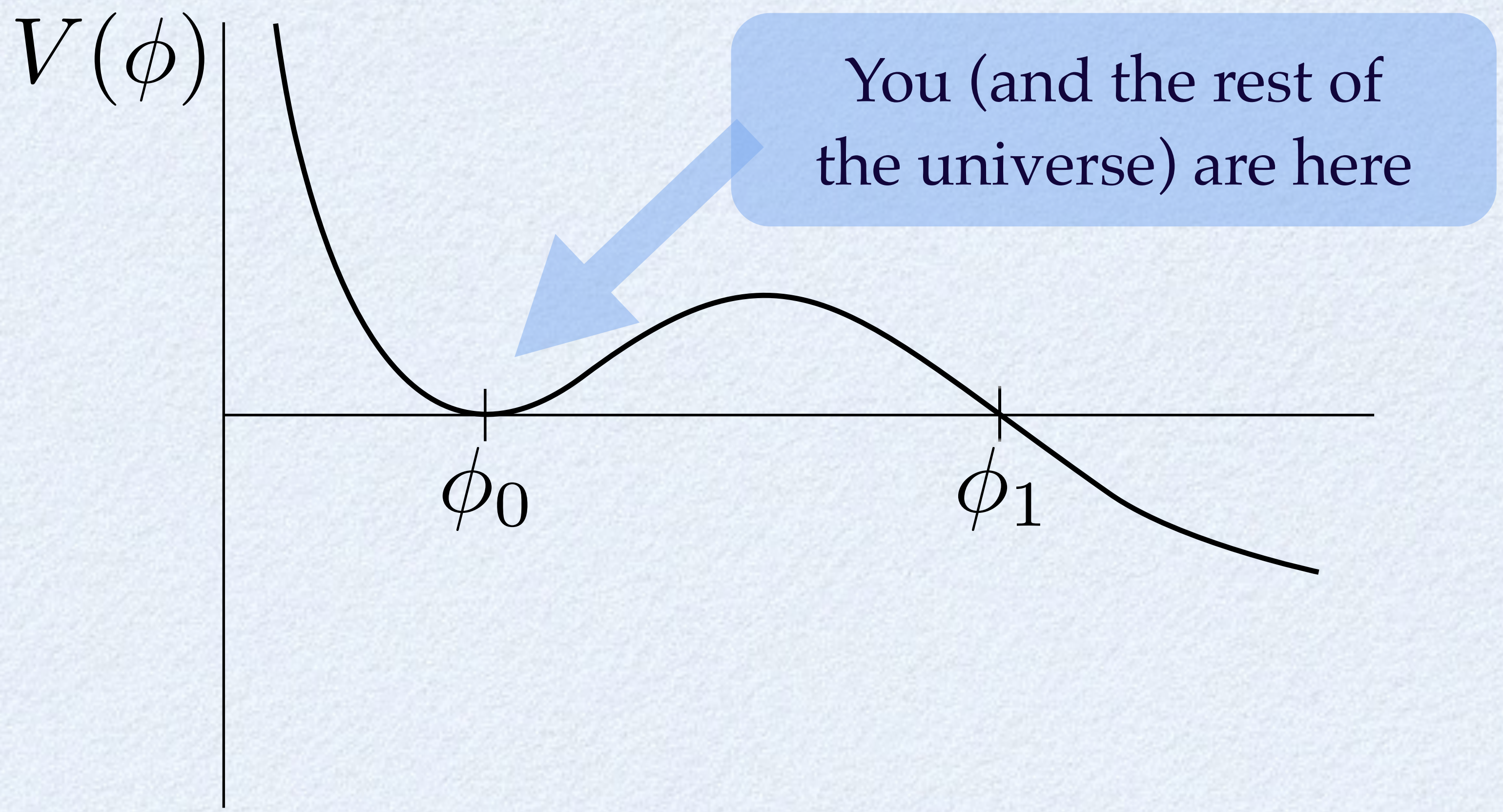

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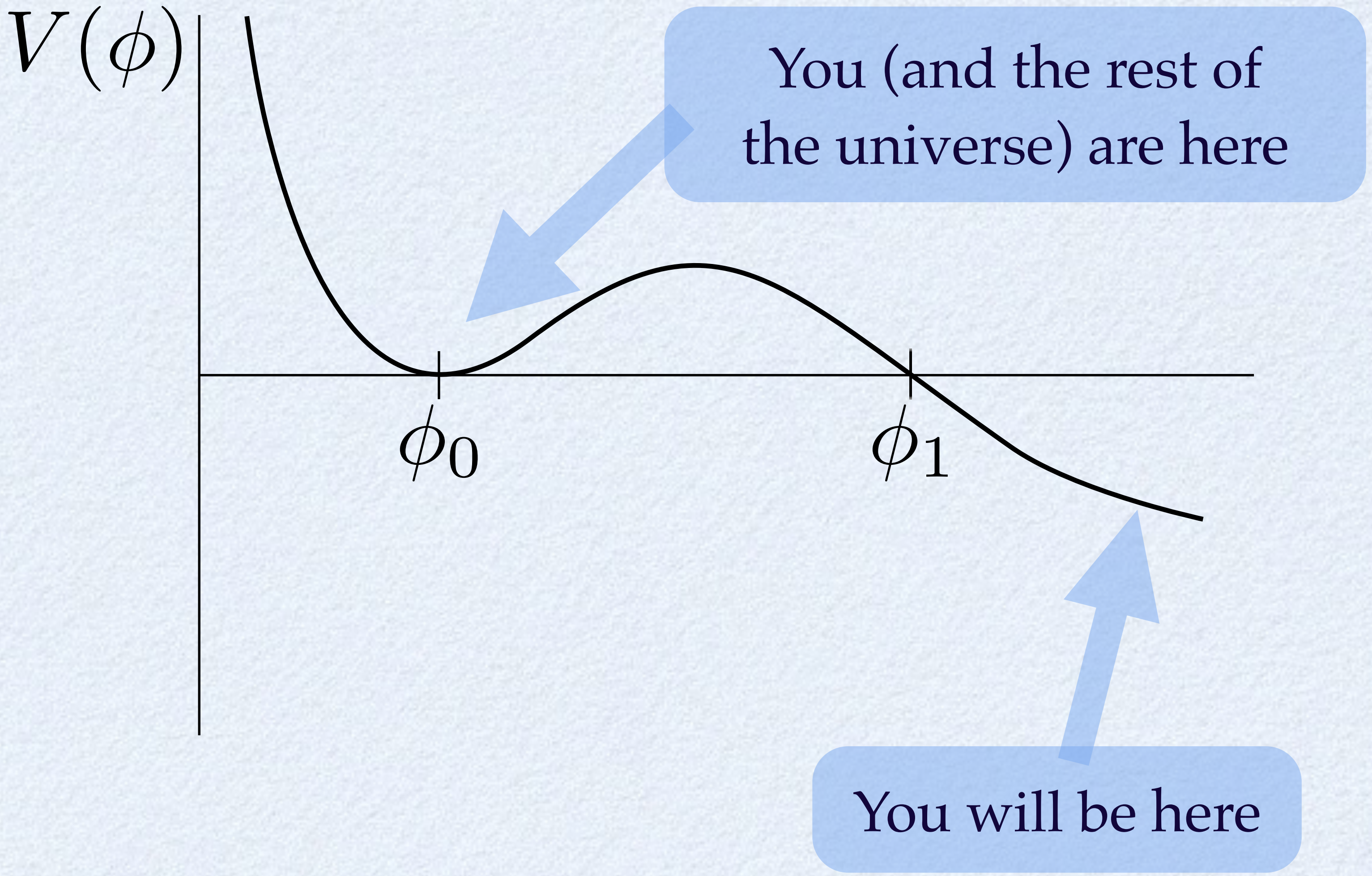

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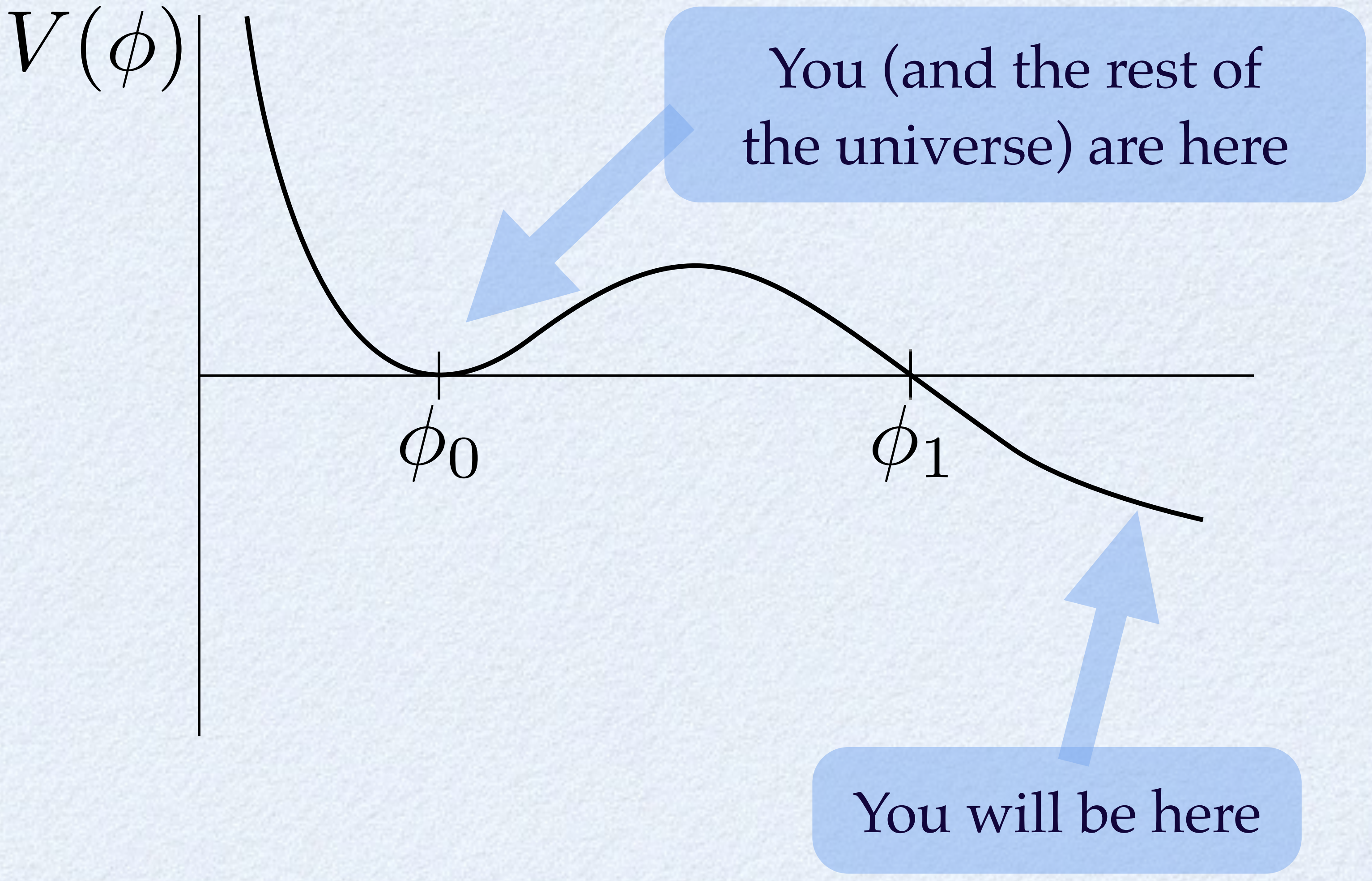
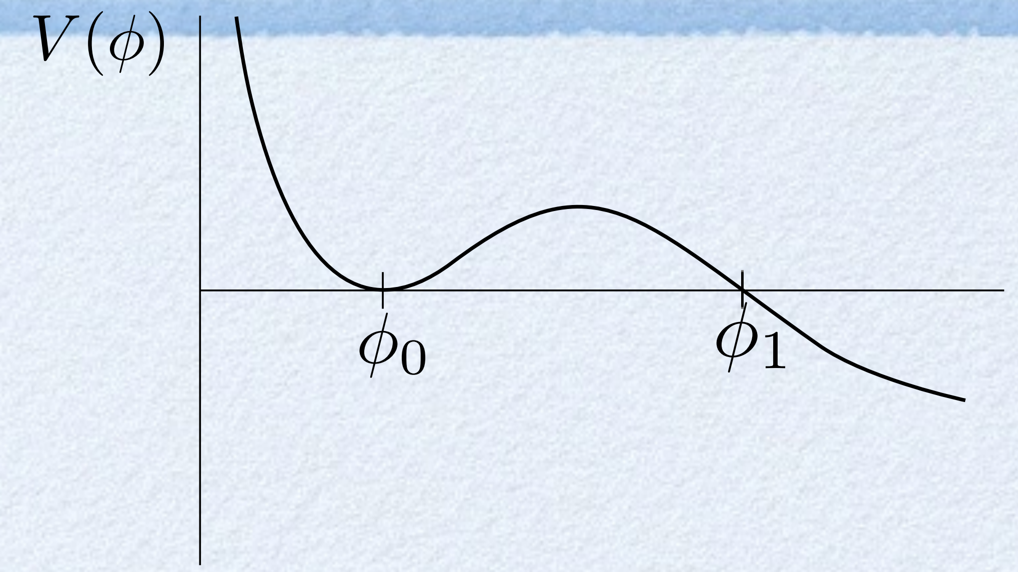


Key reference: S. Coleman, PRD15 2929 (1977)

To compute the decay rate:



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

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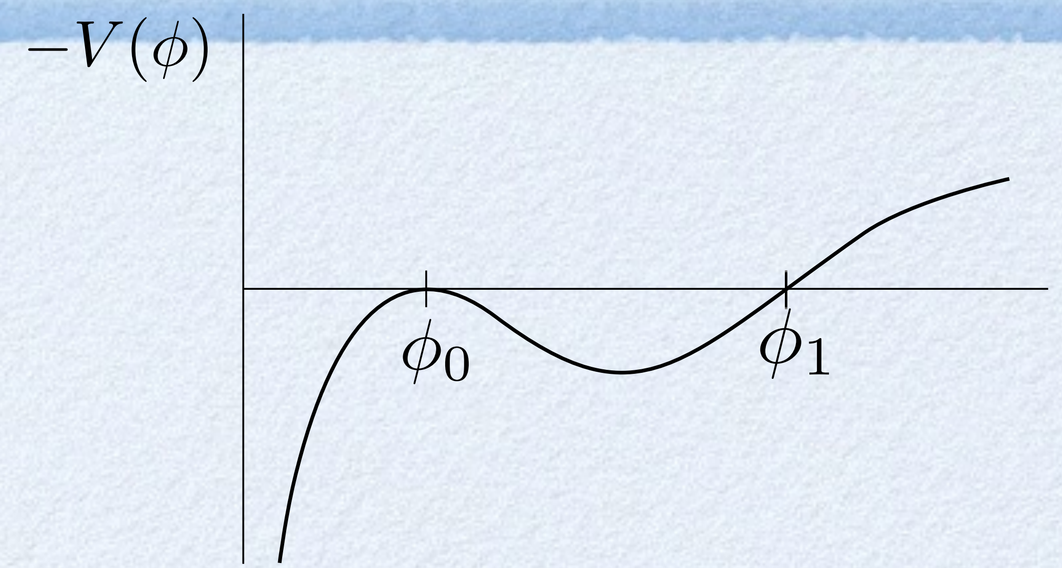
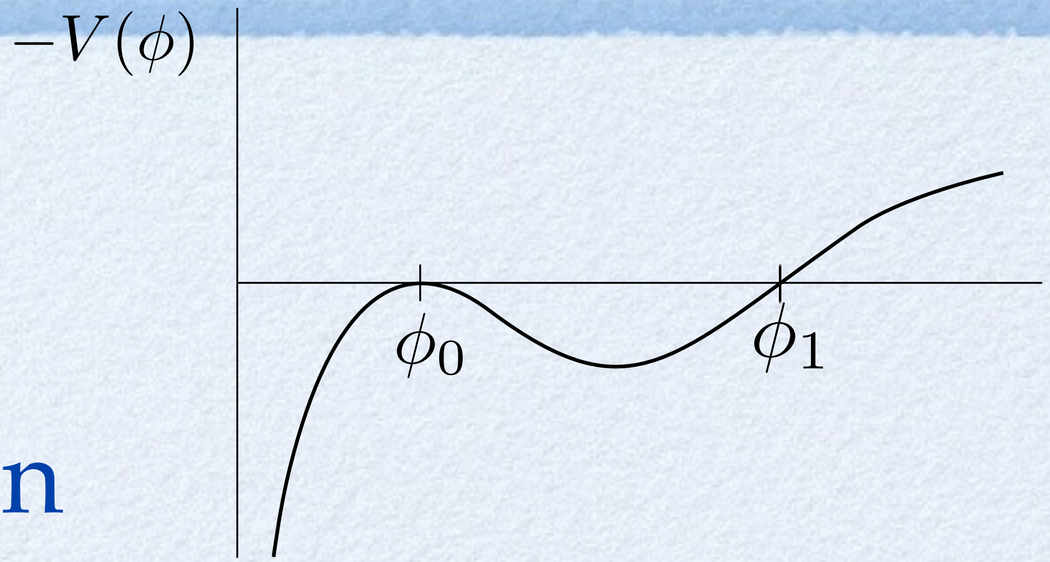


$$\mathcal{L}_E = \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$$



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$$\phi \to \phi_0 \text{ as } r, \tau \to \infty, \qquad \left.\frac{\partial \phi}{\partial \tau}\right|_{\tau=0} = 0$$

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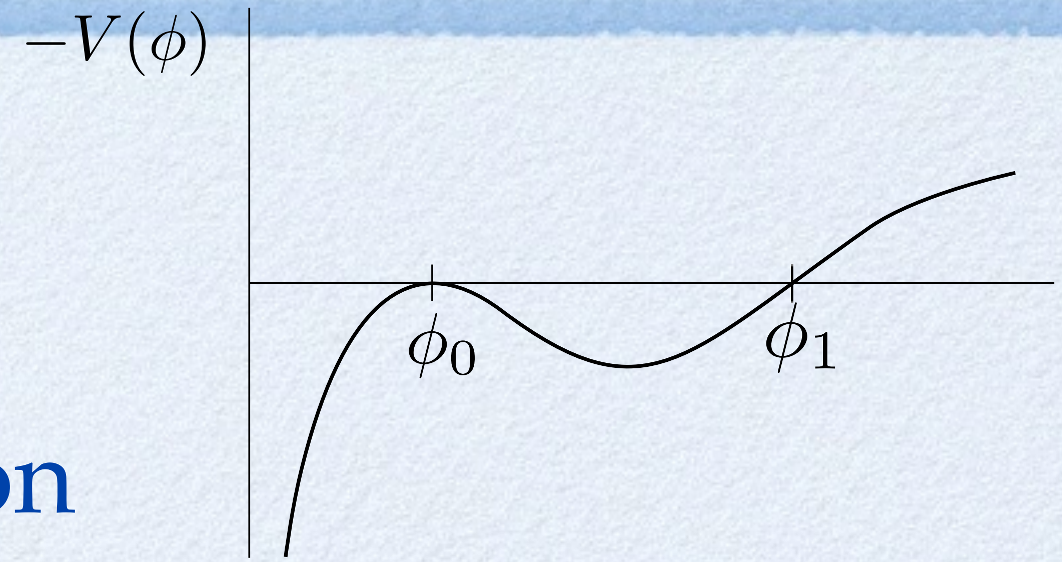

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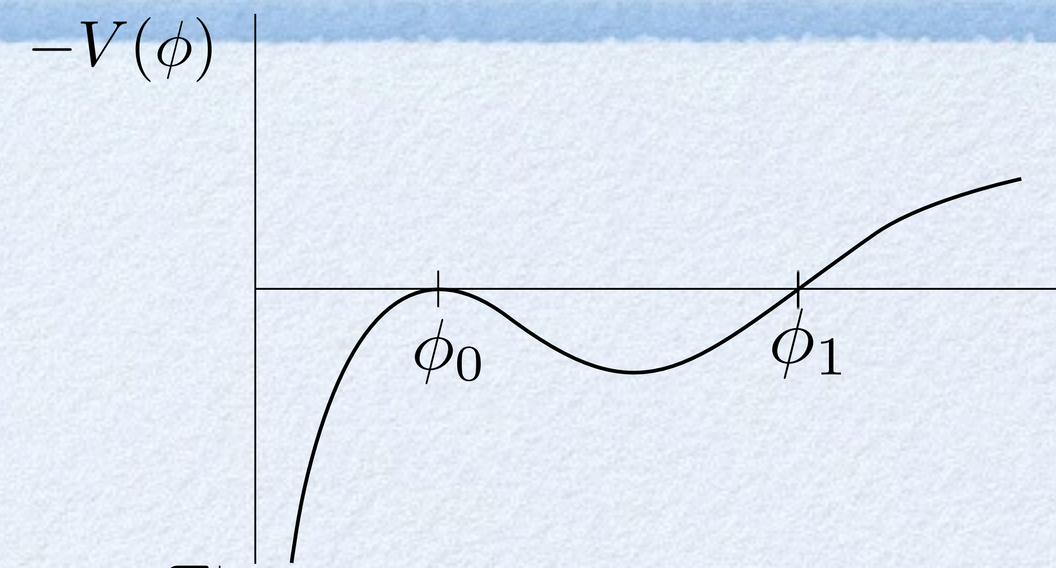
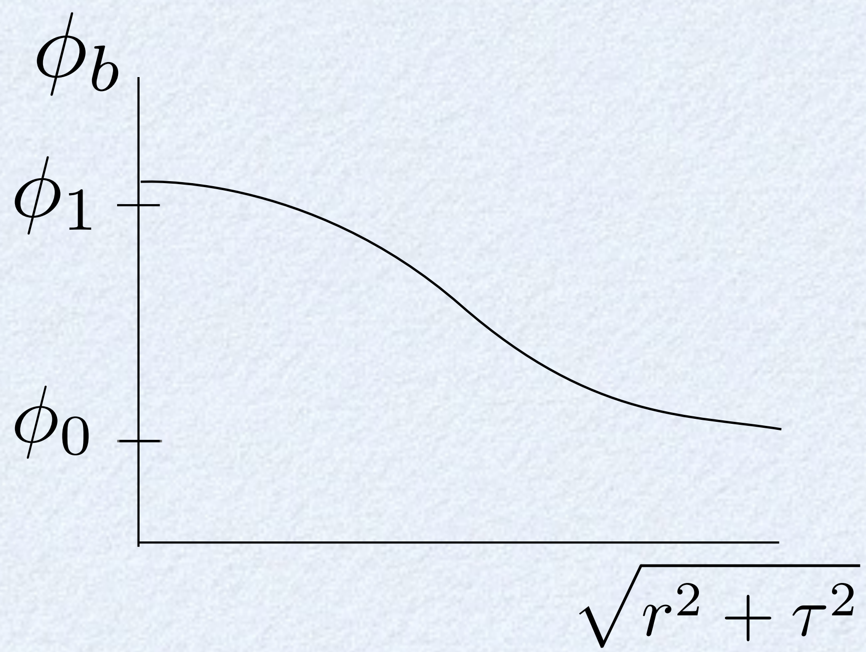
3. The lowest-action nontrivial solution ϕ_b determines the decay rate:

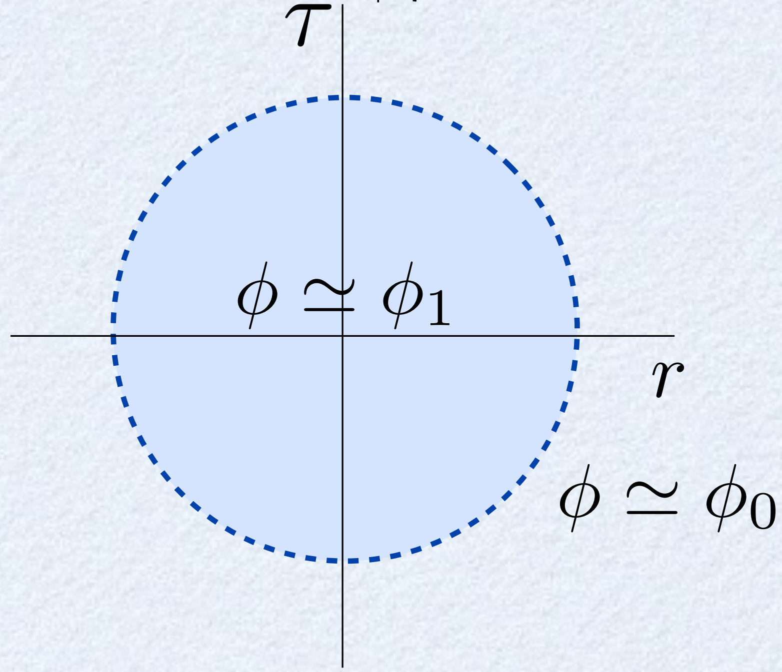
$$\Gamma/V = A e^{-S_E[\phi_b]/\hbar}$$



Bounce (O(4)-symmetric)

$$\phi_b(r,\tau) = \phi_b(\sqrt{r^2+\tau^2})$$

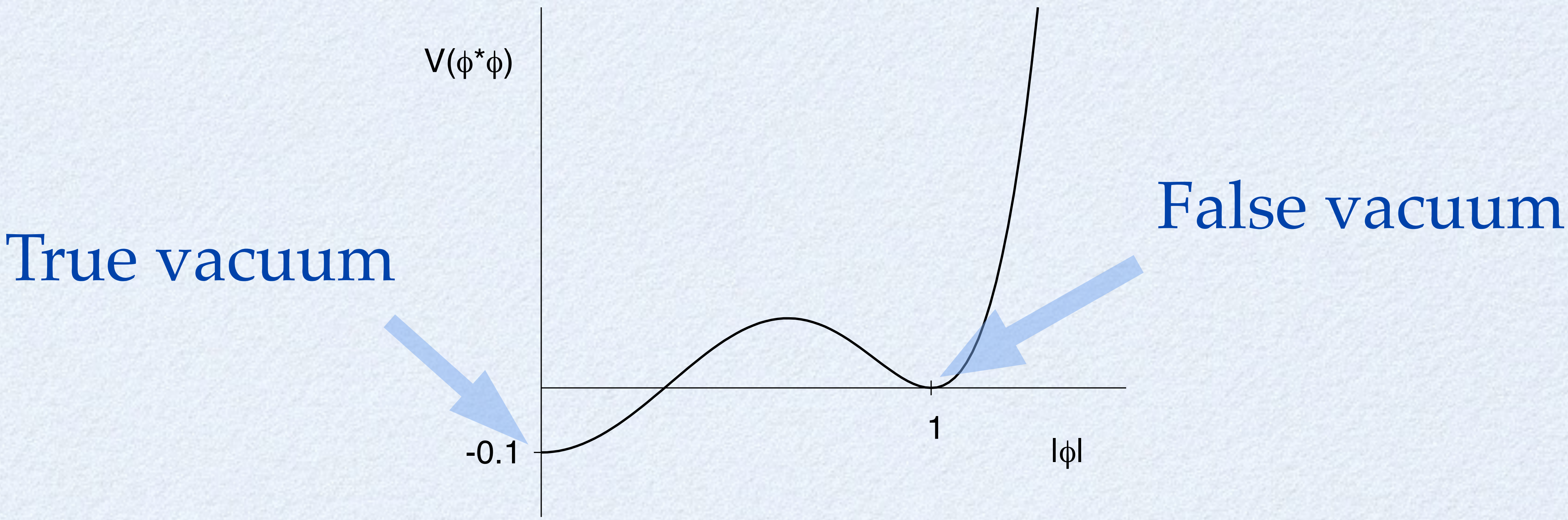




2. Case of interest

symmetric true vacuum

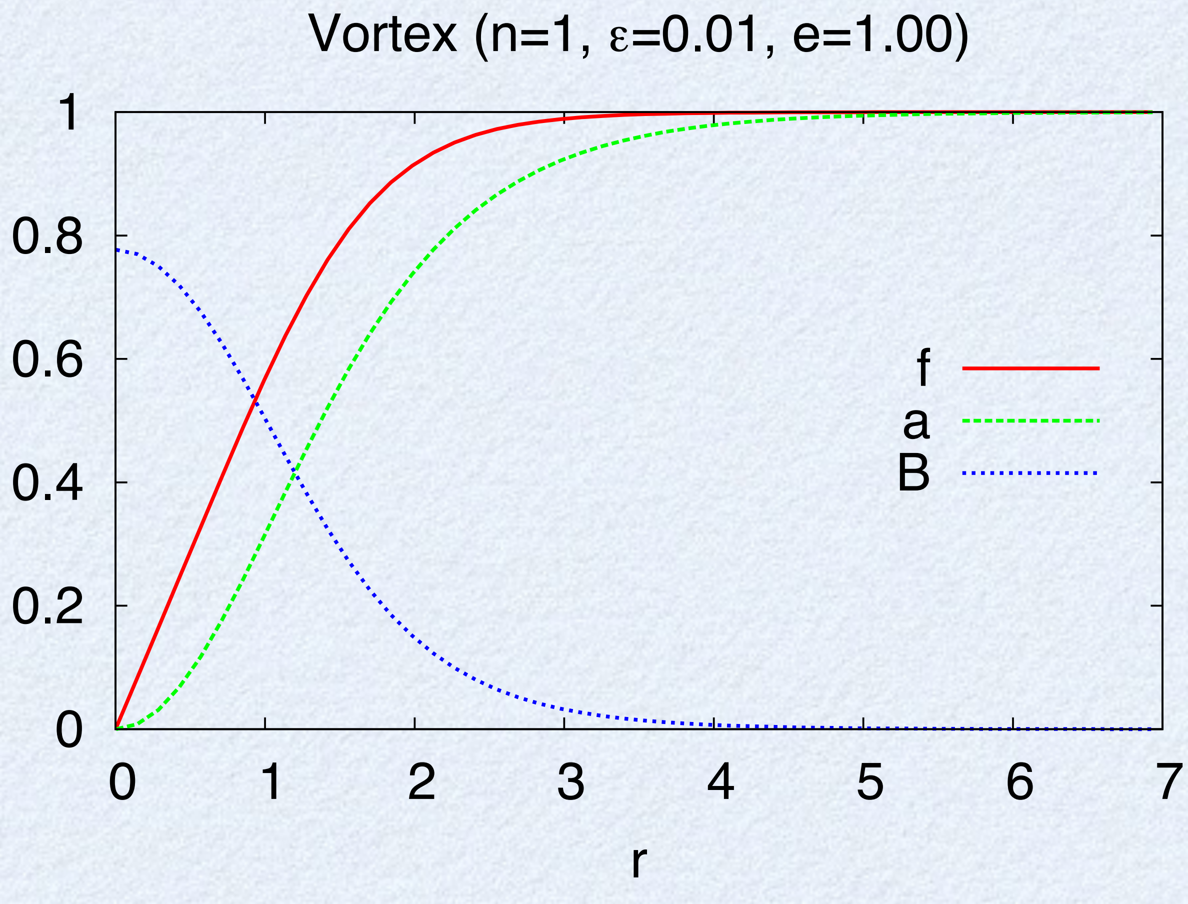
symmetry-breaking false vacuum

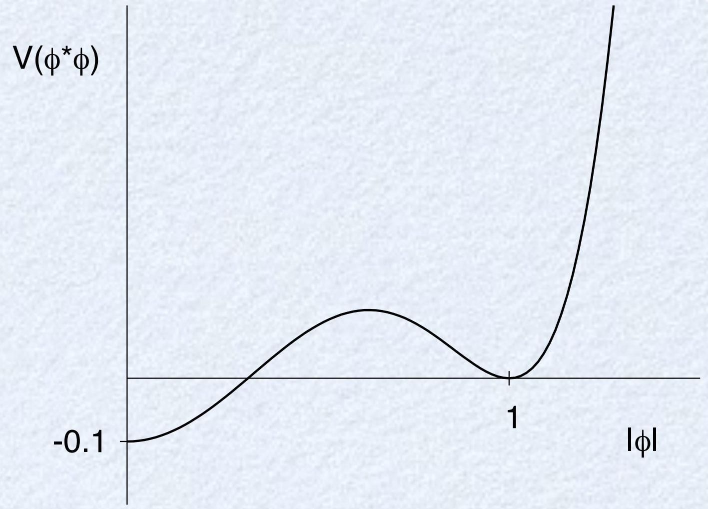


3. Really Quick Review : Vortices

Monopoles (Kumar, et al., PRD82 025022 (2010), vortices (Lee, et al., PRD88, 085031 (2013)) and cosmic strings (Lee, et al., PRD88, 105008 (2013)) have been considered previously.

E.g. vortex:





$$\phi(r,\theta) = f(r)e^{in\theta}$$

$$A_i(r,\theta) = -\frac{n}{e}\frac{\varepsilon^{ij}r_j}{r^2}a(r)$$

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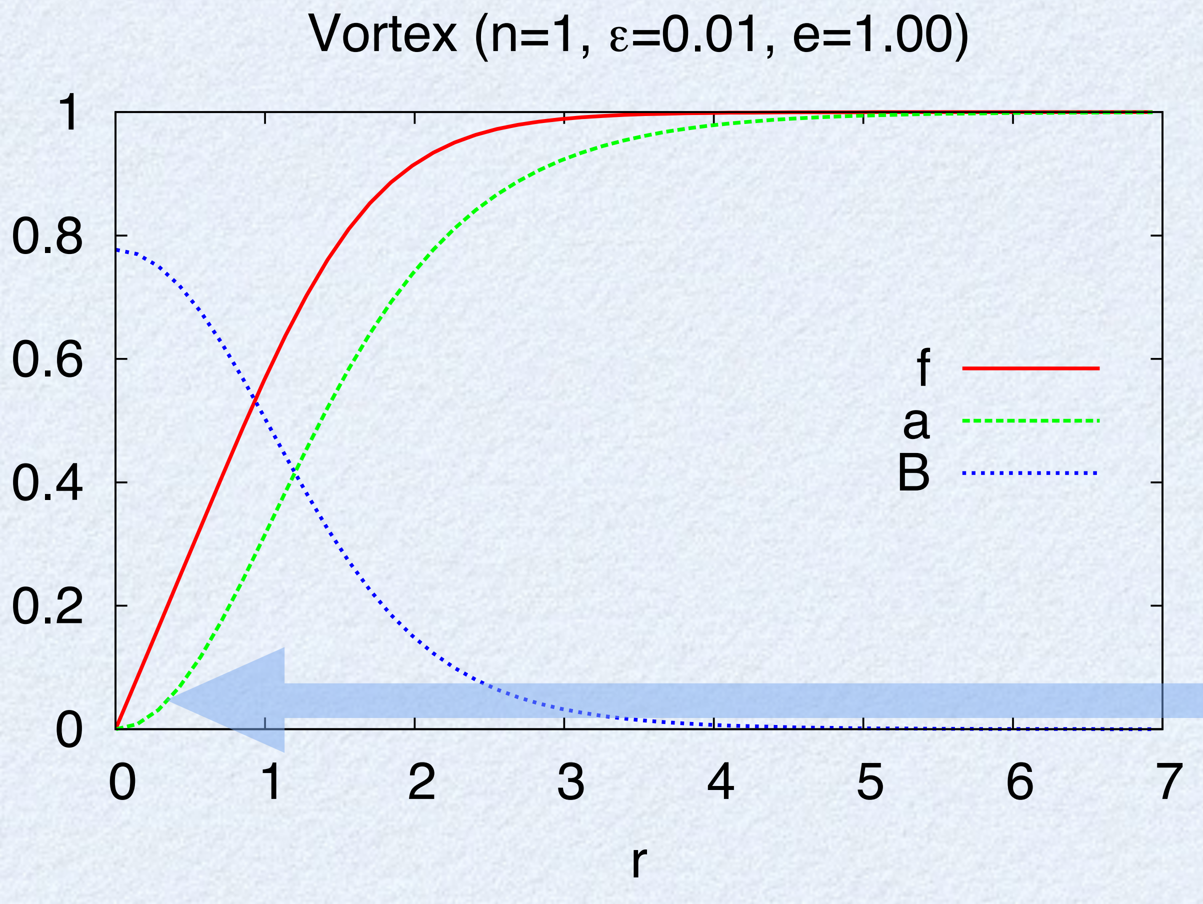

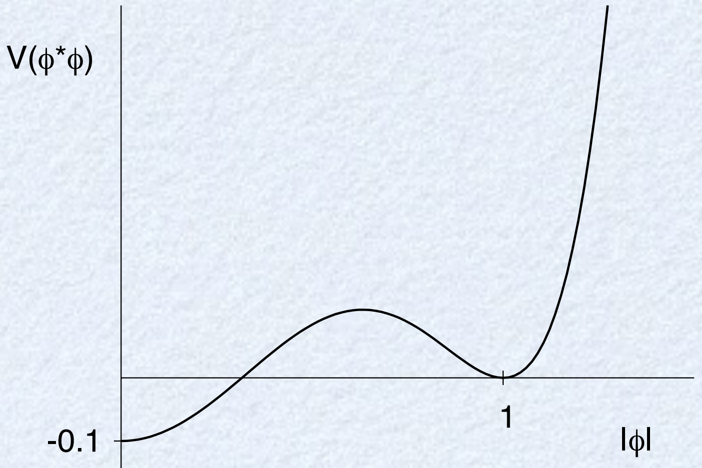

$$\phi(r,\theta) = f(r)e^{in\theta}$$

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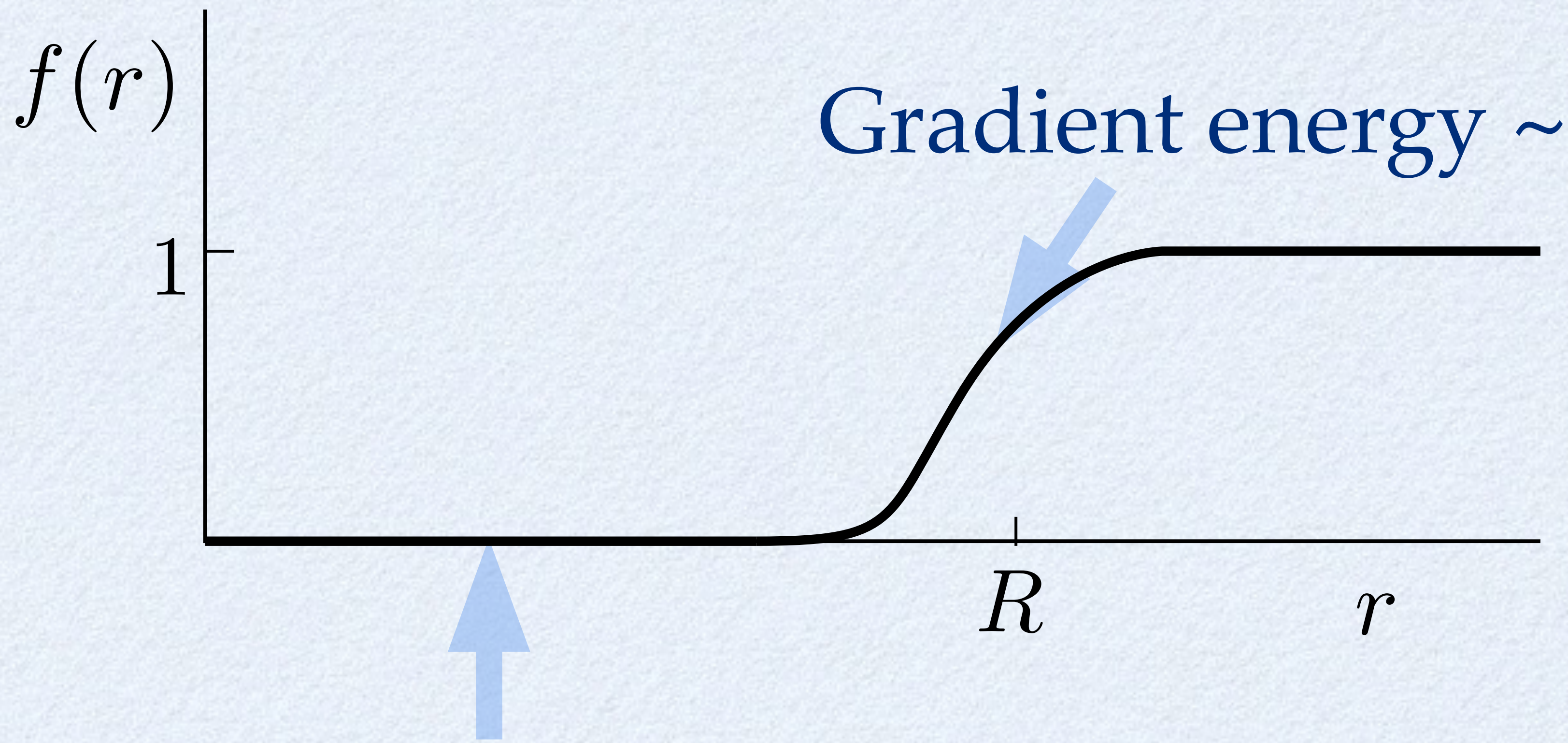
Note: true vacuum

This configuration is classically stable.
But it is NOT, quantum mechanically.
Large (size R) thin-wall vortex:



Gradient energy ~ R

Potential energy ~ $-R^2$
Magnetic energy ~ R^{-2}

Q: Can vortices or cosmic strings speed up (catalyze) vacuum decay?

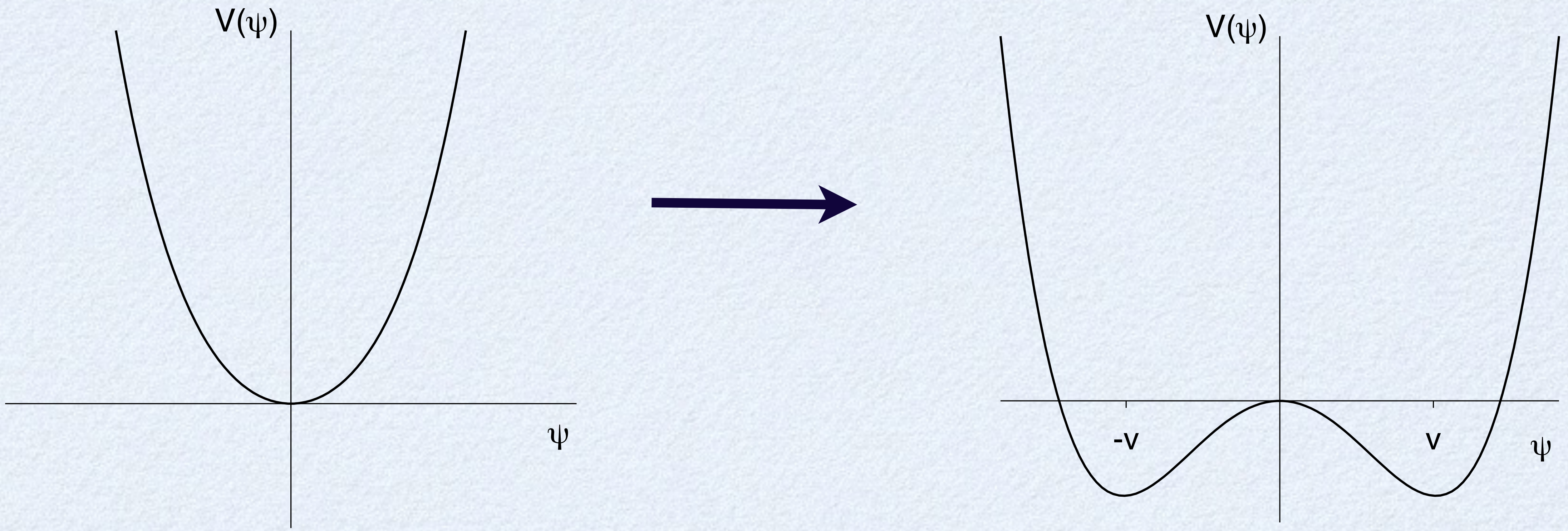
Really quick review : Vortices

Q: Can vortices or cosmic strings speed up (catalyze) vacuum decay?

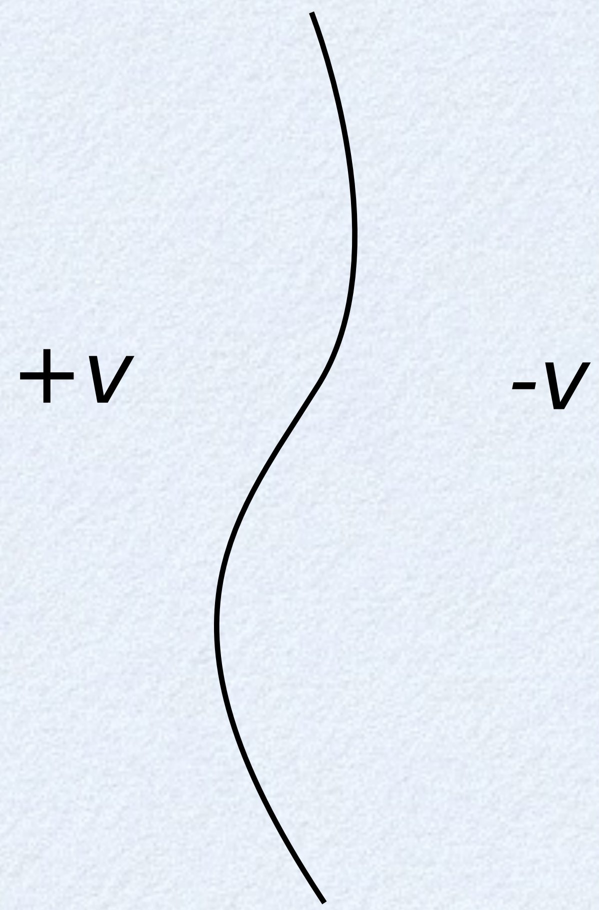
A: Yes! This occurs near the "dissociation limit" of the vortex. (See papers for details.)

4. DOMAIN WALLS

Real scalar field



Topological defects created by spontaneous symmetry breaking:



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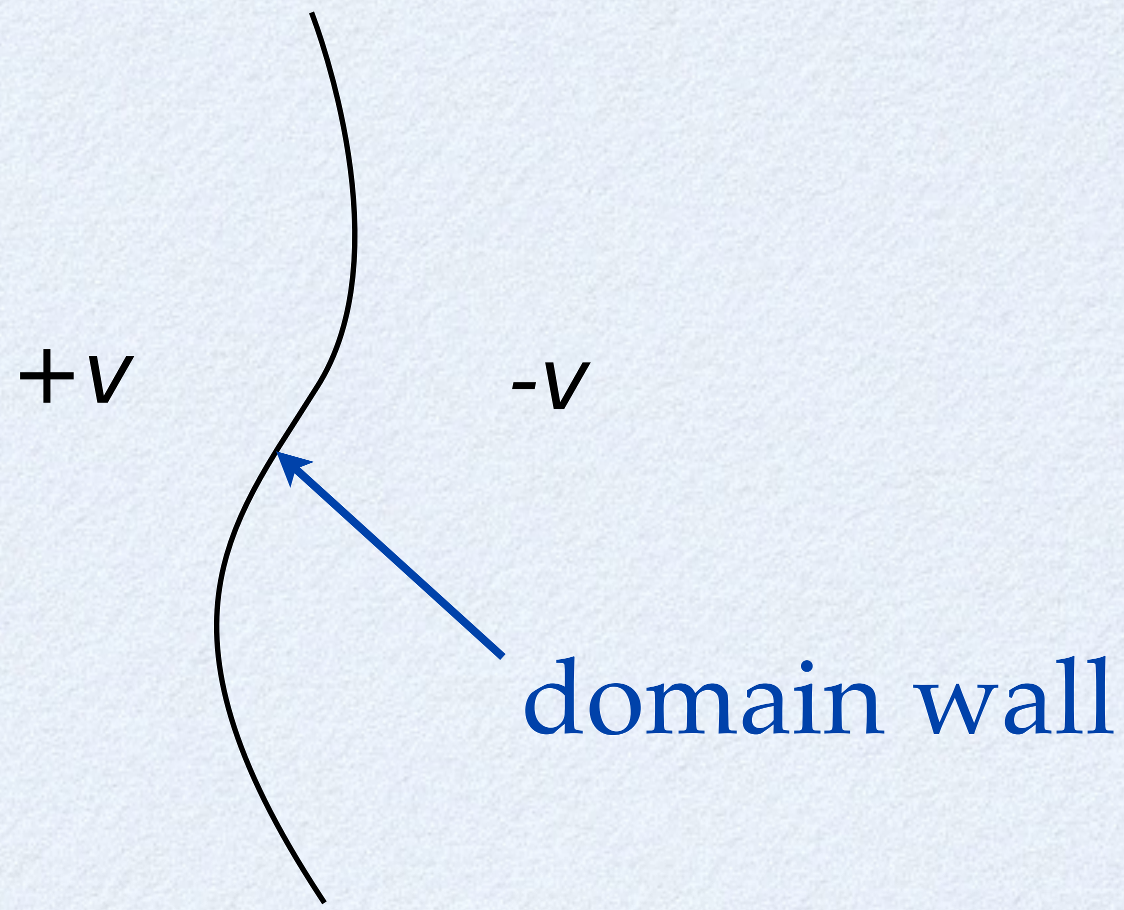
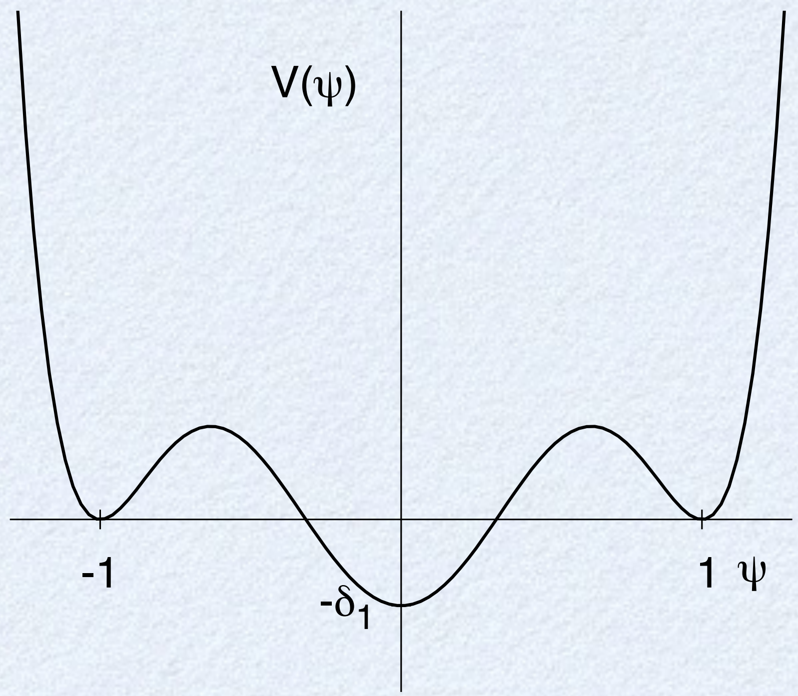


Modify the potential:

$$V(\psi) = (\psi^2 - 1)^2(\psi^2 - \delta_1)$$



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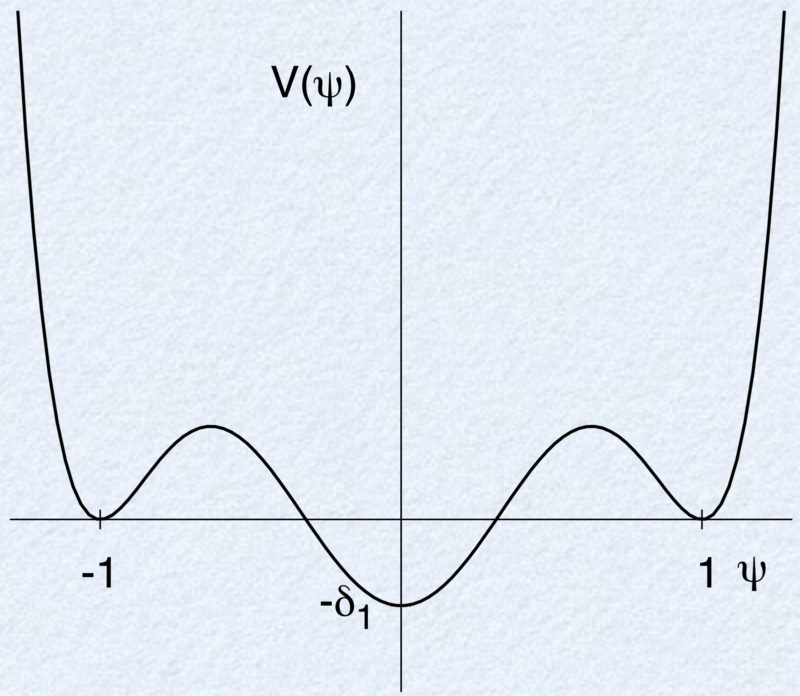
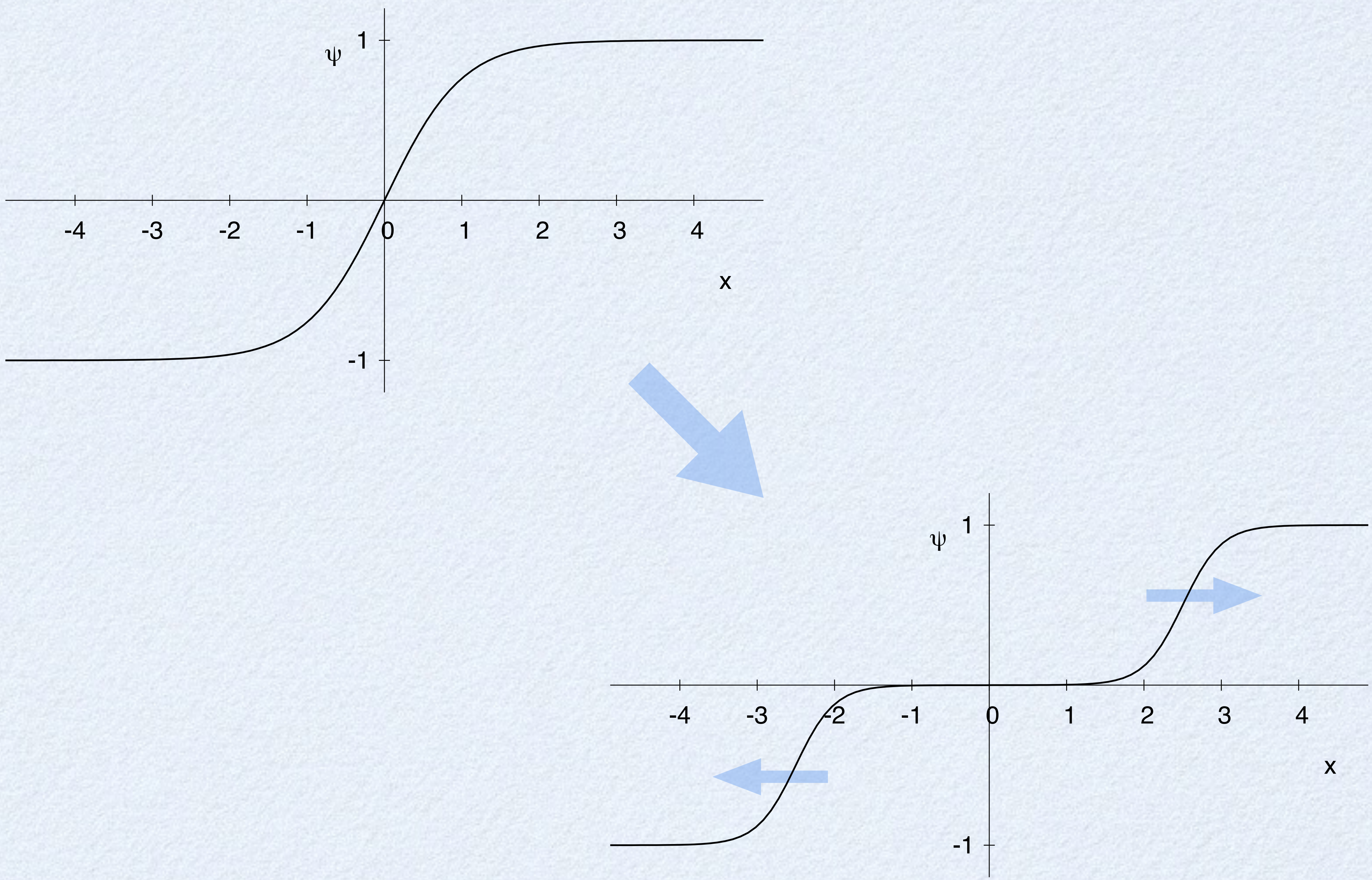


But no stable domain wall…

Domain walls



Domain Walls

To get metastable domain walls with the type of potential we are considering, add a second scalar field (and a bizarre potential)

$$\mathcal{L} = \frac{1}{2}(\partial\psi)^2 + \frac{1}{2}(\partial\phi)^2 - V(\psi,\phi)$$

where

$$\begin{aligned} V(\psi,\phi) &= (\psi^2-1)^2(\psi^2-\delta_1) \\ &\quad + \frac{1}{\psi^2+\gamma}\left((\phi^2-1)^2 - \frac{\delta_2}{4}(\phi-2)(\phi+1)^2\right) \end{aligned}$$

Domain Walls

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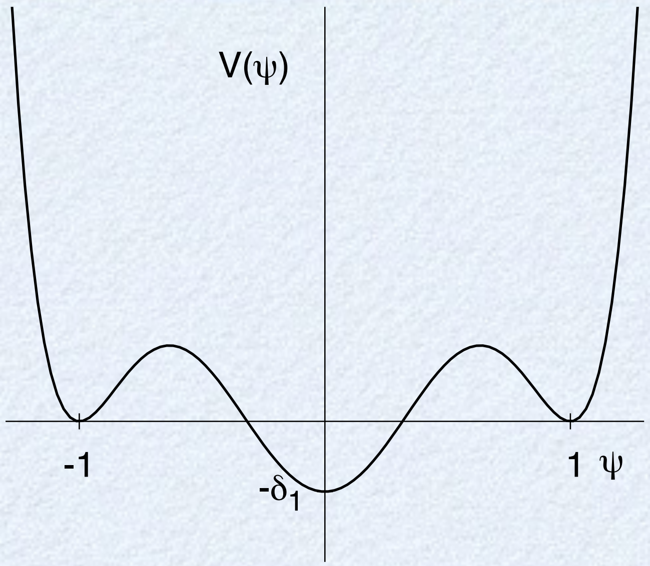
$$

Domain Walls



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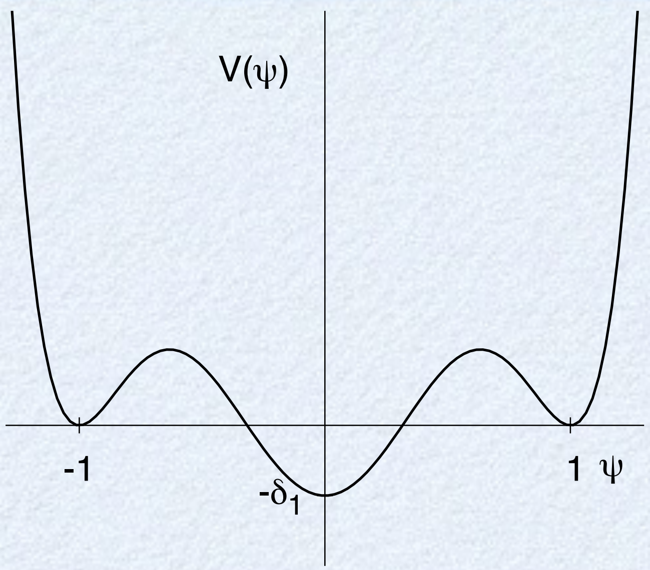
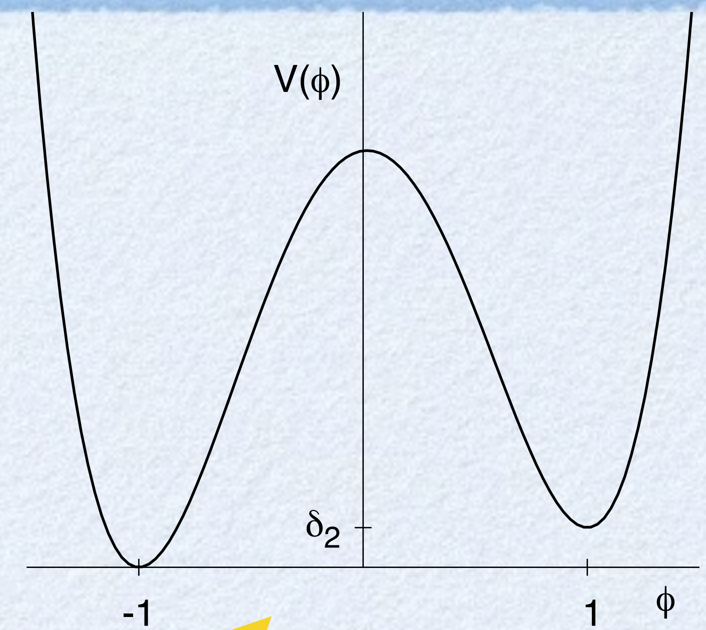
Domain walls





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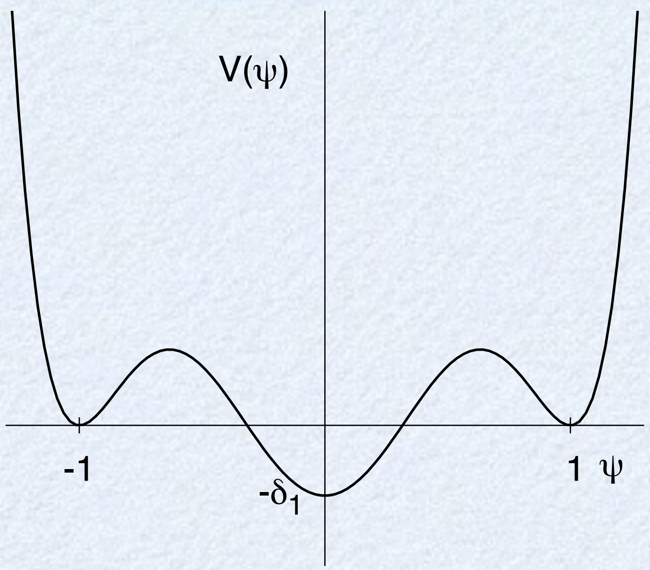
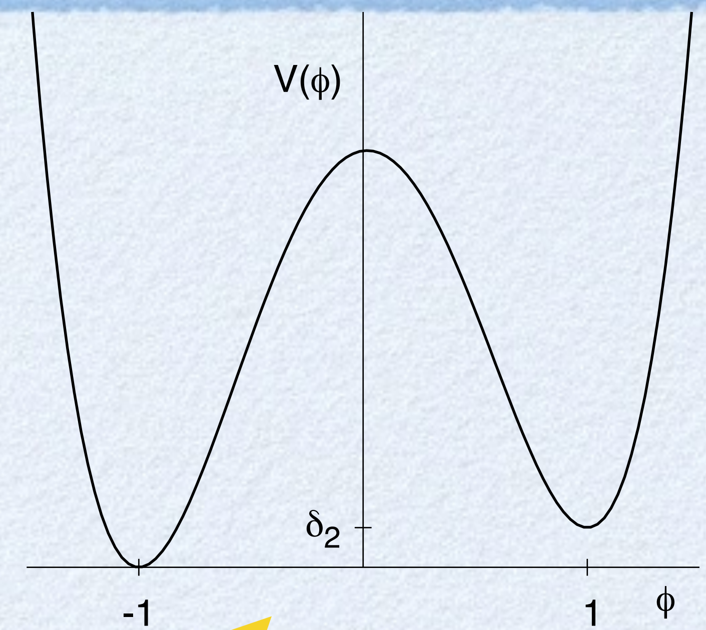
DOMAIN WALLS





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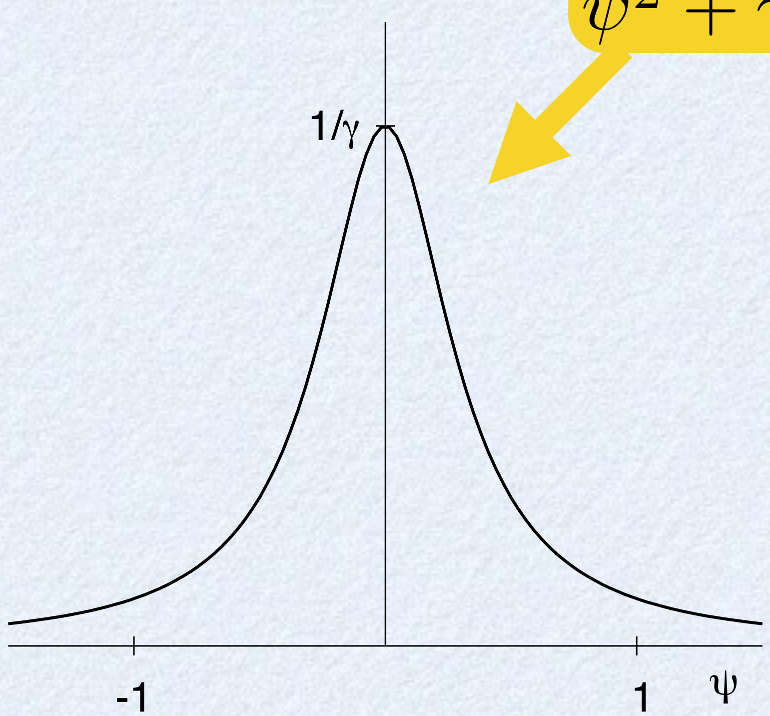

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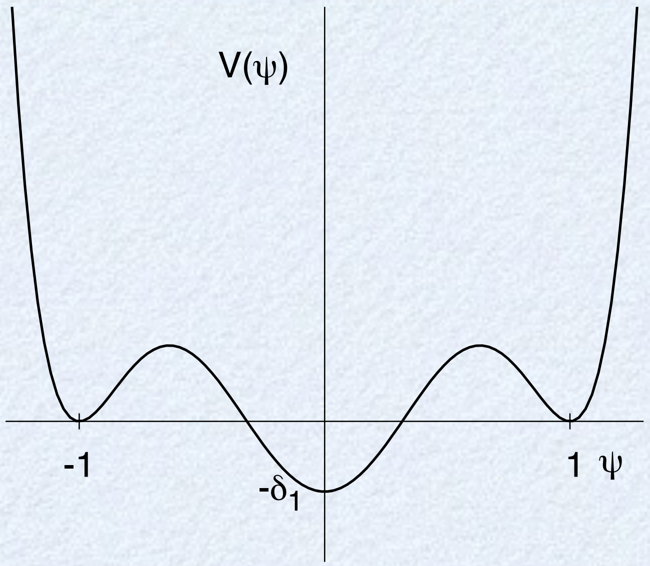

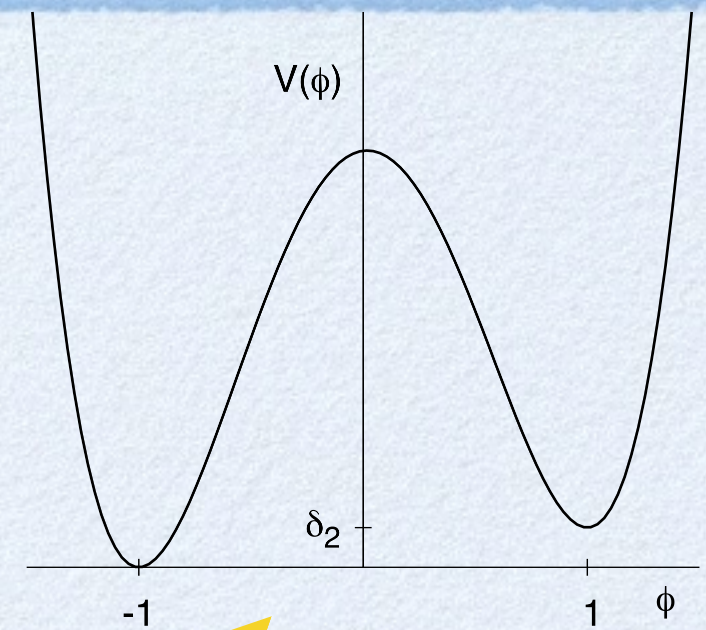

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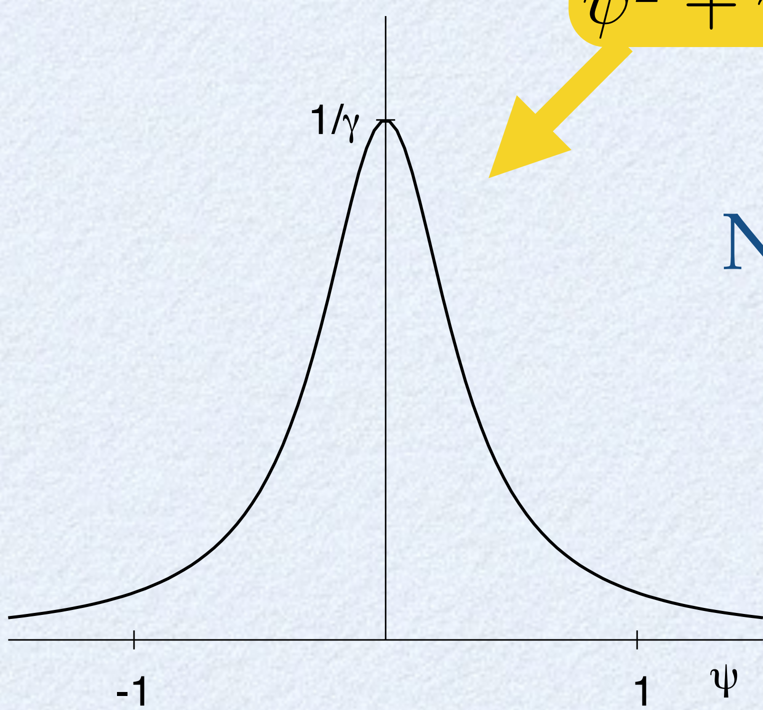

Notes: true vacuum: $(\psi,\phi) = (0,-1)$

false vacua: $(\psi,\phi) = (\pm 1,-1)$

(among others)

Domain Walls

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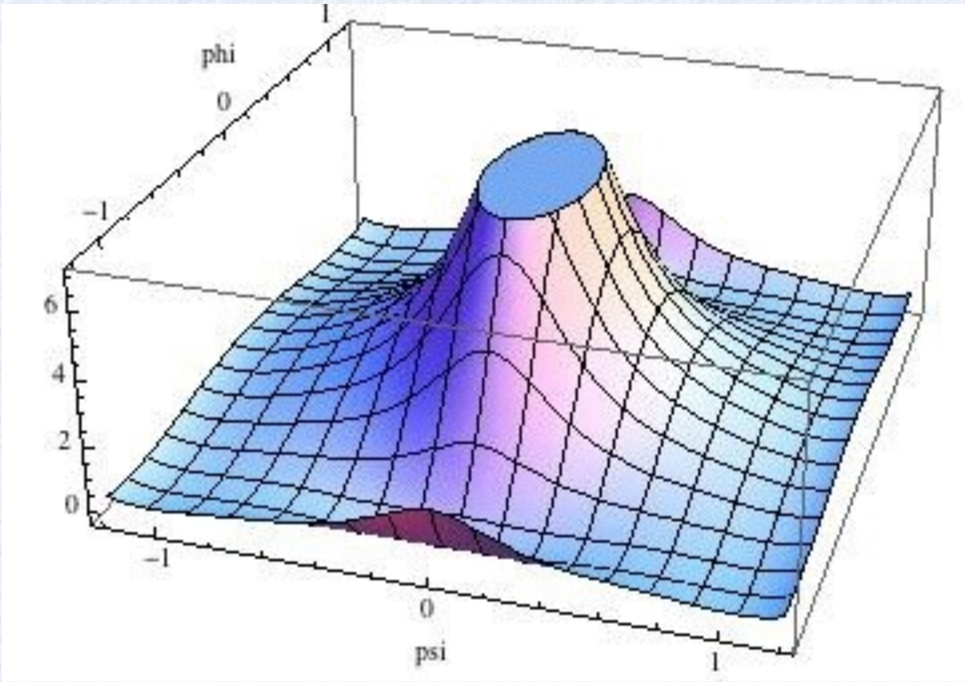

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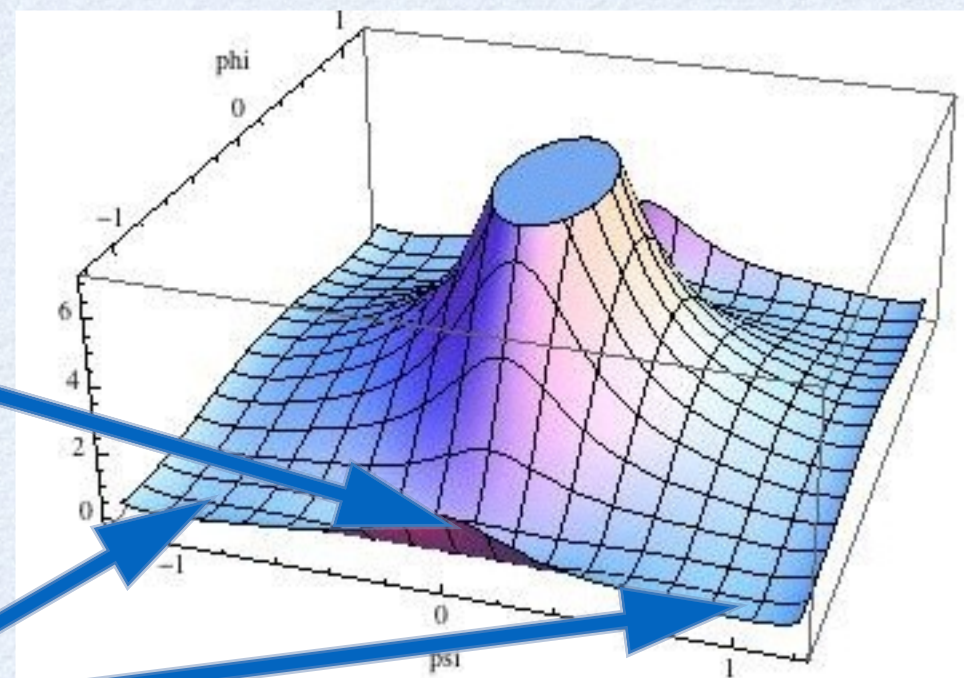

true vacuum

false vacua

Domain walls

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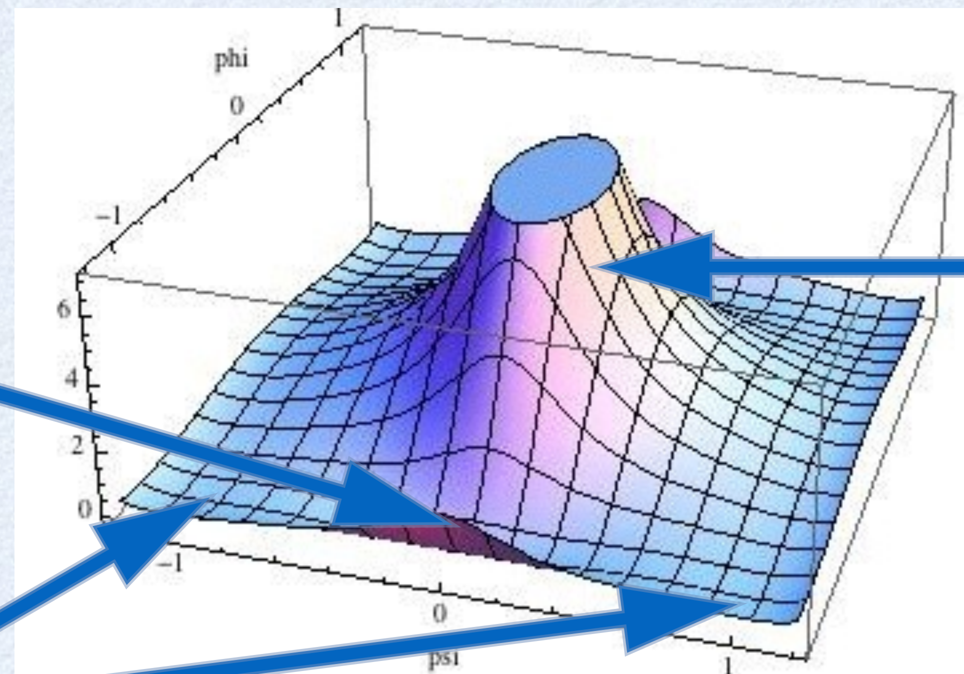


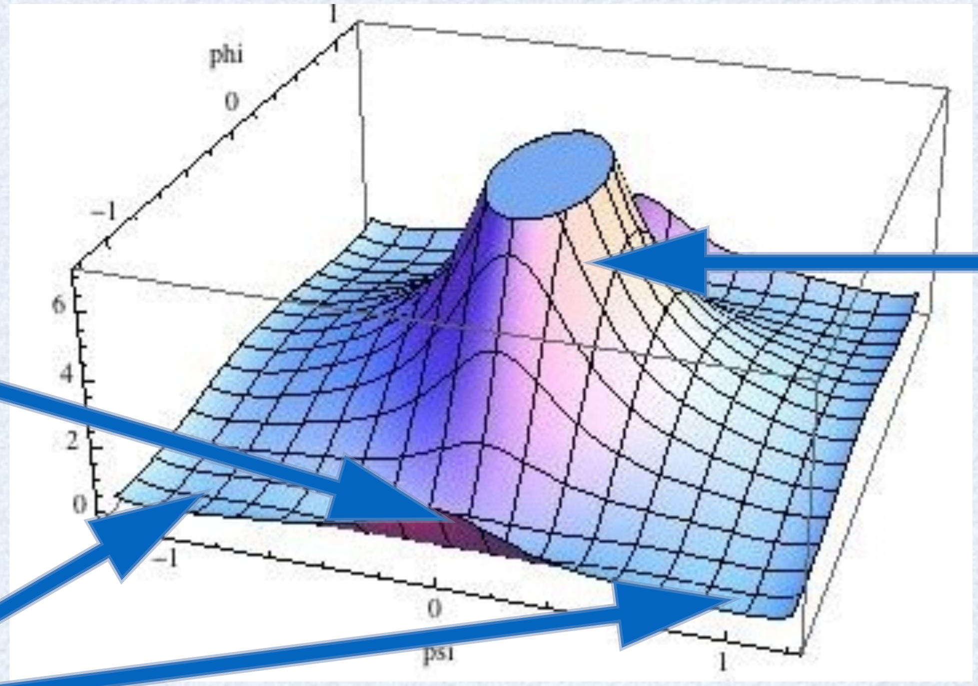
large energy barrier

true vacuum

false vacua

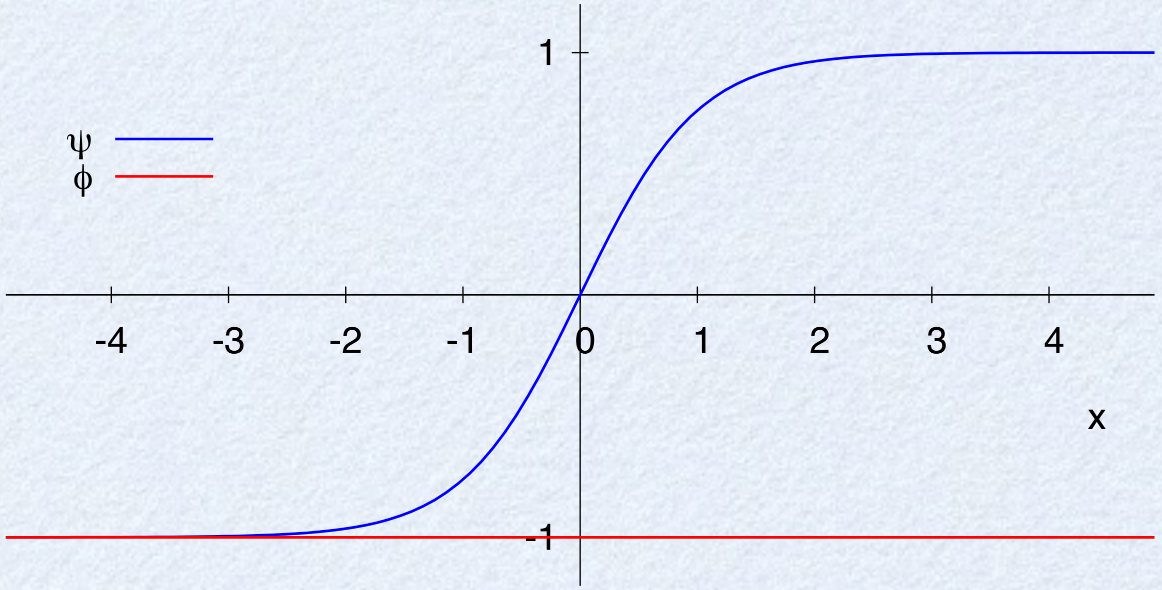
Domain walls

$$V(\psi,\phi) = (\psi^2-1)^2(\psi^2-\delta_1) + \frac{1}{\psi^2+\gamma}\left((\phi^2-1)^2 - \frac{\delta_2}{4}(\phi-2)(\phi+1)^2\right)$$



Look for static solutions interpolating between the false vacua

First attempt:



Domain walls

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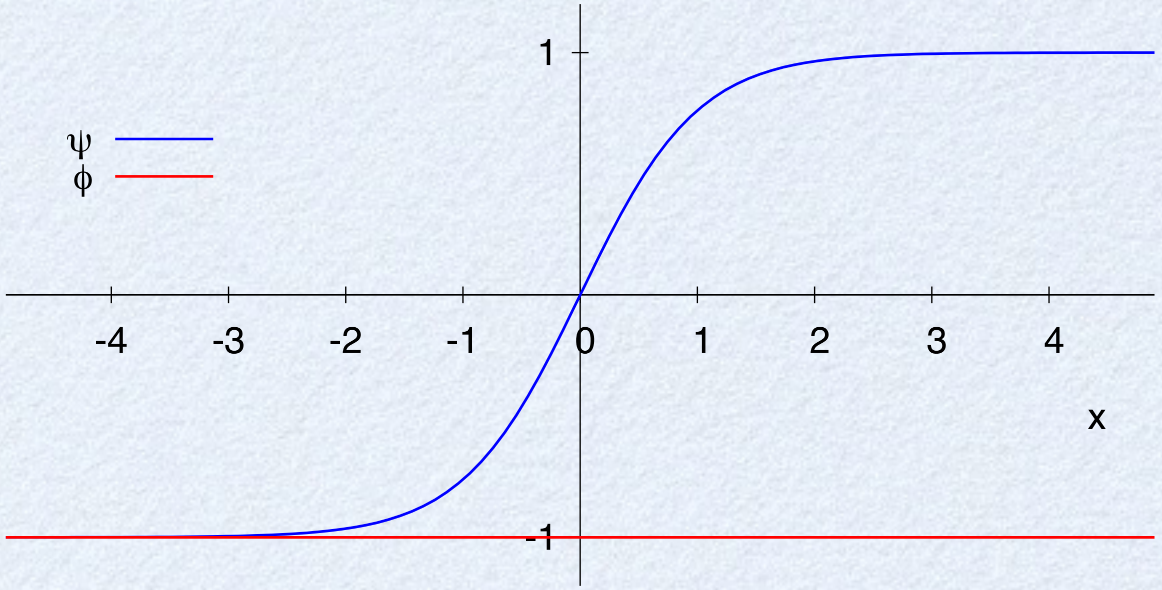
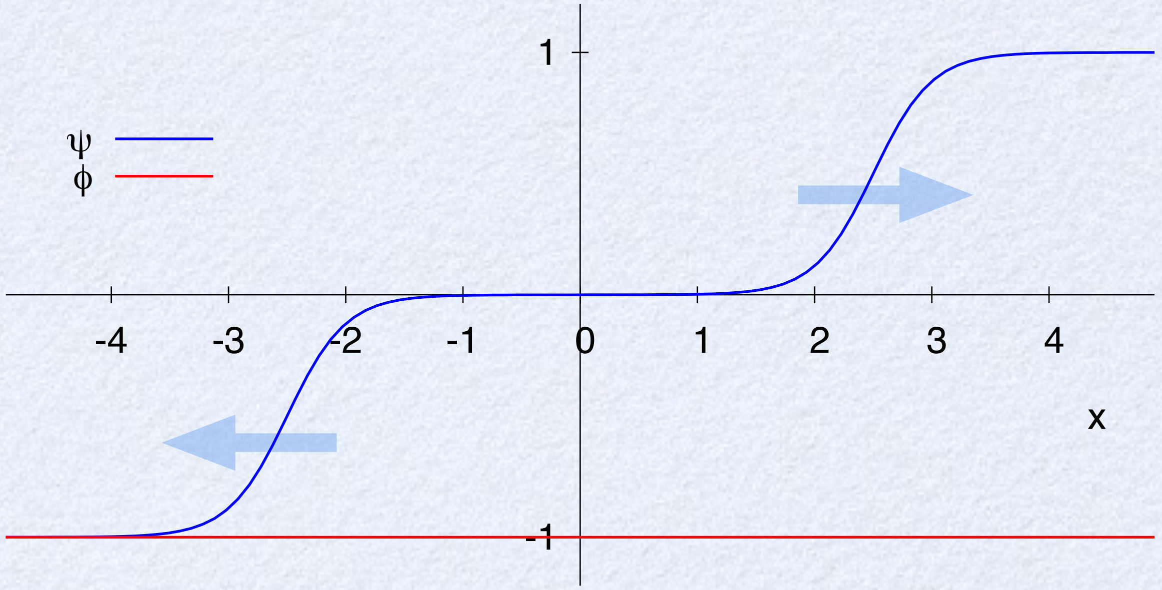
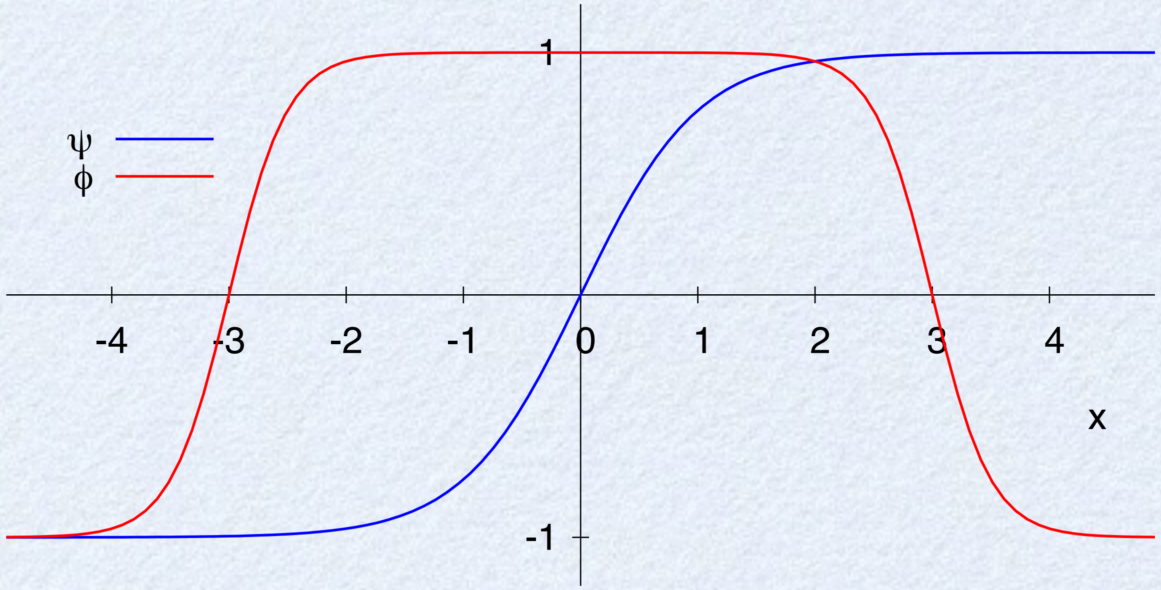


No!



Second attempt:



Domain Walls

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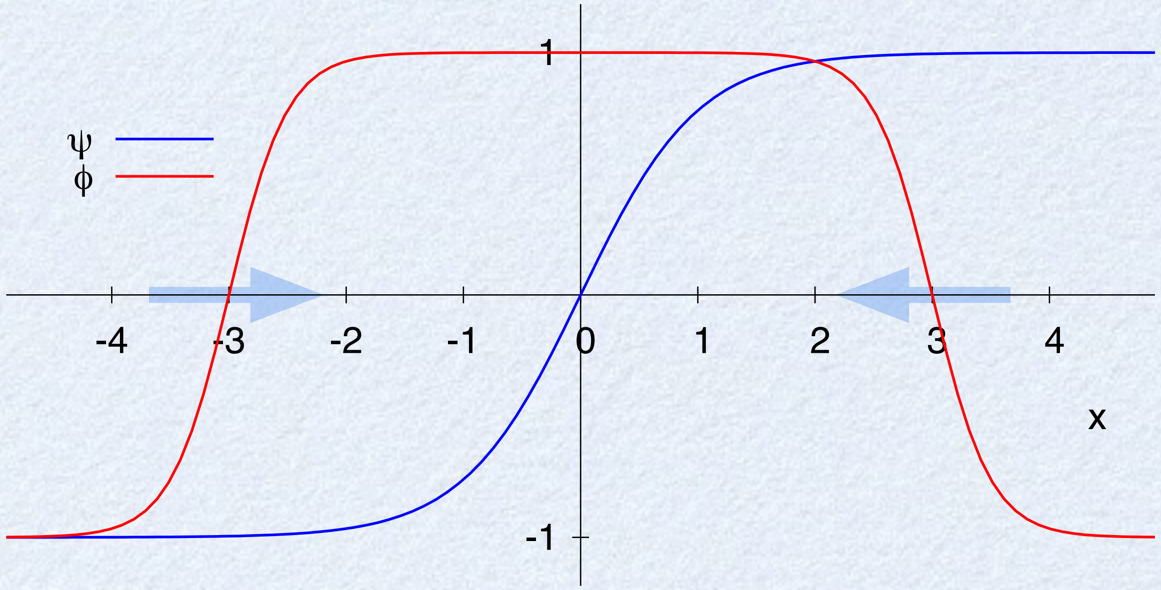
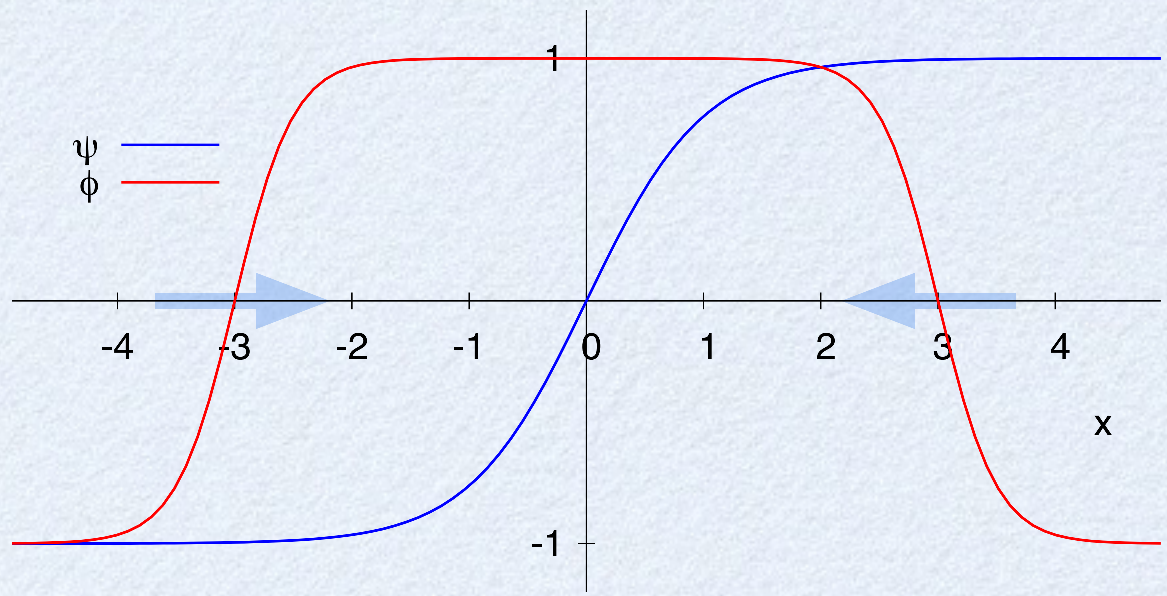


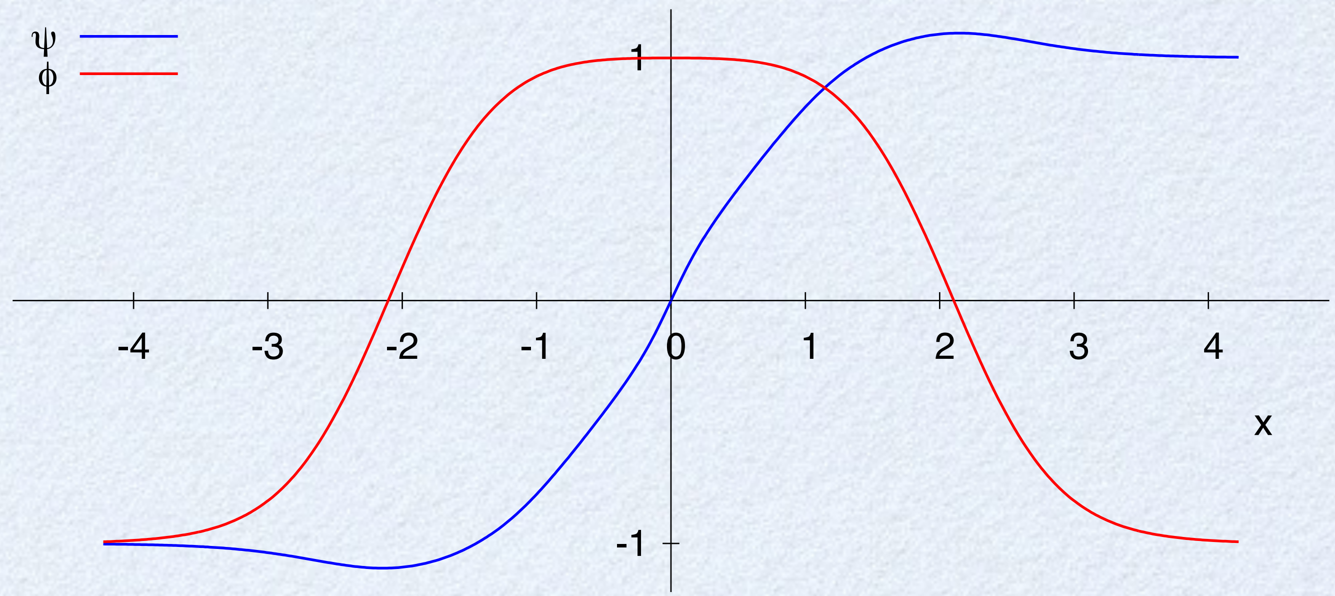
Yes! ϕ acts as a sort of "enveloping function" for ψ, preventing it from spreading.

Domain Walls

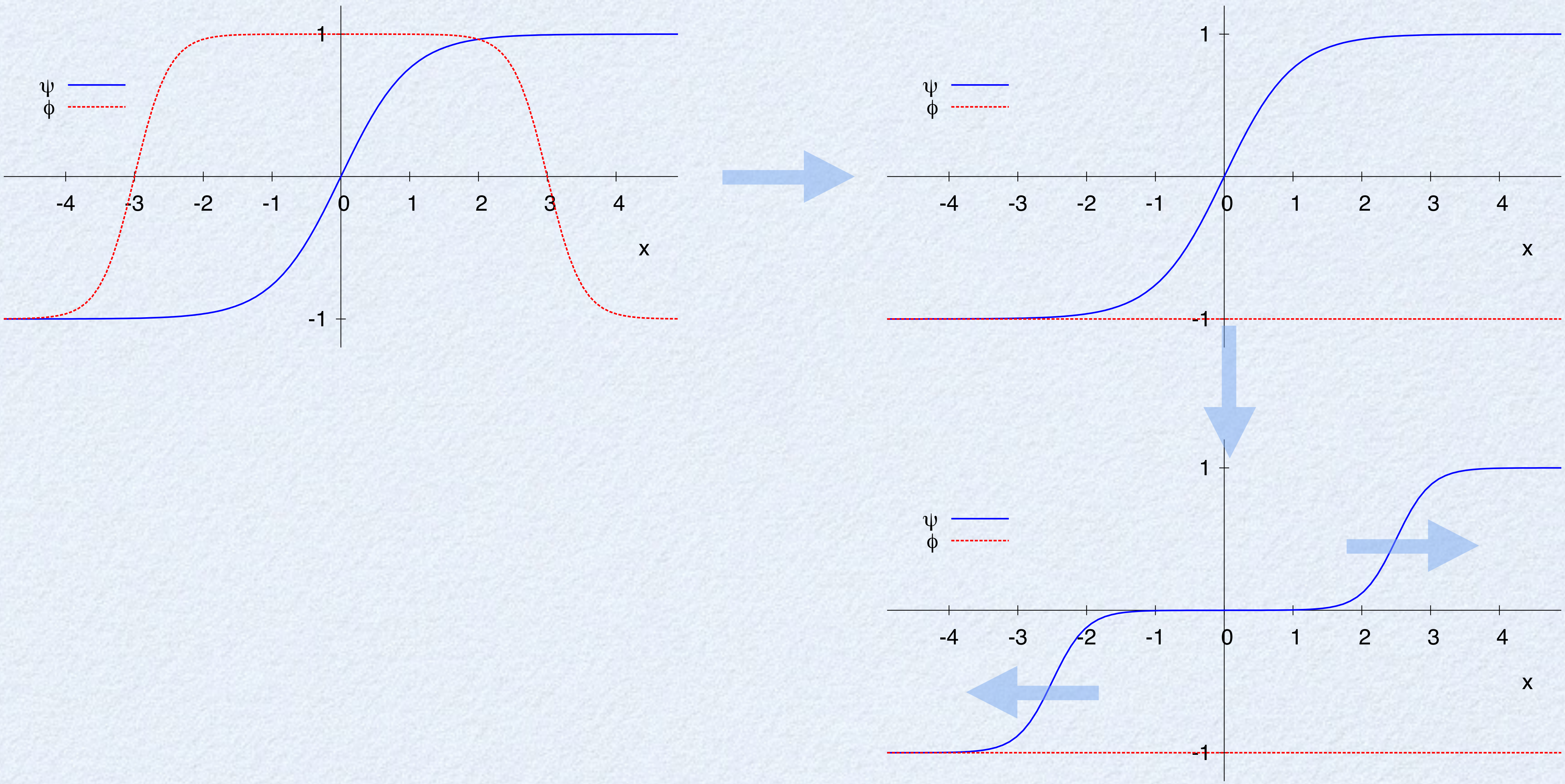
Second attempt:



Numerical solution:



The solution is classically stable, but will tunnel to an unstable solution:



We would like to calculate the decay rate of the domain wall. To do this:

- Euclideanize
- Find solution of least action interpolating between the static solution and an unstable configuration of the same energy
- Solution of least action dominates the decay: $\Gamma \sim e^{-S_E/\hbar}$

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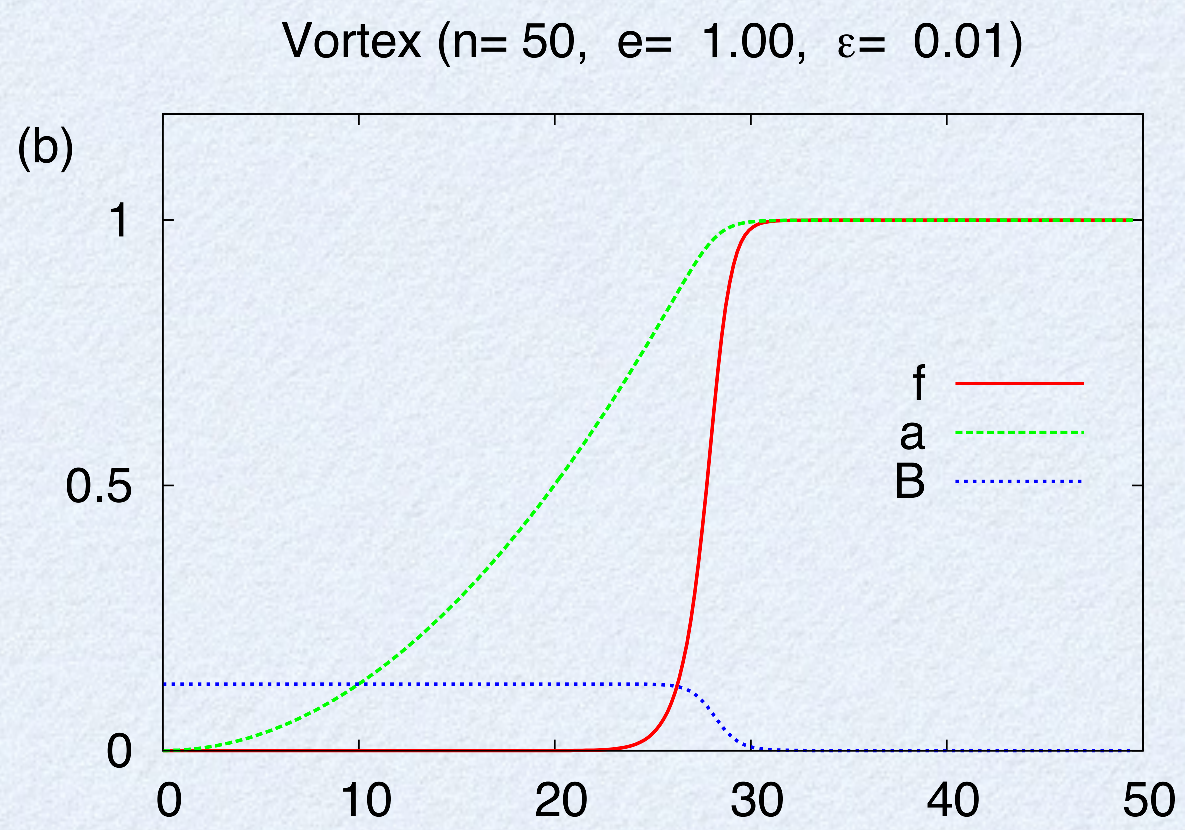
- Euclideanize
- Find solution of least action interpolating between the static solution and an unstable configuration of the same energy
- Solution of least action dominates the decay: $\Gamma \sim e^{-S_E/\hbar}$

THIS IS HARD!

Domain walls

Normal approach: look for a one-parameter family of configurations which interpolate between the stable and unstable configurations.
Typically: thin-wall approximation, eg vortex:



For the domain wall we create the following family of configurations: if the solution is

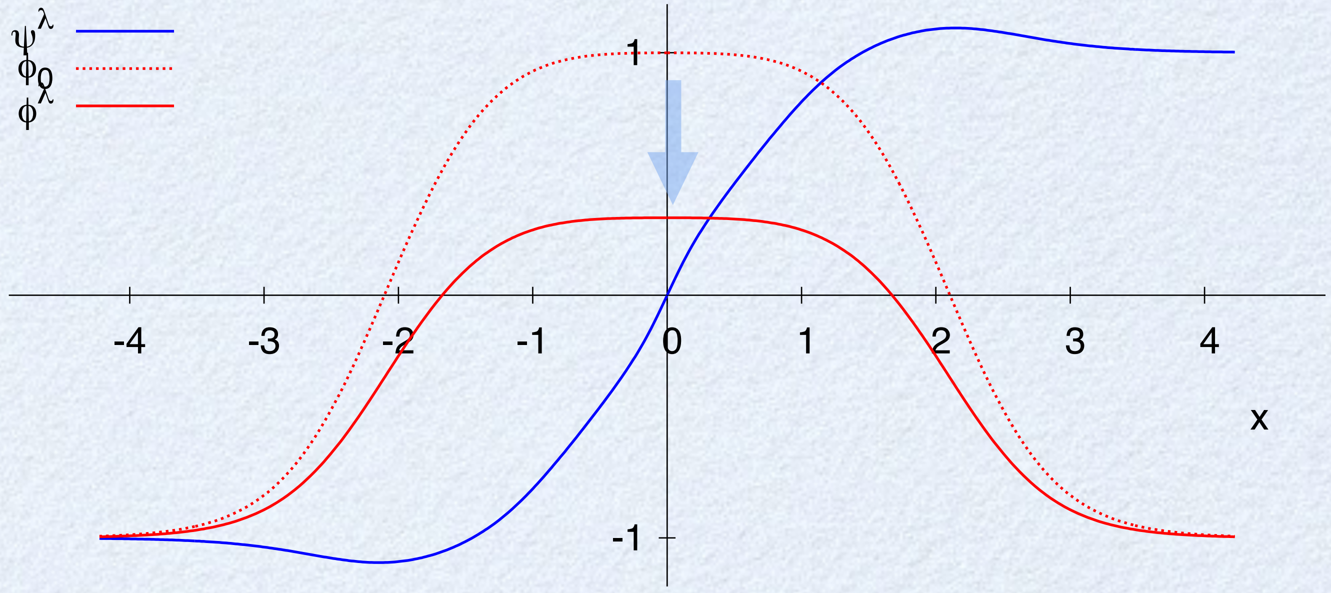
$$(\psi, \phi) = (\psi_0(x), \phi_0(x))$$

then we take

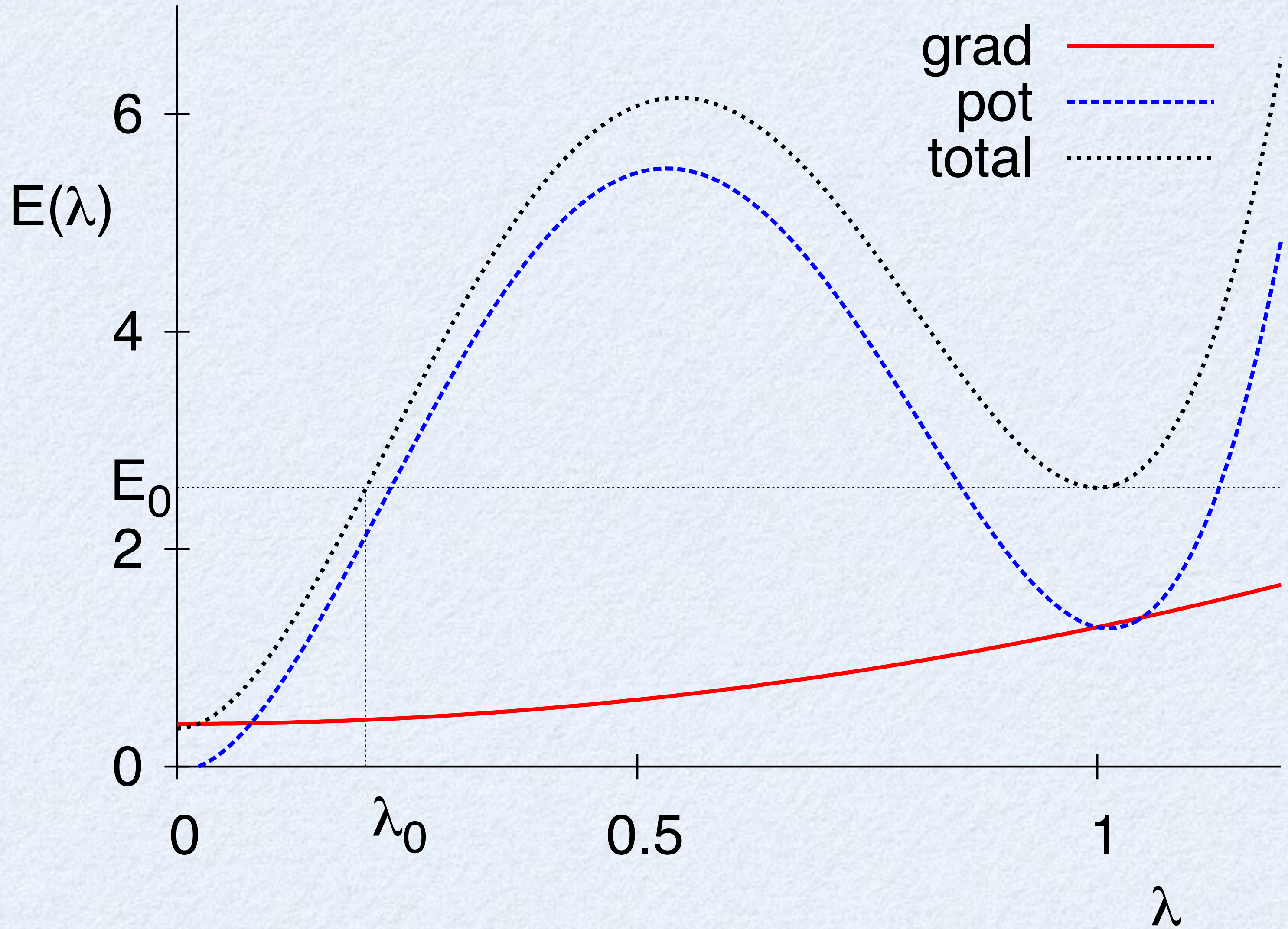
$$(\psi^\lambda, \phi^\lambda) = \Big(\psi_0(x), \lambda(t)(\phi_0(x) + 1) - 1\Big)$$

$$\lambda = 1 \quad : \quad \phi^\lambda = \phi_0$$

$$\lambda = 0 \quad : \quad \phi^\lambda = -1$$



Energy as a function of λ:



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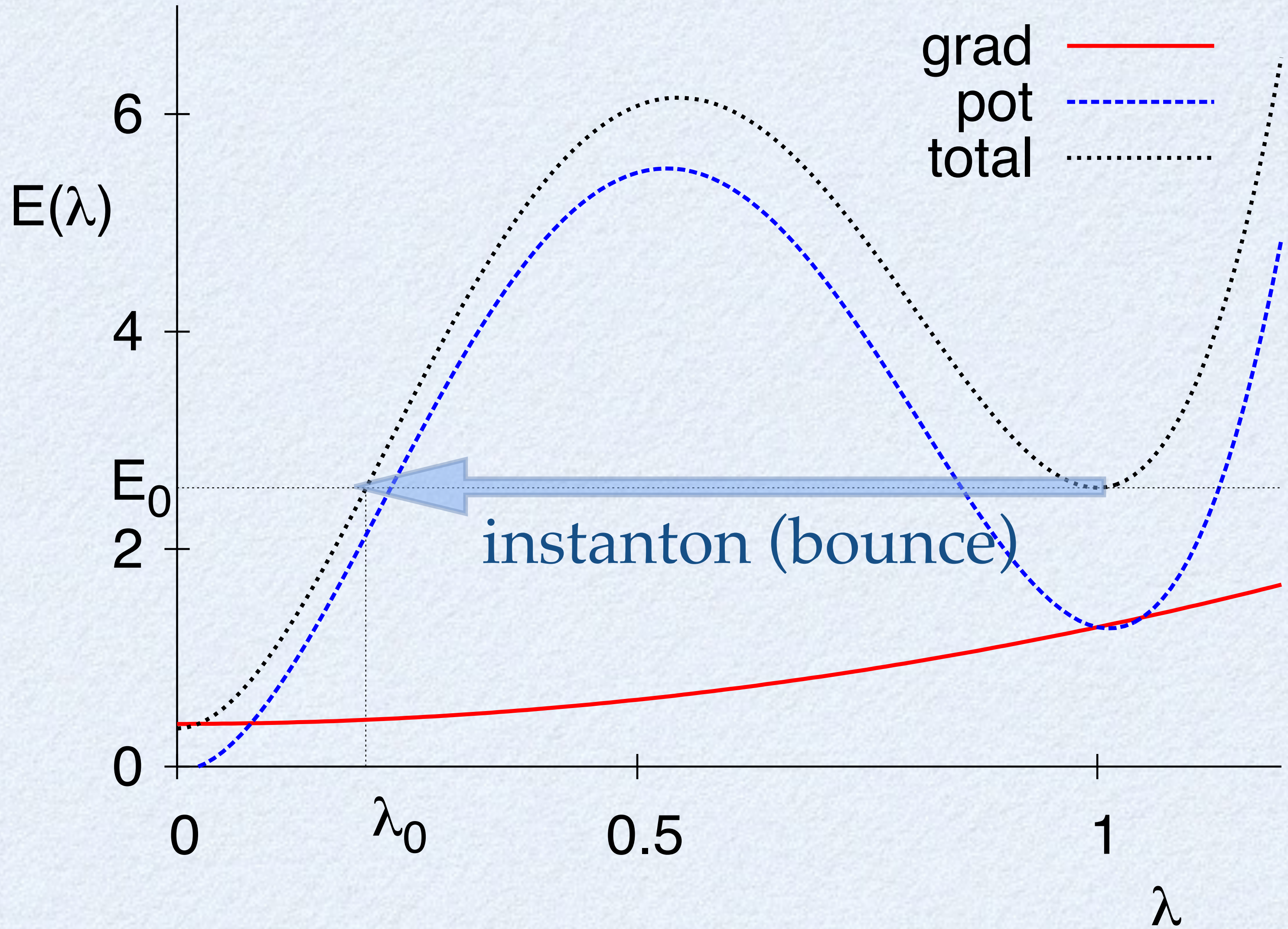

Effective action for $\lambda(t)$:

Euclideanize the field theory action:

$$S_E[\psi,\phi] \quad = \quad \int d^2x \Big\{ \frac{1}{2} \left[(\partial_t \psi)^2 + (\partial_x \psi)^2 + (\partial_t \phi)^2 + (\partial_x \phi)^2 \right] + V(\psi,\phi) \Big\}$$

with $$V(\psi,\phi) \quad = \quad (\psi^2-1)^2(\psi^2-\delta_1) + \frac{1}{\psi^2+\gamma}\left((\phi^2-1)^2 - \frac{\delta_2}{4}(\phi-2)(\phi+1)^2 \right)$$

Substitute $(\psi,\phi) = (\psi^\lambda, \phi^\lambda)$, integrate over x.

This gives:

$$S_E[\lambda(t)] = \int dt \left(\frac{1}{2} M \dot{\lambda}^2 + (E(\lambda) - E_0) \right)$$

where:

$$M = \int dx (\phi_0 + 1)^2$$

$E(\lambda)$ is the static energy of $(\psi^\lambda, \phi^\lambda)$

(It is a quartic function of λ;
coefficients can be evaluated numerically
(not terribly enlightening).)

$$S_E[\lambda(t)] = \int dt \left(\frac{1}{2} M \dot{\lambda}^2 + (E(\lambda) - E_0) \right)$$

The bounce satisfies the Euclidean equation of motion: $$M\ddot{\lambda} = \frac{dE(\lambda)}{d\lambda}$$

First integral: $$\frac{1}{2} M \dot{\lambda}^2 = E(\lambda) - E_0$$

Then the bounce action can be written:

$$S_E[\lambda(t)] = \int_{\lambda_0}^{1} d\lambda \sqrt{2M\Big(E(\lambda) - E_0\Big)}$$

5. Conclusions and outlook

- Studied a model with metastable kinks in 1+1 dimensions
- Found a 1-parameter family of configurations which (hopefully) accurately describe tunnelling
- Obtained an expression for the bounce action (related to the decay rate of the kink)

Future work:

- Explore parameter space (and/or look for a less artificial/contrived model)
- look specifically for regions where the bounce action is small (analog of dissociation limit of vortices)
- analog of thin-wall configurations for which a more realistic bounce may be found
- examine the same model in higher dimensions (so, domain walls in 3+1d)

Future work:

- Explore parameter space (and/or look for a less artificial/contrived model)
- look specifically for regions where the bounce action is small (analog of dissociation limit of vortices)
- analog of thin-wall configurations for which a more realistic bounce may be found
- examine the same model in higher dimensions (so, domain walls in 3+1d)

Thank you!