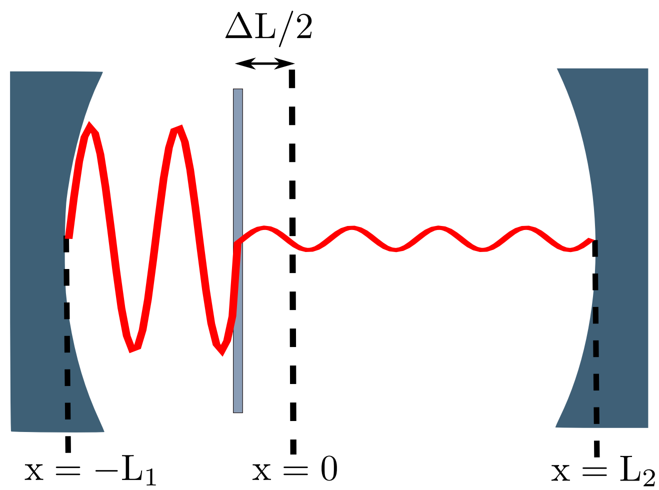

Adiabatic Transfer of Light in a double cavity

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NSERC CRSNG

Membrane in the middle experiment (Yale)

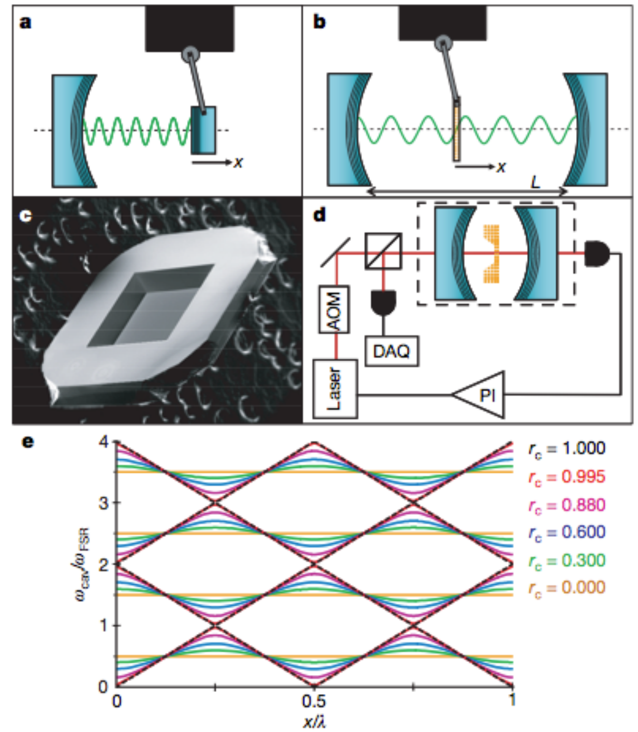
- 50 nm thick SiN membrane

$$\omega = 2\pi \times 100\ \mathrm{kHz}$$

$$Q = 10^6$$

$$R = 20\%$$

$$R = 99.8\%??$$ (if patterned)



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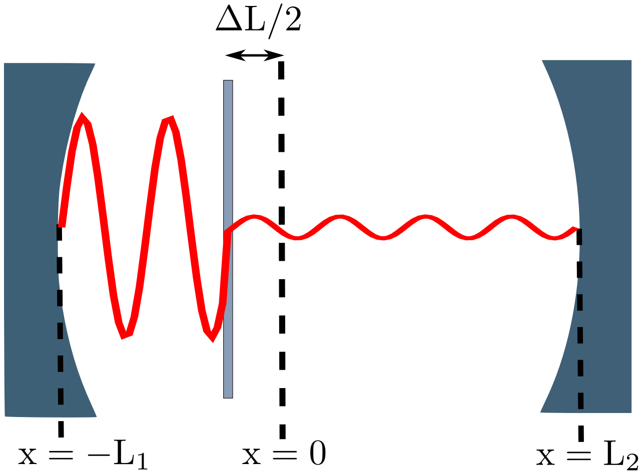
LETTERS

Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane

J. D. Thompson[1], B. M. Zwickl[1], A. M. Jayich[1], Florian Marquardt[2], S. M. Girvin[1,3] & J. G. E. Harris[1,3]

δ-function mirror model

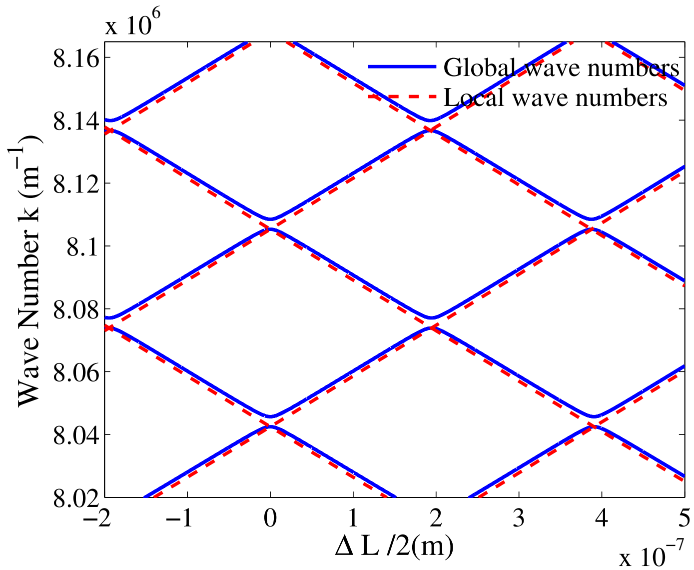


$$\frac{\partial^2 \mathcal{E}}{\partial x^2} - \mu_0 \epsilon(x) \frac{\partial^2 \mathcal{E}}{\partial t^2} = 0$$

$$\epsilon(x) = \begin{cases} \epsilon_0[1 + \alpha\delta(x)], & -L_1 < x < L_2 \\ \infty, & x > L_2,\ x < -L_1 \end{cases}$$

$$\Delta L = L_1 - L_2$$

Wavenumbers of normal modes



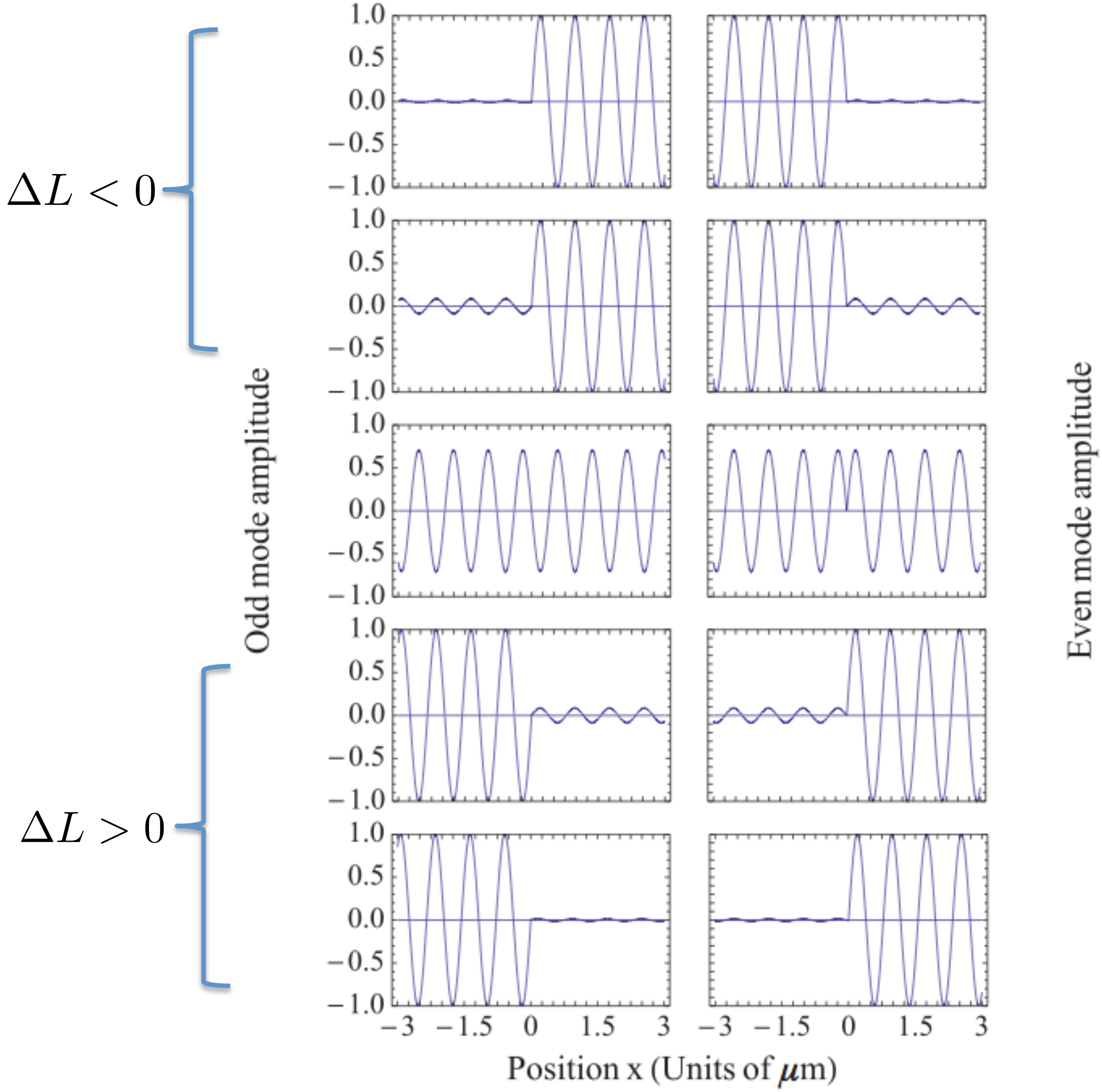
Stationary mirror:

$$\mathcal{E}(x,t) = U(x) e^{-i\omega t}$$

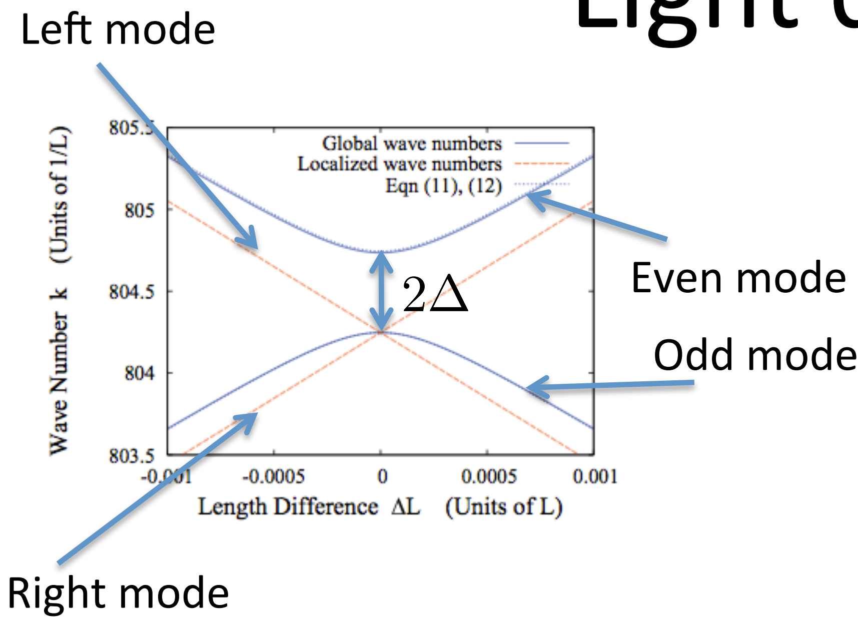
$$\frac{\partial^2 U}{\partial x^2} + k^2 [1 + \alpha\delta(x)]\ U = 0$$

Dependence of light amplitudes on mirror position

$$\Delta L = L_1 - L_2$$



Light dynamics



$$H = \hbar\omega_e b_e^\dagger b_e + \hbar\omega_o b_o^\dagger b_o$$

$$H = (\hbar\omega_{\rm av} + E)a_R^\dagger a_R + (\hbar\omega_{\rm av} - E)a_L^\dagger a_L + \Delta(a_R^\dagger a_L + a_L^\dagger a_R)$$

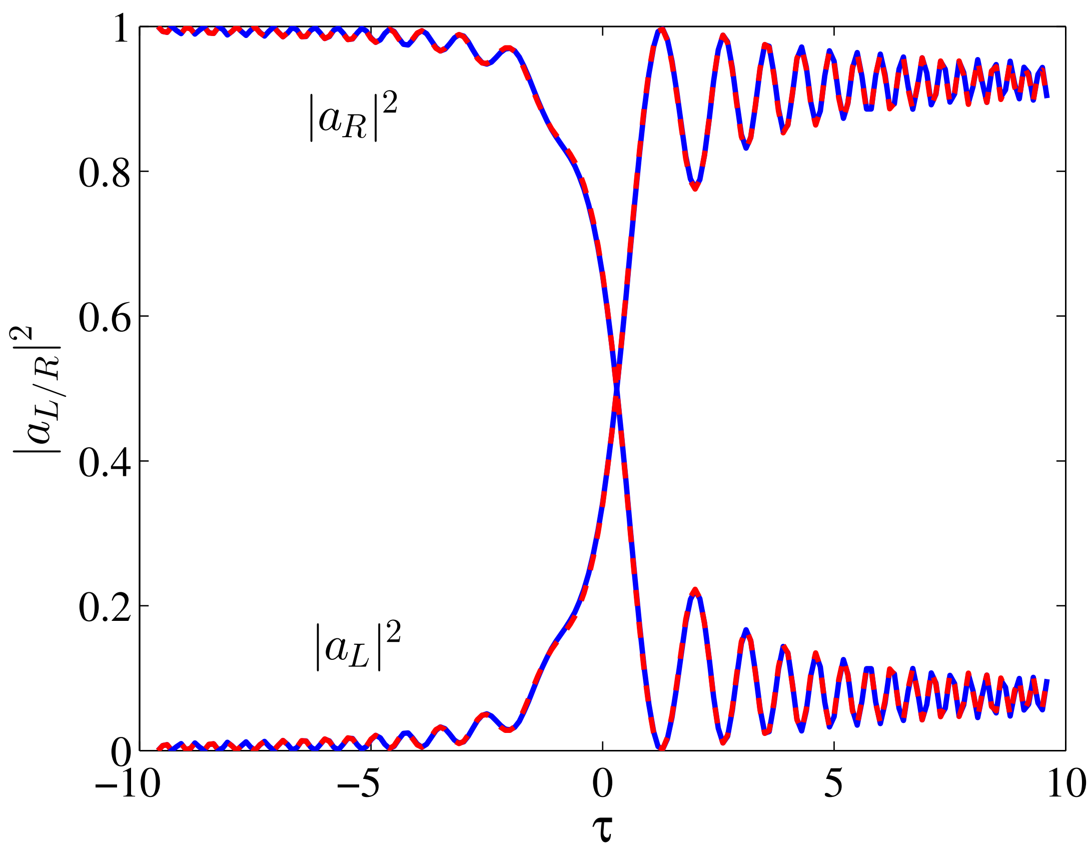
Questions:

1) Can we ignore time dependence of modes?

2) Can we reduce 1st order time dynamics $\dot{a}_i = \frac{1}{i\hbar}[a_i, H]$?

If 2) true, then we can use Landau-Zener theory: $P_{LZ} = \exp(-2\pi\Delta^2/\hbar\vartheta)$ where $E = \vartheta t/2$

2^{nd} order time evolution with and without time-dependence of modes



98% reflectivity, v=5,000 ms^{-1} , $\Delta L=10^{-7}$m

1st vs 2nd order dynamics

2nd order

$$-\begin{pmatrix} \ddot{a}_L \\ \ddot{a}_R \end{pmatrix} = \begin{pmatrix} [\omega_{av} + E(t)]^2 + \Delta^2 & 2\Delta\omega_{av} \\ 2\Delta\omega_{av} & [\omega_{av} - E(t)]^2 + \Delta^2 \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix}.$$

1st order

put: $a_{L/R} = \tilde{a}_{L/R} \exp\left\{-i\int_{t_0}^{t} \beta_{L/R}(t')dt'\right\}$ where $\beta_{L/R}(t) = \sqrt{(E(t) \pm \omega_{av})^2 + \Delta^2}$

$$\Rightarrow \quad i\begin{pmatrix} \dot{a}_L \\ \dot{a}_R \end{pmatrix} \approx \begin{pmatrix} \omega_{av} + E(t) & \Delta \\ \Delta & \omega_{av} - E(t) \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix}.$$

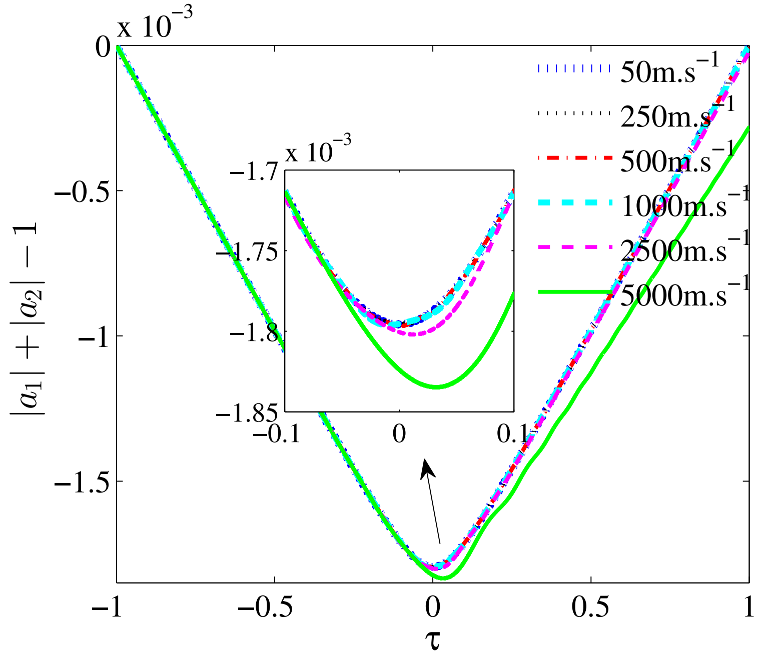
(Landau-Zener form!)

i.e. $\ddot{\tilde{a}} \approx 0, \quad , \dot{\beta}\tilde{a} \approx 0, \quad \beta\dot{\tilde{a}} \neq 0, \quad \beta^2\tilde{a} \neq 0$

N. Miladinovic, F. Hasan, N. Chisholm, I.E. Linnington,
E.A. Hinds, & D.O'D, PRA **84**, 043822 (2011)

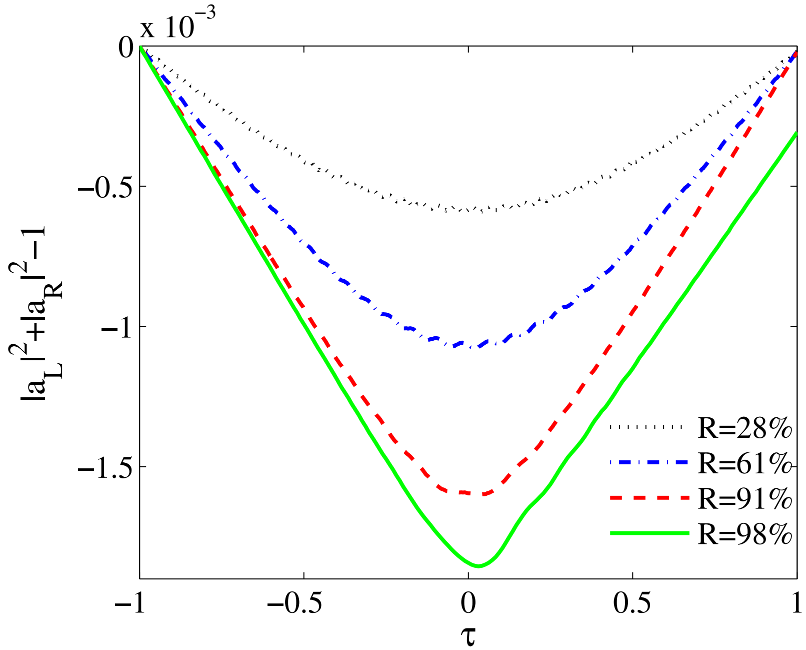
1st vs 2nd order dynamics

Dependence on mirror speed



R=98%

Dependence upon reflectivity

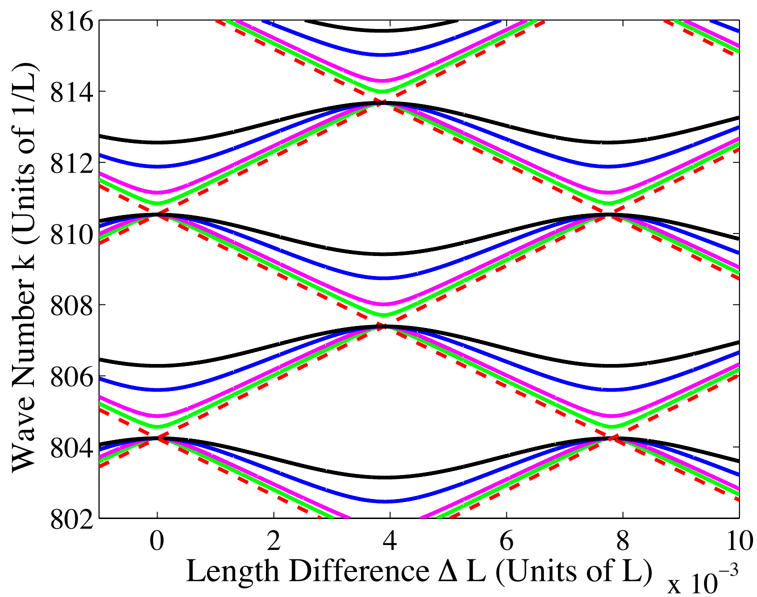


v=5000 ms^{-1}

$$H_{\text{Maxwell}} = \frac{1}{2}\int \epsilon(x)|\mathcal{E}(x,t)|^2 dx = \frac{\epsilon_0}{2}\left(|b_e|^2 + |b_o|^2\right) = \frac{\epsilon_0}{2}\left(|a_L|^2 + |a_R|^2\right)$$

Interesting feature

- As we decrease mirror speed the agreement between 1st and 2nd does **not** keep getting better and better, but instead saturates.
- Saturation point depends upon Δ (~1/Reflectivity)



$$\omega_{e/o} = \omega_{av} \pm \sqrt{\Delta^2 + \gamma \Delta L^2}$$

Better agreement between 1st and 2nd order when reflectivity is small

Measure for difference between 1st and 2nd order time dynamics

Define: $\vec{a} = \begin{pmatrix} a_L \\ a_R \end{pmatrix}$ & $M = -i \begin{pmatrix} \omega_{av} + E(t) & \Delta \\ \Delta & \omega_{av} - E(t) \end{pmatrix}.$

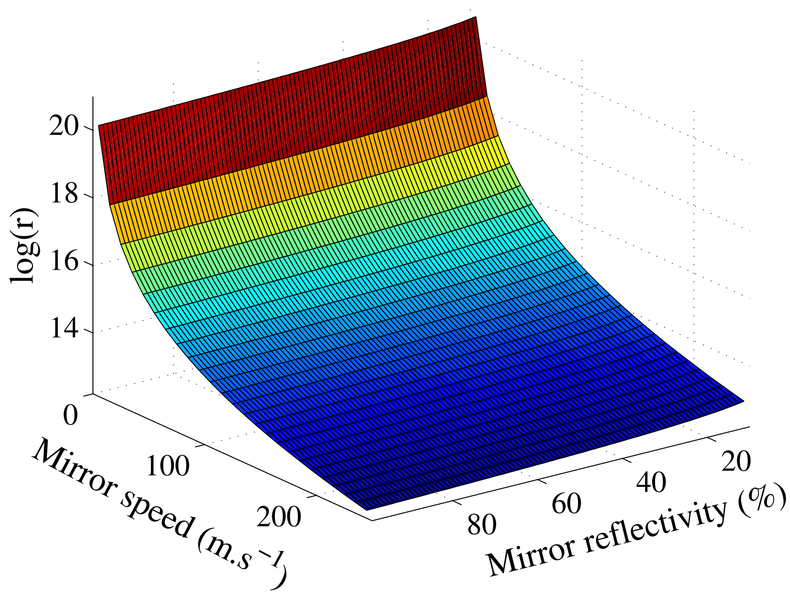
then: $\frac{d^2}{dt^2}\vec{a} = M^2\vec{a}.$ $\frac{d}{dt}\vec{a} = M\vec{a}$

Take derivative of 1st order equation $\frac{d^2}{dt^2}\vec{a} = M^2\vec{a} + \dot{M}\vec{a}$

$$r = \frac{||M||_F}{||\dot{M}||_F} = \frac{(\gamma^2\Delta L^4 + \Delta^4 + \omega_{av}^4) + 6\omega_{av}^2(\Delta^2 + \gamma\Delta L^2) + 2\gamma\Delta^2\Delta L^2}{\gamma\Delta\dot{L}^2}$$

When *r* is small the agreement is good

Dependence of difference between 1st and 2nd order time dynamics upon mirror speed and reflectivity



$$r = \frac{||M||_F}{||\dot{M}||_F} = \frac{(\gamma^2 \Delta L^4 + \Delta^4 + \omega_{av}^4) + 6\omega_{av}^2(\Delta^2 + \gamma \Delta L^2) + 2\gamma \Delta^2 \Delta L^2}{\gamma \Delta \dot{L}^2}$$

When *r* is small the agreement is good

Summary

- For fixed range of ΔL, agreement between 1st and 2nd order time dynamics is best for **slow** and **weakly reflective** mirrors.
- Time dependence of modes can be ignored

Other points (not shown)

- If time is scaled so that $\tau=\Delta t$, then 1st and 2nd agree when Δ/ω_{av} ->0 (weak coupling) i.e. high reflectivity (opposite to here!)
- Energy difference can be related to work done by mirror on light. Connection to dynamical Casimir effect?