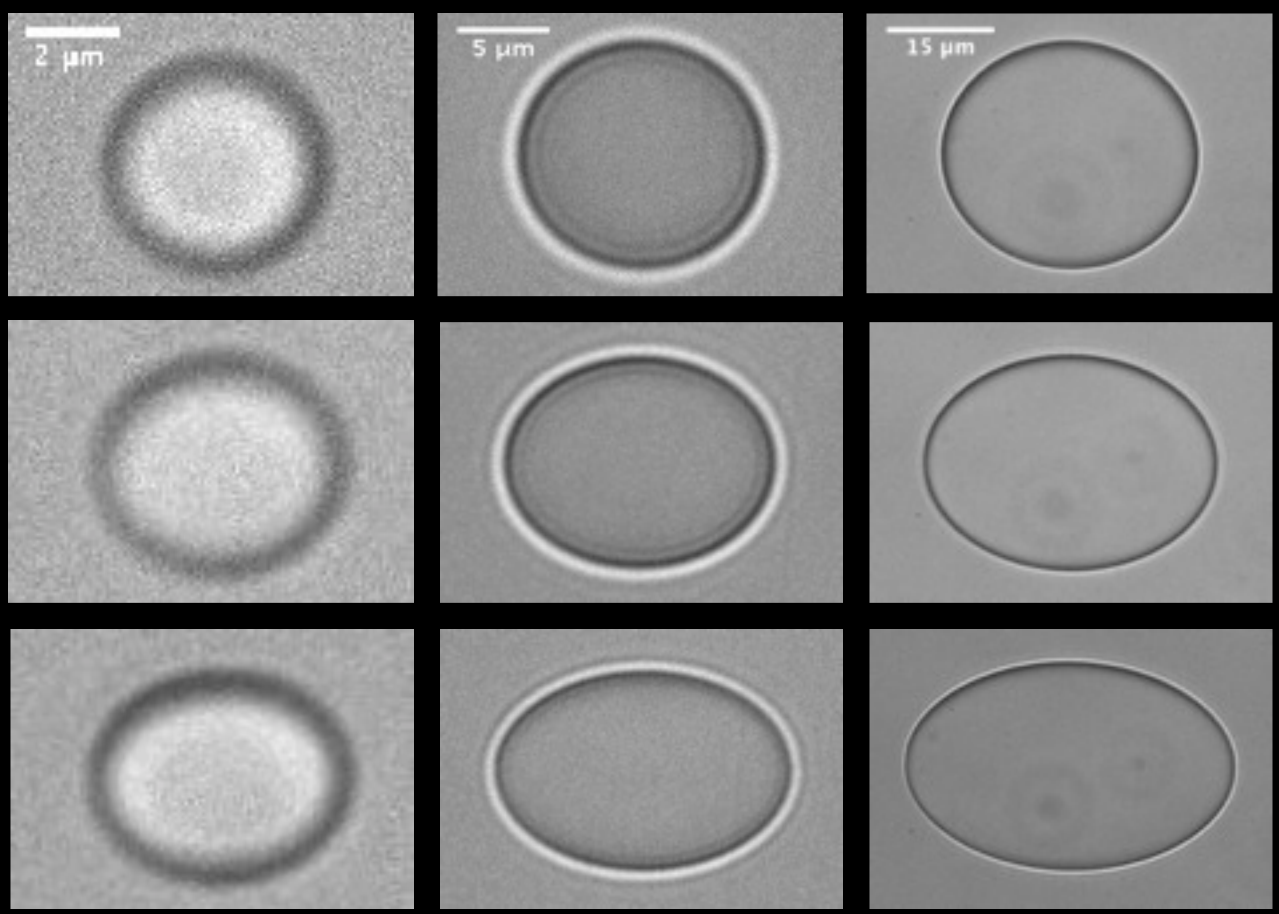
The mechanics of soft composites

Rob Style, Yale University

17/6/2014



With thanks to...

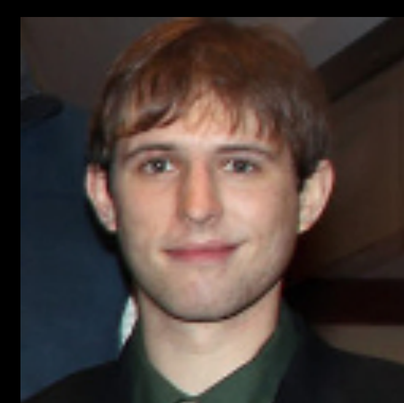
Eric Dufresne

John Wettlaufer

Ross Boltyanskiy

Ben Allen

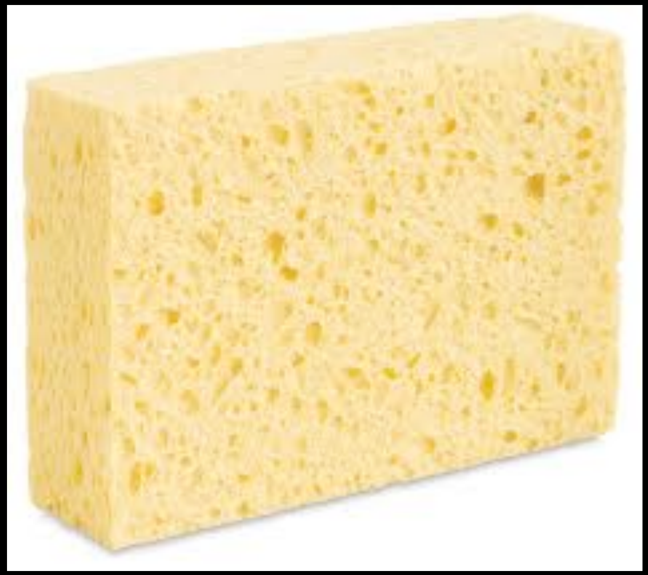
Youngwoo Choo



Jon Singer

Kate Jensen

Do soft composites behave like we'd expect?

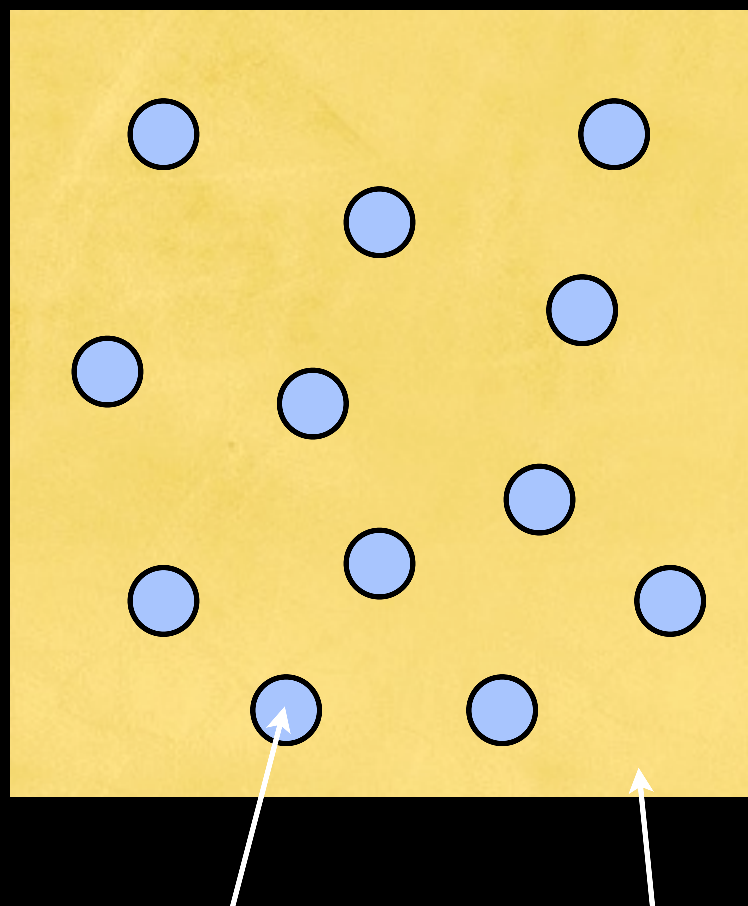


Law of mixtures

$$E_{eff} = E_1(1-\phi) + E_2\phi$$

Does this work for soft materials?

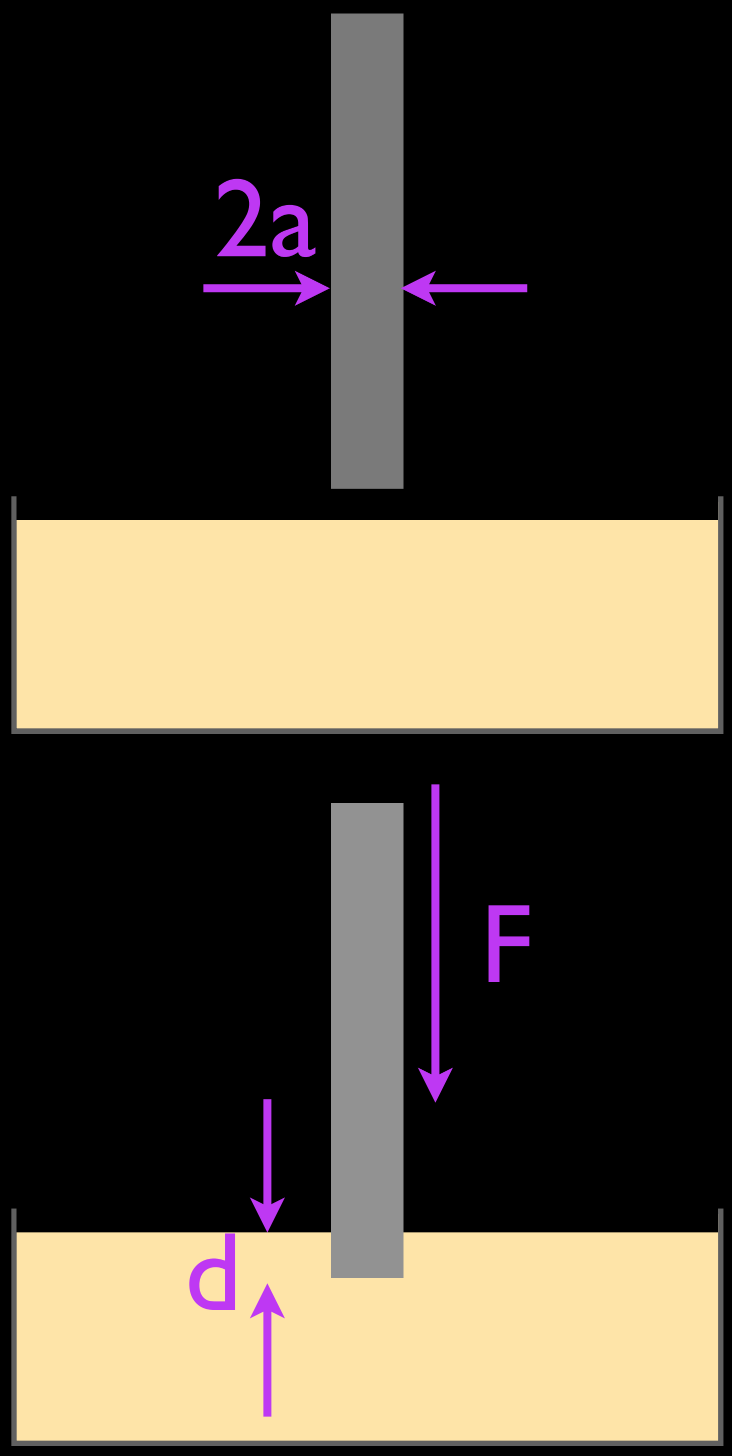
Making holey soft solids



<u>Emulsion:</u>
silicone (PDMS)
silicone surfactant
glycerol

0% ⟶ glycerol content ⟶ 19%

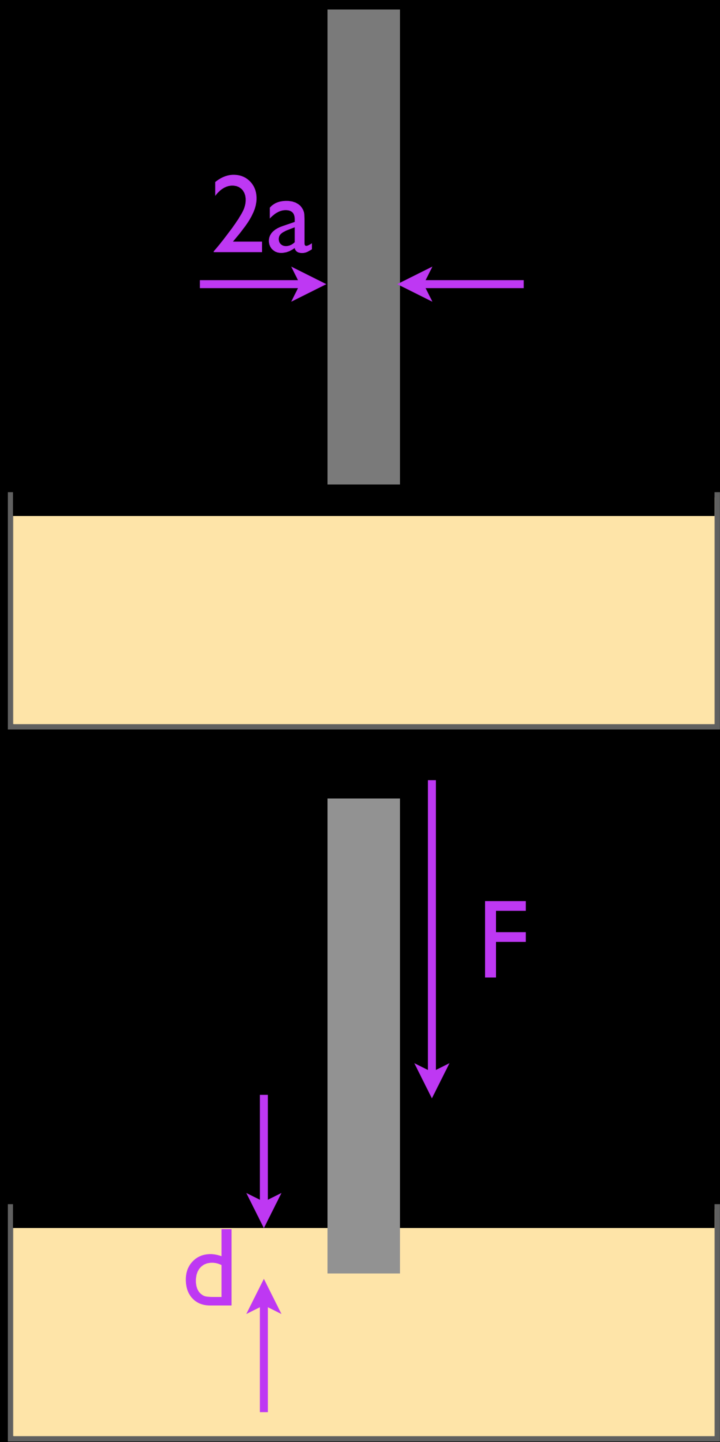
Poking the composite to measure stiffness

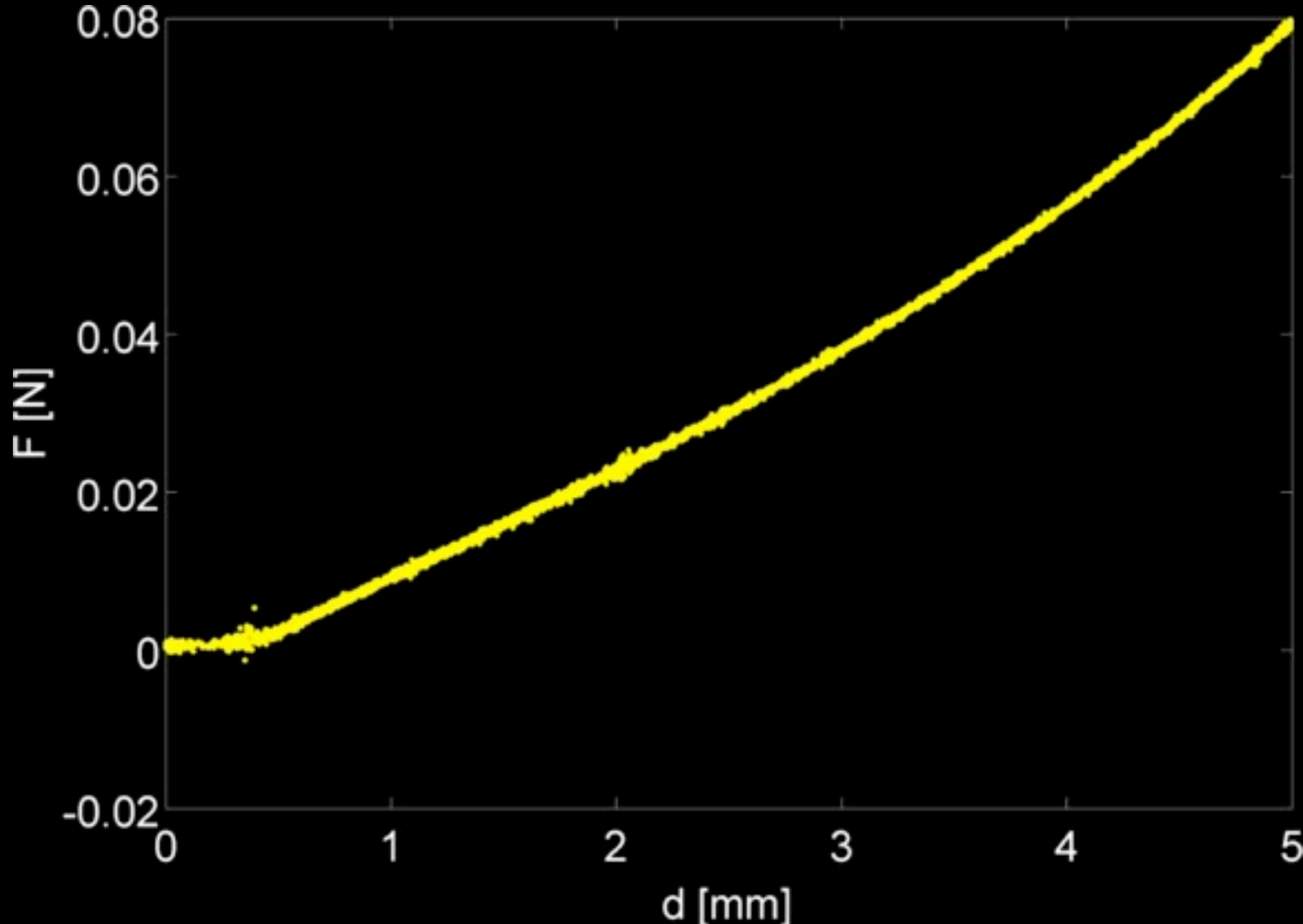


$$F = 2a\left(\frac{E}{1-\nu^2}\right)d$$

E = Young's modulus
ν = Poisson's ratio

Poking the composite to measure stiffness

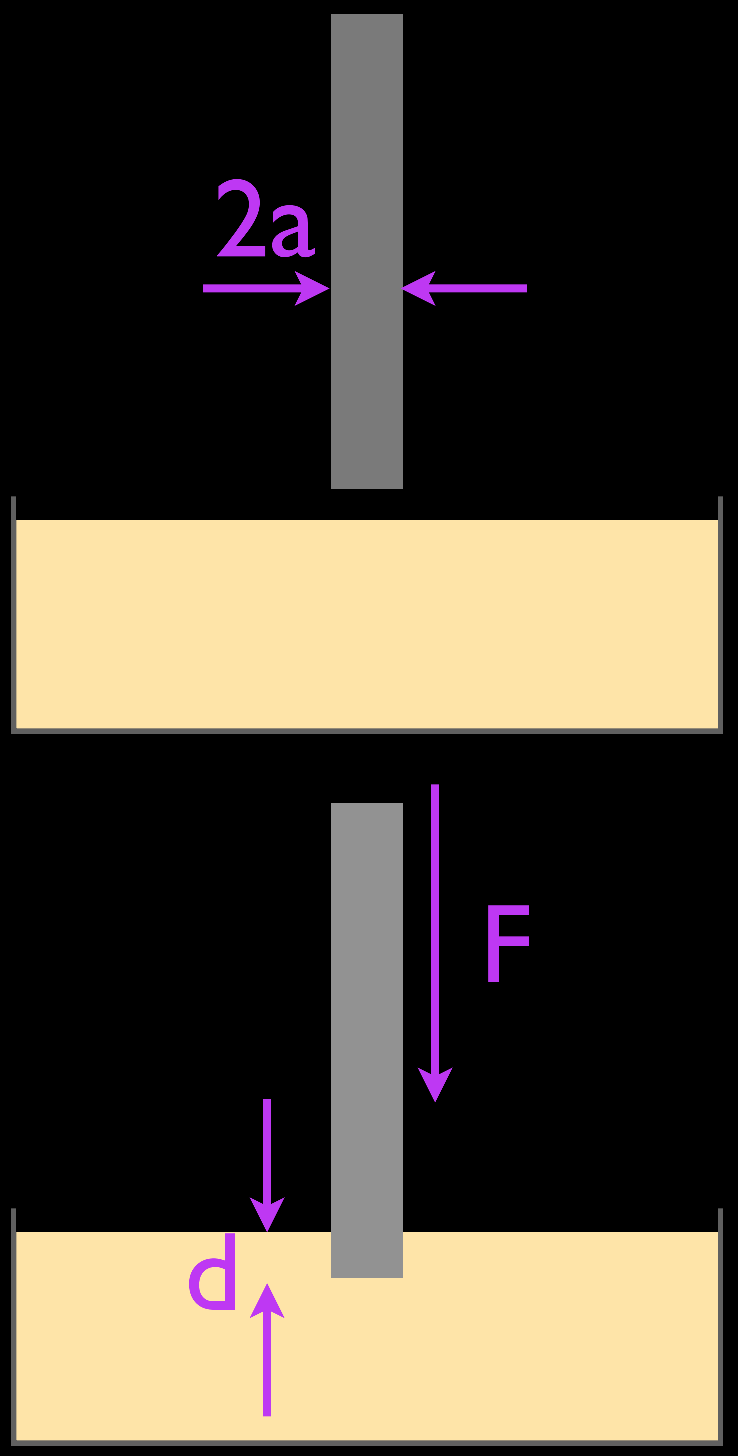


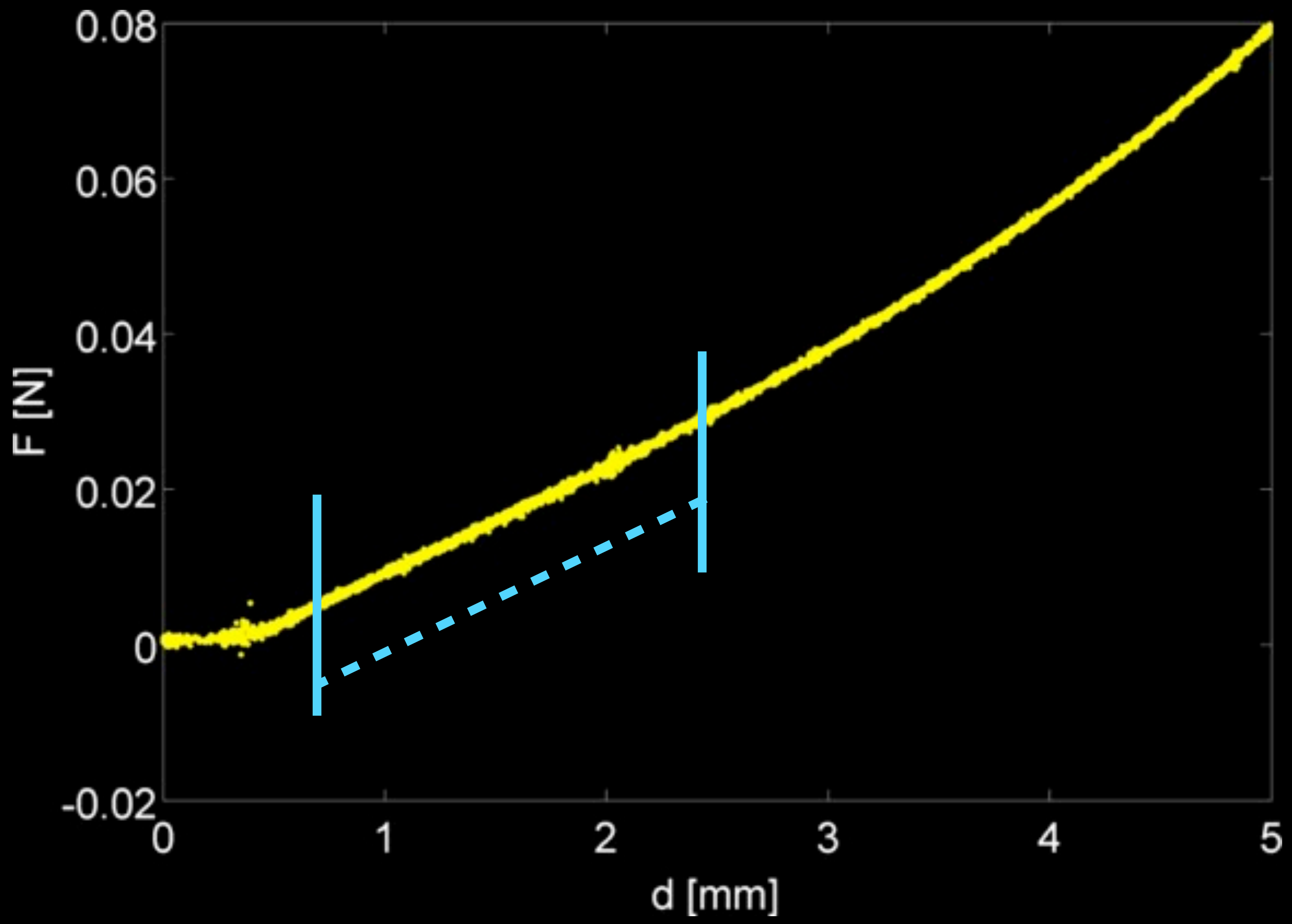


$$F = 2a \left(\frac{E}{1 - \nu^2} \right) d$$

E = Young's modulus
ν = Poisson's ratio

Poking the composite to measure stiffness



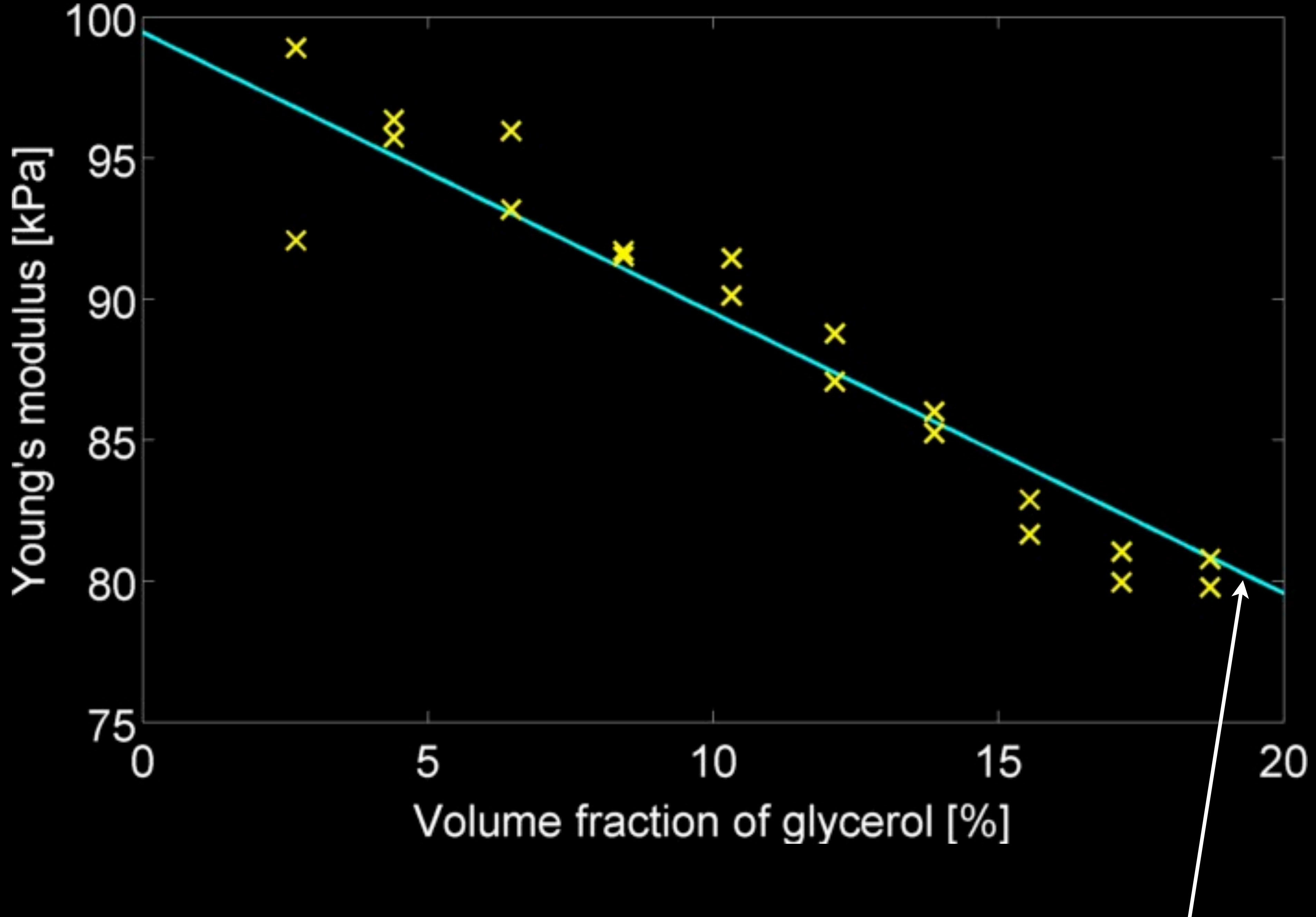


$$F = 2a\left(\frac{E}{1-\nu^2}\right)d$$

E = Young's modulus

ν = Poisson's ratio

Stiff composites get softer with increasing liquid content

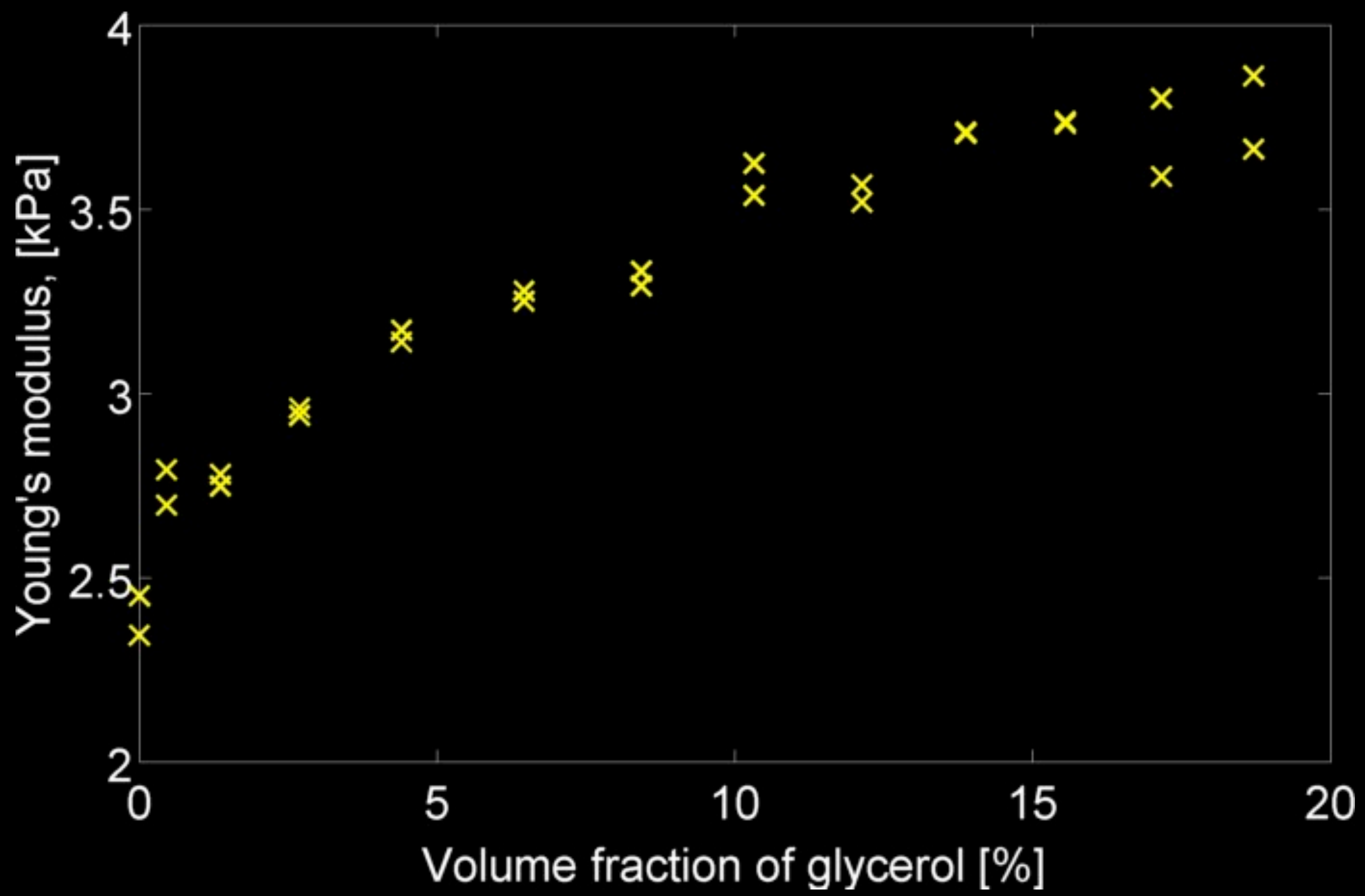


Young's modulus of pure silicone ~ 100kPa

Law of mixtures

$$E_{eff} = E(1-\phi)$$

Softer composites gets stiffer with increasing liquid content

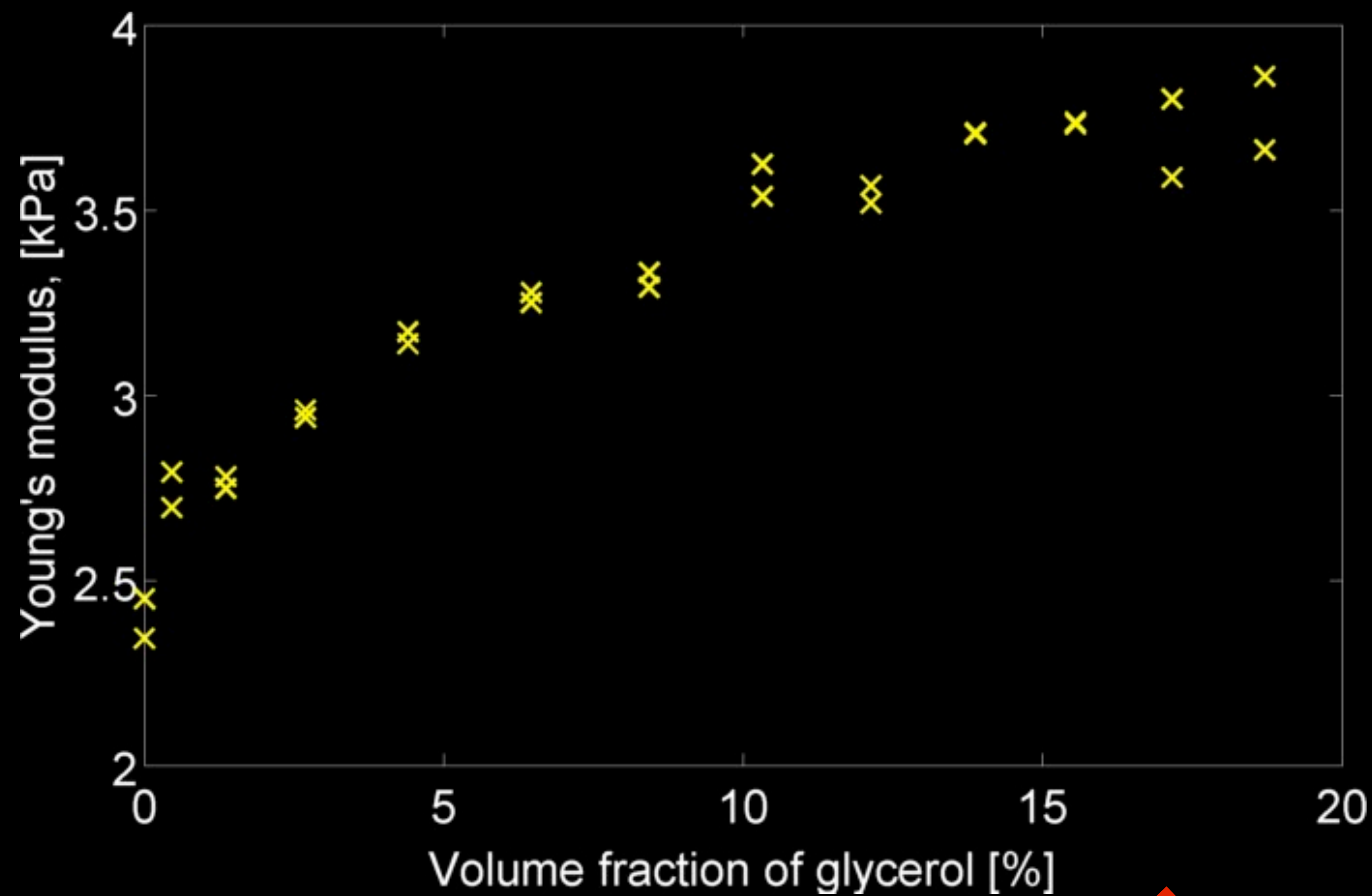


Young's modulus of pure silicone ~ 2.5kPa

Law of mixtures

$$E_{eff} = E(1 - \phi)$$

Softer composites gets <u>stiffer</u> with increasing liquid content

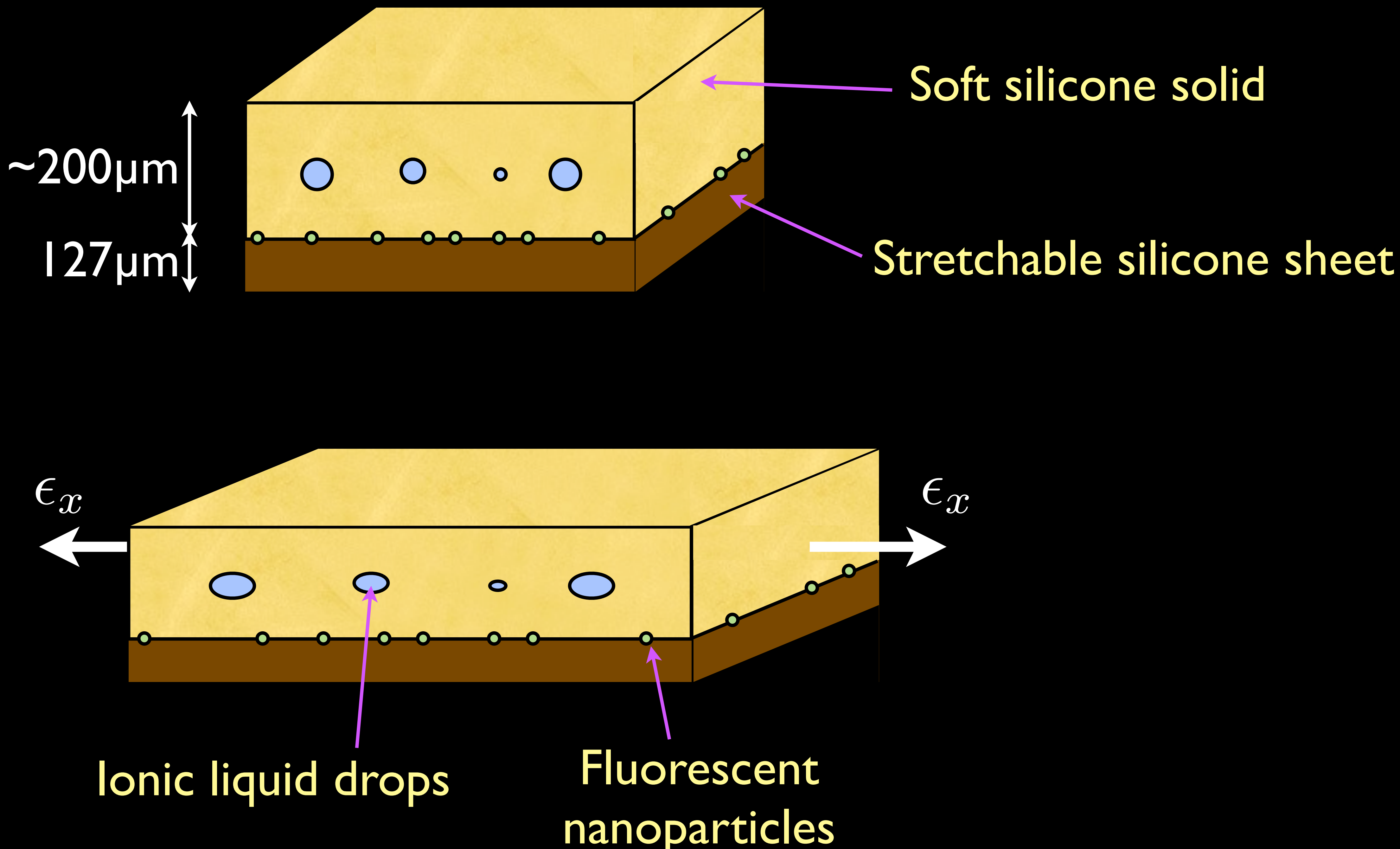


Young's modulus of pure silicone ~ 2.5kPa

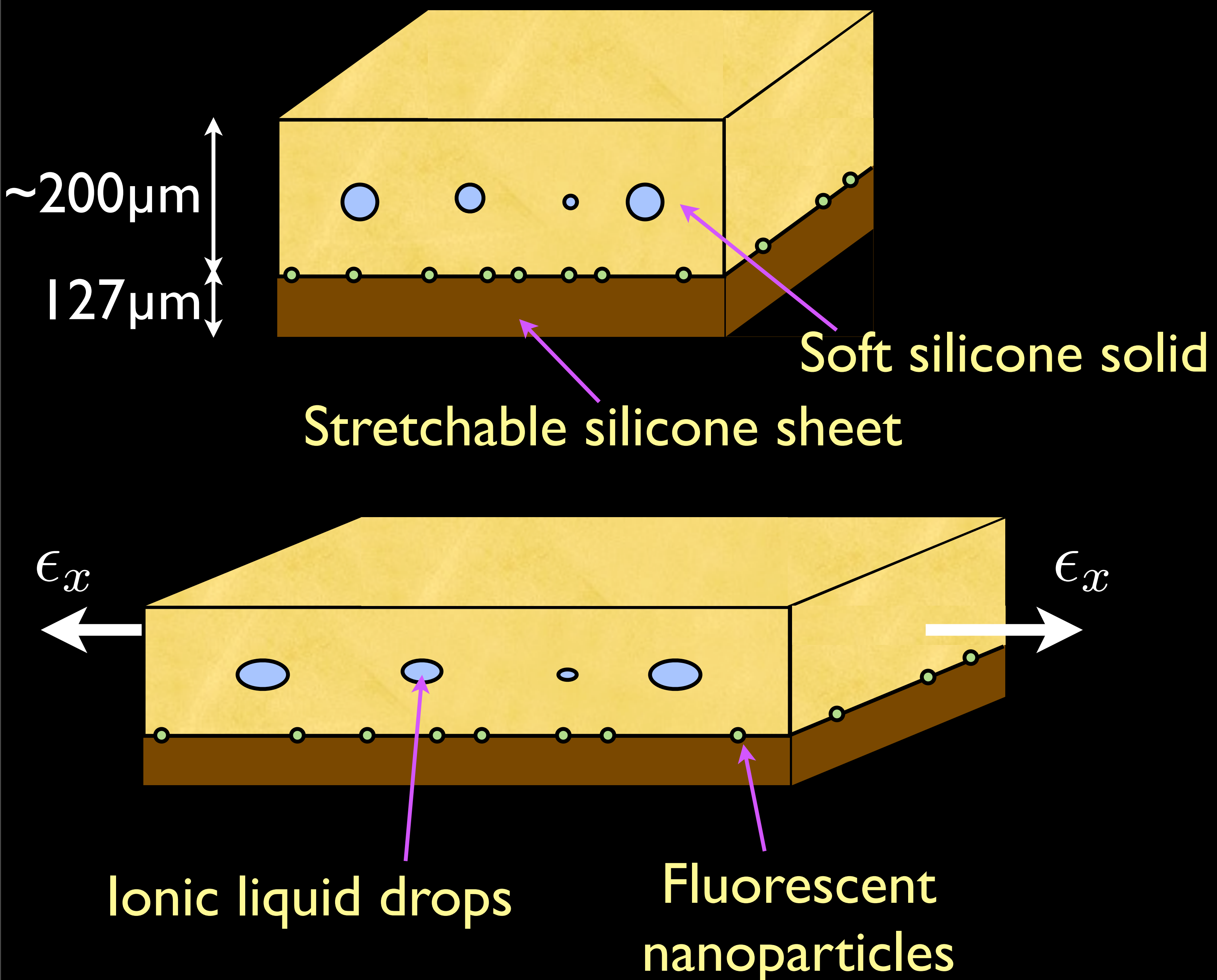
Law of mixtures

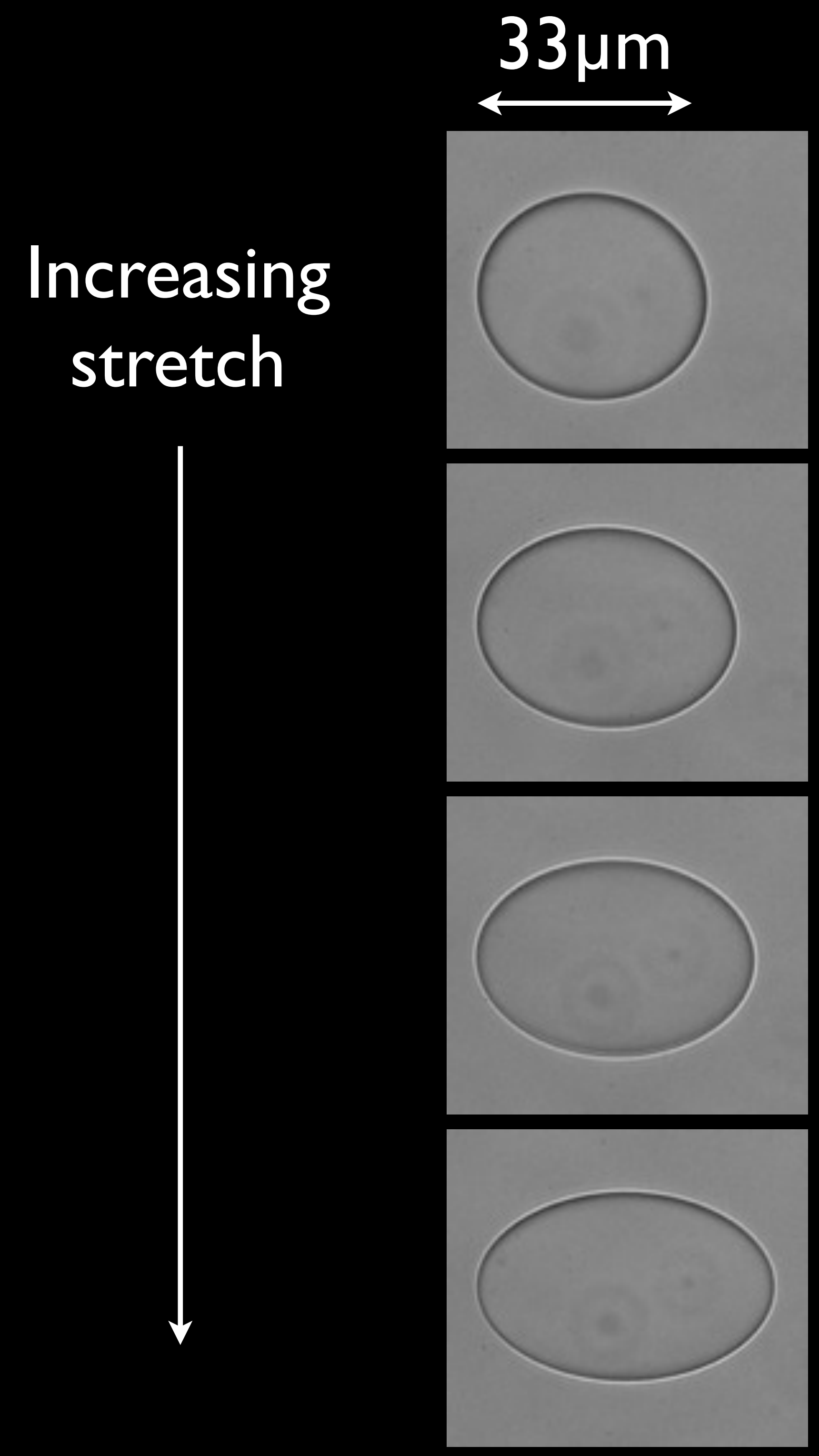
$$E_{eff} = E(1-\phi)$$

What's going on? Let's look at the individual drop scale

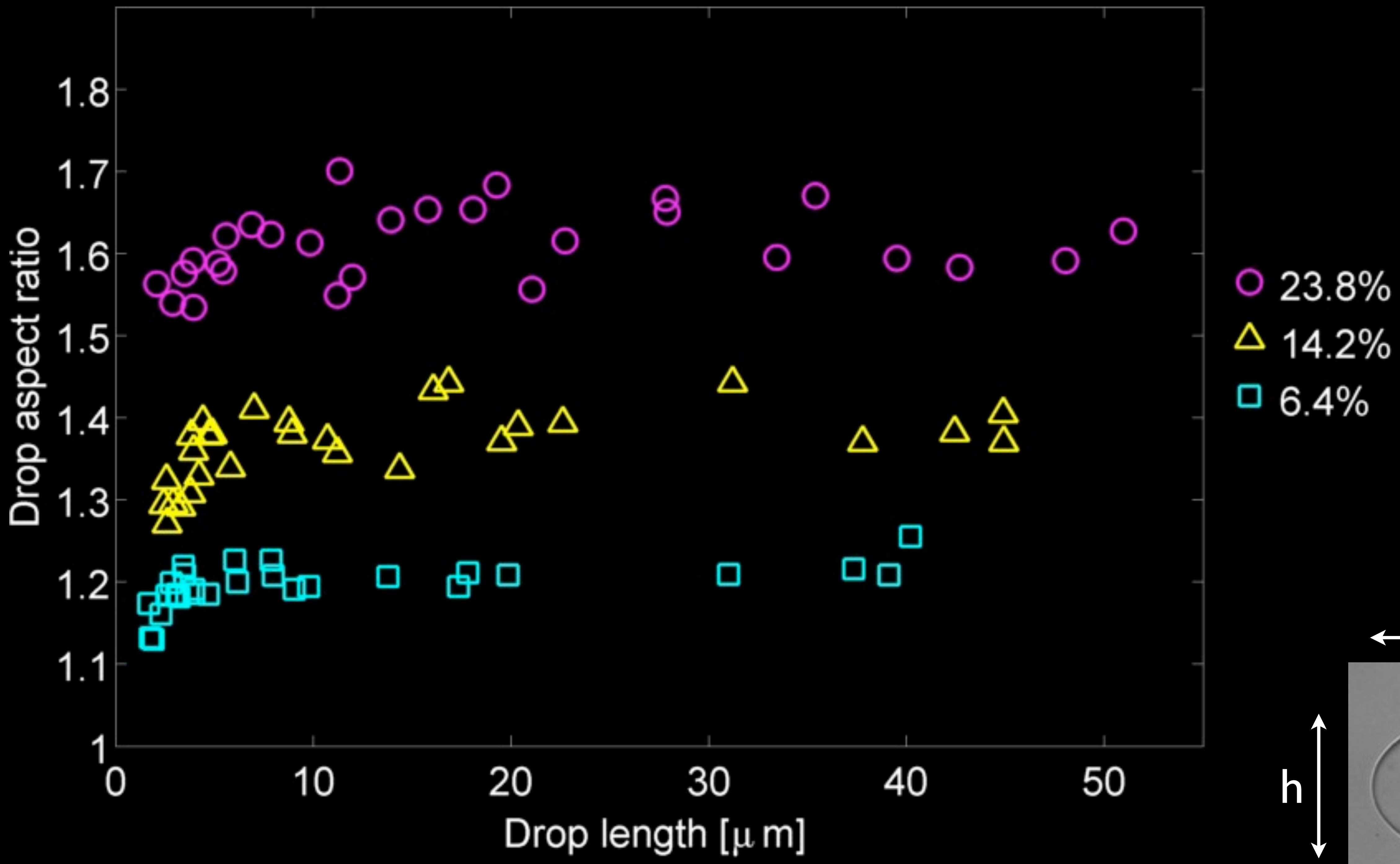


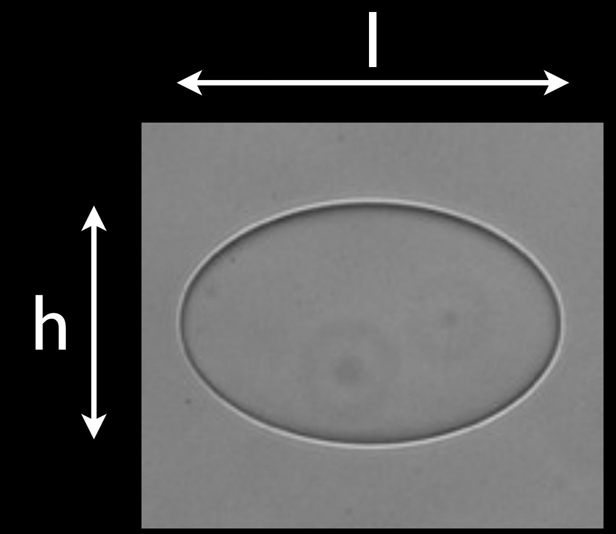
What's going on? Let's look at the individual drop scale





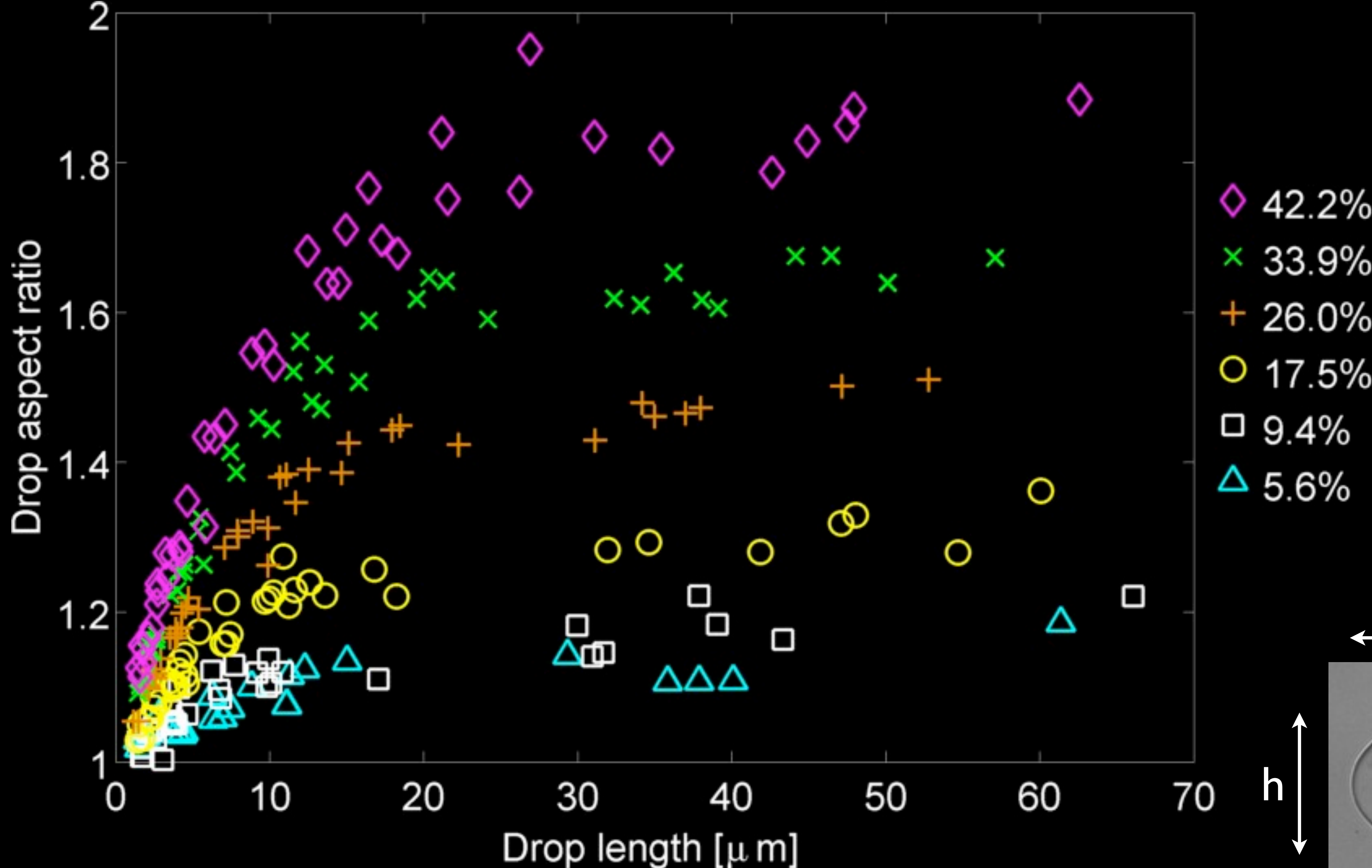
Aspect ratios of droplets at different stretches - stiff solid

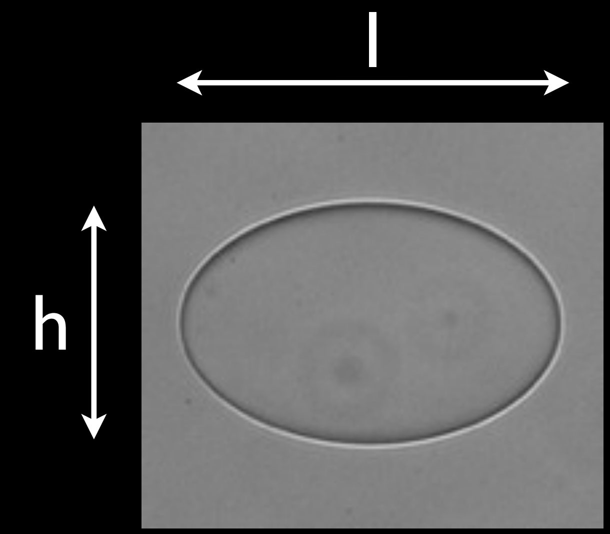




aspect ratio=l/h

Aspect ratios of droplets at different stretches - soft solid



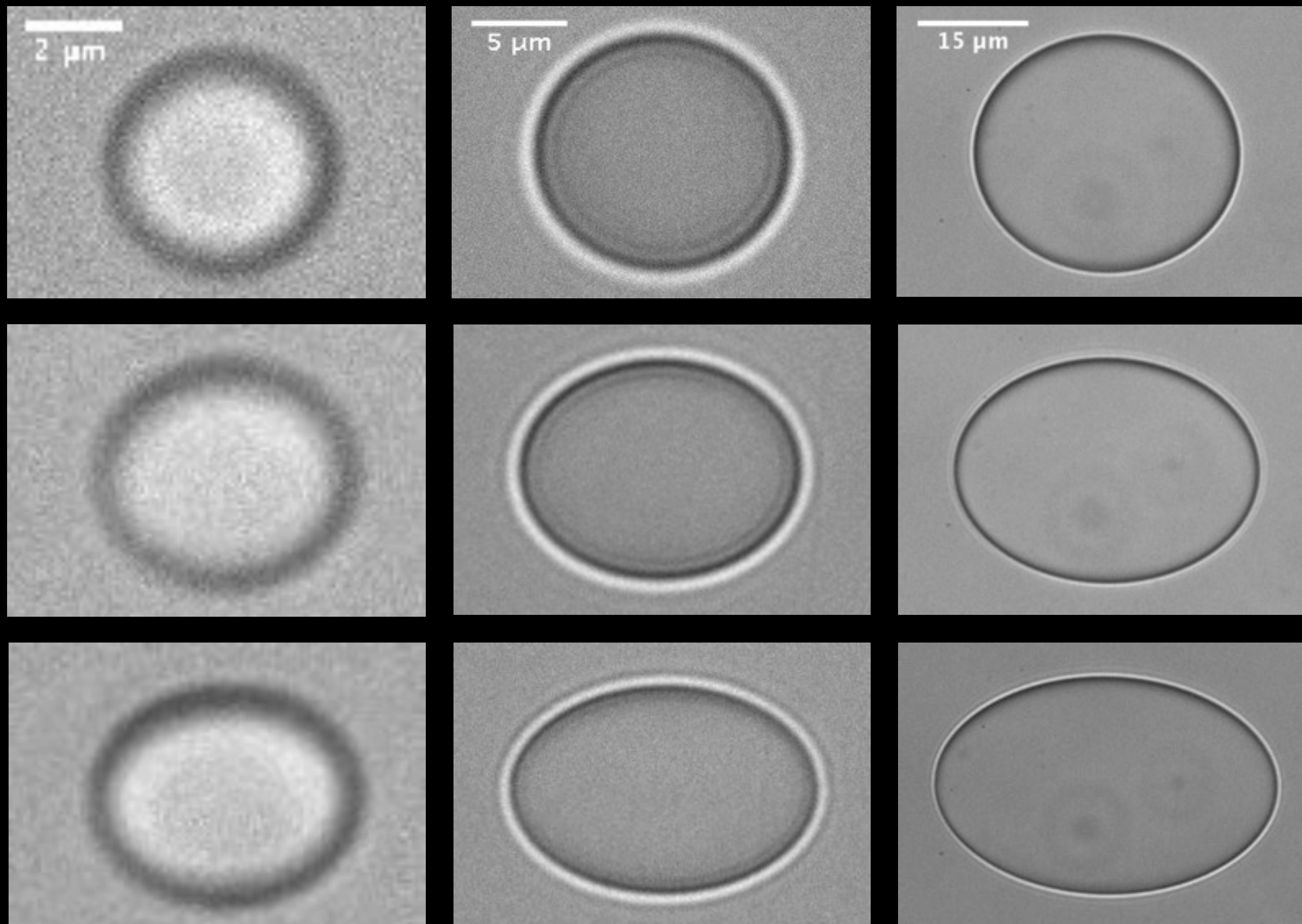


aspect ratio=l/h

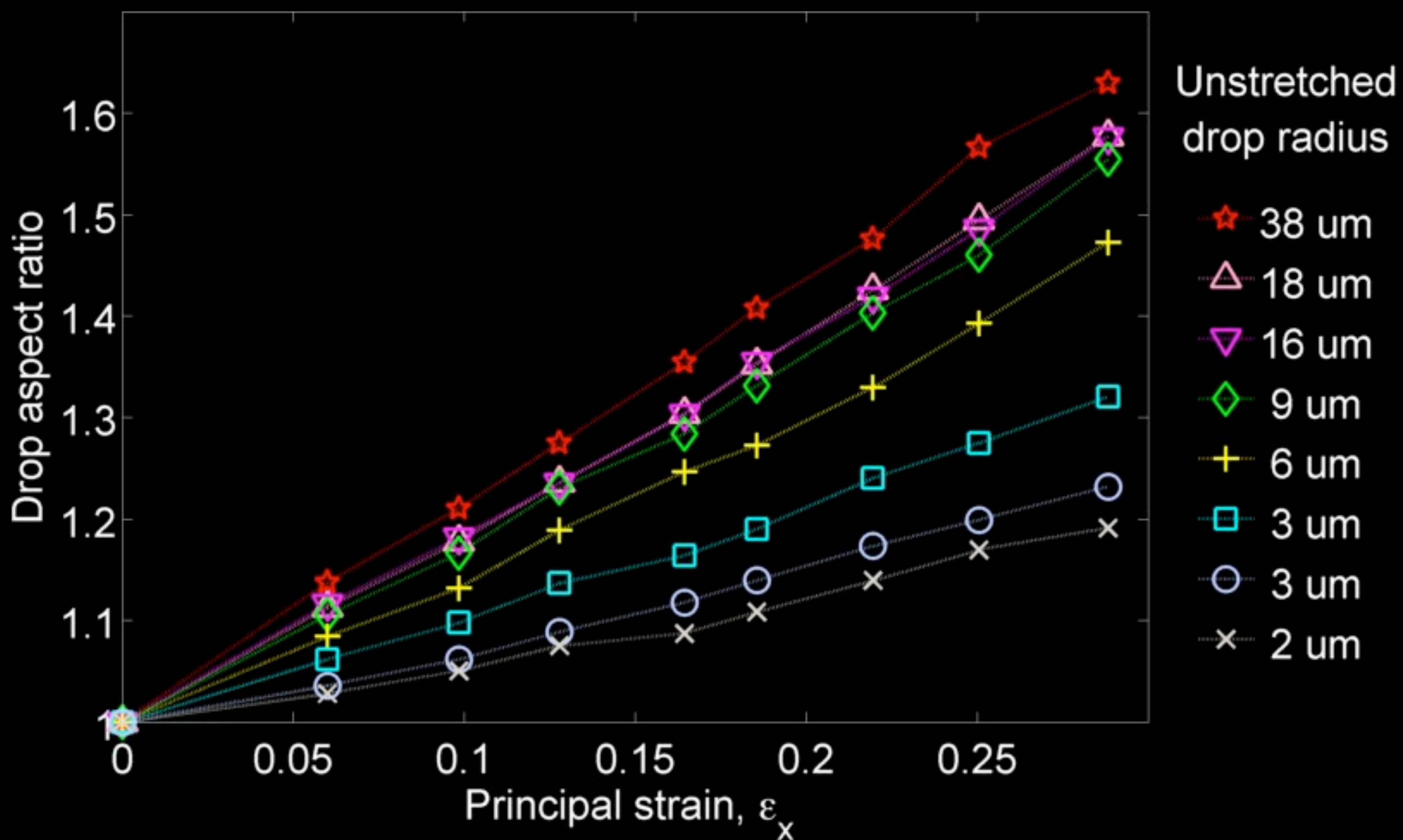
Smaller droplets stay more round under stretch

Droplet size: 2.5μm | 6.4μm | 16.5μm

Strain: 6% | 19% | 29%

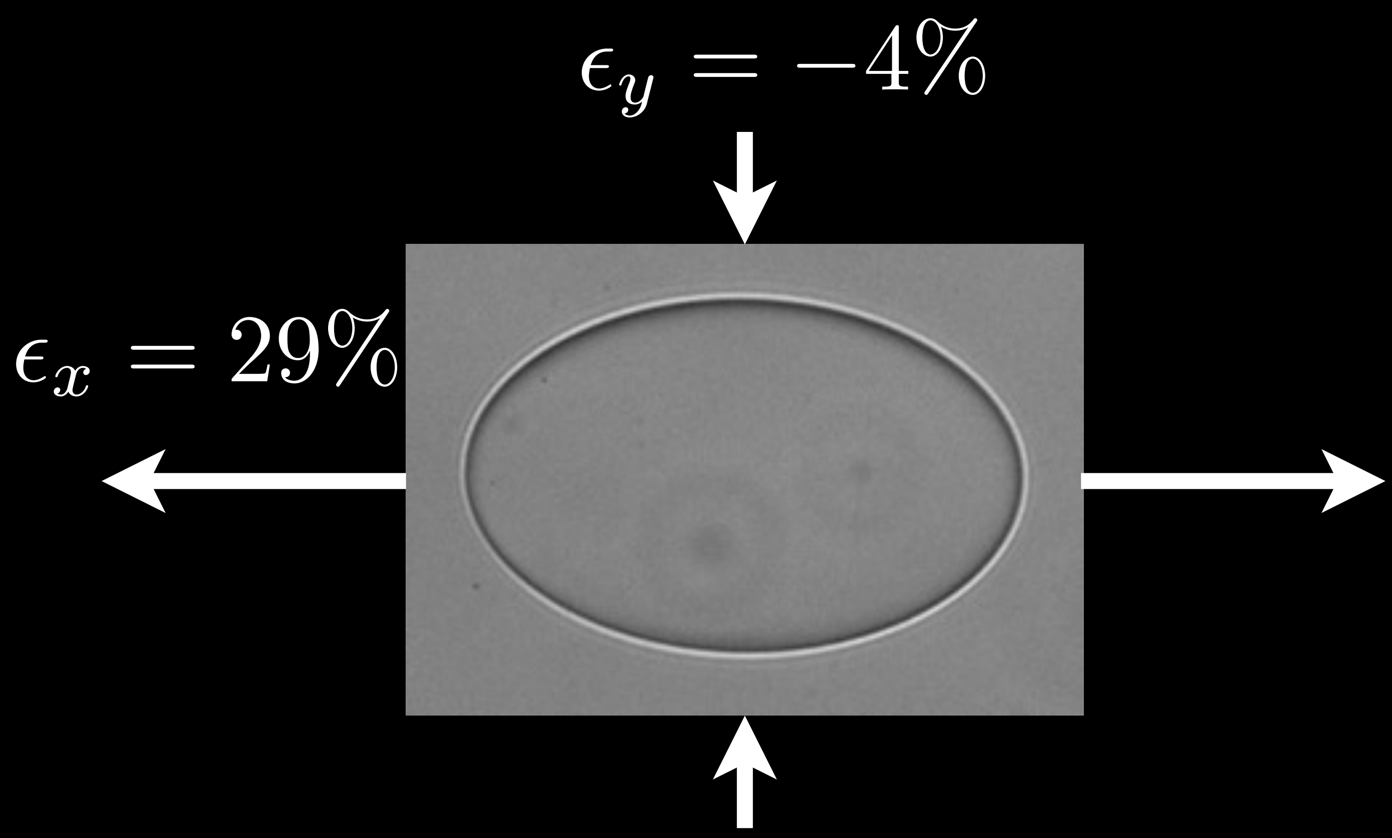


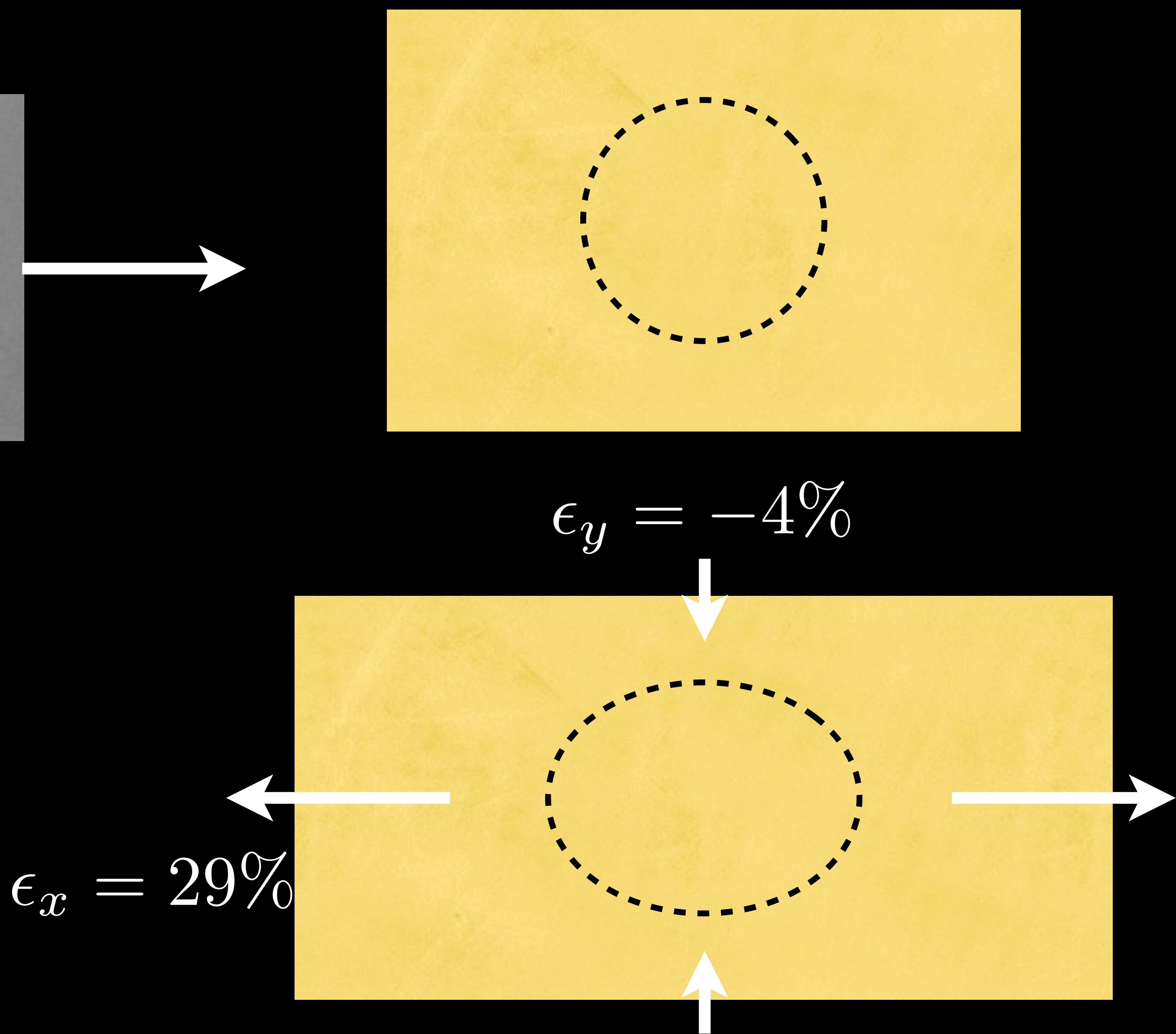
Aspect ratio depends on strain and droplet size



Comparing drop stiffness with solid stiffness

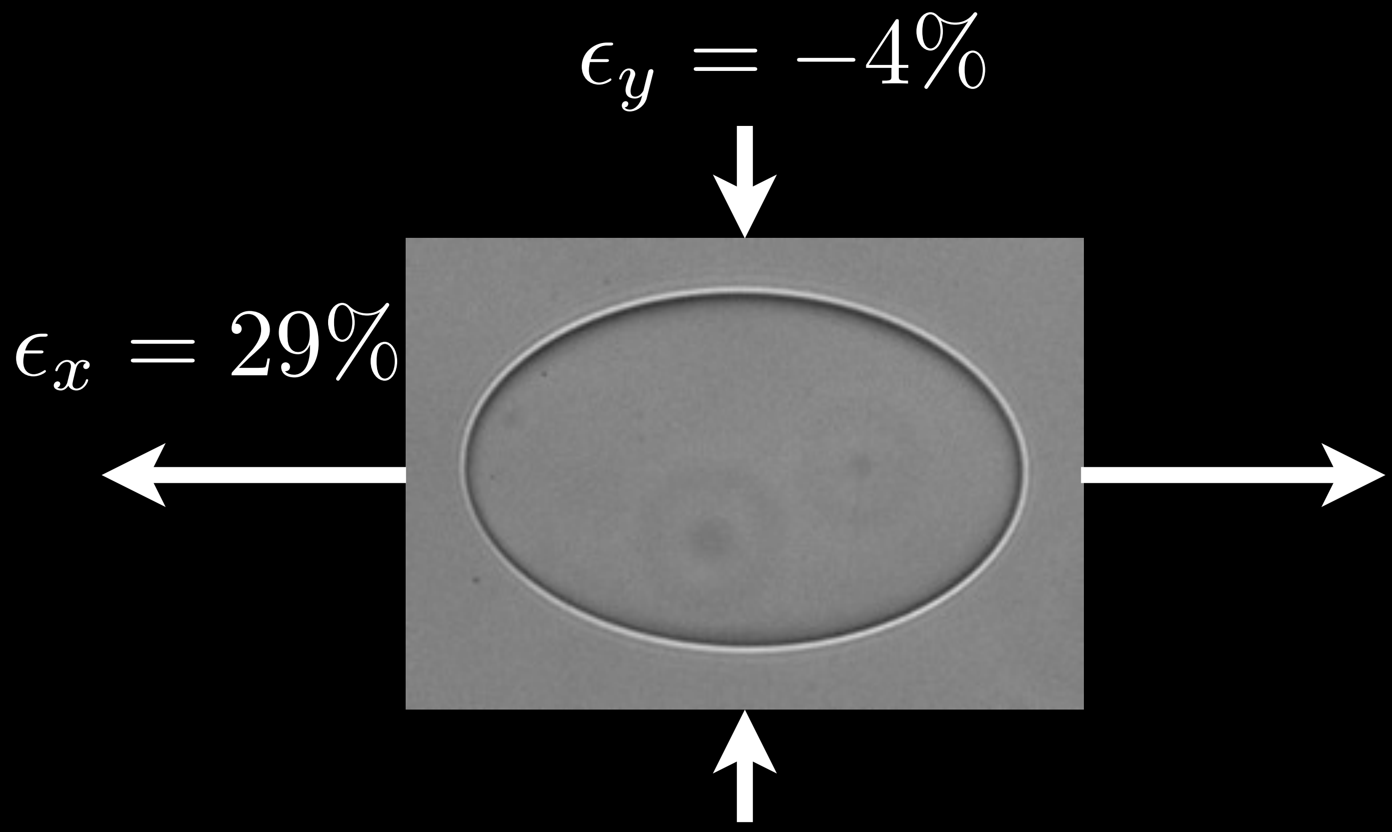
Drop radius 16.5μm

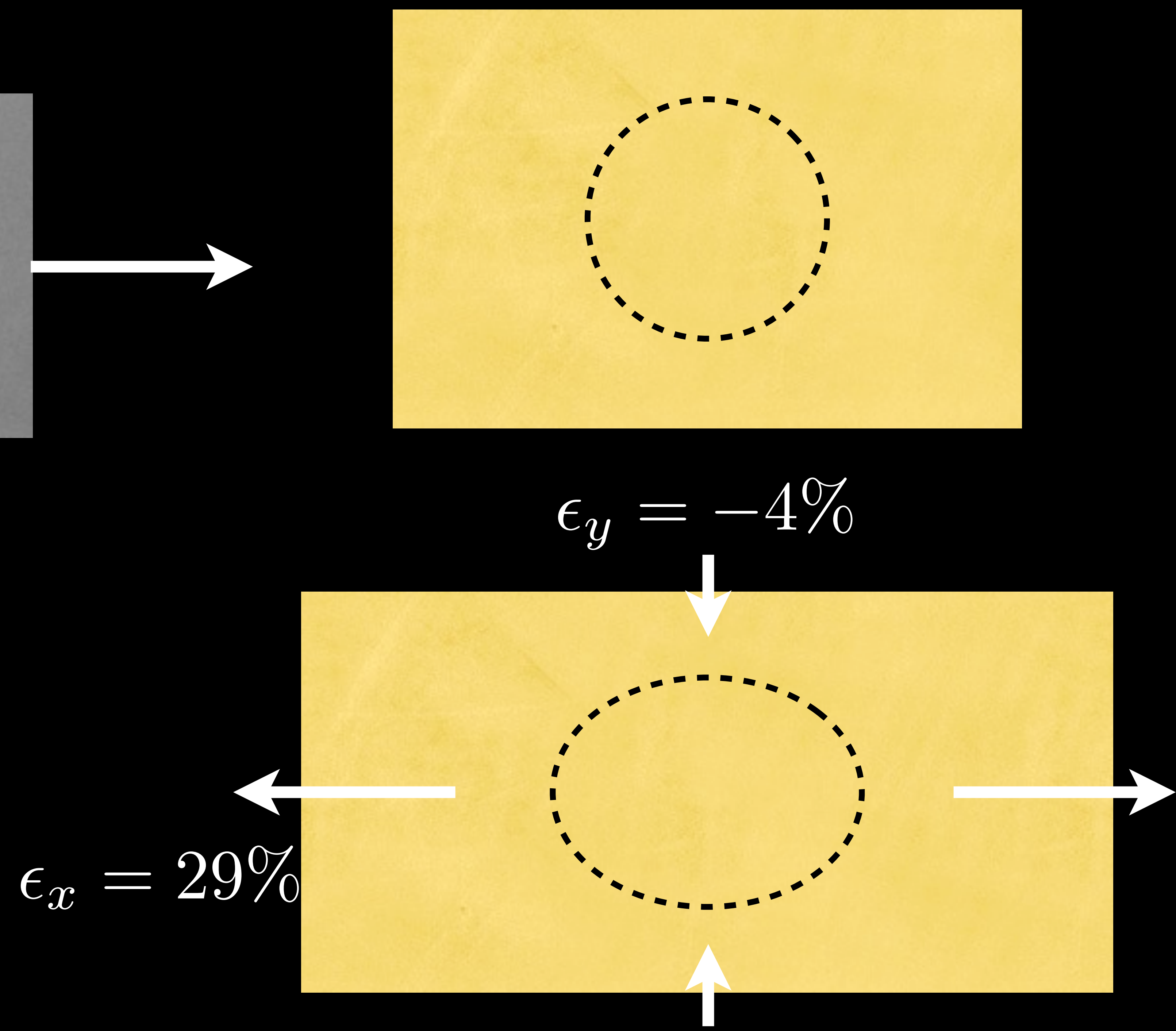




Comparing drop stiffness with solid stiffness

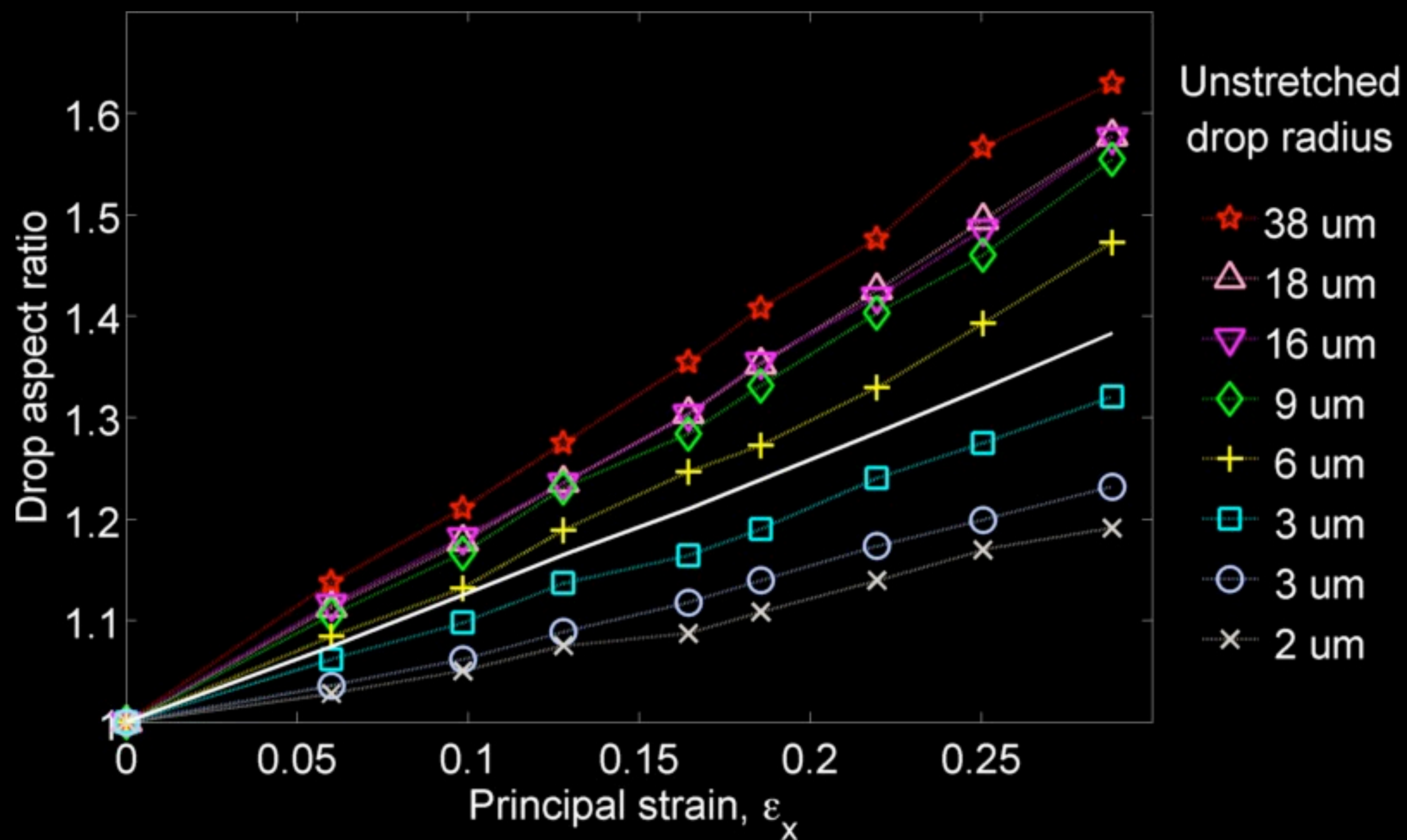
Drop radius 16.5μm





Here, droplet is 'softer' than the solid

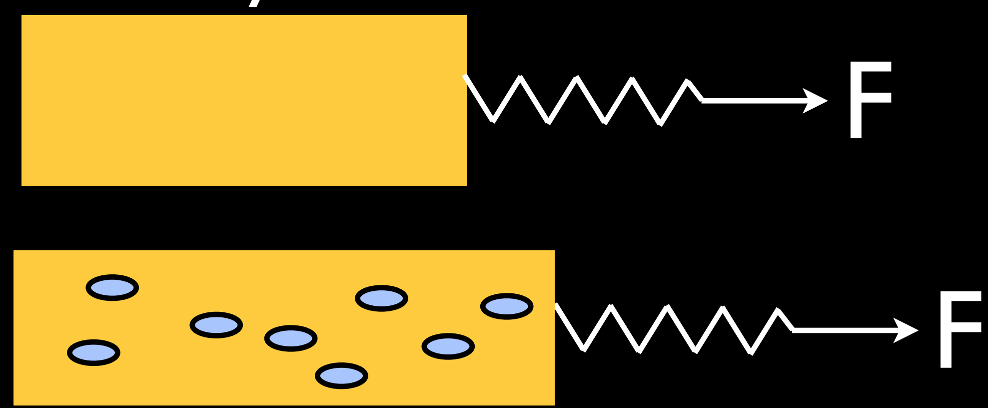
Small droplets are ‘stiffer’ than the surrounding solid



Quick summary of data

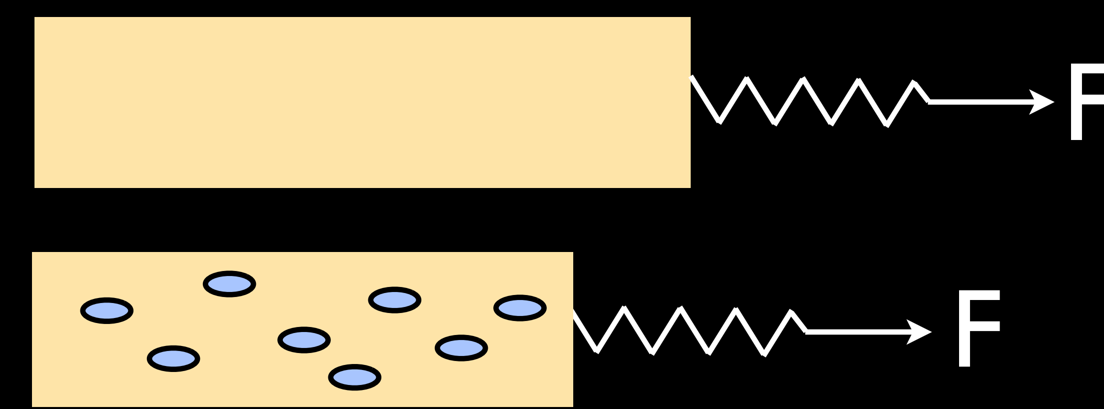
Stiffer silicone (E ~ 100kPa)

Soften as you make small holes in it



Softer silicone (E ~ 2.5kPa)

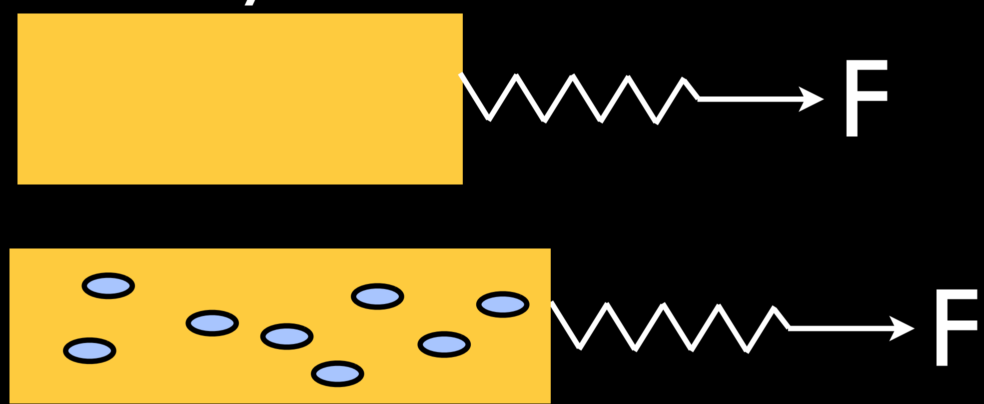
Stiffen as you make small holes in it



Quick summary of data

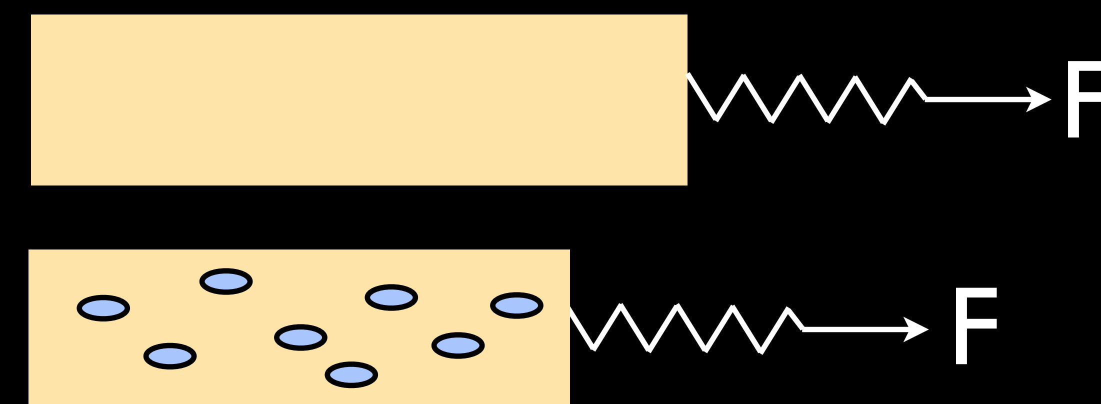
Stiffer silicone (E ~ 100kPa)

Soften as you make small holes in it

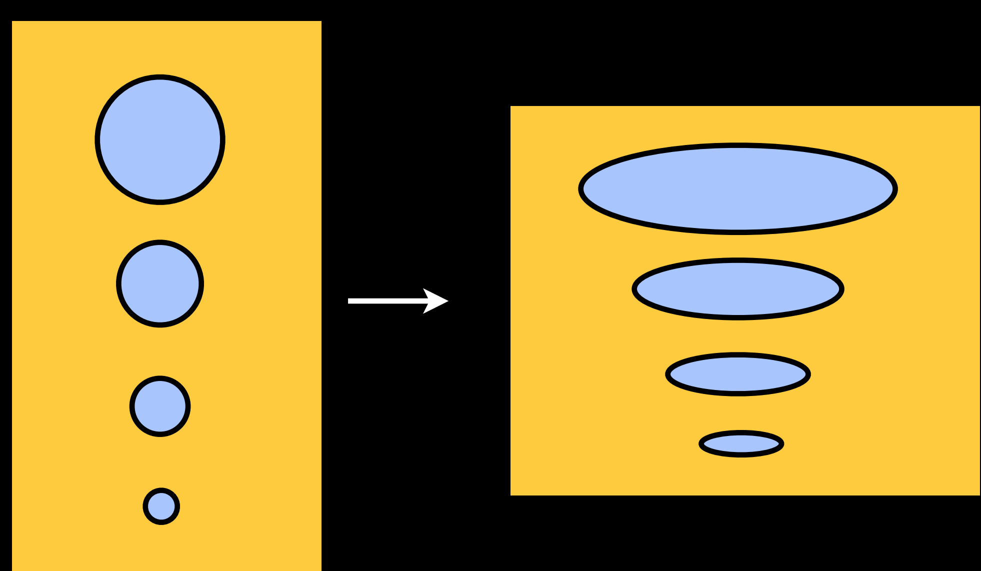


Softer silicone (E ~ 2.5kPa)

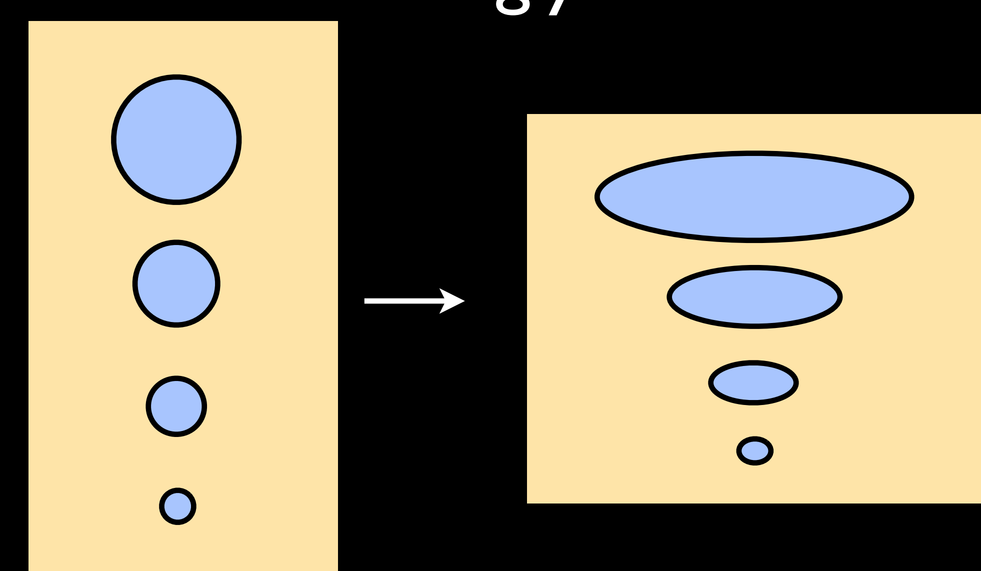
Stiffen as you make small holes in it



Droplet shape is independent of size



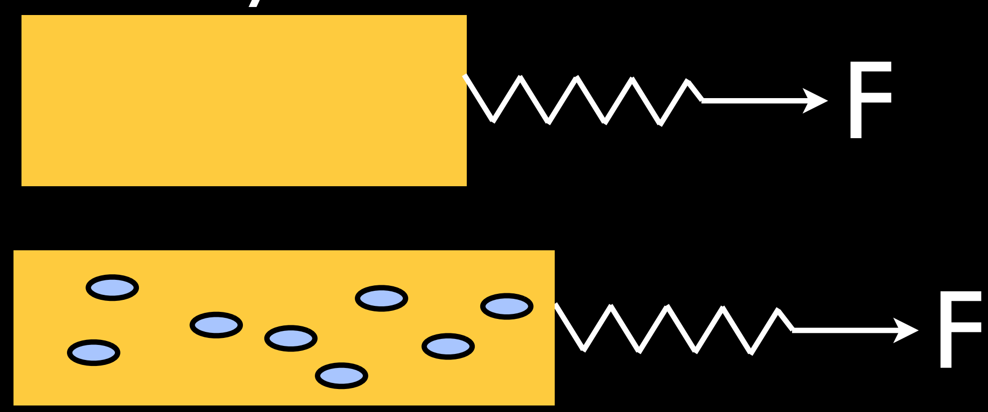
Droplet shape depends strongly on size



Quick summary of data

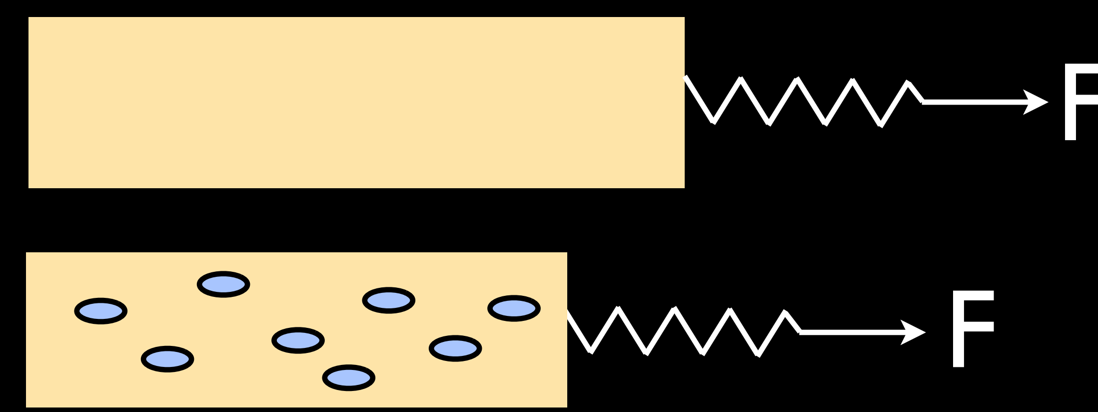
Stiffer silicone (E ~ 100kPa)

Soften as you make small holes in it

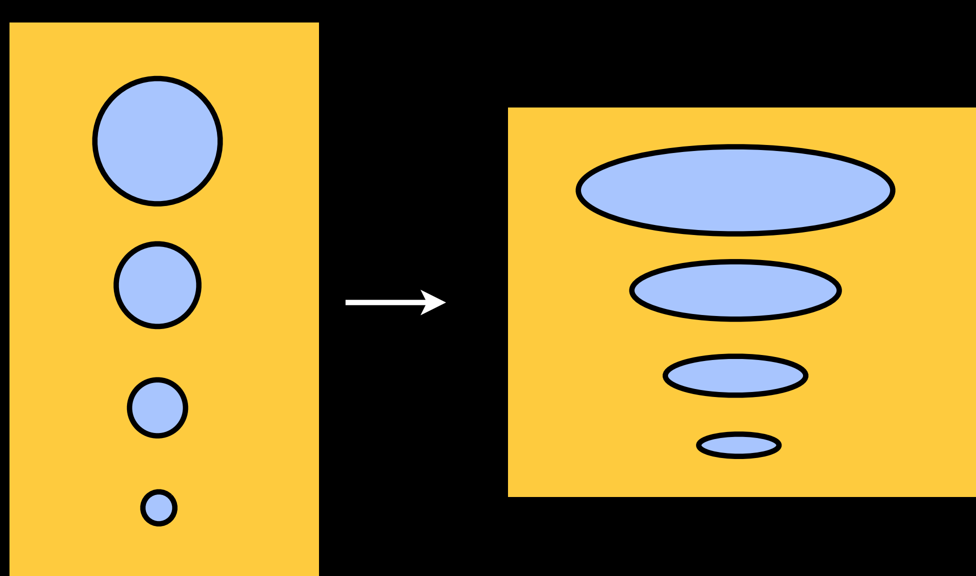


Softer silicone (E ~ 2.5kPa)

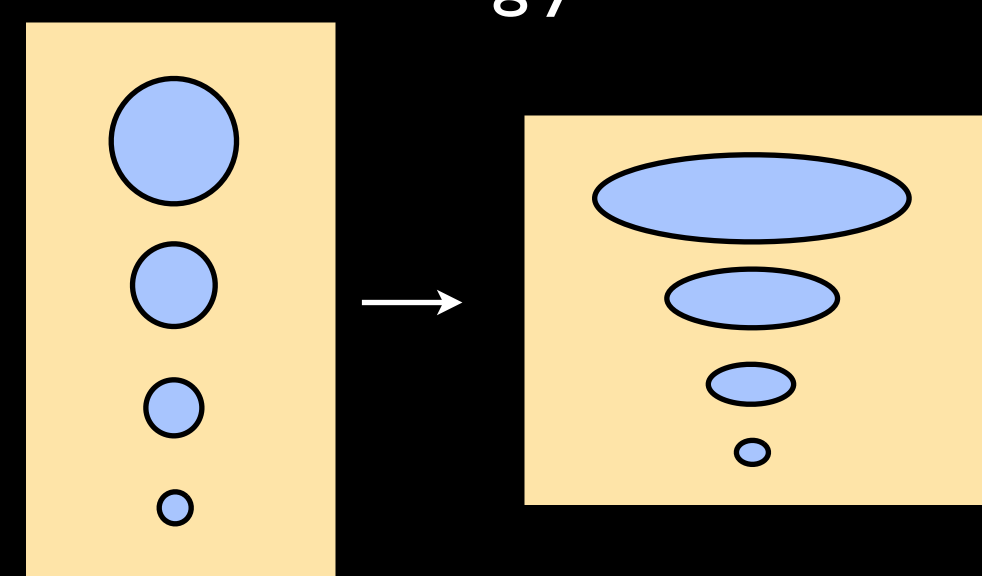
Stiffen as you make small holes in it



Droplet shape is independent of size



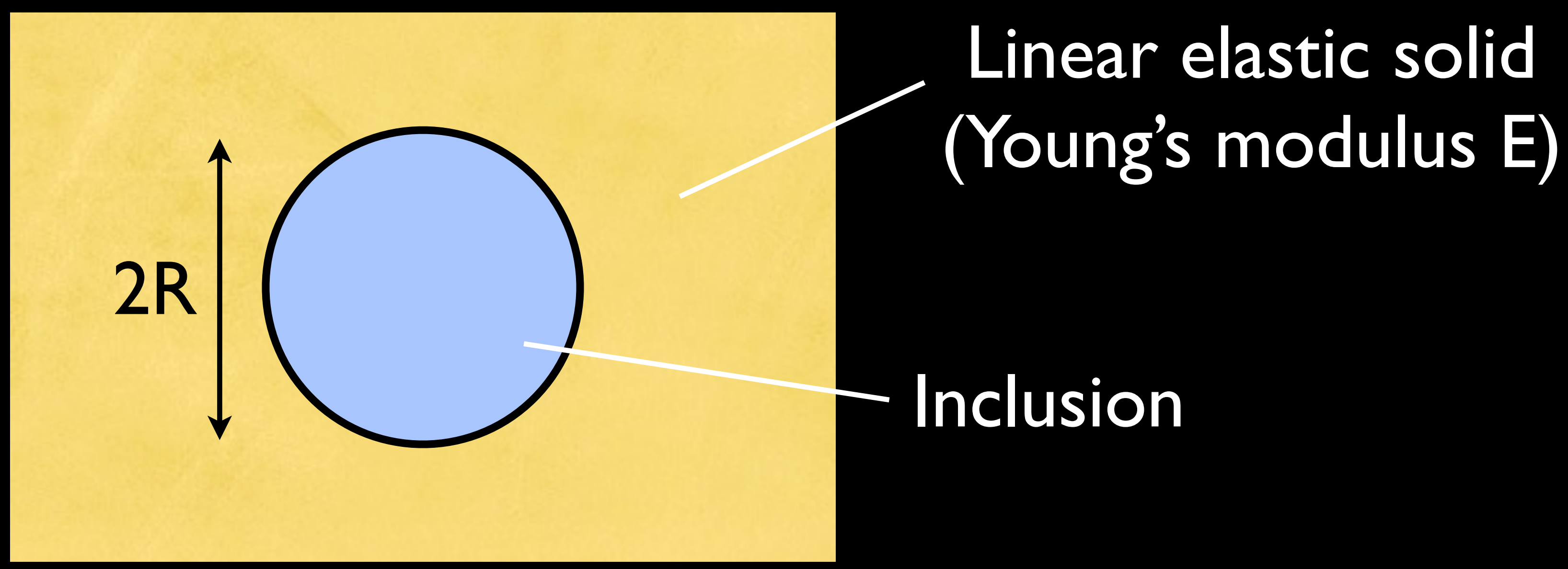
Droplet shape depends strongly on size



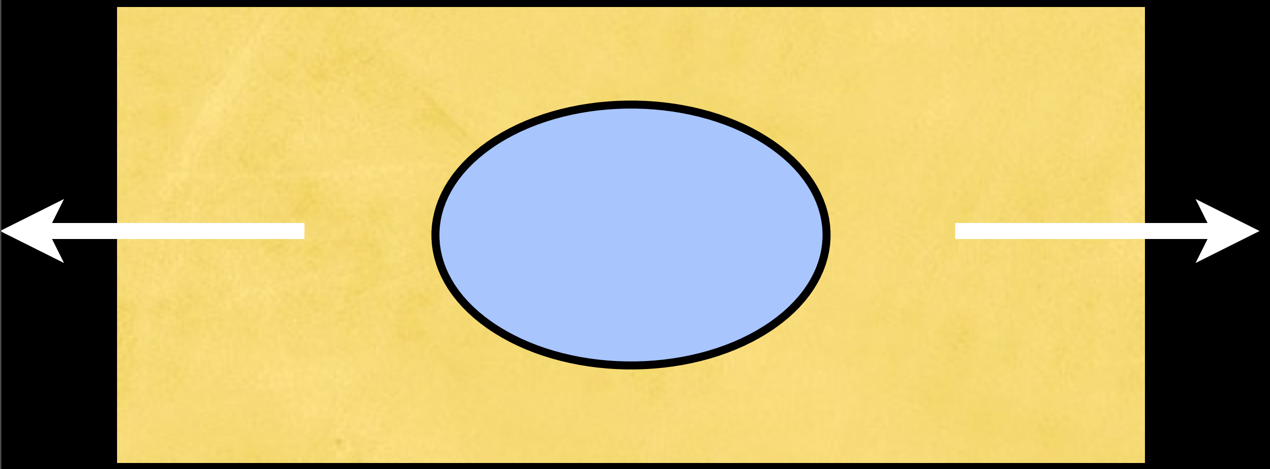
Droplets are 'softer' than the surrounding material

Small droplets are 'stiffer' than the surrounding material

Eshelby theory explains stiff solid results

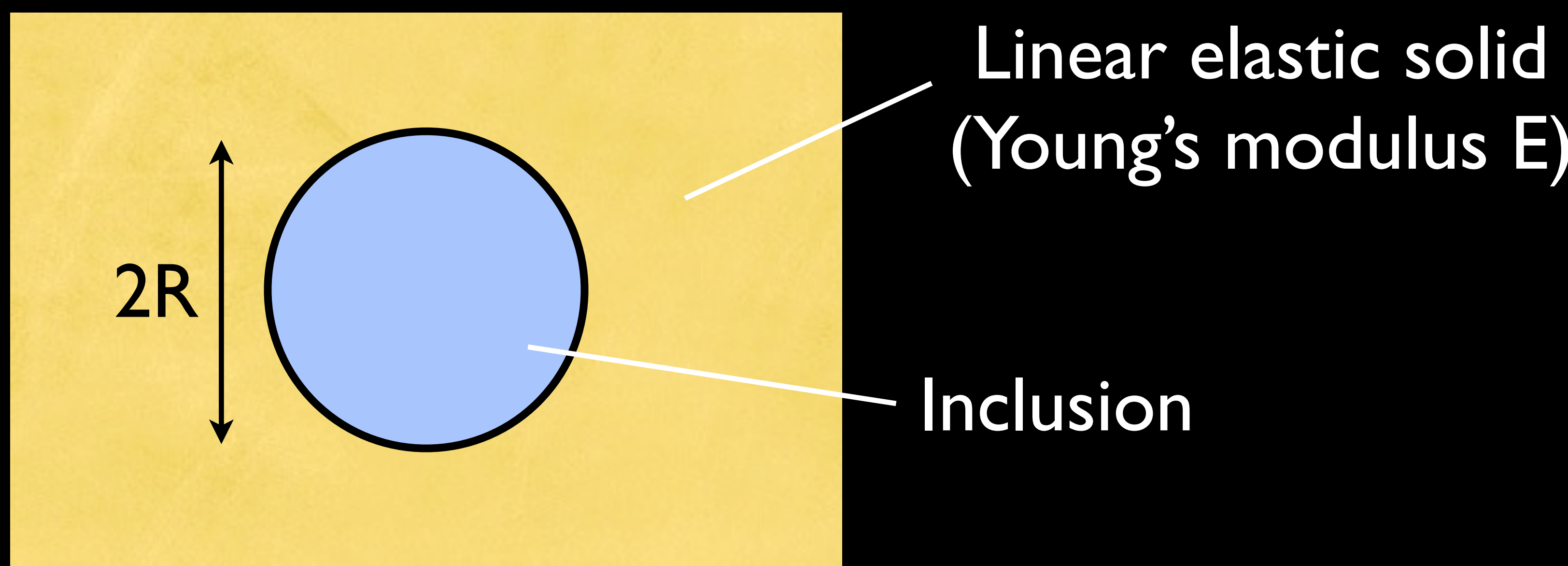


Apply strain ϵ



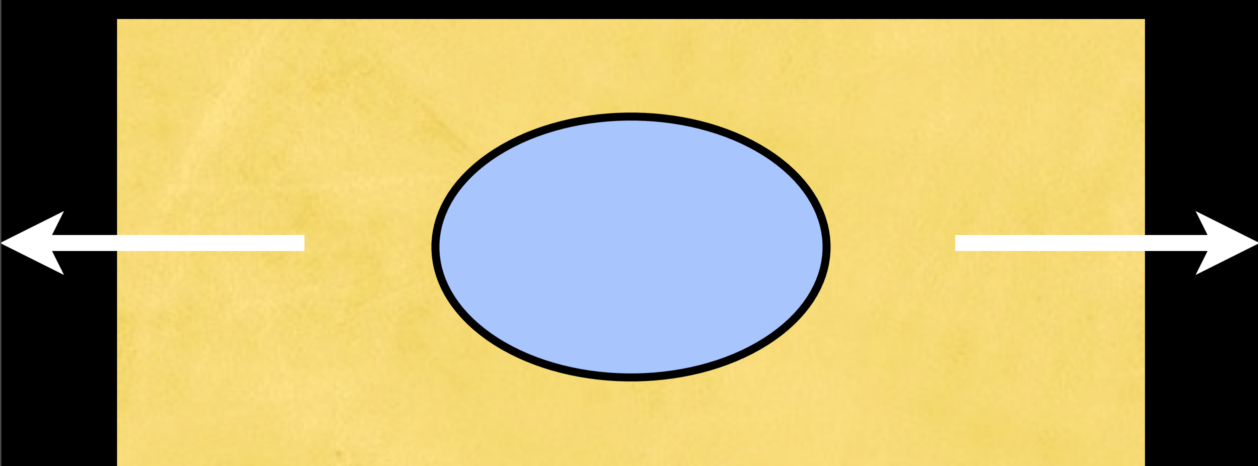
(Eshelby, 1957)

Eshelby theory explains stiff solid results



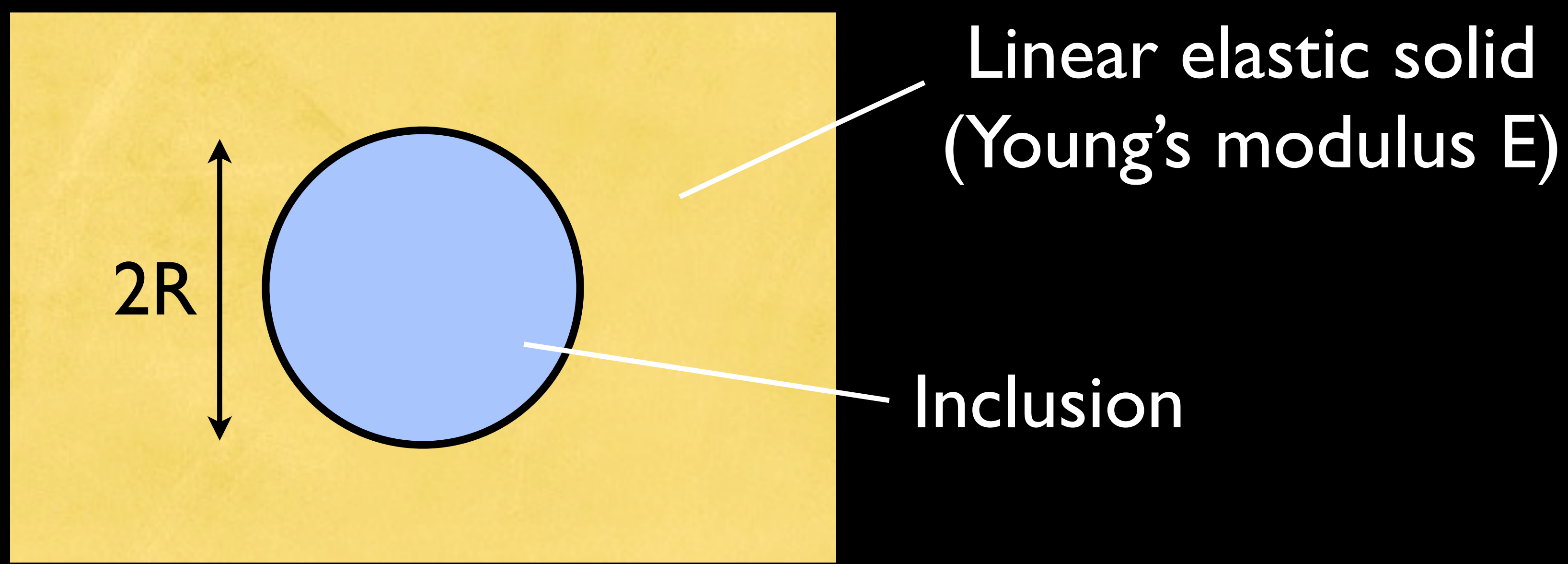
- Shape independent of droplet size

Apply strain ϵ



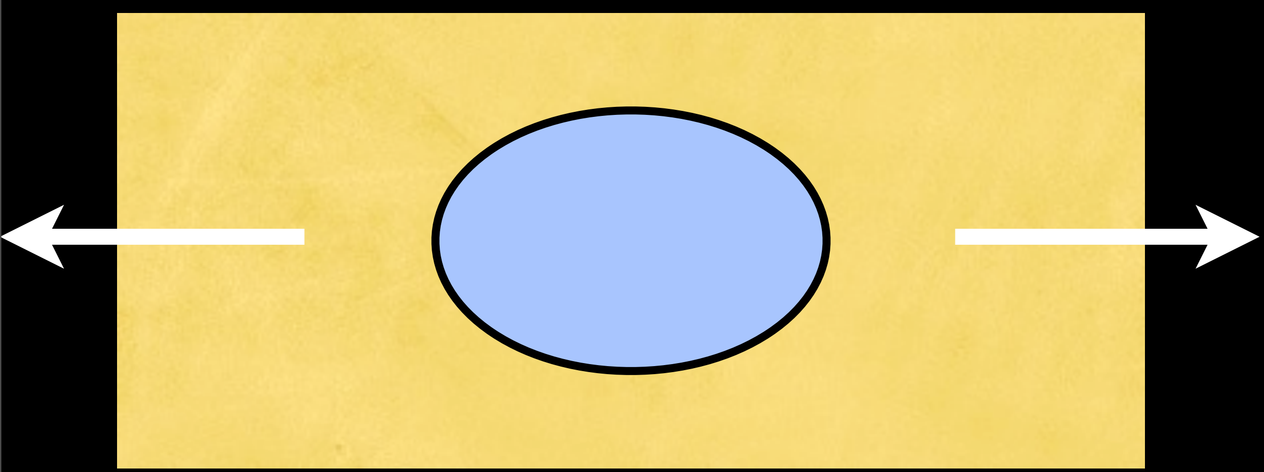
(Eshelby, 1957)

Eshelby theory explains stiff solid results



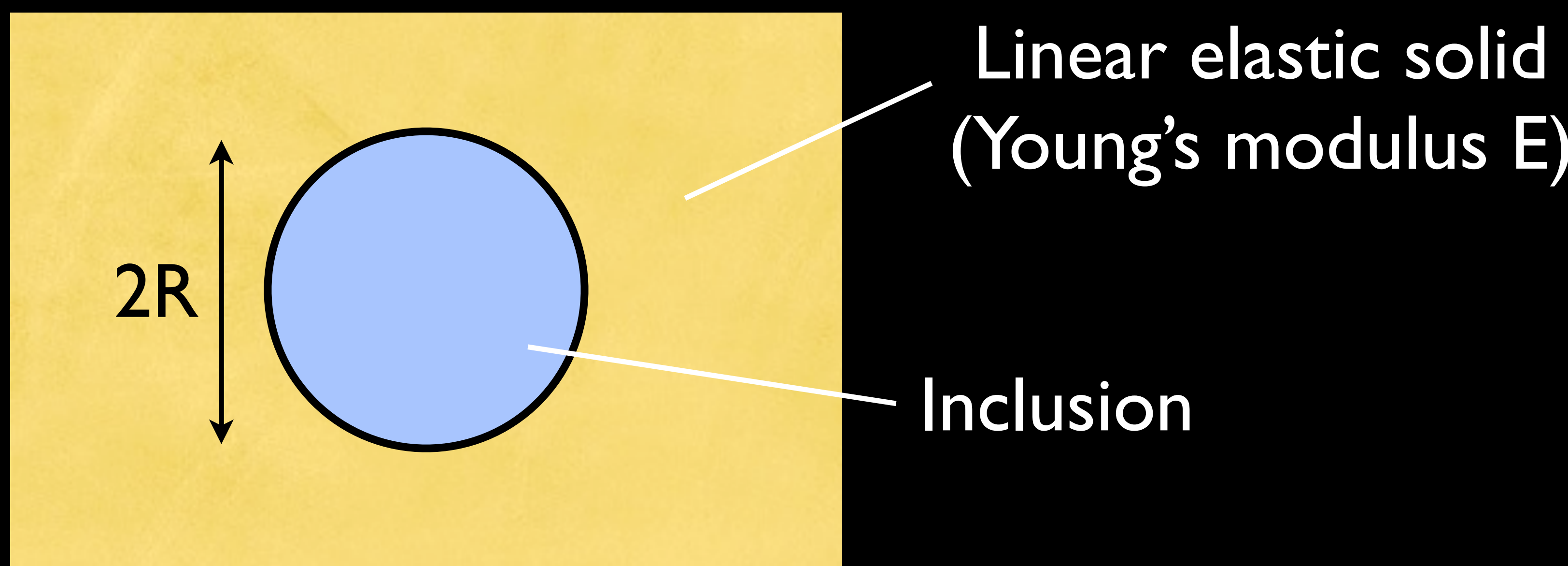
- Shape independent of droplet size
- Droplets 'softer' than the solid

Apply strain ϵ



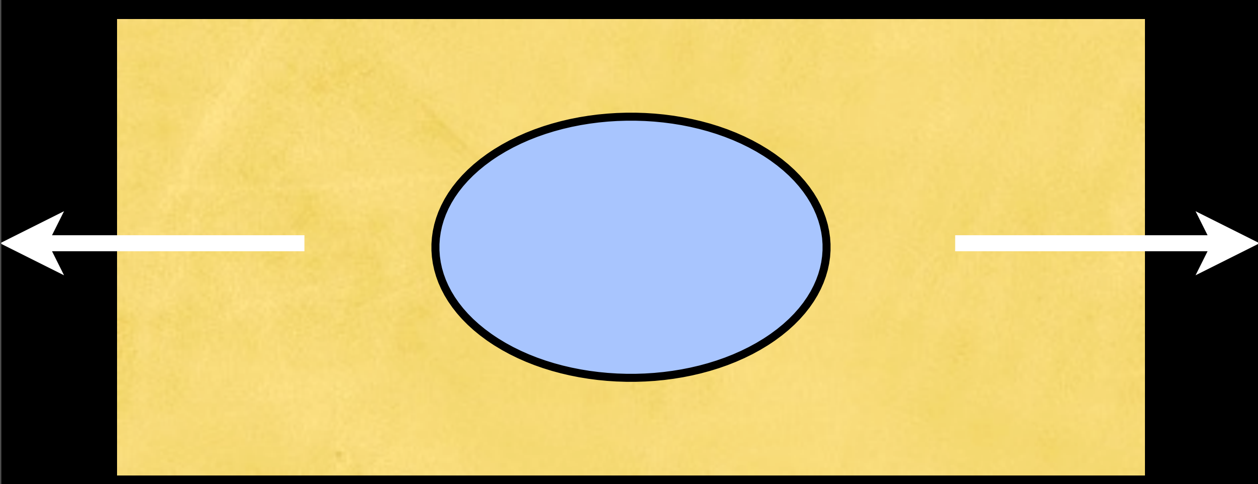
(Eshelby, 1957)

Eshelby theory explains stiff solid results



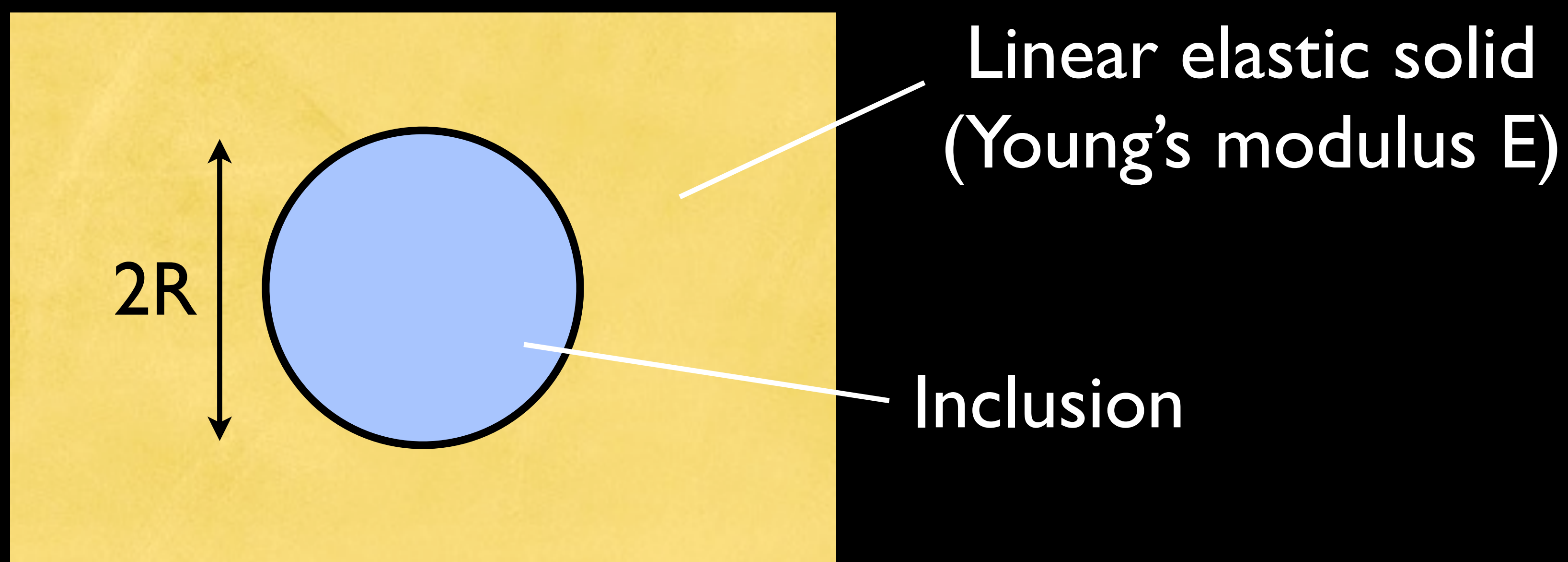
- Shape independent of droplet size
- Droplets 'softer' than the solid
- Composite stiffness ~ law of mixtures

Apply strain ϵ

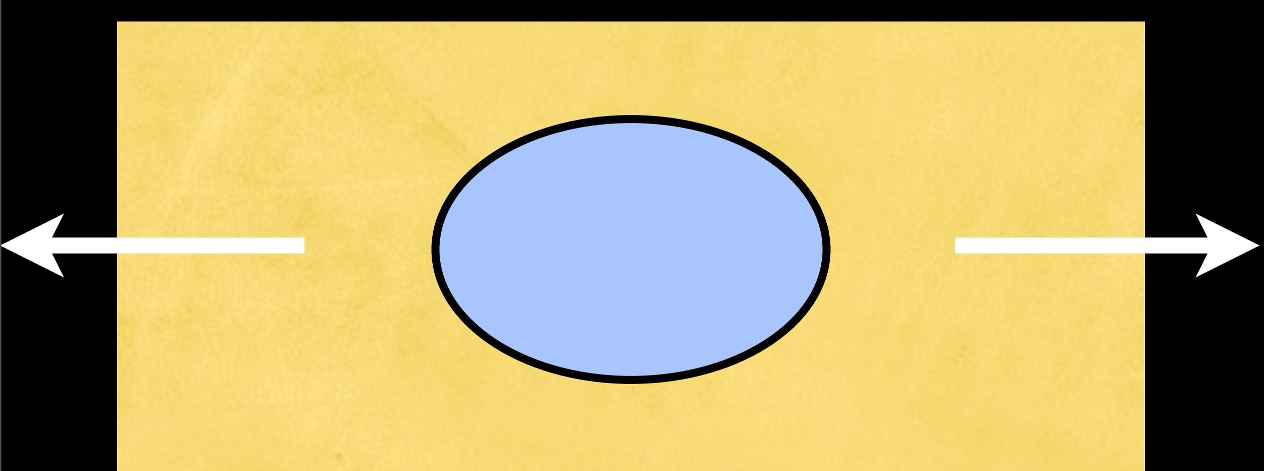


(Eshelby, 1957)

Eshelby theory explains stiff solid results



Apply strain ϵ



- Shape independent of droplet size
- Droplets 'softer' than the solid
- Composite stiffness ~ law of mixtures

Something additional is needed to explain the soft composites

(Eshelby, 1957)

Solid surface tension?

There seems to be a force that's acting to keep small embedded droplets spherical...

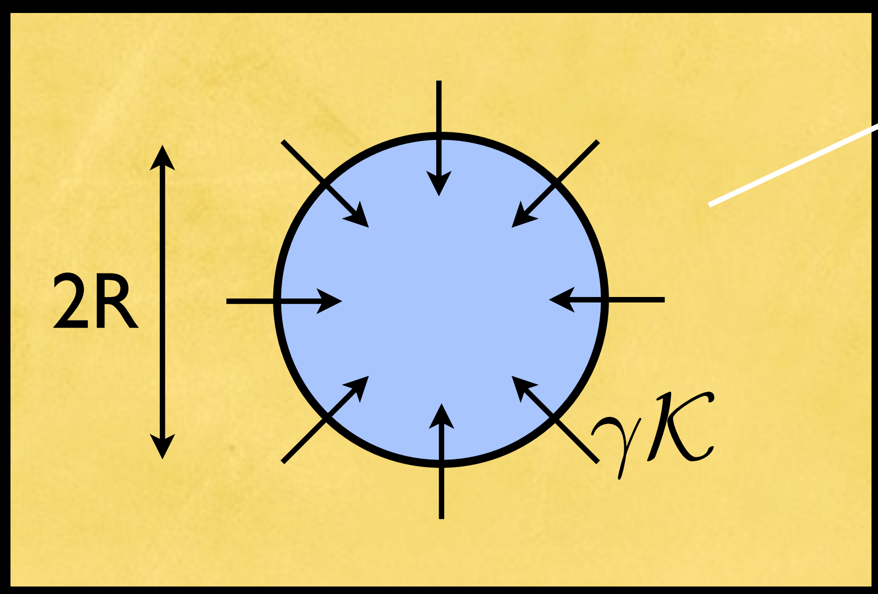
Liquid surface tension

keeps small things spherical

small effects at larger scales

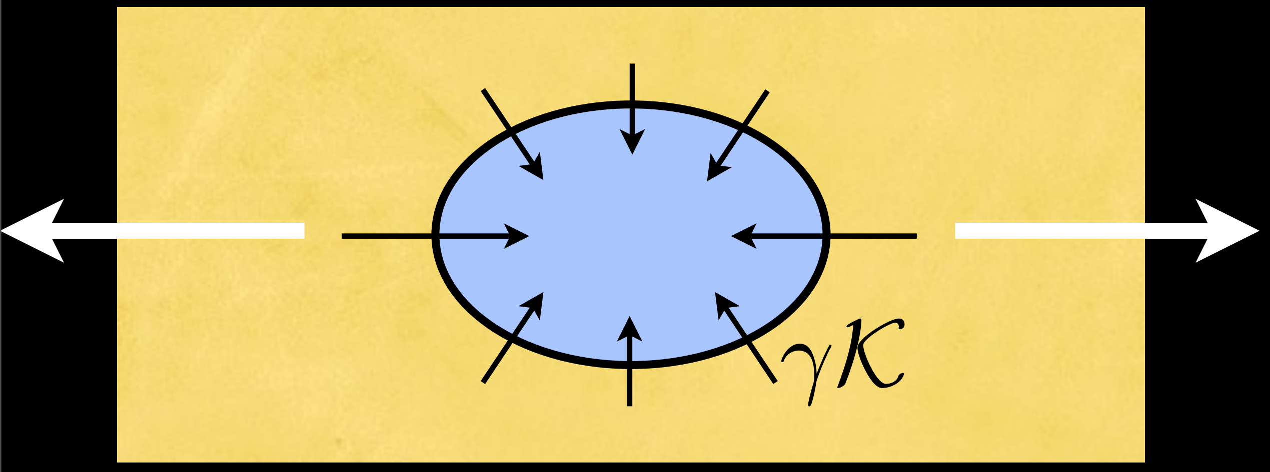
Capillary length: $L_\gamma = \sqrt{\frac{\rho g}{\gamma}}$

Eshelby theory with surface tension

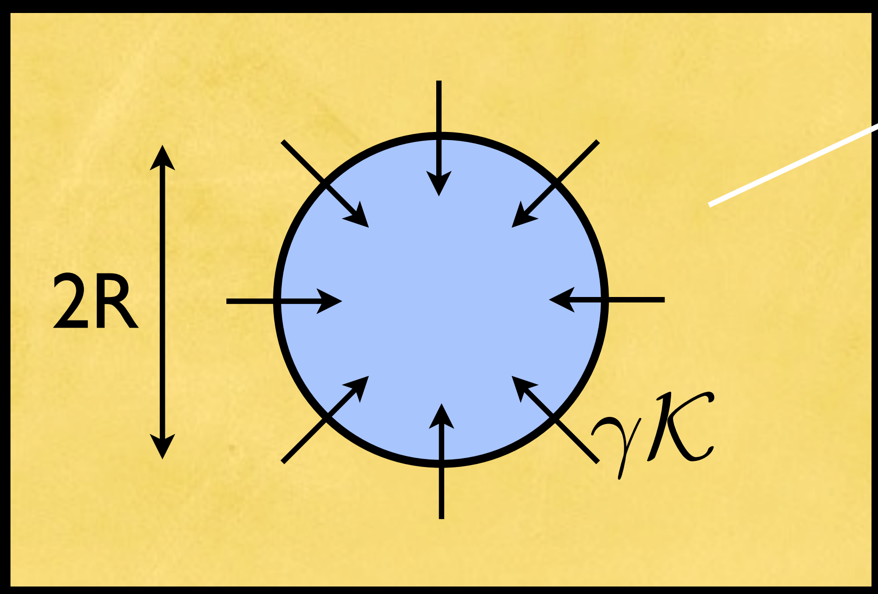


Linear elastic solid
(Young's modulus E)

Apply strain ϵ



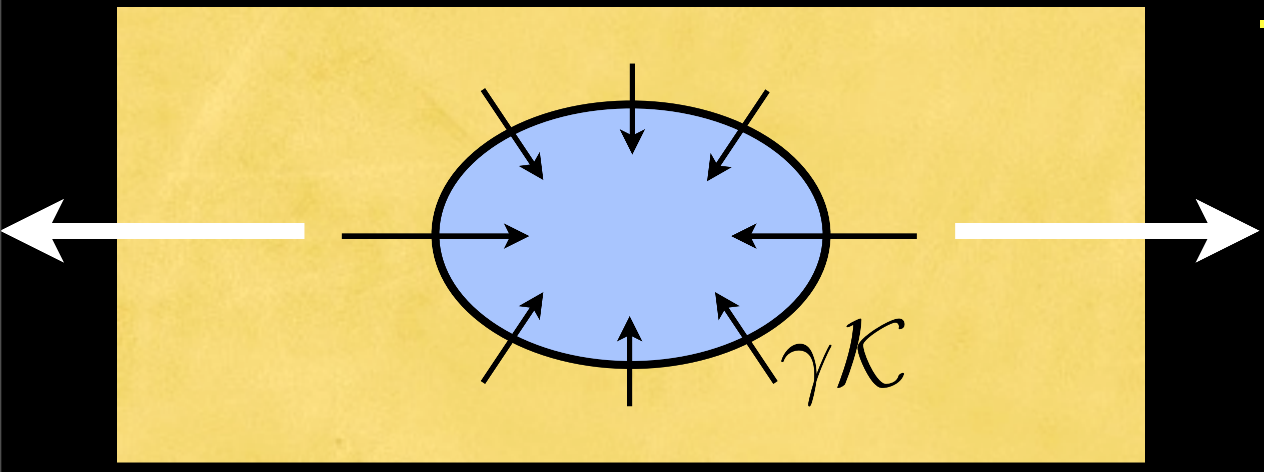
Eshelby theory with surface tension



Linear elastic solid
(Young's modulus E)

- Shape depends on the parameter $\dfrac{R}{\gamma/E}$
- Large $\dfrac{R}{\gamma/E}$ $\rightarrow$ Eshelby theory
- Small $\dfrac{R}{\gamma/E}$ $\rightarrow$ droplet stays spherical

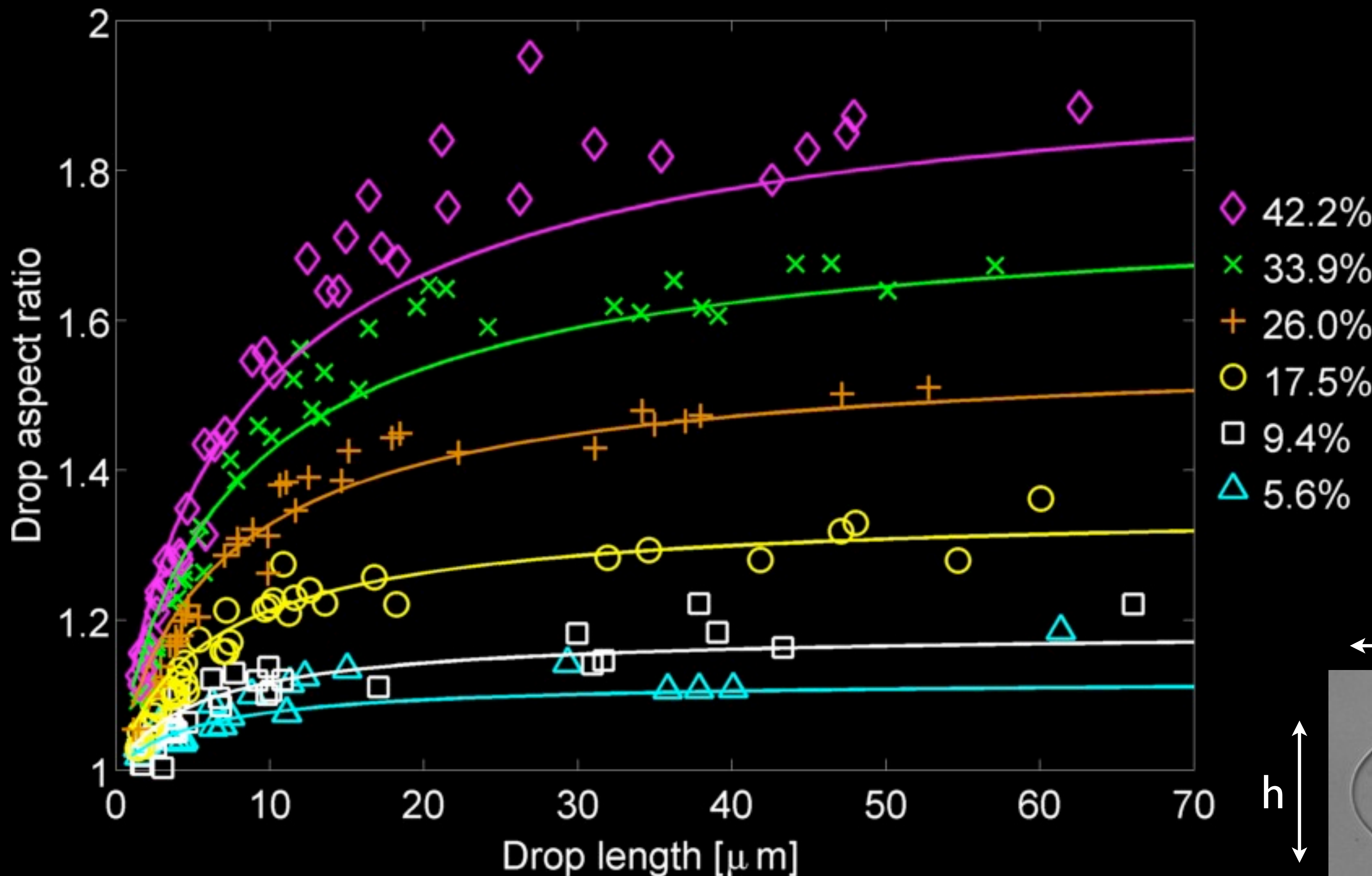
Apply strain ϵ

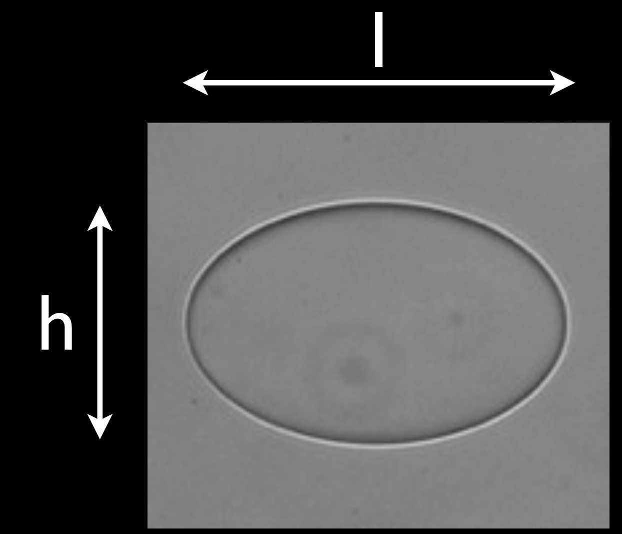


γ/E = elastocapillary length

Surface tension theory agrees with the data

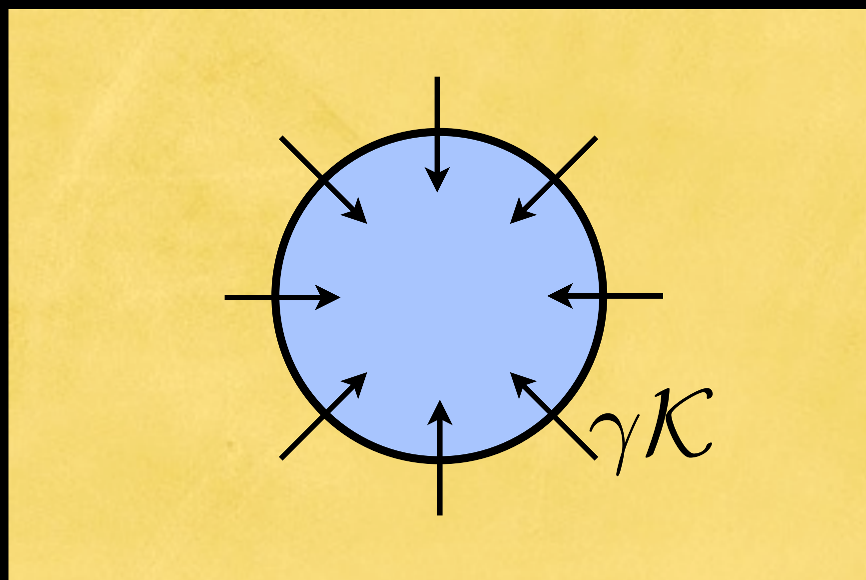
Fit the surface tension: $\gamma = 2\mathrm{mN/m}$

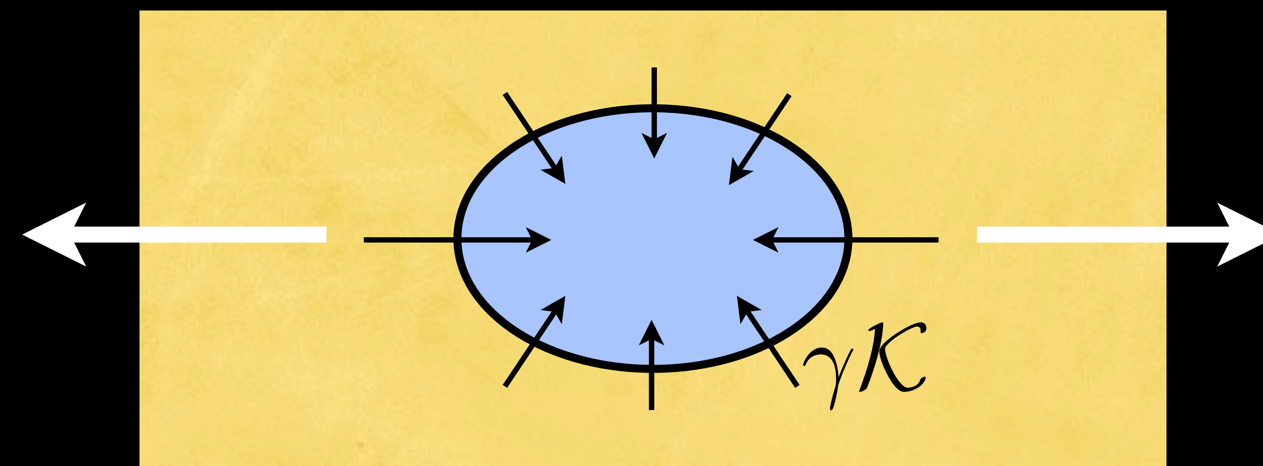




aspect ratio=l/h

Solid surface tension stiffens soft solids

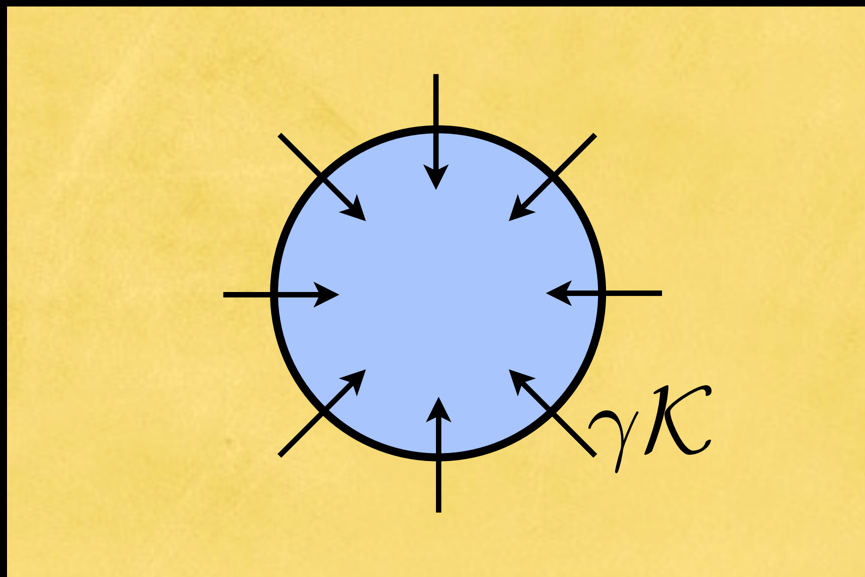


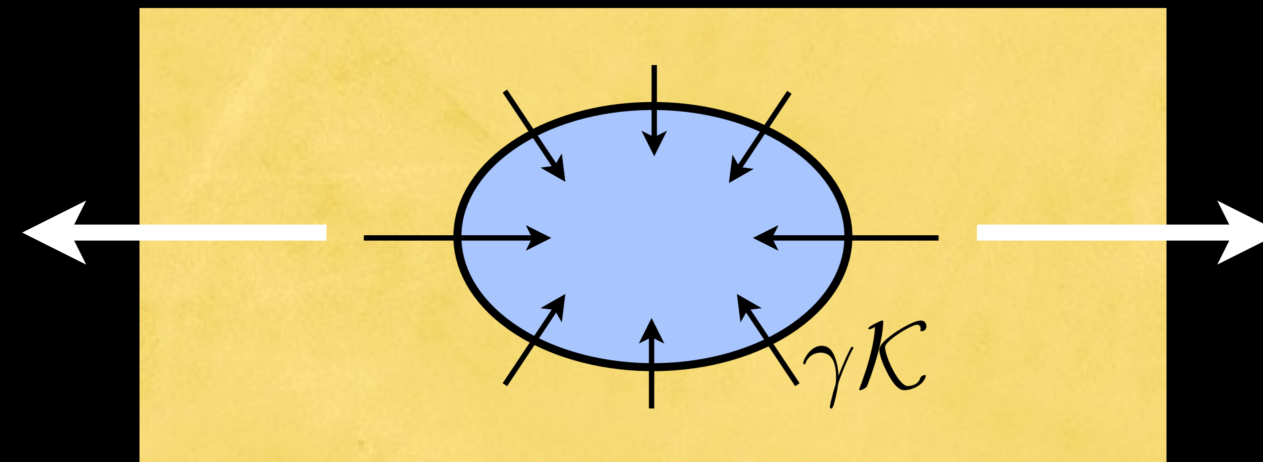


Tricky to apply the theory directly to the composite stiffness data, but:

- Large droplets/stiff solids with $\dfrac{R}{\gamma/E} > 1.5 \rightarrow$ softening
- Small droplets/soft solids with $\dfrac{R}{\gamma/E} < 1.5 \rightarrow$ stiffening

Solid surface tension stiffens soft solids





Tricky to apply the theory directly to the composite stiffness data, but:

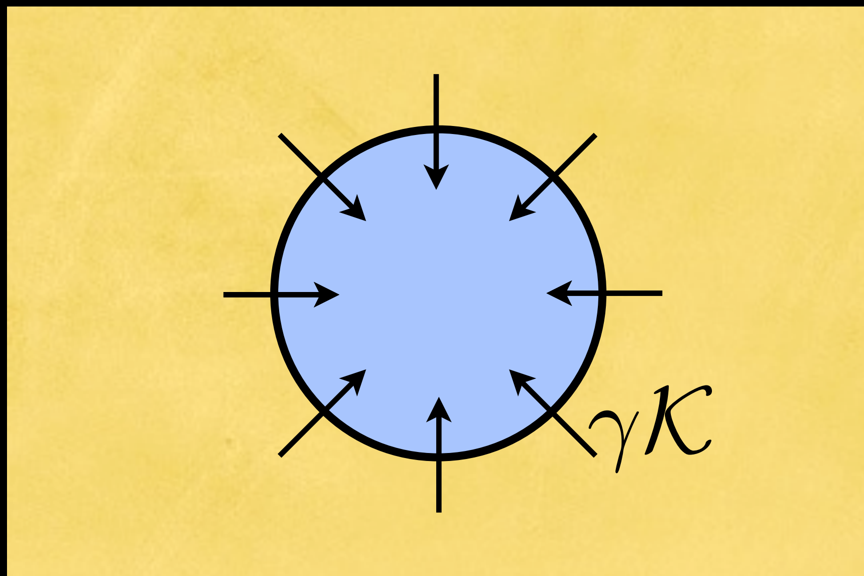
- Large droplets/stiff solids with $\dfrac{R}{\gamma/E} > 1.5 \rightarrow$ softening
- Small droplets/soft solids with $\dfrac{R}{\gamma/E} < 1.5 \rightarrow$ stiffening

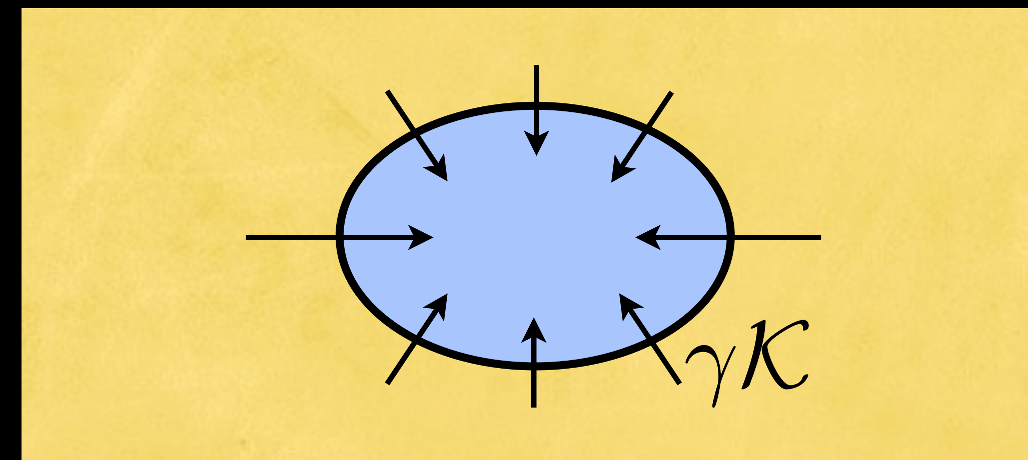
---

For glycerol in silicone:

$$\begin{aligned}\gamma &\sim 20\mathrm{mN/m}\\ E &= 2.5\,/\,100\mathrm{kPa} \longrightarrow \frac{R}{\gamma/E} \sim 0.1\,/\,4\\ R &\sim 1\mu\mathrm{m}\end{aligned}$$

What's the relevance of the elastocapillary length?





Pressure in droplet ~ $\dfrac{\gamma}{R}$

Significant elastic deformations occur if stress ~ *E*

Surface tension can deform the solid when $\dfrac{\gamma}{R} \gtrsim E$ or $R \lesssim \dfrac{\gamma}{E}$

When will surface tension effects affect composite stiffness?

(Estimates using typical liquid surface tensions ~ 20 mN/m)

	E	γ/E (Critical inclusion size)	
Glass	1 GPa	< 1 Angstrom	*No problem!*
Elastomer	1 MPa	~ 30 nanometres	*Nanocomposites...*
Gels	1-10 kPa	~ 3-30 microns	*Soft composites, biological solids, foods, colloidal inclusions...*

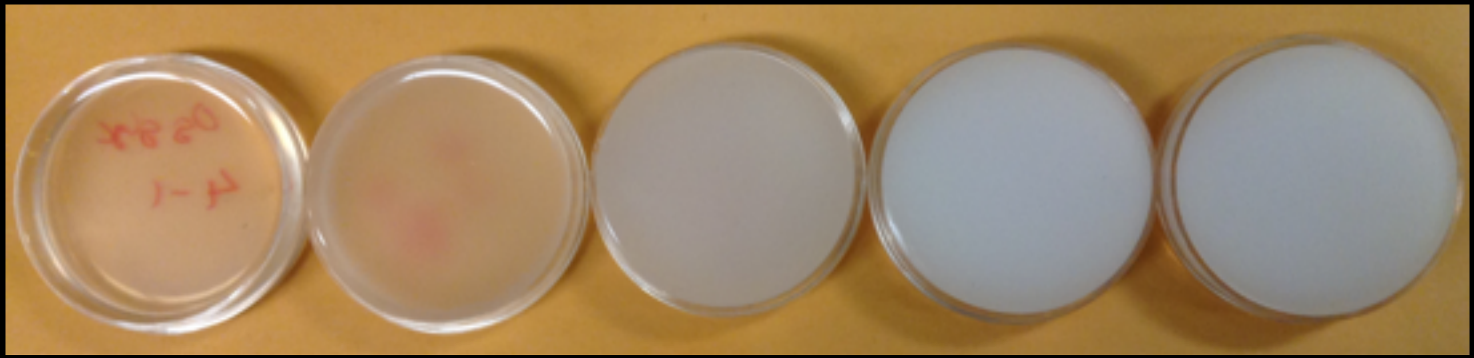
Conclusions

• Surface tension can play a big role in determining the mechanics of soft composites.

Law of mixtures

$$E_{eff} = E(1-\phi)$$

• We made solid/liquid composites that stiffen with increasing liquid content



• Surface tension is important when R< γ/E