The Generalized Gell-Mann–Okubo Formalism

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Previous work

- G. Landry (2013). Symétries et nomenclature des baryons. M. Sc. Thesis, Université de Moncton.
- N. Beaudoin, G. Landry, R. Sandapen (2013). Generalized isospin, generalized mass groups, and generalized Gell-Mann–Okubo formalism. arXiv:1309.0517 [hep-ph].

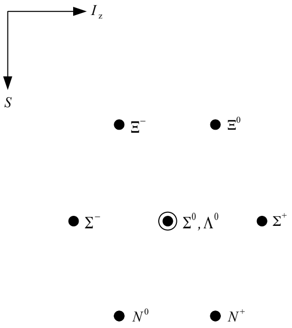
History

- 1909–1947: Early Particle Physics
  - Discovery of the nucleus, neutron, proton
  - Concept of isospin
  - Discovery of pions
- 1951–1964: Strange Particle Physics
  - Discovery of K, Λ, Σ, Ξ, ...
  - Concept of strangeness
  - Eightfold Way, Gell-Mann–Okubo formalism
  - Discovery of Ω
- 1964–Present: Quarks, heavy hadrons
  - Quark model
  - Discovery of light quarks (u, d, s)
  - Discovery of heavy quarks (c, b, t)

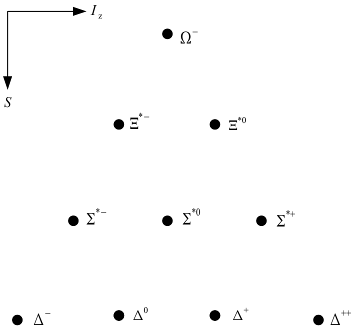
The Eightfold Way

Part I – Representations

Gell-Mann and Ne'eman: Mathematics of SU(3) and their various representations (e.g. $\mathbf{10}$, $\mathbf{8}$, $\mathbf{1}$, ...)



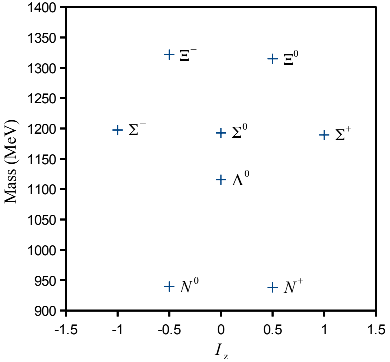
Weight diagram for $\mathbf{8}$.



Weight diagram for $\mathbf{10}$.

Some representations of SU(3) and their weight diagrams.

The light baryon multiplets



$J = \frac{1}{2}^+$ $\qquad$ $J = \frac{3}{2}^+$

The known baryons in 1964.

The Eightfold Way

Part II – Gell-Mann–Okubo Formalism

- Charge

$$Q = I_z + \frac{1}{2}\left(\tilde{B} + S\right)$$

- Isospin

$$\mathrm{mult}(I_z) = 2I + 1$$

- Mass formula

$$M = a_0 - a_1 S + a_2 \left[I\left(I+1\right) - \frac{1}{4}S^2\right]$$

- Equal spacing rule

$$\Omega - \Xi^* = \Xi^* - \Sigma^* = \Sigma^* - \Delta = a_1 - 2a_2$$

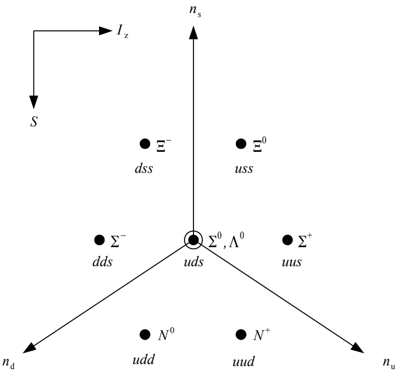
The Quark Model

Part I – Proposal

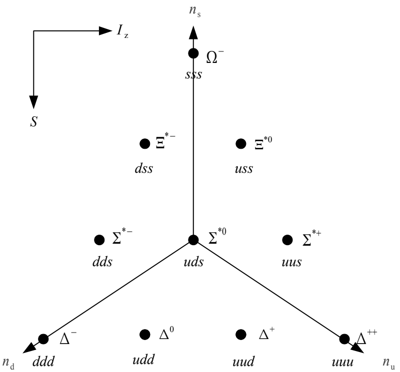
- Gell-Mann, Zweig
- $\mathbf{10}$ and $\mathbf{8}$...
  - $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$
  - $\mathbf{3}$ is the fundamental representation
  - $\mathbf{3}$ corresponds to quarks $(u,\ d,\ s)$

The Quark Model

Part II – Representations



Weight diagram for **8**.



Weight diagram for **10**.

Some representations of SU(3) and their weight diagrams.

The Quark Model

Part III – Flavour quantum numbers

$$\tilde{B} = \frac{1}{3}\left(n_u + n_d + n_s\right)$$

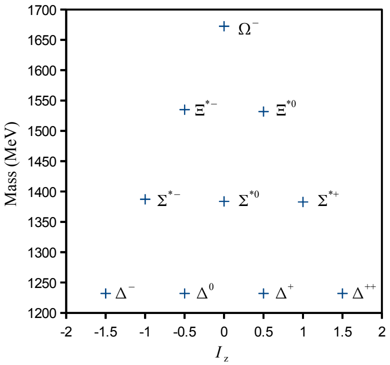
$$\boxed{I_\mathrm{z} = \frac{1}{2}\left(n_u - n_d\right)}$$

$$S = -n_s$$

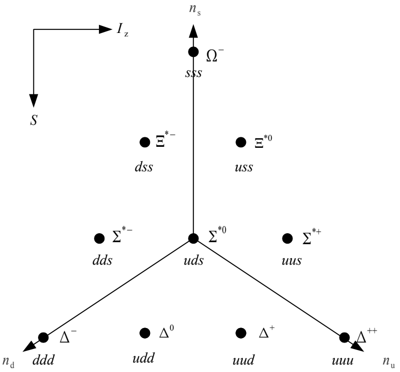
$$\begin{aligned} Q &= I_z + \frac{1}{2}\left(\tilde{B} + S\right) \\ &= +\frac{2}{3}n_u - \frac{1}{3}\left(n_d + n_s\right) \end{aligned}$$

The Quark Model

Part IV – Quark masses



$J = \frac{3}{2}^+$ baryons



Weight diagram for **10**.

$$\text{Equal spacing} = m_s - \frac{1}{2}(m_u + m_d)$$

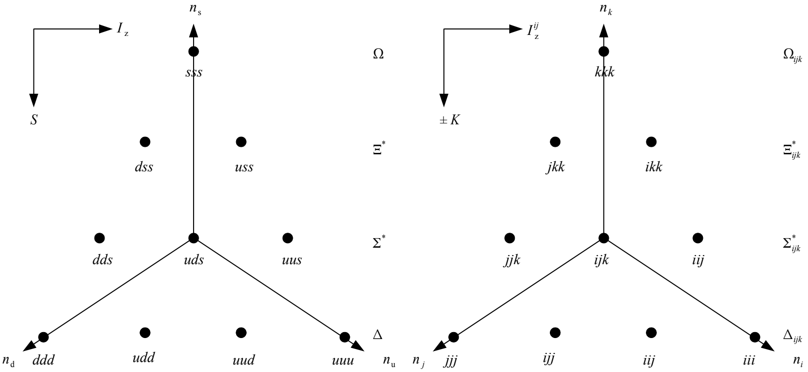
The Quark Model
Part V – Today

- 6 quarks $(u,\ d,\ s,\ c,\ b,\ t)$
- $\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{70} \oplus \mathbf{20}$
- Quantum numbers
  - $I_{\mathrm{z}} = \frac{1}{2}\left(n_u - n_d\right)$
  - $S = -n_s$
  - $C = +n_c$
  - $B = -n_b$
  - $T = +n_t$
  - $\tilde{B} = \frac{1}{3}\left(n_u + n_d + n_s + n_c + n_b + n_t\right)$
- Charge formula
  - $Q = I_{\mathrm{z}} + \frac{1}{2}\left(\tilde{B} + S + C + B + T\right)$
  - $Q = +\frac{2}{3}\left(n_u + n_c + n_t\right) - \frac{1}{3}\left(n_d + n_s + n_b\right)$

Generalized GMO formalism

Part I – The problem

- How do we deal with SU(6)?
- What happens in SU(3) when $u, d, s \to i, j, k$?



uds decuplet

ijk decuplet

Generalized GMO formalism

Part II – Generalized mass groups

uds mass groups

	Mass group	I	n_s
8	N	1/2	0
	Λ	0	1
	Σ	1	1
	Ξ	1/2	2
10	Δ	3/2	0
	Σ^*	1	1
	Ξ^*	1/2	2
	Ω	0	3

ijk mass groups

	Mass group	I^{ij}	n_k
8	N_{ijk}	1/2	0
	Λ_{ijk}	0	1
	Σ_{ijk}	1	1
	Ξ_{ijk}	1/2	2
10	Δ_{ijk}	3/2	0
	Σ^*_{ijk}	1	1
	Ξ^*_{ijk}	1/2	2
	Ω_{ijk}	0	3

Generalized GMO formalism

Part III – Generalized Gell-Mann–Okubo formalism

$$\text{mult}\,(I_\text{z}) = 2I + 1 \quad \rightarrow \quad \text{mult}\left(I_\text{z}^{ij}\right) = 2I^{ij} + 1$$

$$I_\text{z} = \tfrac{1}{2}\,(n_u - n_d) \quad \rightarrow \quad I_\text{z}^{ij} = \tfrac{1}{2}\,(n_i - n_j)$$

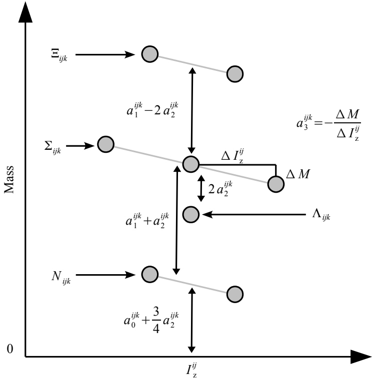
$$M = a_0^{ijk} + a_1^{ijk} n_k + a_2^{ijk}\left[I^{ij}\left(I^{ij}+1\right) - \frac{1}{4}n_k^2\right] \boxed{-a_3^{ijk} I_\text{z}^{ij}}$$

$$a_1^{ijk} - 2a_2^{ijk} = m_k - \frac{1}{2}\,(m_i + m_j)$$

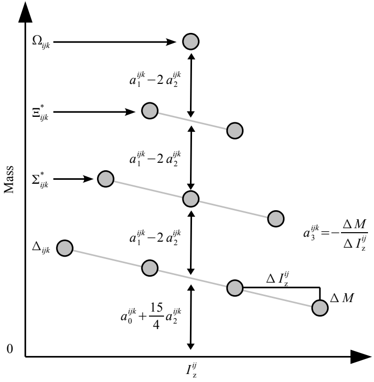
$$\boxed{a_3^{ijk} = -\,(m_i - m_j)}$$

Generalized GMO formalism

Part IV – Parameter significance



Octet parameters



Decuplet parameters

Significance of generalized GMO parameters

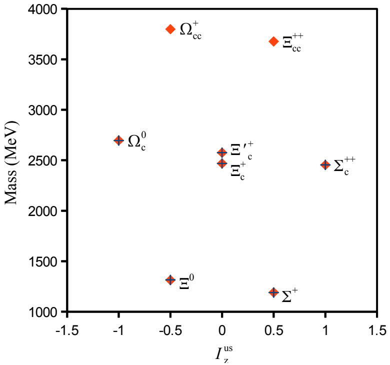
Generalized GMO formalism

Part V – The big question

Does it work?

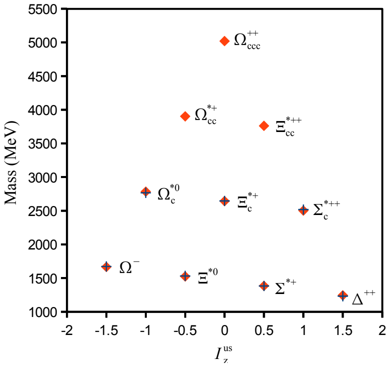
Generalized GMO formalism

Part VI - The worse case



The usc octet.

RMSE $=$ 1.30 MeV



The usc decuplet.

RMSE $=$ 10.67 MeV

PDG masses ($+$), GGMO masses (♦)

Generalized GMO formalism

Parameter values – Octets

Generalized GMO parameters for octets[1]

ijk	a_0^{ijk}	a_1^{ijk}	a_2^{ijk}	a_3^{ijk}	$a_1^{ijk}-2a_2^{ijk}$	$m_k-\frac{1}{2}(m_i+m_j)$	$2a_2^{ijk}$	$\Sigma_{ijk}-\Lambda_{ijk}$	$-(m_i-m_j)$	RMSE
uds	911.33	200.83	44.60	4.05	111.63	91.45	89.2	76.96	2.5	6.95
udc	876.26	1431.08	83.54	0.16	1264.00	1271.45	167.08	166.44	2.5	0.49
udb	866.17	4777.48	97.00	1.94	4583.48	4176.45	194.00	—	2.5	0.23
usc	1211.94	1269.25	53.56	121.59	1162.12	1226.35	107.12	107.80	92.7	1.30
usb	1194.91	4608.41	76.27	125.49	4455.87	4131.35	152.54	—	92.7	1.23
ucb	**3060.85**	**3959.33**	**19.43**	**1242.88**	**3920.47**	3541.35	**38.86**	—	1272.3	—
dsc	1220.31	1263.67	52.37	121.44	1158.93	1225.1	104.74	107.02	90.2	1.46
dsb	1202.52	4607.60	76.07	127.05	4457.19	4130.1	152.14	—	90.2	0.99
dcb	**3061.94**	**3965.65**	**17.08**	**1241.97**	**3931.49**	3540.1	**34.16**	—	1270.2	—
scb	**3219.08**	**3890.75**	**35.74**	**1011.37**	**3819.27**	3495	**71.48**	—	1180.0	—

[1] Plain values were determined using only the PDG baryon masses, while values in bold were estimated by "completing" multiplets.

Generalized GMO formalism

Missing baryon masses

Predicted masses of missing octet baryons[2]

Multiplet	$\Omega^{+}_{\mathrm{ccb}}$	Multiplet	$\Omega^{0}_{\mathrm{cbb}}$	Multiplet	Ξ^{++}_{cc}	Multiplet	Ξ^{+}_{cc}	Multiplet	Ξ^{0}_{bb}	Multiplet	Ξ^{-}_{bb}
ucb	**8297.06**	ucb	**11596.09**	udc	3717.46	udc	3717.62	udb	10395.91	udb	10397.85
dcb	**8299.45**	dcb	**11609.96**	usc	3676.25	dsc	3673.84	usb	10329.92	dsb	10335.18
scb	**8273.75**	scb	**11542.33**								
Average	**8290.09**	Average	**11582.79**	Average	3696.86	Average	3695.73	Average	10362.92	Average	10376.52
σ	**14.20**	σ	**35.72**	σ	29.14	σ	30.96	σ	46.66	σ	30.17

Multiplet	Ω^{+}_{cc}	Multiplet	Ω^{-}_{bb}	Multiplet	Σ^{0}_{b}	Multiplet	Ξ'^{0}_{b}	Multiplet	Ξ'^{-}_{b}	Multiplet	Ξ^{+}_{cb}
usc	3797.85	usb	10455.41	udb	5813.40	usb	5936.79	dsb	5943.24	ucb	**7015.32**
dsc	3795.28	dsb	10463.23								
Average	3796.57	Average	10458.82	Average	5813.40	Average	5936.79	Average	5943.24	Average	**7015.32**
σ	1.82	σ	4.82	σ	—	σ	—	σ	—	σ	—

Multiplet	Ξ'^{+}_{cb}	Multiplet	Ξ^{0}_{cb}	Multiplet	Ξ'^{0}_{cb}	Multiplet	Ω^{0}_{cb}	Multiplet	$\Omega'^{0}_{\mathrm{cb}}$
ucb	**7054.18**	dcb	**7023.32**	dcb	**7057.48**	scb	**7100.90**	scb	**7172.38**
Average	**7054.18**	Average	**7023.32**	Average	**7057.48**	Average	**7100.90**	Average	**7172.38**
σ	—	σ	—	σ	—	σ	—	σ	—

[2] Plain values were determined using only the PDG baryon masses, while values in bold were estimated by "completing" multiplets.

Generalized GMO formalism

Parameter values – Decuplet

Generalized GMO parameters for decuplets[3]

ijk	$a_0^{ijk} + \frac{15}{4} a_2^{ijk}$	Δ_{ijk}	$a_1^{ijk} - 2a_2^{ijk}$	$m_k - \frac{1}{2}(m_i + m_j)$	a_3^{ijk}	$-(m_i - m_j)$	RMSE
uds	1233.73	1232.00	148.37	91.45	0.80	2.5	3.18
udc	1232.00	1232.00	1286.07	1271.45	-0.13	2.5	0.33
udb	1232.00	1232.00	4601.60	4176.45	0.43	2.5	0.73
usc	1454.76	1455.01	1188.47	1226.36	140.45	92.7	10.67
usb	1454.76	1455.01	4506.03	4131.35	143.98	92.7	9.51
ucb	3160.85	—	3957.15	3541.35	1285.90	1272.3	—
dsc	1456.66	1456.66	1186.77	1225.1	140.24	90.2	11.13
dsb	1456.66	1456.66	4507.18	4130.1	143.89	90.2	9.86
dcb	3162.2	—	3959.70	3540.1	1286.80	1270.2	—
scb	**3355.12**	—	**3886.07**	3495	**1137.02**	1180.0	**17.51**

[3] Plain values were determined using only the PDG baryon masses, while values in bold were estimated by “completing” multiplets.

Generalized GMO formalism

Missing decuplet masses

Predicted masses of missing decuplet baryons[4]

Multiplet	Ω_{ccc}^{++}	Multiplet	Ω_{bbb}^{-}	Multiplet	Ξ_{cc}^{*++}	Multiplet	Ξ_{cc}^{*+}	Multiplet	Ξ_{bb}^{*0}	Multiplet	Ξ_{bb}^{*-}
udc	5090.21	udb	15036.80	udc	3804.21	udc	3804.08	udb	10434.99	udb	10435.45
usc	5020.17	usb	15032.30	usc	3761.48	dsc	3760.88	usb	10394.83	dsb	10399.08
ucb	5089.70	ucb	14972.85	ucb	3803.80	dcb	3805.60	ucb	10432.20	dcb	10418.20
dsc	5016.97	dsb	14978.20								
dbc	5092.40	dcb	15011.30								
Average	5061.89	Average	15006.29	Average	3789.83	Average	3790.19	Average	10420.67	Average	10417.58
σ	39.57	σ	29.75	σ	24.55	σ	25.39	σ	22.42	σ	18.19

Multiplet	Ω_{cc}^{*+}	Multiplet	Ω_{b}^{*-}	Multiplet	Ω_{bb}^{*-}	Multiplet	Ω_{ccb}^{*+}	Multiplet	Ω_{cbb}^{*0}	Multiplet	Σ_{b}^{*0}
usc	3901.93	usb	6104.77	usb	10538.81	ucb	8403.90	ucd	11718.10	udb	5833.60
dsc	3900.32	dsb	6107.73	dsb	10542.97	dcb	8398.70	dcb	11705.00		
Average	3901.13	Average	6106.25	Average	10540.89	Average	8401.30	Average	11711.55	Average	5833.60
σ	1.14	σ	2.09	σ	2.94	σ	3.68	σ	9.26	σ	—

Multiplet	Ξ_{cb}^{*+}	Multiplet	Ξ_{b}^{*-}	Multiplet	Ξ_{cb}^{*0}	Multiplet	Ω_{cb}^{*0}
ucb	7118.00	dsb	5936.84	ucb	7111.90	scb	**7241.19**
Average	7118.00	Average	5936.84	Average	7111.90	Average	**7241.19**
σ	—	σ	—	σ	—	σ	—

[4] Plain values were determined using only the PDG baryon masses, while values in bold were estimated by "completing" multiplets.

Conclusions

- Generalized framework is simple and familiar
  - Mass groups exist in other multiplets
  - Quark numbers + generalized isospin
  - No need for flavour quantum numbers
  - Easy to distinguish Σ-likes ($I^{ij} = 1$) from Λ-likes ($I^{ij} = 0$)
- Generalized GMO formalism seems to works very well
  - RMSE $\sim$10 MeV reproduction of existing masses
  - Consistant ($\sigma < 50$ MeV) predictions from independant fits
  - Could be trivial agreement (need doubly-heavy baryons)
  - Could reduce number of parameters via quark mass relations
- Covers all $\mathbf{70}$ (e.g. $J^P = \frac{1}{2}^+$) and $\mathbf{56}$ (e.g. $J^P = \frac{3}{2}^+$) baryons
- Silent on $\mathbf{20}$ (e.g. $J^P = \frac{1}{2}^-$) baryons

Acknowledgments

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  - Université de Moncton
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- arxiv:1309.0517 [hep-ph]