The little we know of LOCC

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Local Operations with Classical Communication

LOCC: set of multiparty quantum operations that can be implemented if each party can apply local operations, and they can communicate classical information to one another.

- Operationally motivated

  class communication much easier than quantum

- Connection to fundamental quantum information

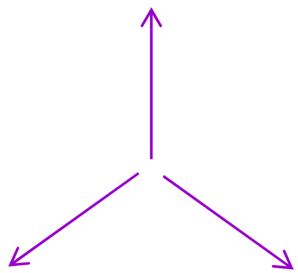
  class communication is generated by measurement which extracts information and disturbs the q state

- Complementary study to entanglement theory

- Applications: quantum error correcting codes, data hiding, ...

Local Operations with Classical Communication

e.g. Discrimination of the double trine states [Peres-Wootters 94]



For a random $k \in \{0,1,2\}$,
Alice and Bob each receives
$\cos(2\pi/3)k\ |0\rangle + \sin(2\pi/3)k\ |1\rangle$

- Optimal global strategy known
  (accessible info, min error, unambiguous discrimination)

- LOCC discrimination was conjectured suboptimal, recently proved for some measures of success [Chitambar-Hsieh 13]

- Optimal LOCC strategy still unknown !

Local Operations with Classical Communication

Why so difficult:

- "Information gain implies disturbance" is not fully understood
  (cf QKD with bound entangled states, locking, bounded-storage crypto models, nonlocality without entanglement)

- No succinct mathematical description for LOCC

Often use: LOCC $\subset$ SEP $\subset$ PPT

SEP: quantum operations with tensor product Kraus operators
PPT: quantum operations preserving PPT states

Useful relaxations ... except for studying SEP\LOCC.

- Potentially unbounded # of communication rounds or intermediate results

Plan:

- LOCC 101

  (from 1210.4583 & Lo-Fortescu 06)

- State discrimination by LOCC

  (largely survey)

LOCC:

- m parties

- For fixed k (# measurement outcomes), an instrument $\mathcal{J}$ is a k-tuple $(E_1, E_2, \cdots, E_k)$ where each E_i is a completely positive map and $\sum_{i=1}^{k} E_i$ is trace preserving.

  $\mathcal{J}$ is associated with the TCP map: $\rho \to \sum_{i=1}^{k} E_i(\rho) \otimes |i\rangle\langle i|$

- Distance on the set of instruments is induced by the diamond norm on the associated TCP maps

- In 1 round of communication, one party can broadcast unlimited amount of classical data to all other parties

LOCC:

finite

- $LOCC_r$: instruments realized with LO and r rounds of comm
- $LOCC_N := \cup_r LOCC_r$
-
- $cl(LOCC_N)$:= topological closure of $LOCC_N$

Operationally:

$cl(LOCC_N)$ includes all m-party instruments that can be approx to arbitrary precision by a sequence $\{\mathcal{J}_1,\mathcal{J}_2,\ldots\}$ in $LOCC_N$

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Operationally:

$cl(LOCC_N)$ includes all m-party instruments that can be approx to arbitrary precision by a sequence $\{\mathcal{J}_1, \mathcal{J}_2, \ldots\}$ in $LOCC_N$

$LOCC_\infty$: instrument that can be approx by adding more and more rounds of communication without changing earlier steps.

LOCC:

- LOCC_r : instruments realized with LO and r rounds of comm
- $\mathrm{LOCC}_{\mathbb{N}} := \bigcup_r \mathrm{LOCC}_r$
- $\mathrm{LOCC}_\infty := \mathrm{LOCC}_{\mathbb{N}}$ + limit points of special sequences
- $\mathrm{cl}(\mathrm{LOCC}_{\mathbb{N}})$:= topological closure of $\mathrm{LOCC}_{\mathbb{N}}$

$$\mathrm{LOCC}_r \underset{\neq}{\subset} \mathrm{LOCC}_{r+1} \underset{\neq}{\subset} \mathrm{LOCC}_\infty \underset{\neq}{\subset} \mathrm{cl}(\mathrm{LOCC}_{\mathbb{N}}) \underset{\neq}{\subset} \mathrm{SEP}$$

r=1: QKD, ent distillation, ...

$\forall$ r, m=2: Duan-Xin 08, perfect discrimination of $O(r^2)$ states in $C^r \otimes C^r$

$\forall$ r, m$\geq$3: Chitambar11, random distillation of 3-qubit states ("gapped")

m$\geq$3: Chitambar, Cui, Lo 12
rand dist

m=2: nonlocality w/o ent

m=2: Chitambar, L, Mancinska, Ozols, Winter 12, 2-qubit ∅

Thus, when considering whether certain instrument can be "performed" in LOCC, we have to distinguish all 3 possible cases:

- instrument can be performed in finitely many LOCC steps

- instrument cannot be performed in finitely many LOCC steps but can approximated better and better

- instrument cannot even be approximated by LOCC

What's in $\mathrm{cl}(\mathrm{LOCC}_N) \setminus \mathrm{LOCC}_\infty$:

Random distillation of W states [Chitambar-Cui-Fortescu-Lo]:

3 parties: A, B, and C

Let $|W\rangle = (|001\rangle + |010\rangle + |100\rangle) / \sqrt{3}$

$|EPR\rangle = (|00\rangle + |11\rangle) / \sqrt{2}$

Goal: convert $|W\rangle$ on ABC to $|EPR\rangle$ on AB, BC, or AC

(Known: cannot convert W-state to EPR pair on two specific parties, here, doesn't care which two parties receive the EPR pair)

What's in cl($LOCC_N$) \ $LOCC_\infty$:

Goal: convert $|W\rangle$ on A,B,C to $|EPR\rangle$ on AB, BC, or AC

Consider $\{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r, \ldots\}$:

A, B, C each applies to their qubit the measurement with POVM:

$$M_0 := \begin{pmatrix} \sqrt{\varepsilon} & 0 \\ 0 & 0 \end{pmatrix} \quad M_1 := \begin{pmatrix} \sqrt{1-\varepsilon} & 0 \\ 0 & 1 \end{pmatrix}$$

Outcome	Postmeas state	Action	Prob
011, 101 ,011	$\|0\rangle\|EPR\rangle$ etc	Success	$2\epsilon(1-\epsilon)$
001, 010, 100	$\|001\rangle$ etc	Failure	ϵ^2
111	$\|W\rangle$	Try again	$(1-\epsilon)^2$

$\mathcal{P}_r$: try r times, pr[success] = $(1-(1-\epsilon)^{2r})\, 2(1-\epsilon)/(2-\epsilon) < 1 \;\; \forall r$.

Optimize ϵ based on r, pr[success] $\to$ 1 as r $\to \infty$.

So exact random distillation is in cl($LOCC_N$) but not in $LOCC_\infty$

2-qubit non-closure of LOCC_N example:

We consider a 3-outcome instrument $\mathcal{J} = \{\mathcal{E}_{00}, \mathcal{E}_{01}, \mathcal{E}_{10}\}$ with

$$\mathcal{E}_{00}(\rho) := |11\rangle\langle 11|\rho|11\rangle\langle 11|,$$

$$\mathcal{E}_{01}(\rho) := \sum_{i=1}^{2} (T_i \otimes |0\rangle\langle 0|)\rho(T_i^\dagger \otimes |0\rangle\langle 0|),$$

$$\mathcal{E}_{10}(\rho) := \sum_{i=1}^{2} (|0\rangle\langle 0| \otimes T_i)\rho(|0\rangle\langle 0| \otimes T_i^\dagger),$$

and

$$T_1 := \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 \\ 0 & \frac{1}{\sqrt{3}} \end{pmatrix}, \qquad T_2 := \begin{pmatrix} \frac{1}{\sqrt{6}} & 0 \\ 0 & \sqrt{\frac{2}{3}} \end{pmatrix}.$$

Precise definition not needed for the understanding ...
What's needed: (1) can be approx (similar to FL06)
(2) why not in LOCC_∞

2-qubit non-closure of LOCC_N example:

Why $\mathcal{J} \notin \text{LOCC}_\infty$:

$$|W\rangle \xrightarrow{\mathcal{J}_{AB} \otimes \mathcal{I}_C} \begin{cases} |0\rangle\langle 0|_A \otimes \omega_{BC} & \text{w. prob. } \frac{1}{2} \\ \omega_{AC} \otimes |0\rangle\langle 0|_B & \text{w. prob. } \frac{1}{2} \end{cases} \quad \text{where } \omega := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 4/9 & 0 \\ 0 & 4/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We found a mixed state tripartite entanglement monotone (nonincreasing under LOCC) which strictly decreases after any nontrivial local measurement, yet unchanged by the above state transformation.

... a contradiction if $\mathcal{J} \in \text{LOCC}_\infty$.

Thus, when considering whether certain instrument can be "performed" in LOCC, we have to distinguish all 3 possible cases:

- instrument can be performed in finitely many LOCC steps

- instrument cannot be performed in finitely many LOCC steps but can approximated better and better

- instrument cannot even be approximated by LOCC

even for the smallest possible remote system (2 qubits).

Thm. For instruments $(E_1, E_2, \cdots, E_k)$ with fixed k, $LOCC_r$ is compact.

(i.e., infinitely many intermediate measurement outcomes gives no advantage.)

Plan:

- LOCC 101
- State discrimination by LOCC

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- LOCC 101

- State discrimination by LOCC

- can be performed in finitely many LOCC steps	many examples
- cannot be performed in finitely many LOCC steps but can approximated better and better	no known examples
- cannot even be approximated by LOCC	many examples

State Discrimination Problem (bipartite case)

Let $S = \{|\psi_1\rangle, \ldots, |\psi_n\rangle\} \subset \mathbb{C}^{d_A} \otimes \mathbb{C}^{d_B}$ be a known set of quantum states. Suppose that $k \in \{1, \ldots, n\}$ is selected uniformly at random and Alice and Bob are given the corresponding parts of state $|\psi_k\rangle \in S$. Their task is to determine the index k.

Qn: can any LOCC operation discriminate the states as well as the best global operation?

The answer can depend on the measure of success.

As a first step, focus on orthogonal sets S and ask whether LOCC achieves perfect discrimination.

LOCC state discrimination:

- Sometimes an LOCC op achieves global optimum:

  e.g., Any 2 orthogonal pure states can be discriminated perfectly by $LOCC_N$ [Walgate, Short, Hardy, Vedral 00]

- More often, $cl(LOCC_N)$ ops can't achieve global optimum:

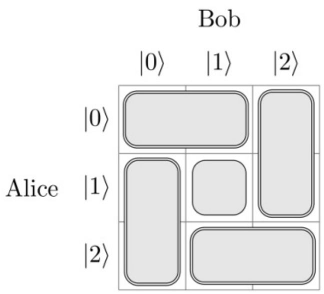
  e.g., a basis including at least one entangled state cannot be discriminated by $cl(LOCC_N)$.

  e.g., quantum data hiding states are nearly completely indistinguishable to parties performing $cl(LOCC_N)$. In fact, there are sets of near orthogonal separable states that are indistinguishable by PPT operations).

  e.g., nonlocality without entanglement – sets of separable states that can be perfectly distinguishable by SEP but not by $cl(LOCC_N)$.

Bennett, DiVincenzo, Fuchs, Mor, Rains, Shor, Smolin, Wootters 98

Quantum Nonlocality Without Entanglement



Theorem . Any LOCC protocol for discriminating states

$|1\rangle|1\rangle$

$|0\rangle|0 \pm 1\rangle$ $\quad$ $|1 \pm 2\rangle|0\rangle$

$|2\rangle|1 \pm 2\rangle$ $\quad$ $|0 \pm 1\rangle|2\rangle$

has mutual information deficit at least 0.00000531 bits.

Constant deficit implies $cl(LOCC_N) \subsetneq SEP$

What's the operational difference?

How different are they?

What product basis can be discriminated perfectly by $cl(LOCC_N)$?

Improve the gap?

How was it proved?

$\exists$ measurements that cannot create entanglement but requires entanglement to perform

In 1206.5822 & 1306.5992, we:

- replace original proof by a simpler and more systematic one

- extend the result to more product bases

- demonstrate an operational difference between LOCC and SEP in perfect discrimination of product basis

- Understand the limitations of similar proof techniques

Plan:

- LOCC 101

- State discrimination by LOCC

  - Nonlocality without entanglement

  - when cl($LOCC_N$) gives no advantage over $LOCC_N$

  - Operational difference between LOCC and SEP
    for perfect discrimination of product bases

When cl($LOCC_N$) gives no advantage over $LOCC_N$

Kleinmann, Kampermann, Bruss 11:

- A necessary condition for a set of states to be perfectly discriminated in cl($LOCC_N$)

- A product basis can be perfectly discriminated in cl($LOCC_N$), iff it can be perfectly discriminated in $LOCC_N$

Also, there is an algorithm to determine [in time $O((d_A d_B)^3)$] if a basis can be perfectly discriminated by $LOCC_N$.

[Rinaldis 04, Mancinska thesis 13]

When cl($LOCC_N$) gives no advantage over $LOCC_N$

Kleinmann, Kampermann, Bruss 11:

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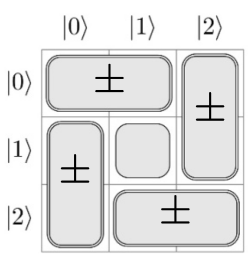
A small extension:

Consider any projective measurement M on a bipartite system, such that all but one POVM elements are tensor product operators, and the last one separable. Then, $M \in LOCC_N$ iff $M \in cl(LOCC_N)$.

e.g.,



even if we don't care to distinguish between the + states, the (6) states still cannot be distinguished in cl($LOCC_N$). A bit more work gives the same conclusion for discrimination of the +, –, and 11 states.

Plan:

- LOCC 101
- State discrimination by LOCC
  - Nonlocality without entanglement reproof
  - when cl($LOCC_N$) gives no advantage over $LOCC_N$
  - Operational difference between LOCC and SEP for perfect discrimination of bipartite product bases

Interpolatability distinguishes LOCC from SEP <u>for discrimination of bipartite product bases</u>

Recall that if a measurement M is too informative, it can be performed in 2 steps: the first step extracts a controlled amount of information, and together, the 2 steps implements M. We call the 2-step process an “interpolation” of M.

One can interpolate with respect to any reasonable measure of information gain (continuous, monotonic wrt coarse-graining, etc).

One can also interpolate any LOCC measurement, since it’s a composition of local measurements which can be interpolated locally.

Thm: Let M $\in$ SEP is a measurement along a product basis. Then M can be interpolated as 2 separable measurements iff M has initial steps in LOCC_N.

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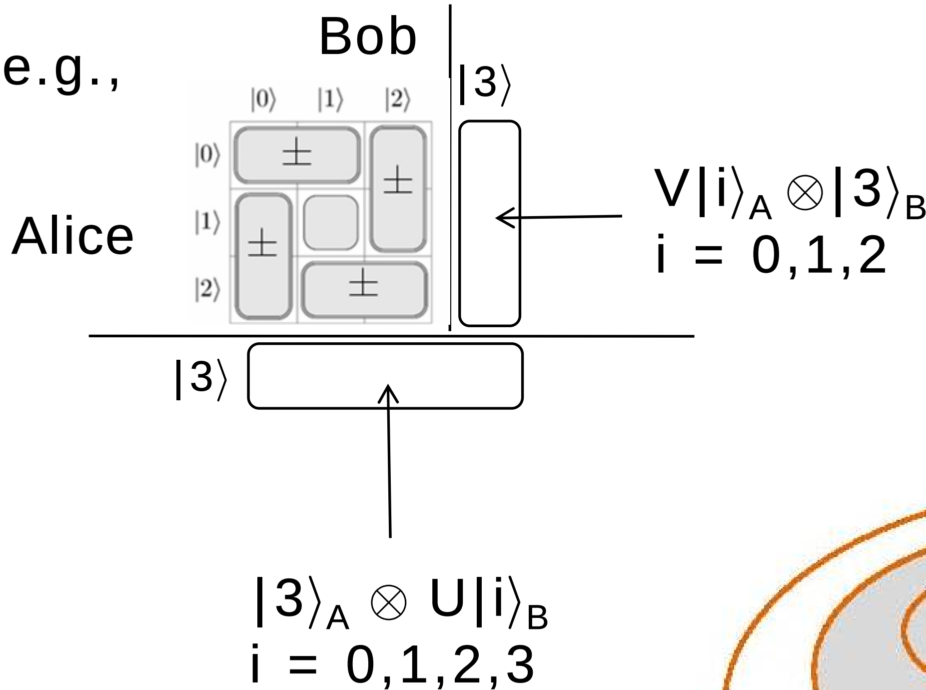

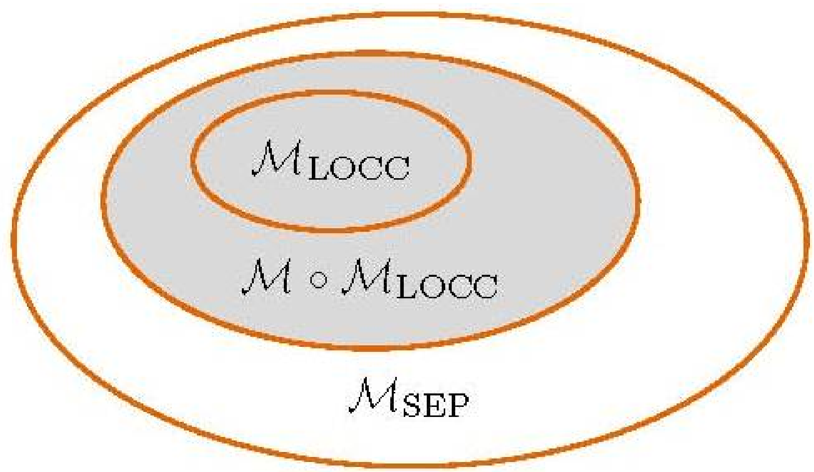

- admits ε-interpolation in SEP for small $\varepsilon > 0$.

Recent development: Chitambar-Hsieh 13, Chitamber-Duan-Hsieh 13

- LOCC discrimination of the double trine states does not achieve global optimal discrimination (both in the min error setting and the unambiguous discrimination setting)

- Example of two 2-qubit states, mutually orthogonal and both separable, with ranks 1 and 2 respectively, that cannot be perfectly discriminated by LOCC.

- Almost all sets of 3 pure states on 2-qubit cannot be discriminated optimally in LOCC (in min error setting), and some sets consist of 3 pure product states.

...

Thus providing a variety of new examples for nonlocality without entanglement.

Recent development: Fu-L-Mancinska 13

- Unextendable product basis (set of mutually orthogonal product states) in two-qutrits cannot be discriminated in cl(LOCC).

Open problems

1. Extend the Kleinmann, Kampermann, and Bruss 11 result to more general measurement, such as all separable projective measurements, or to the discrimination of incomplete orthogonal sets.

2. Other operational difference between LOCC and SEP (e.g, Koashi noted that LOCC measurement is a sequence of refinement of a measurement where Alice and Bob take turn)

3. Investigate the difference between LOCC and SEP for tasks other othan state discrimination (e.g., random state distillation by Cui, Chitambar, Lo)

4. Investigate round complexity in LOCC

5. Better understanding of information gain implies disturbance.