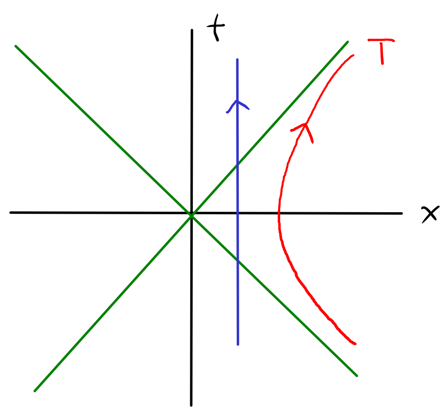
The Unruh Effect in a Cavity

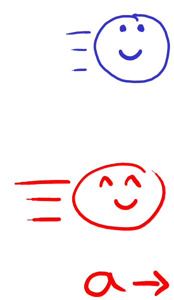
To the Continuum and Beyond

Wilson Brenna
June 18, 2014

The Unruh Effect

$$T = \frac{a}{2\pi}$$





The Unruh Effect

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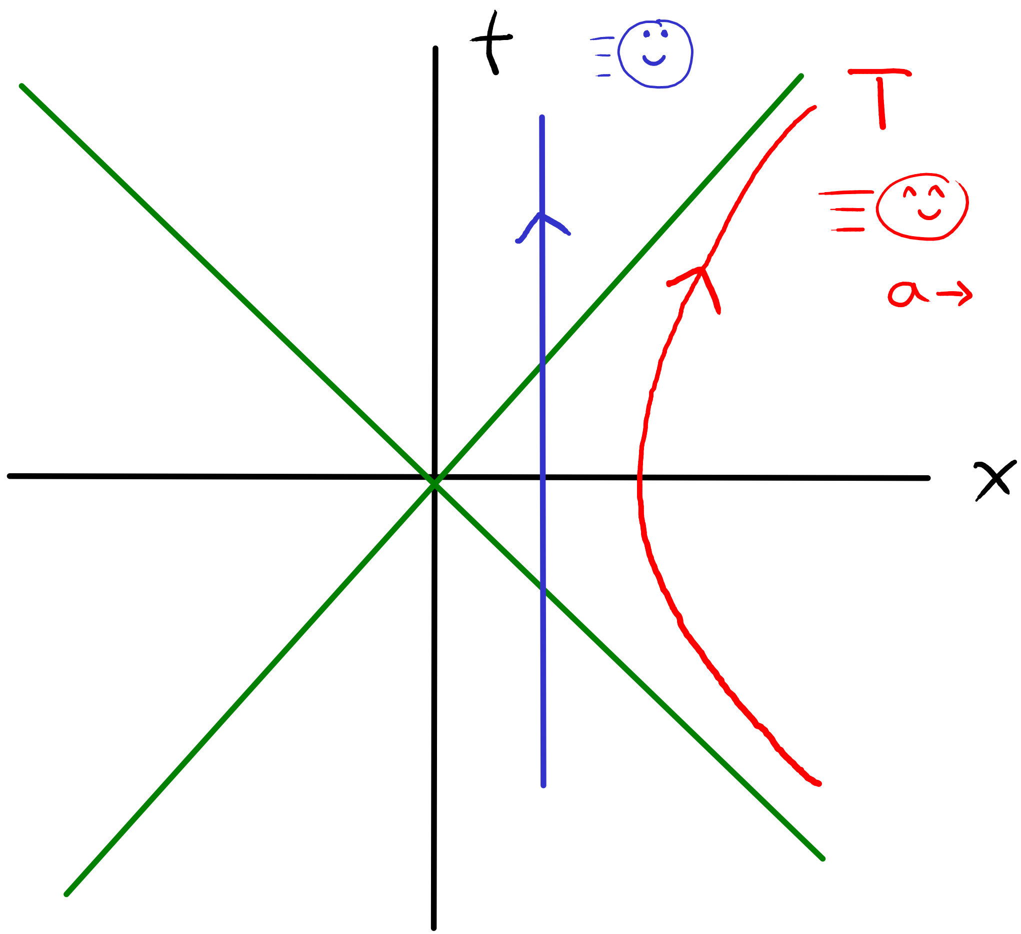
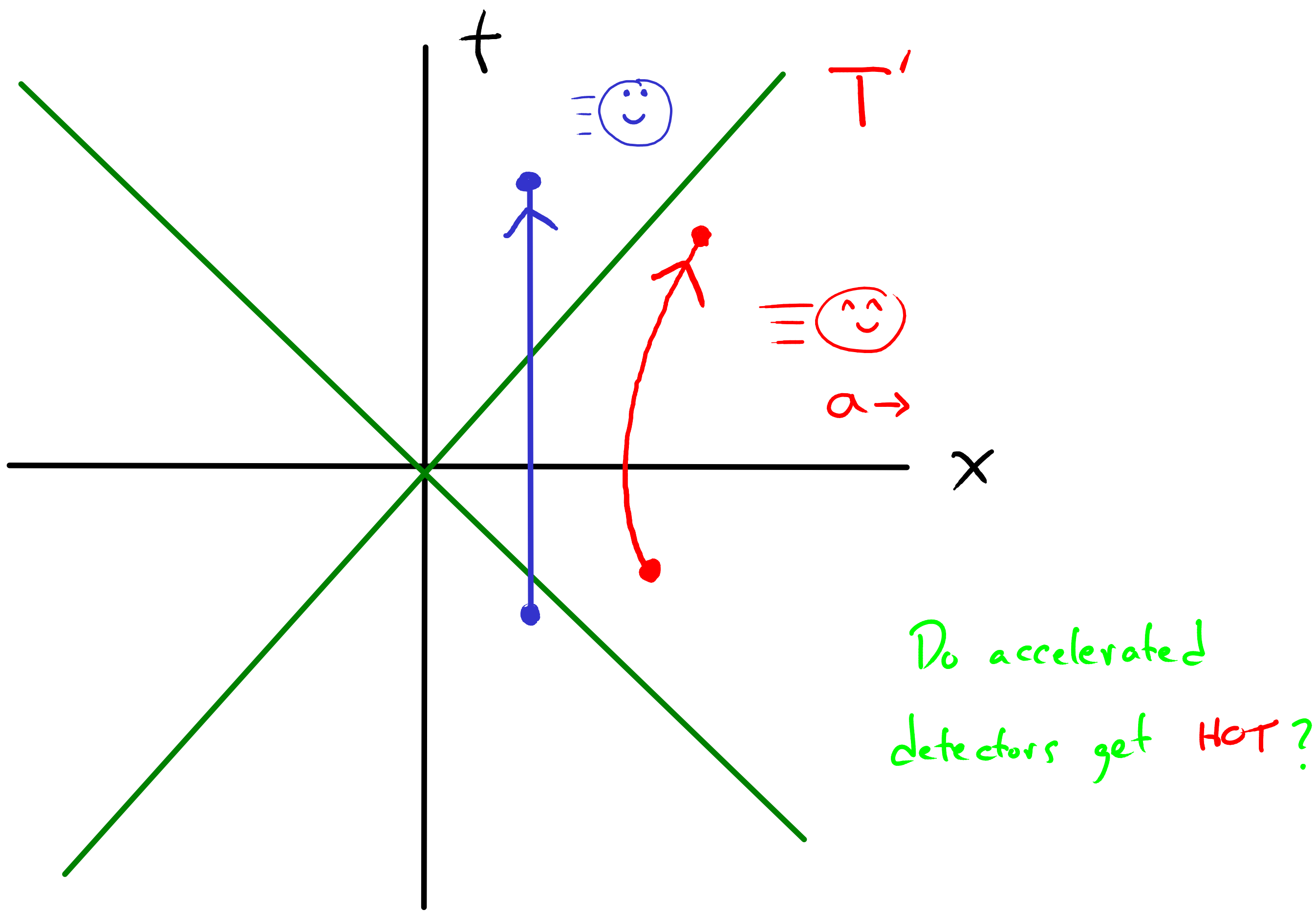


The Unruh Effect

$$T \stackrel{?}{=} \frac{a}{2\pi}$$



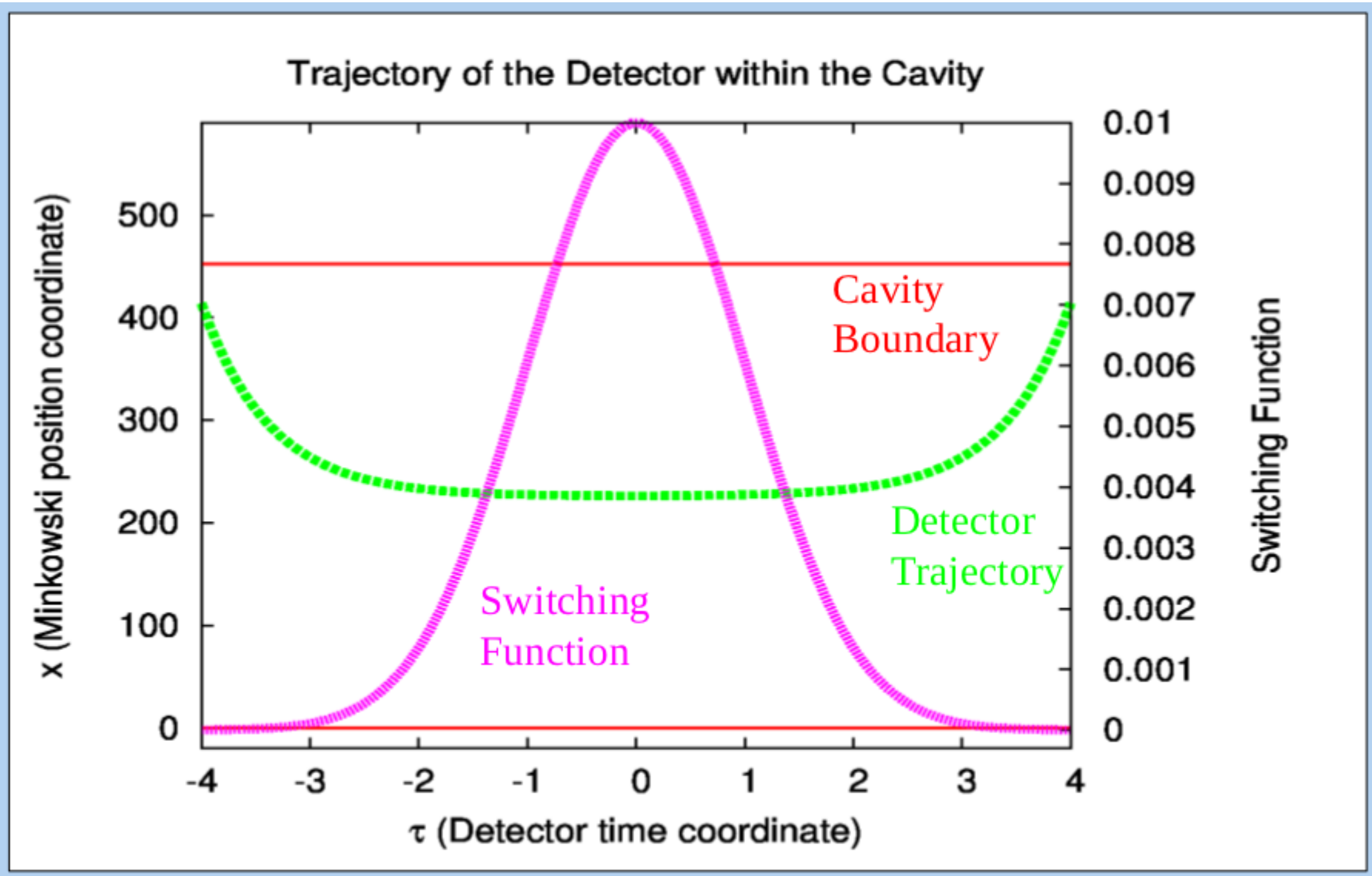
Outline

1. Review: The Unruh-DeWitt Detector in 2D and Gaussian Quantum Detectors

2. Temperature and Acceleration Plots
    a. Perturbative
    b. Non-Perturbative

3. Conclusion

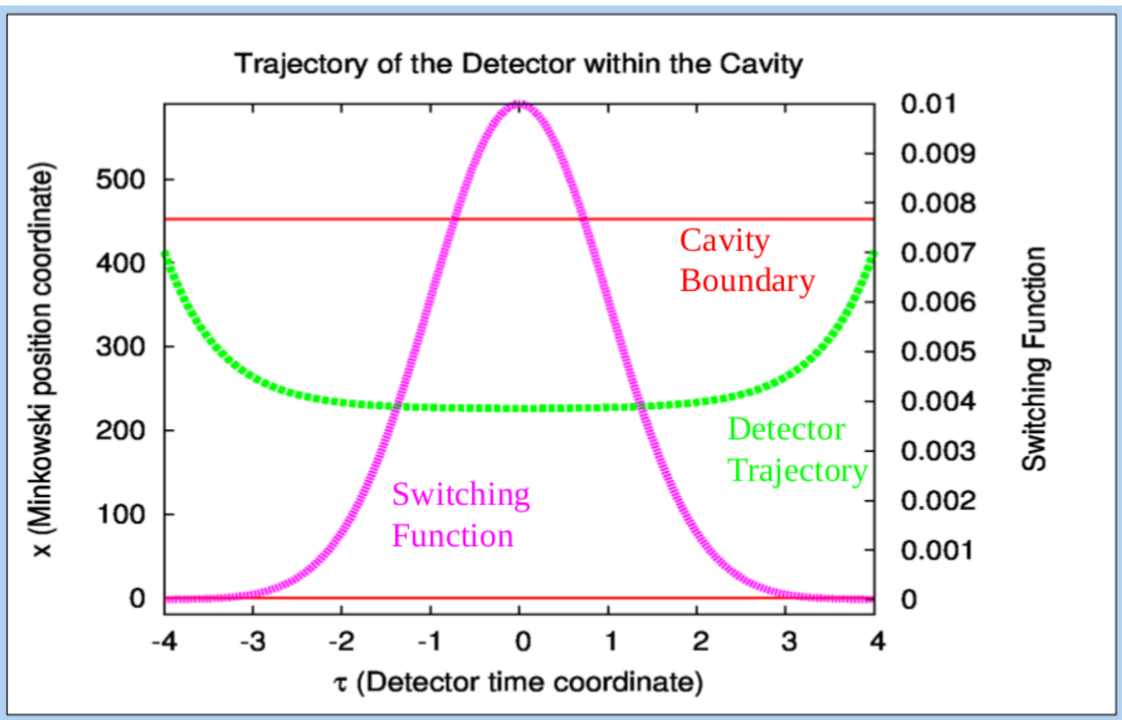
1. Review of UdW and GQI Detectors



1. Review of UdW and GQI Detectors

Notation

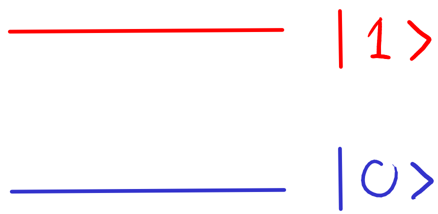
- $\lambda(\tau)$: Switching Function $= \lambda_0 e^{-\tau^2/2\sigma^2}$ ← width of switching σ
- L : length of cavity
- $\Delta\tau$: time of detector's trajectory
- N : number of field modes
- Ω : energy gap of detector



1. Review of UdW and GQI Detectors

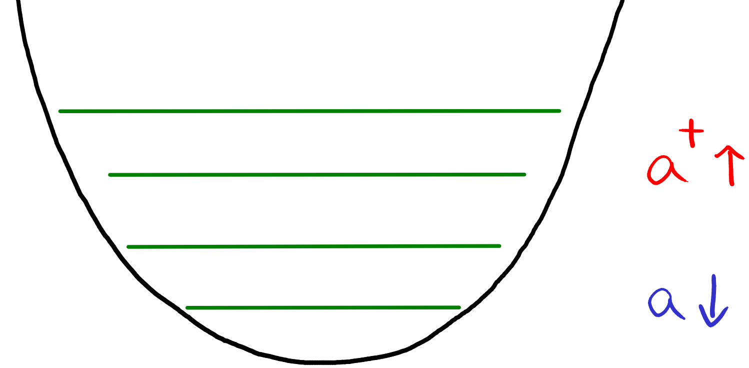
UD (Minkowski)

"Perturbative"



Gaussian

"Non-Perturbative"



$\hat{\rho}$: Density Matrix

σ : Covariance Matrix (State)

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\sigma = \begin{bmatrix} \langle x^2 \rangle & \langle xp + px \rangle \\ \langle xp + px \rangle & \langle p^2 \rangle \end{bmatrix}$$

1. Review of UdW and GQI Detectors

$$\dot{\mathcal{F}}_{\Delta t}(\Omega) = \int_{-\tau+\tau_0}^{\tau-\tau_0} ds \cdot e^{-i\Omega s} \frac{1}{2\pi} \int \frac{dk}{2\omega} e^{-i\omega t(s)+ik(x(s)-1)}$$

$$\dot{\mathcal{F}}_{\infty}(\Omega) = \int_{-\infty}^{\infty} ds \cdot e^{-i\Omega s - \frac{|s|}{\kappa}} \frac{1}{2\pi} \int \frac{dk}{2\omega} e^{-i\omega\alpha \sinh(s/\alpha)+ik(\alpha\cosh(s/\alpha)-1)}$$

After some mathematical tricks, this evaluates to

$$\dot{\mathcal{F}}_{\infty} = \frac{1}{\Omega\left(e^{2\pi\alpha\Omega}-1\right)}$$

~> Boltzmann Dist. at $T = \frac{1}{2\pi\alpha} = \frac{a}{2\pi}$

→ We numerically evaluate the above integrals, after adding UV and IR cutoffs:

$$k = \frac{2\pi n}{L} \quad n = [1..N]$$

1. Review of UdW and GQI Detectors

$$\boldsymbol{\sigma}_d = \begin{pmatrix} \langle \hat{q}_d^2 \rangle & \langle \hat{q}_d \hat{p}_d + \hat{p}_d \hat{q}_d \rangle \\ \langle \hat{q}_d \hat{p}_d + \hat{p}_d \hat{q}_d \rangle & \langle \hat{p}_d^2 \rangle \end{pmatrix}$$

$$\hat{H} = \Omega_d \hat{a}_d^\dagger \hat{a}_d + \frac{dt}{d\tau} \sum_n \omega_n \hat{a}_n^\dagger \hat{a}_n + \hat{H}_I(\tau)$$

$$\hat{H}_I = \lambda(\tau) \cdot \hat{m} \cdot \hat{\phi}[x(\tau)]$$

$$\hat{H} = \hat{\boldsymbol{x}}^T \mathbb{F}(\tau) \hat{\boldsymbol{x}}$$

$$\hat{\boldsymbol{x}}^T = (\hat{q}_d, \hat{p}_d, \hat{q}_1, \hat{p}_1, \ldots, \hat{q}_N, \hat{p}_N)$$

States are Gaussian, i.e. they can be squeezed, shifted, thermal, etc.
Quadratic Hamiltonians preserve Gaussianity.

Brown, Martín-Martínez, Menicucci, Mann
PRD 87, 2013; arXiv:1212.1973

1. Review of UdW and GQI Detectors

- Symplectic form Ω_{ij} from $[\hat{\boldsymbol{a}}_i, \hat{\boldsymbol{a}}_j] = \Omega_{ij} = \begin{pmatrix} 0 & 1 & & & \\ -1 & 0 & 1 & & 0 \\ & -1 & 0 & 1 & \\ 0 & & -1 & 0 & \\ & & & & \ddots \end{pmatrix}$

where $\hat{\boldsymbol{a}} \equiv \left(\hat{a}_d, \hat{a}_d^\dagger, \hat{a}_1, \hat{a}_1^\dagger, \hat{a}_2, \hat{a}_2^\dagger, \ldots, \hat{a}_N^\dagger\right)^T$

- Gaussian states and quadratic Hamiltonian[4]

$$\sigma(t) = S(t)\sigma_0 S(t)^T \quad \text{(Unitary Evolution)}$$

$$\frac{d}{dt}S(t) = \Omega F^{sym}(t) \cdot S(t)$$

F^{sym} : symmetrization of the Hamiltonian in matrix form, $\mathbb{F}(t)$

Numerically evolve the first order DE from initial conditions!

1. Review of UdW and GQI Detectors

Detector Covariance Matrix

$$\sigma_d = \begin{pmatrix} (1+\delta)e^{-r} & 0 \\ 0 & (1+\delta)e^{r} \end{pmatrix}$$

Unruh Effect

$$T = \Omega \left[\ln\left(1 + \frac{2}{\delta}\right) \right]^{-1}$$

$$T \sim a$$

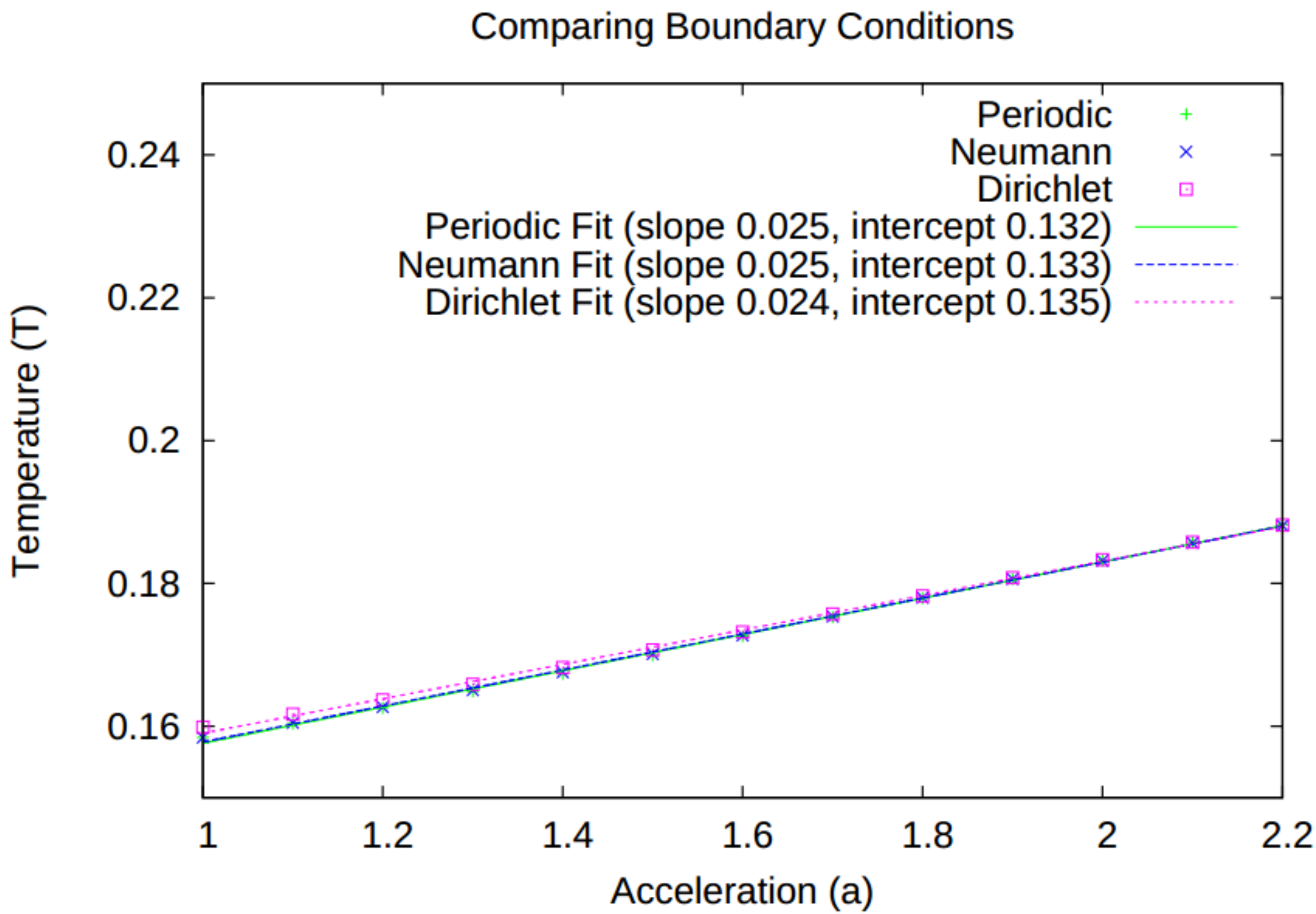
- linearity
- slope

Thermality

$$E \approx \Omega \left(1 + \delta + \frac{1}{2} r^2\right)$$

$$\therefore r^2 << \delta \implies \text{thermality}$$

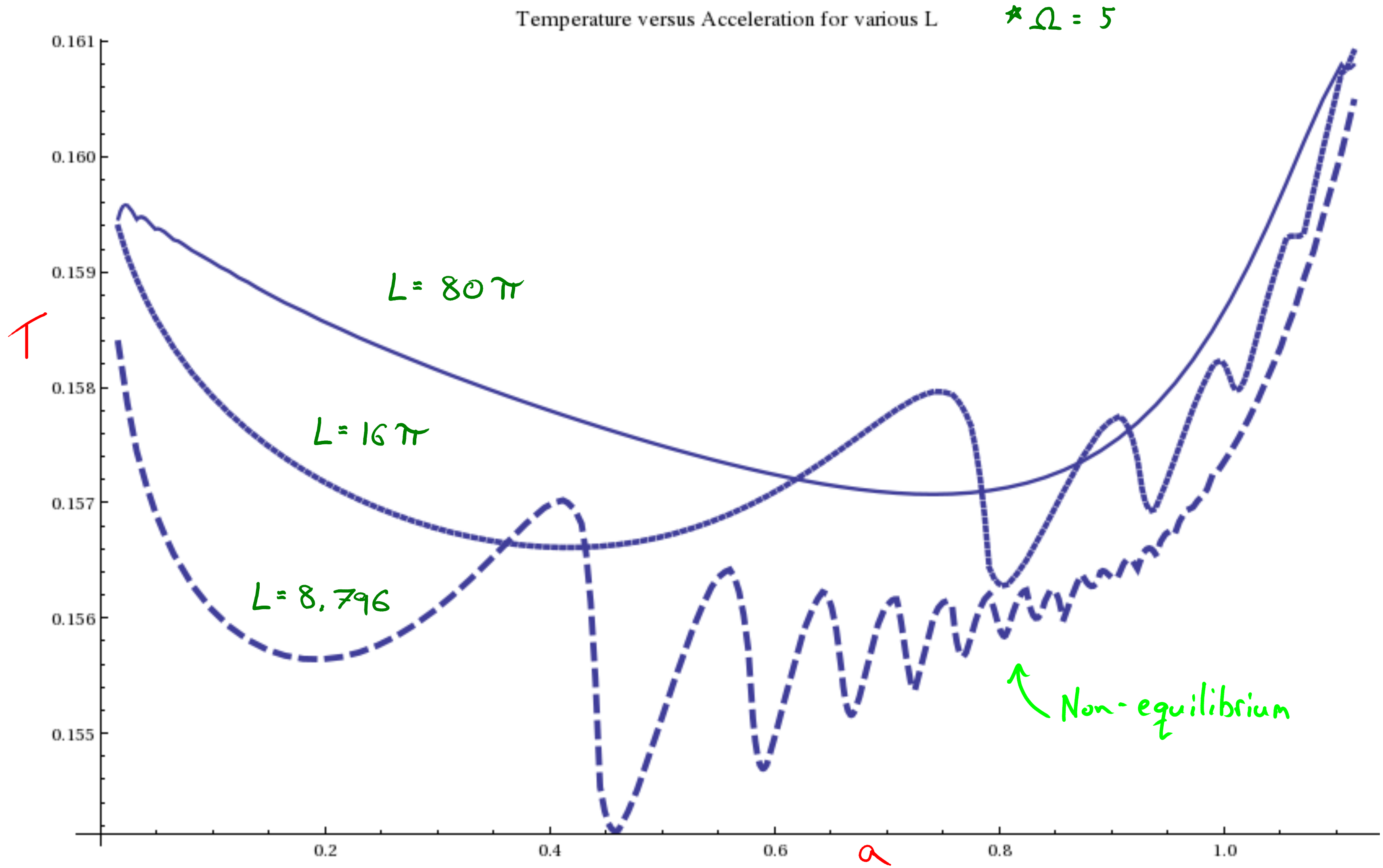
1. Review of UdW and GQI Detectors



WGB, E Brown, RB Mann, E M-Martinez
PRD 88 (2013) 064031

2. Temperature and Acceleration Plots

a. Perturbative (Unruh-DeWitt)



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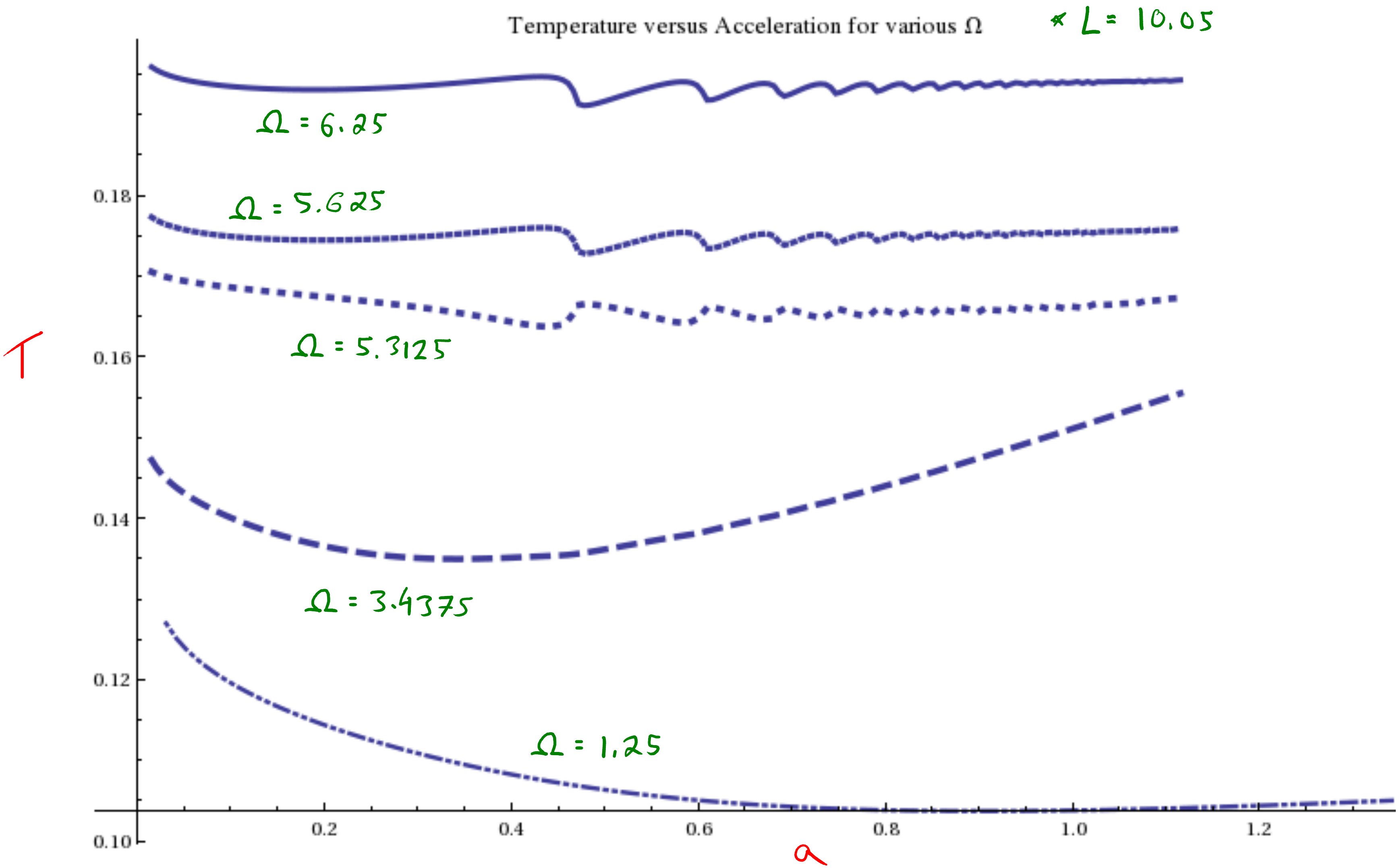

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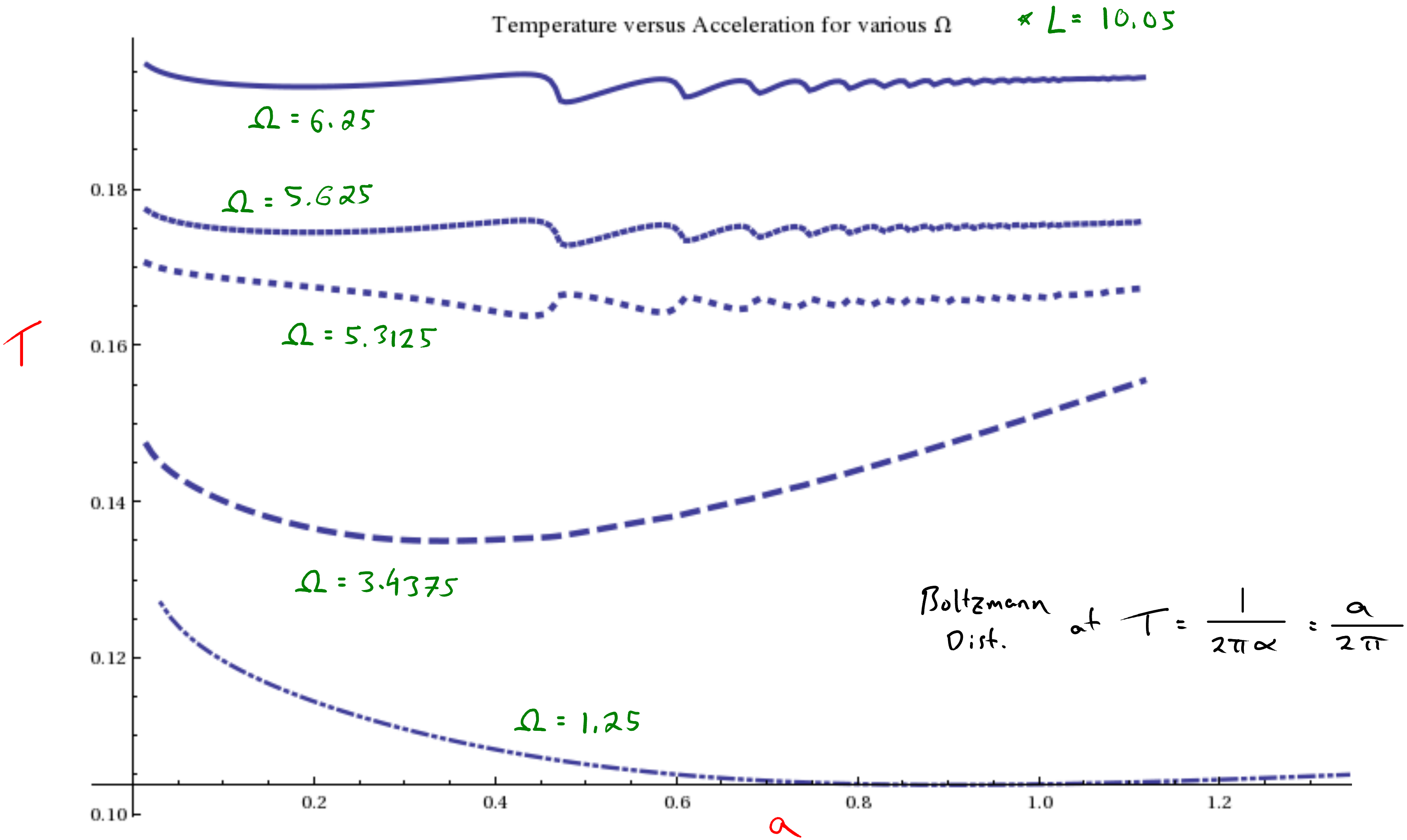

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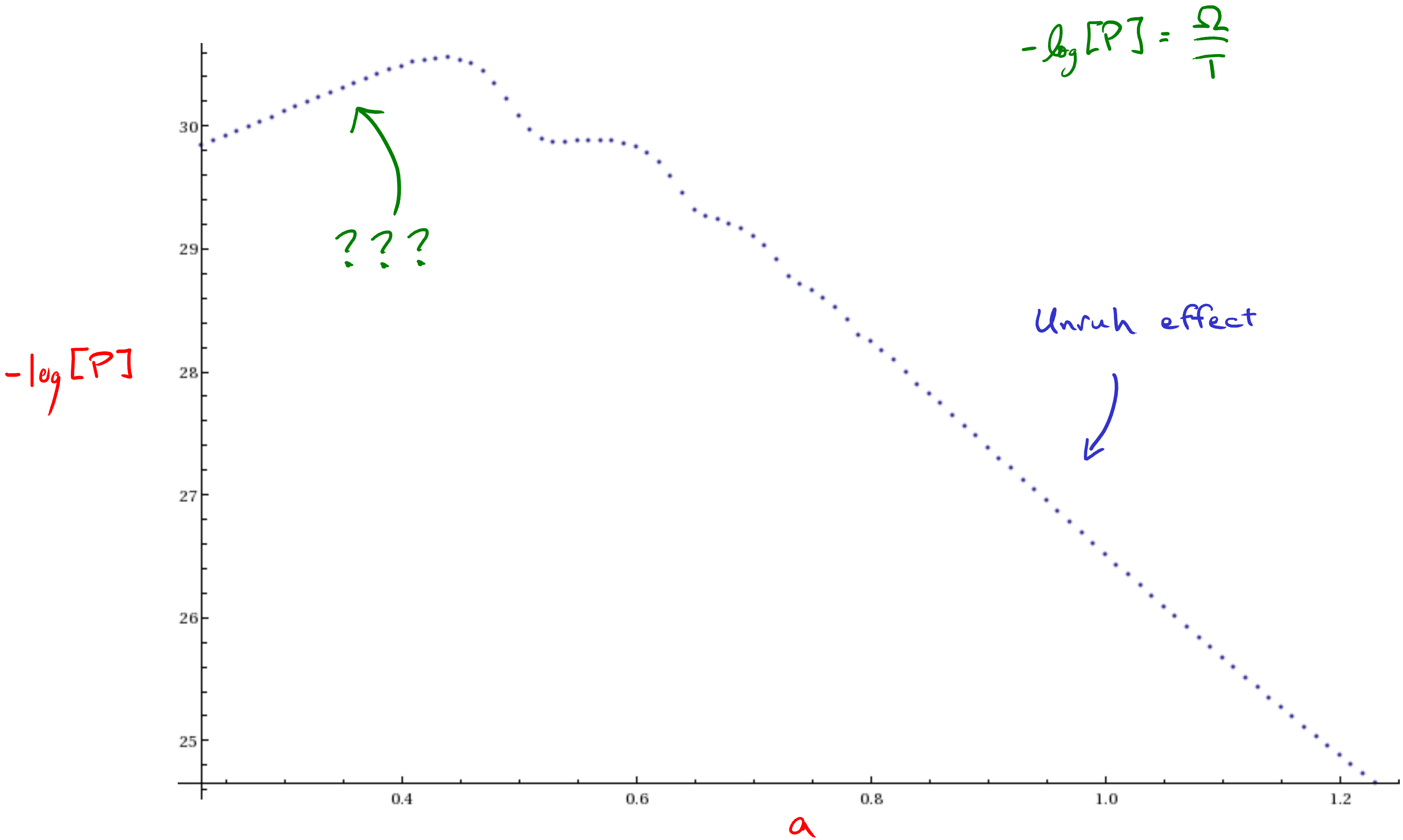

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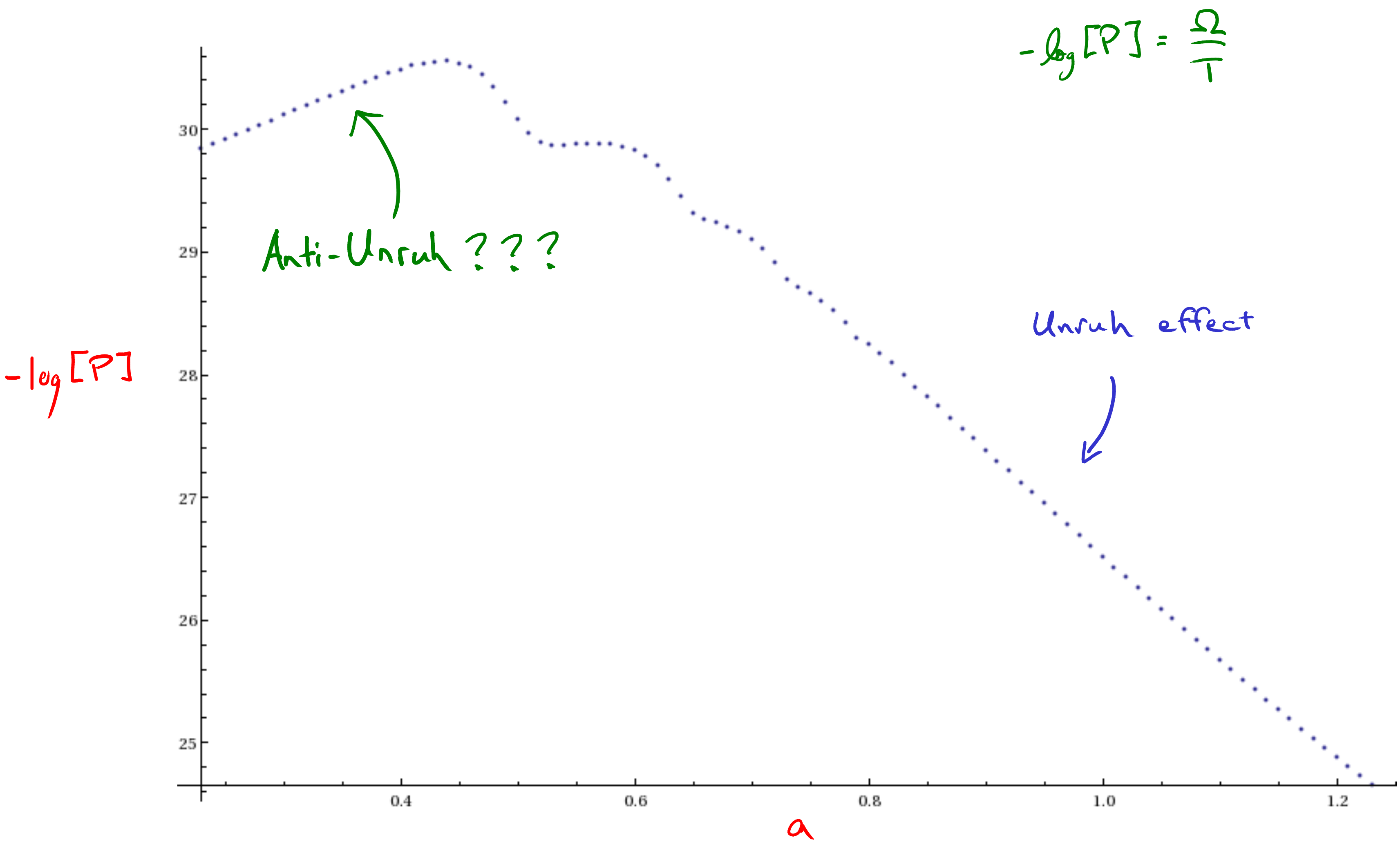
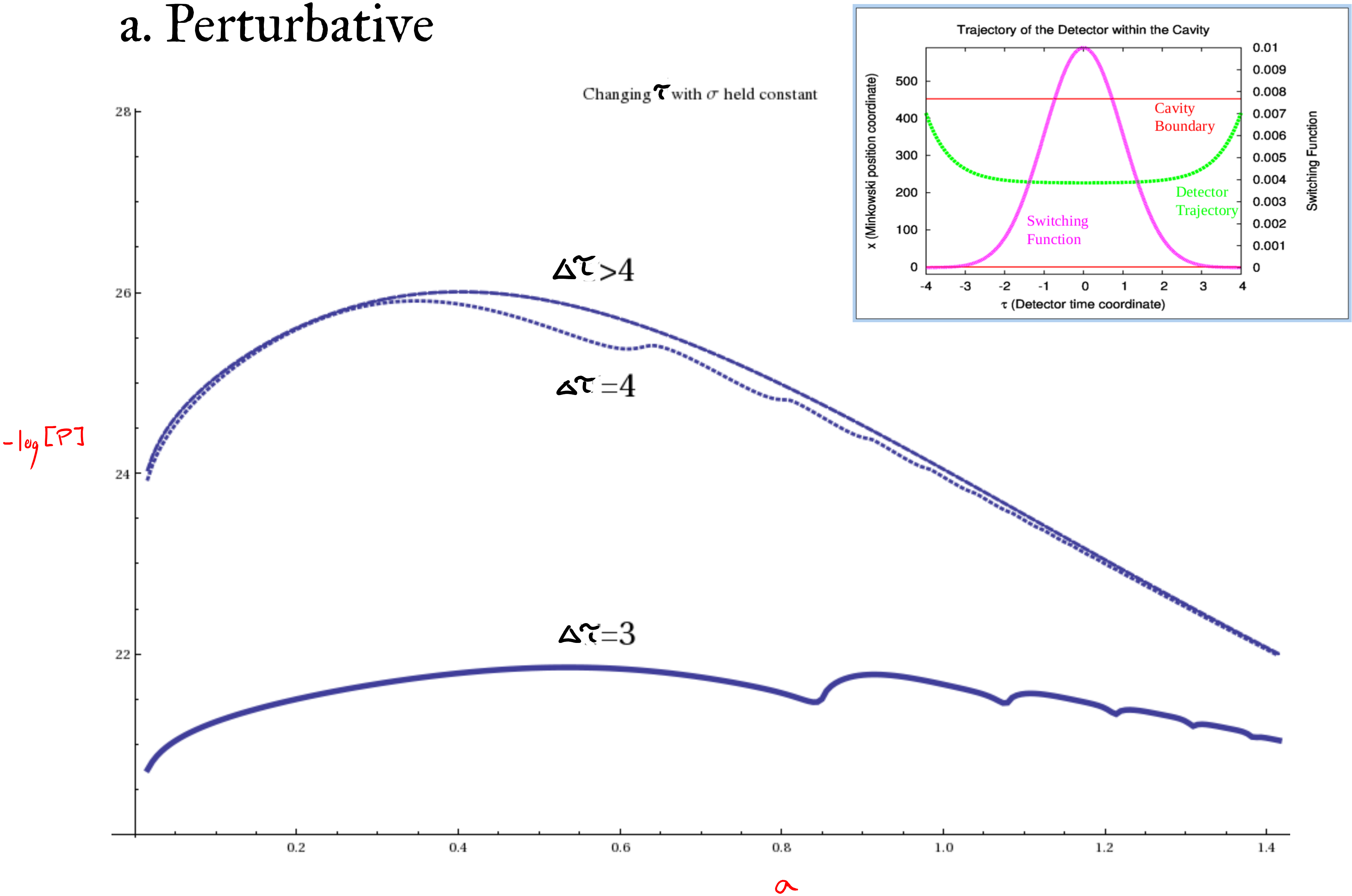


2. Temperature and Acceleration Plots

$L = 4\pi \quad \Omega = 4$

a. Perturbative



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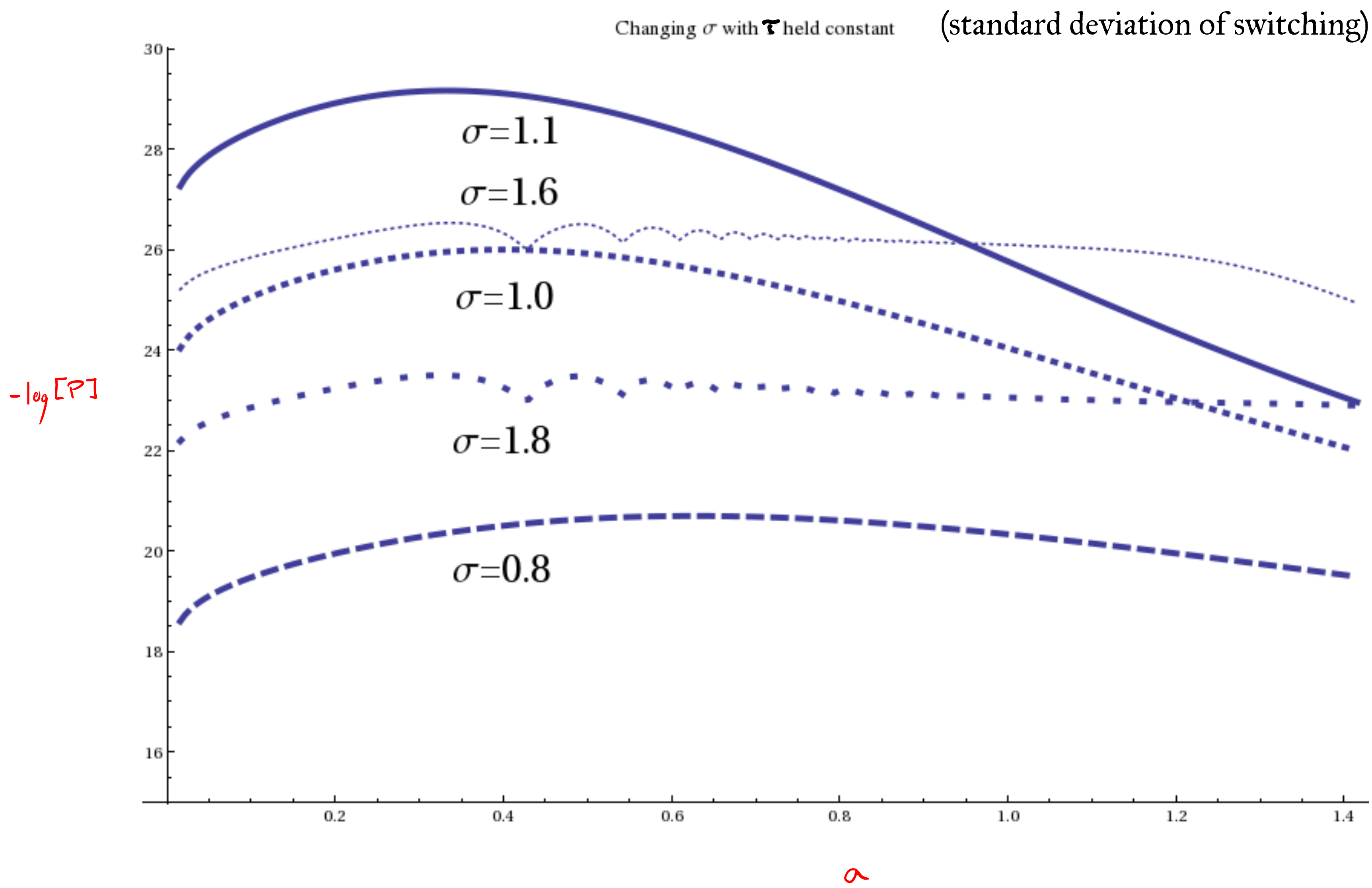
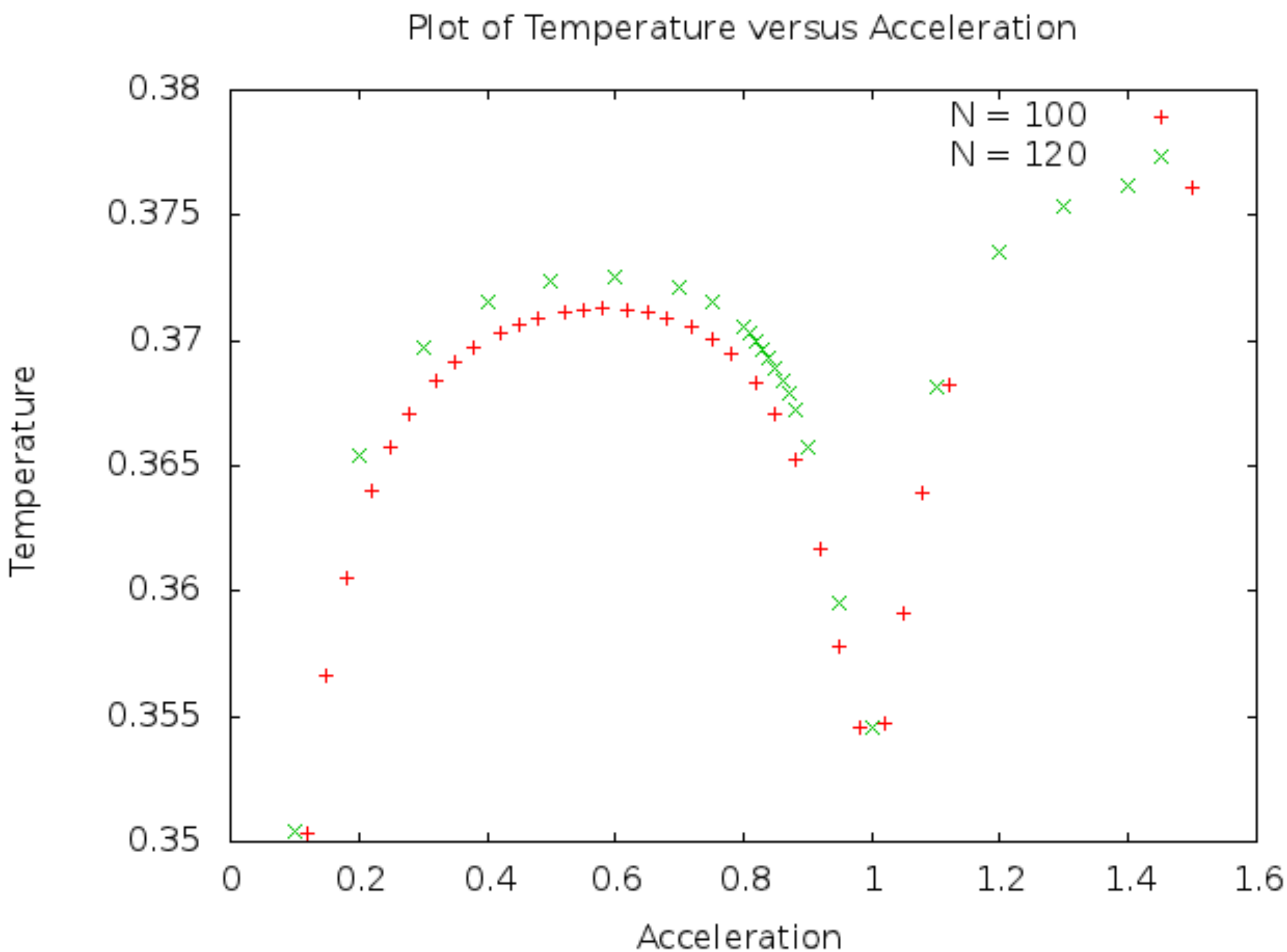


2. Temperature and Acceleration Plots

b. Non-Perturbative (Gaussian Quantum Detector)



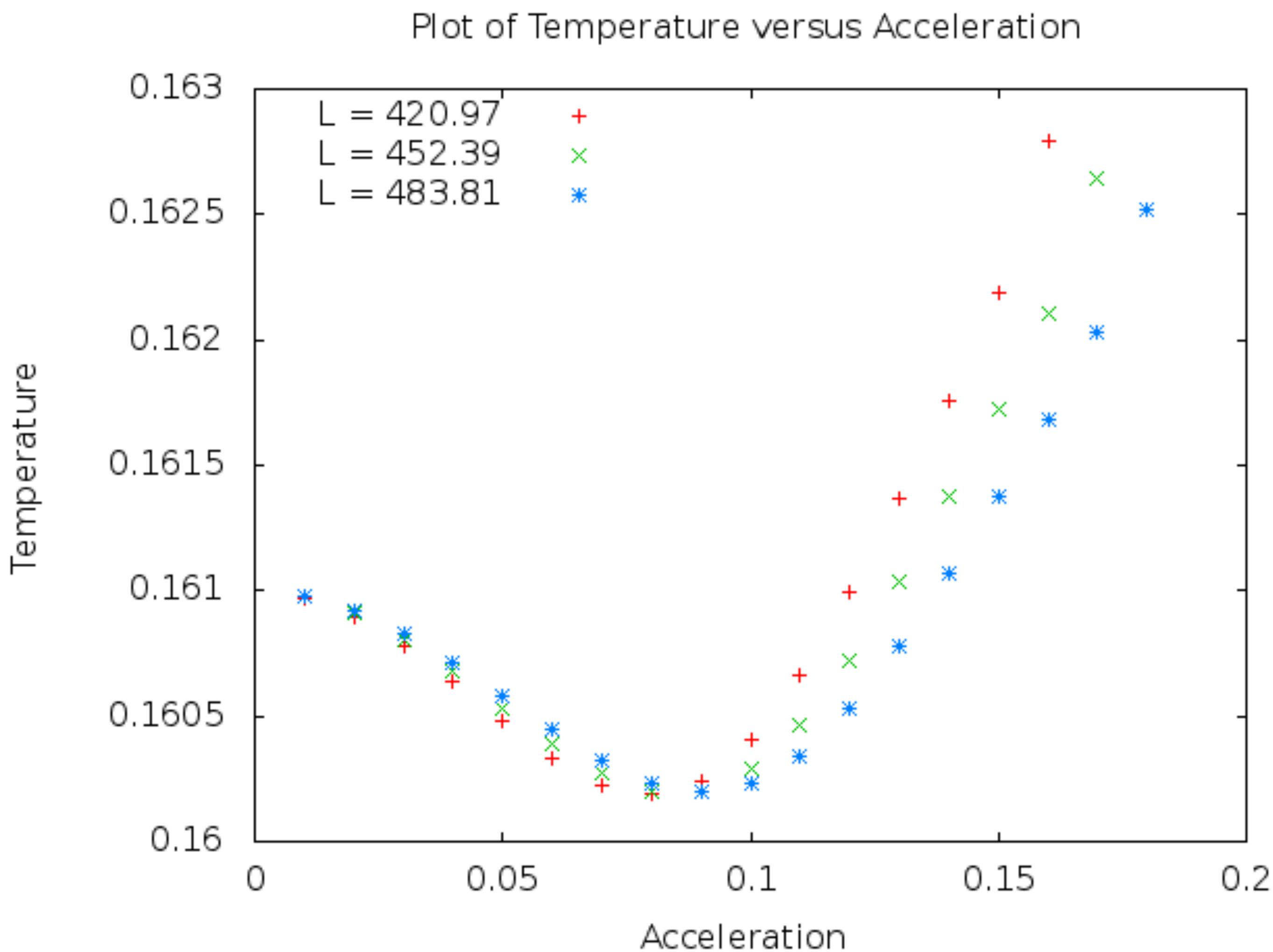
L=12.566

$\Omega = 4$

$\Delta\tau = 10$

2. Temperature and Acceleration Plots

b. Non-Perturbative



N = 200

Ω = 4

Δτ = 8

Conclusion

Taking the cavity to infinity will remain difficult.
Does a detector get hot when you accelerate it?

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Does a detector get hot when you accelerate it?

Not always! Detectors can cool down as we accelerate them.

The negative linear temperature vs acceleration behaviour?

It's a non-equilibrium effect!
Appears to be due to a finitely sized cavity/trajectory!

Conclusion

Taking the cavity to infinity will remain difficult.
Does a detector get hot when you accelerate it?

Not always! Detectors can cool down as we accelerate them.

The negative linear temperature vs acceleration behaviour?

It's a non-equilibrium effect!
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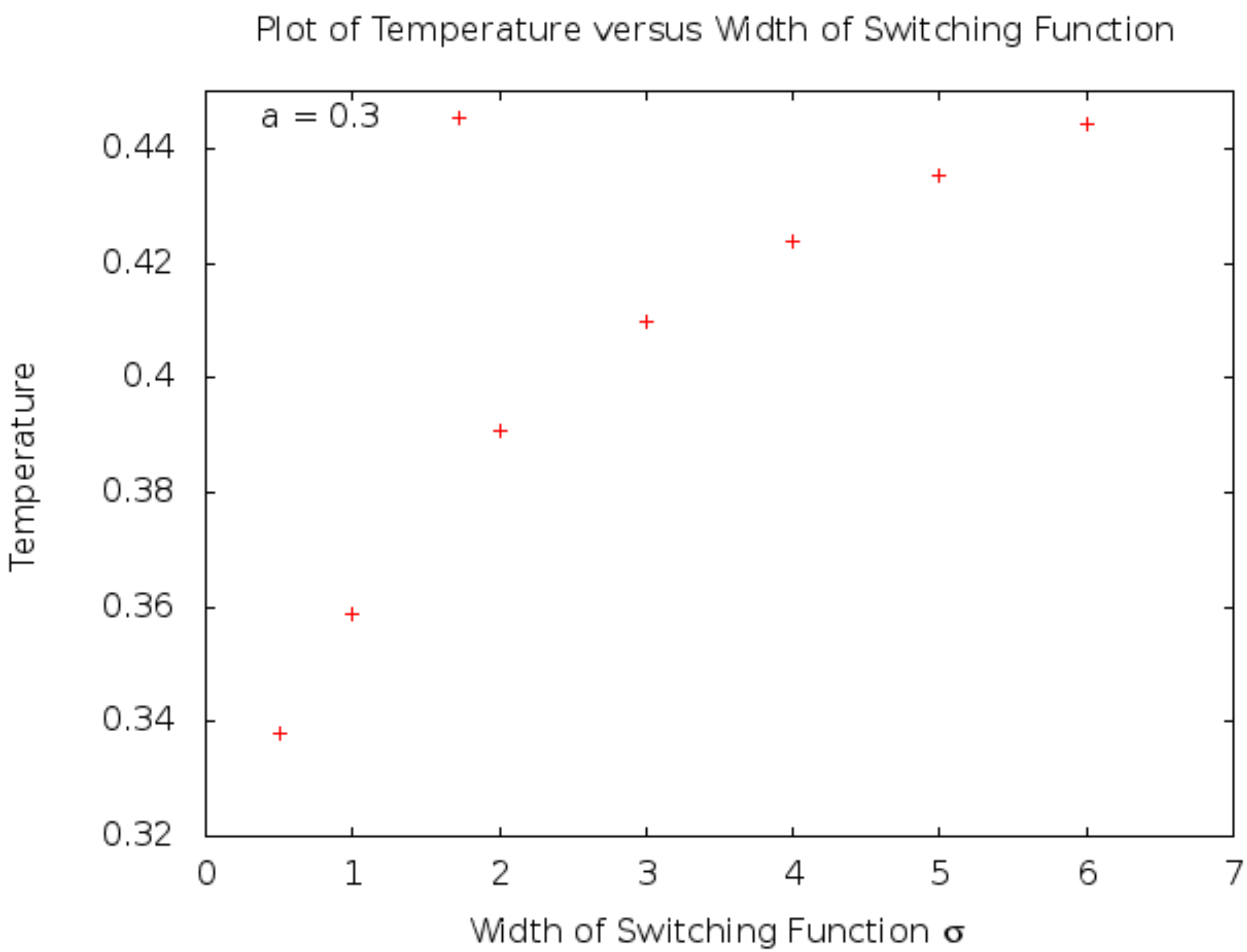
Anti-Unruh...?

Questions?

2. Temperature and Acceleration Plots

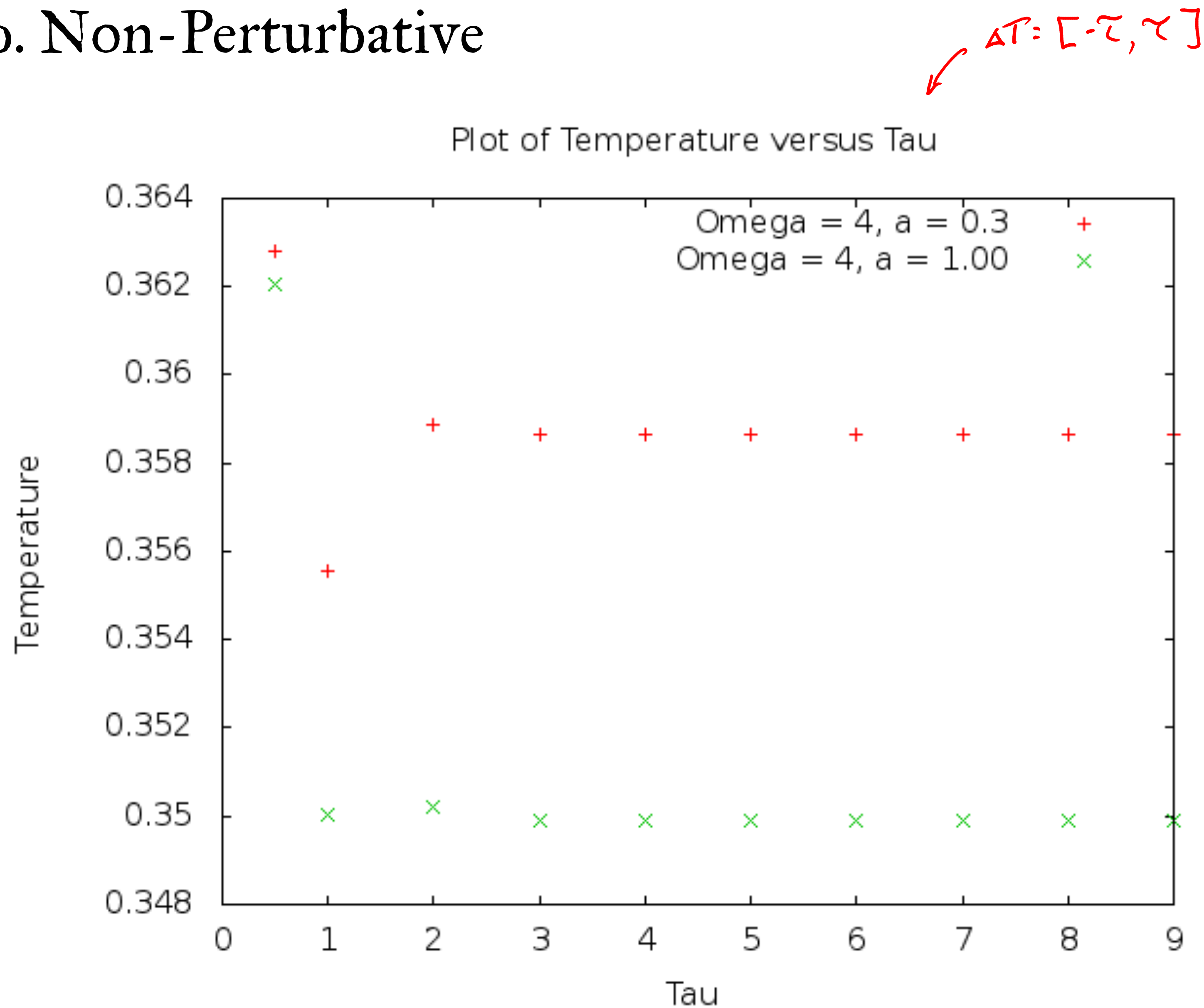
b. Non-Perturbative

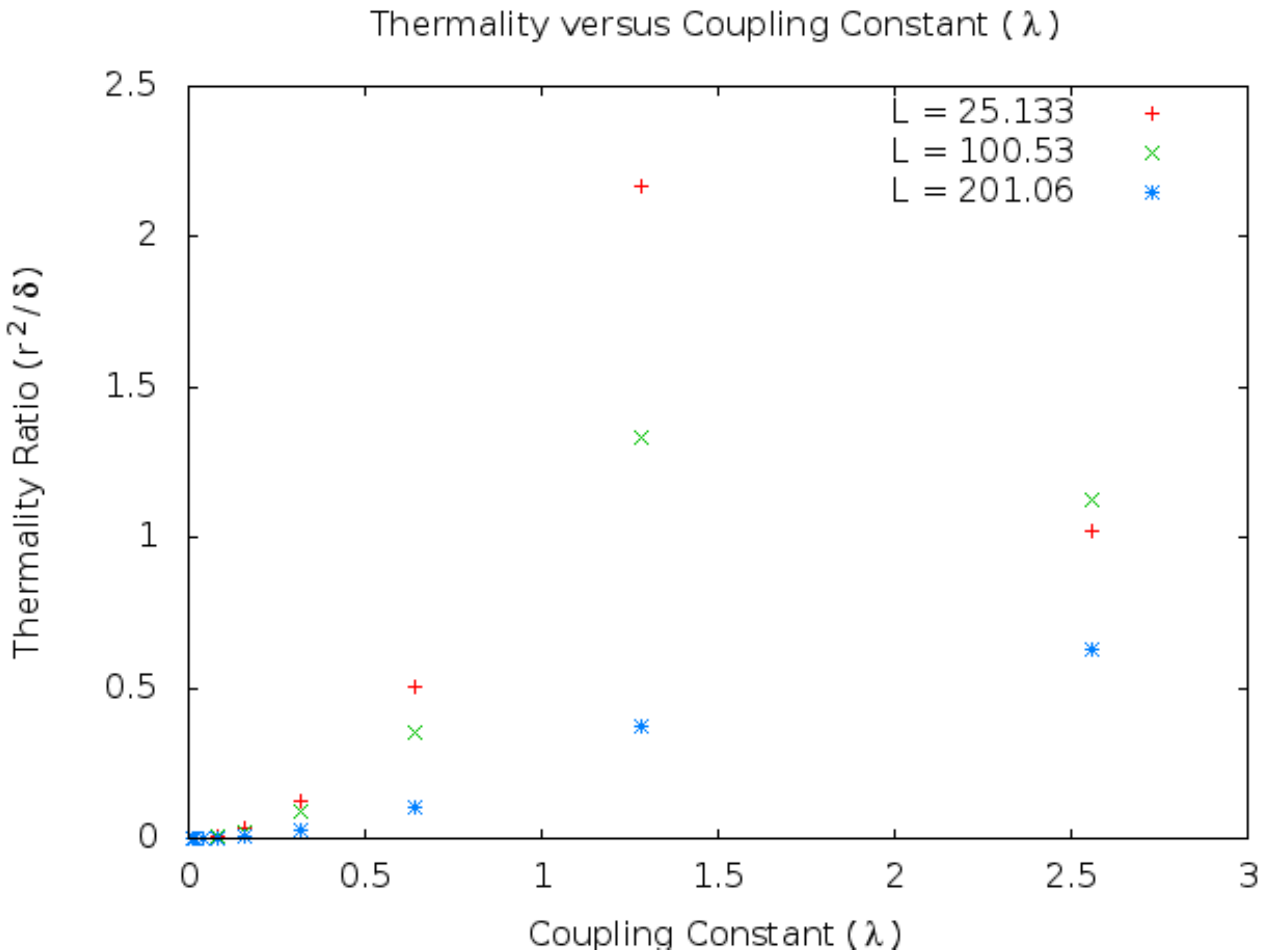
ΔT increases too, to maintain ΔT = 12σ

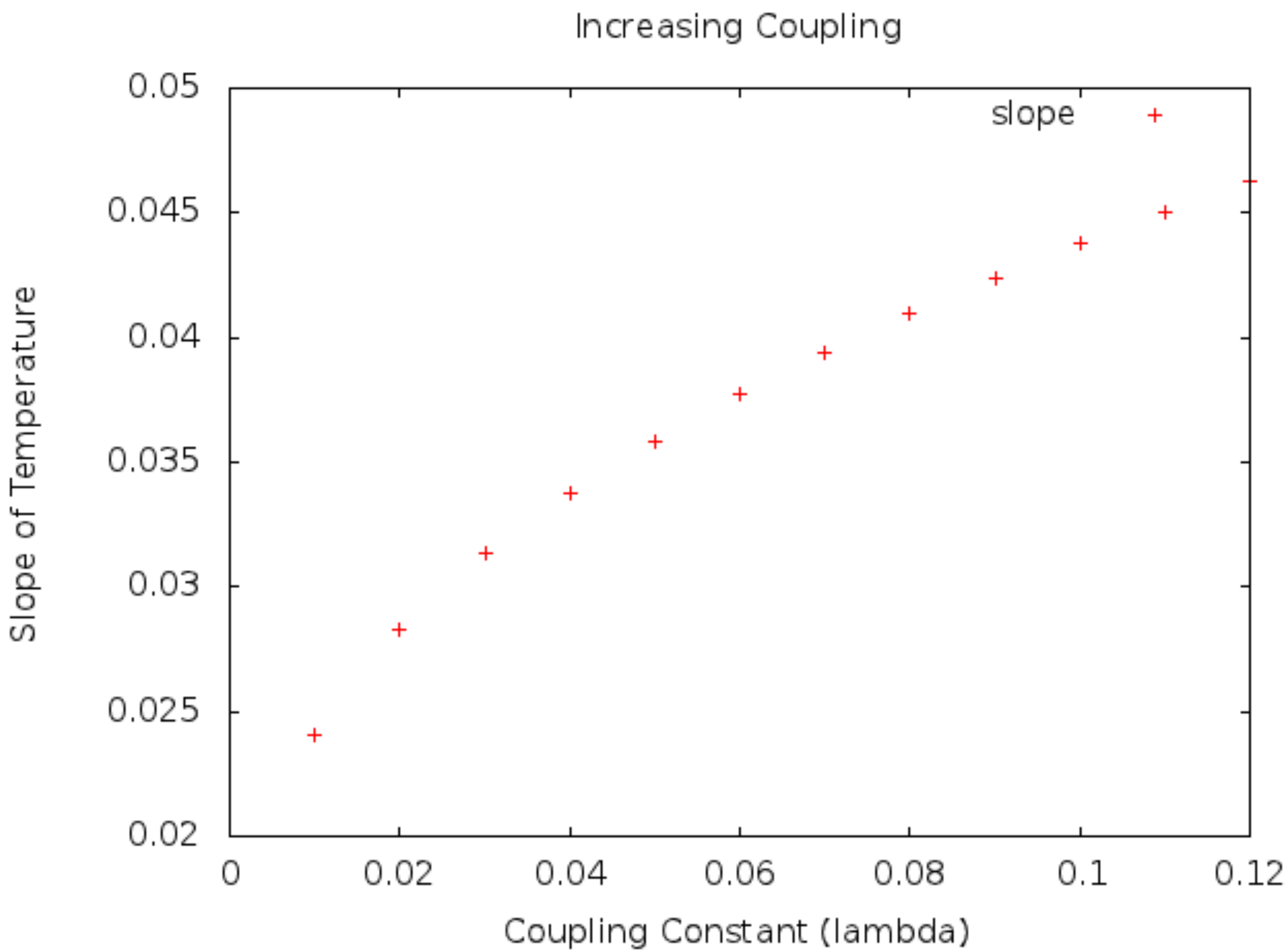


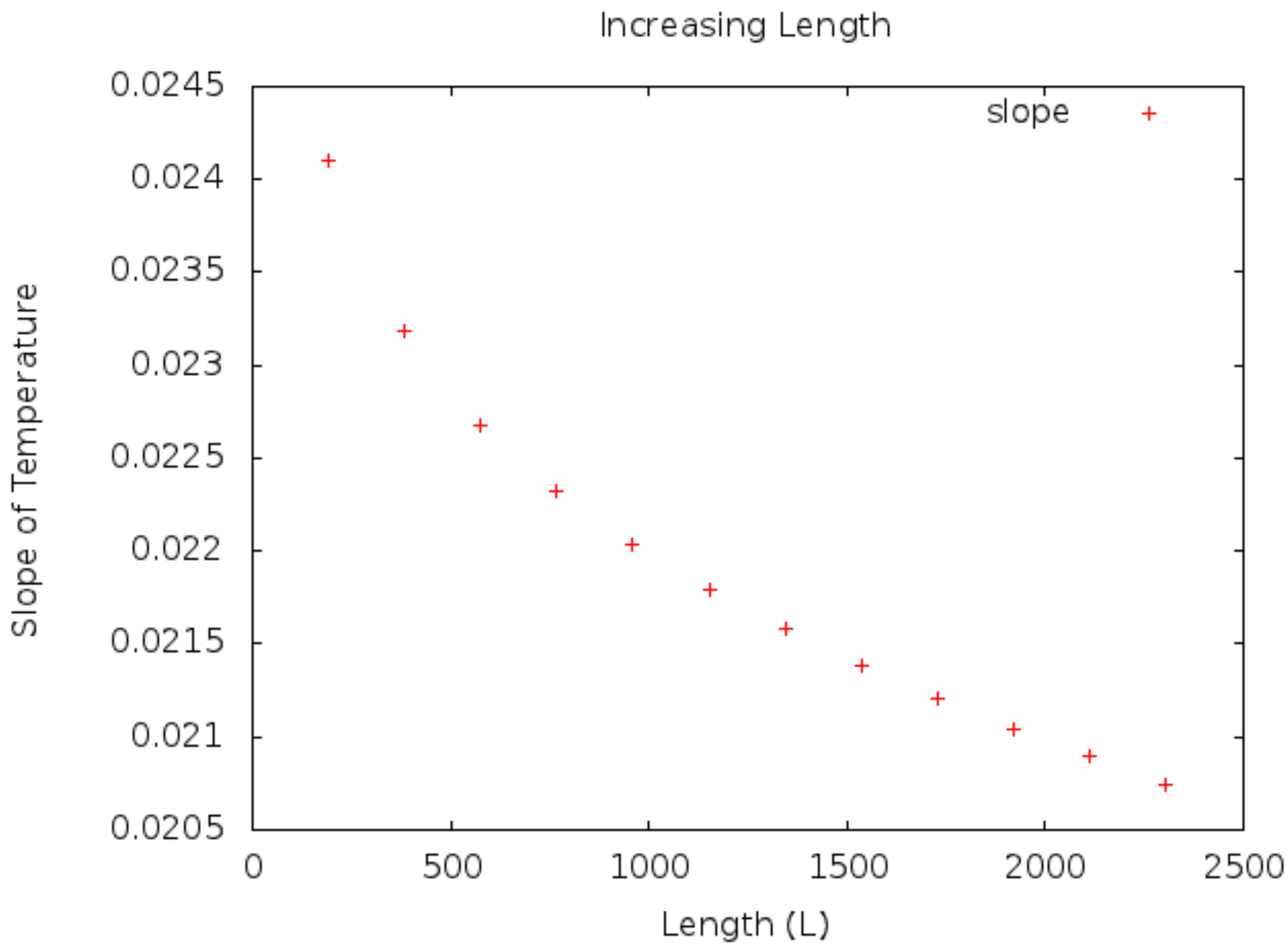
2. Temperature and Acceleration Plots

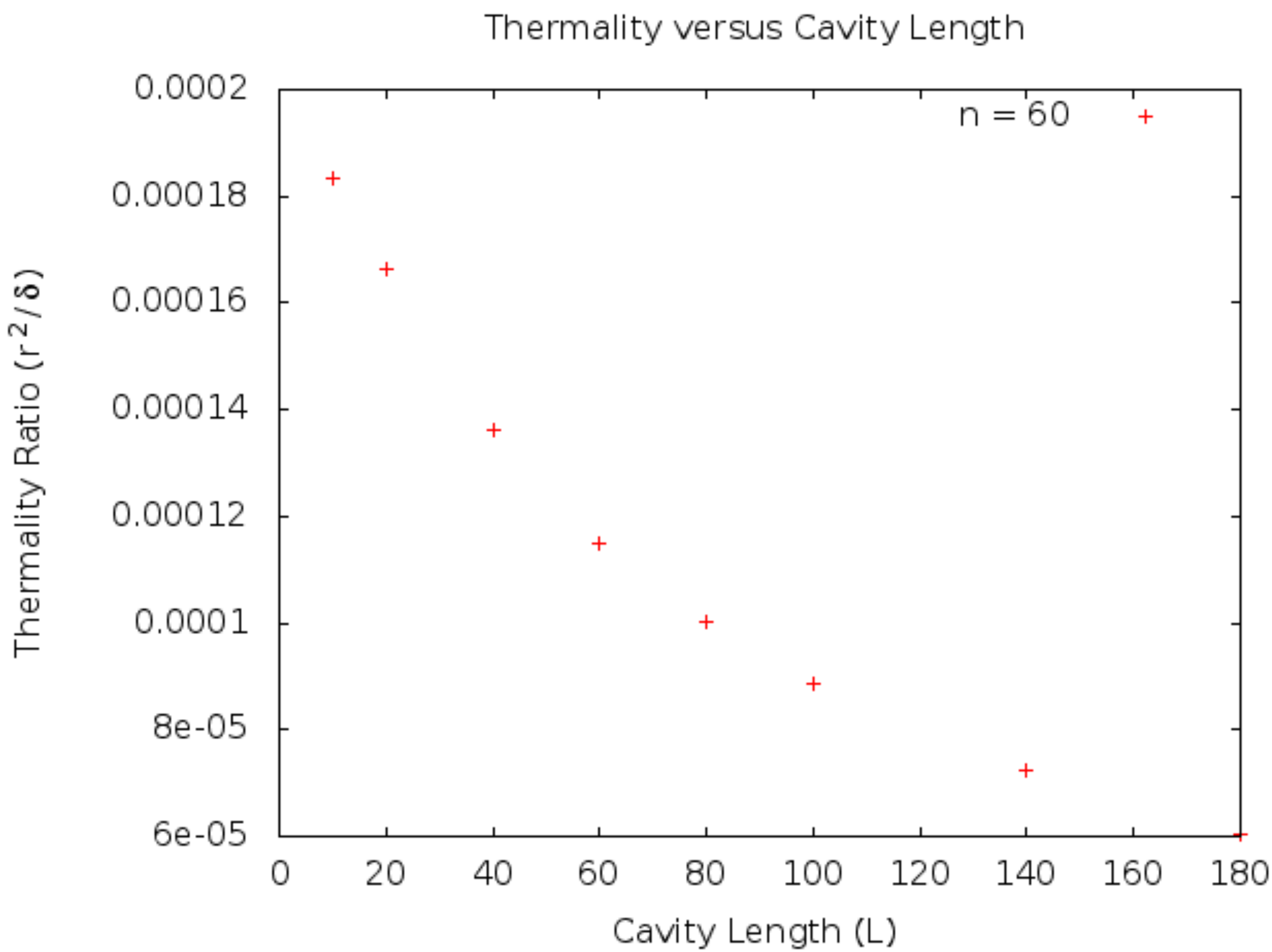
b. Non-Perturbative

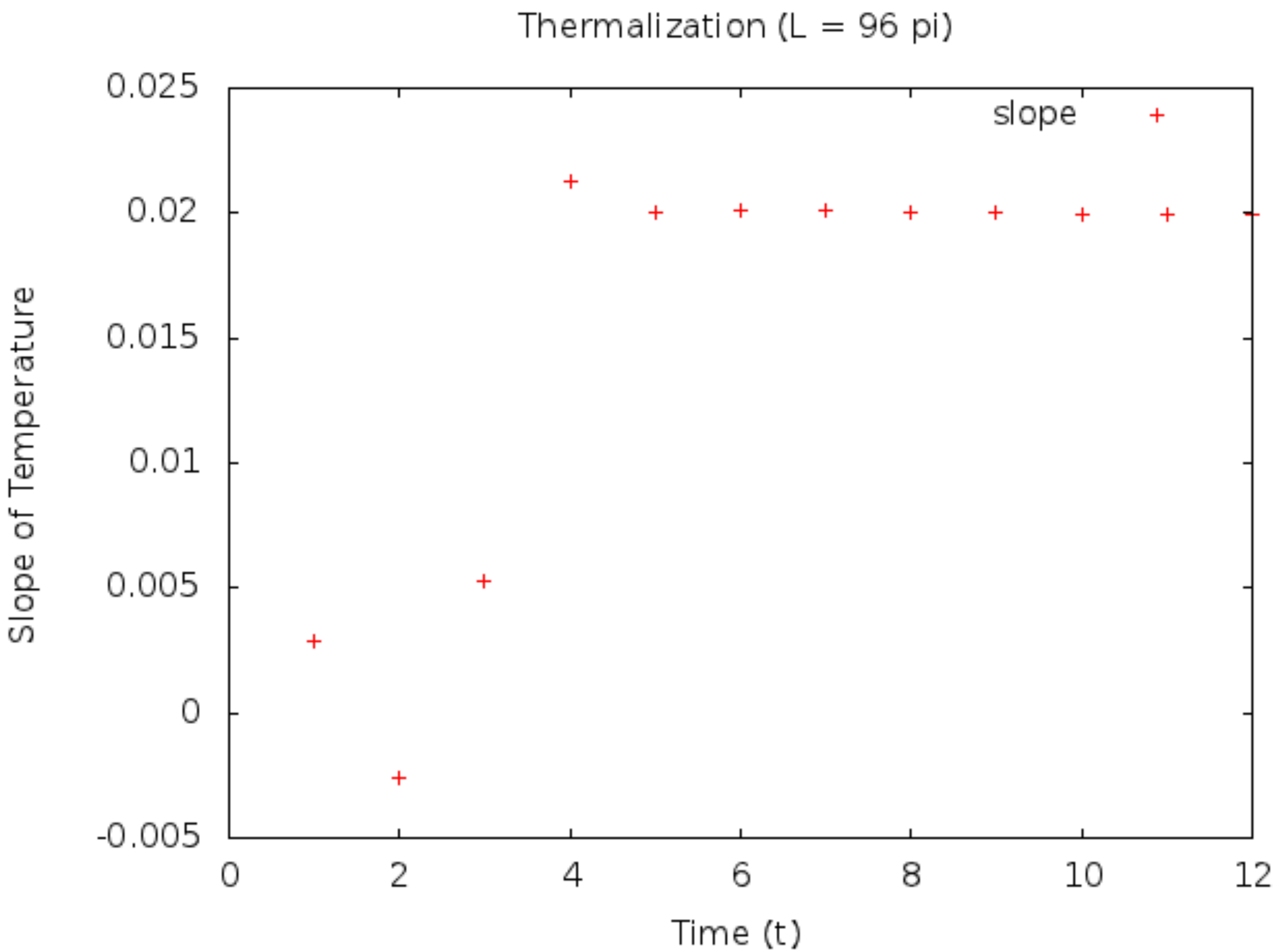



Thermality versus Coupling Constant (λ)
L = 25.133
L = 100.53
L = 201.06
Thermality Ratio (r²/δ)
Coupling Constant (λ)
0
0.5
1
1.5
2
2.5
3

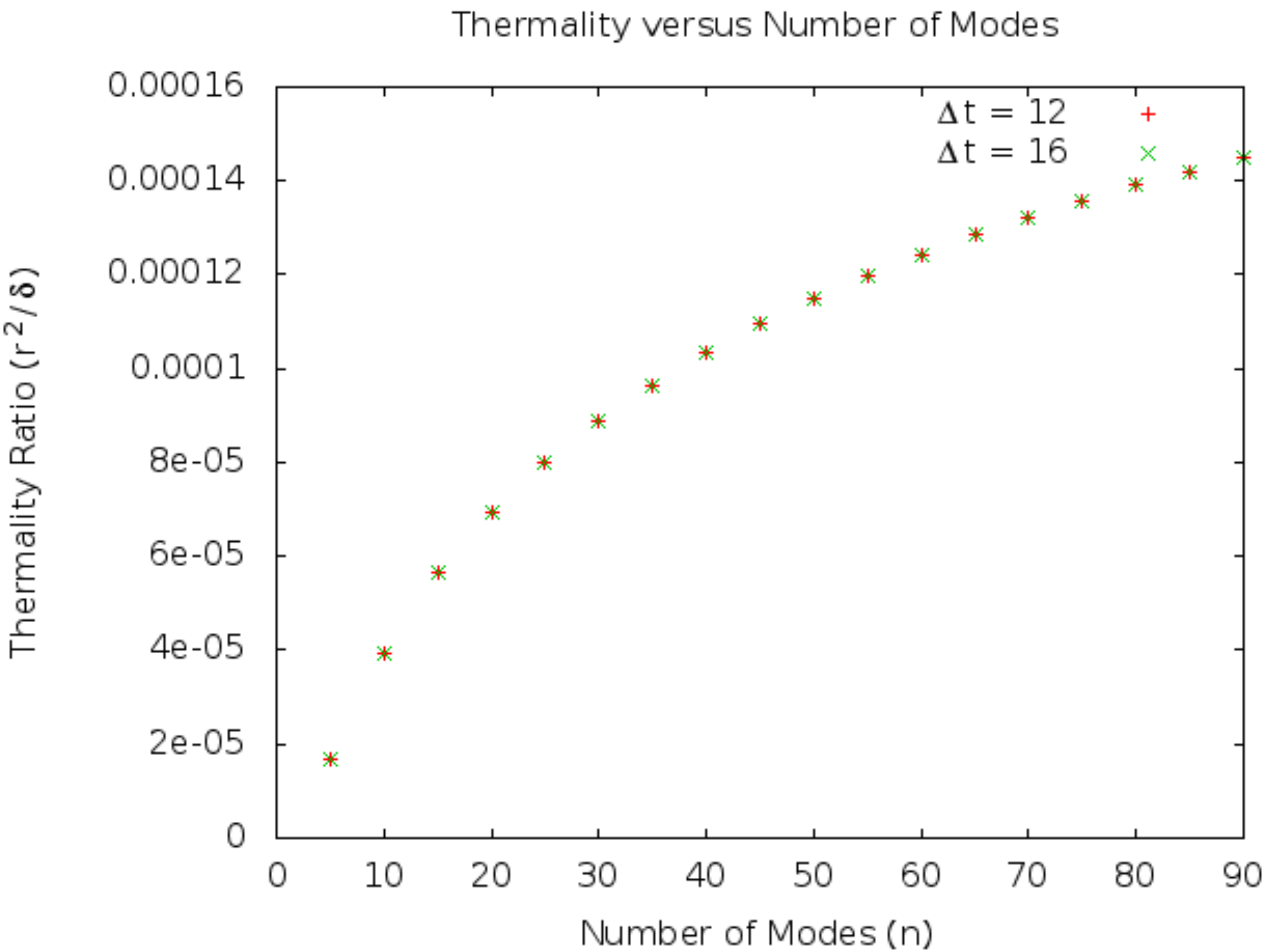

Increasing Coupling
slope
Slope of Temperature
Coupling Constant (lambda)
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0
0.02
0.04
0.06
0.08
0.1
0.12


Increasing Length
slope
0.0245
0.024
0.0235
0.023
0.0225
0.022
0.0215
0.021
0.0205
Slope of Temperature
0
500
1000
1500
2000
2500
Length (L)

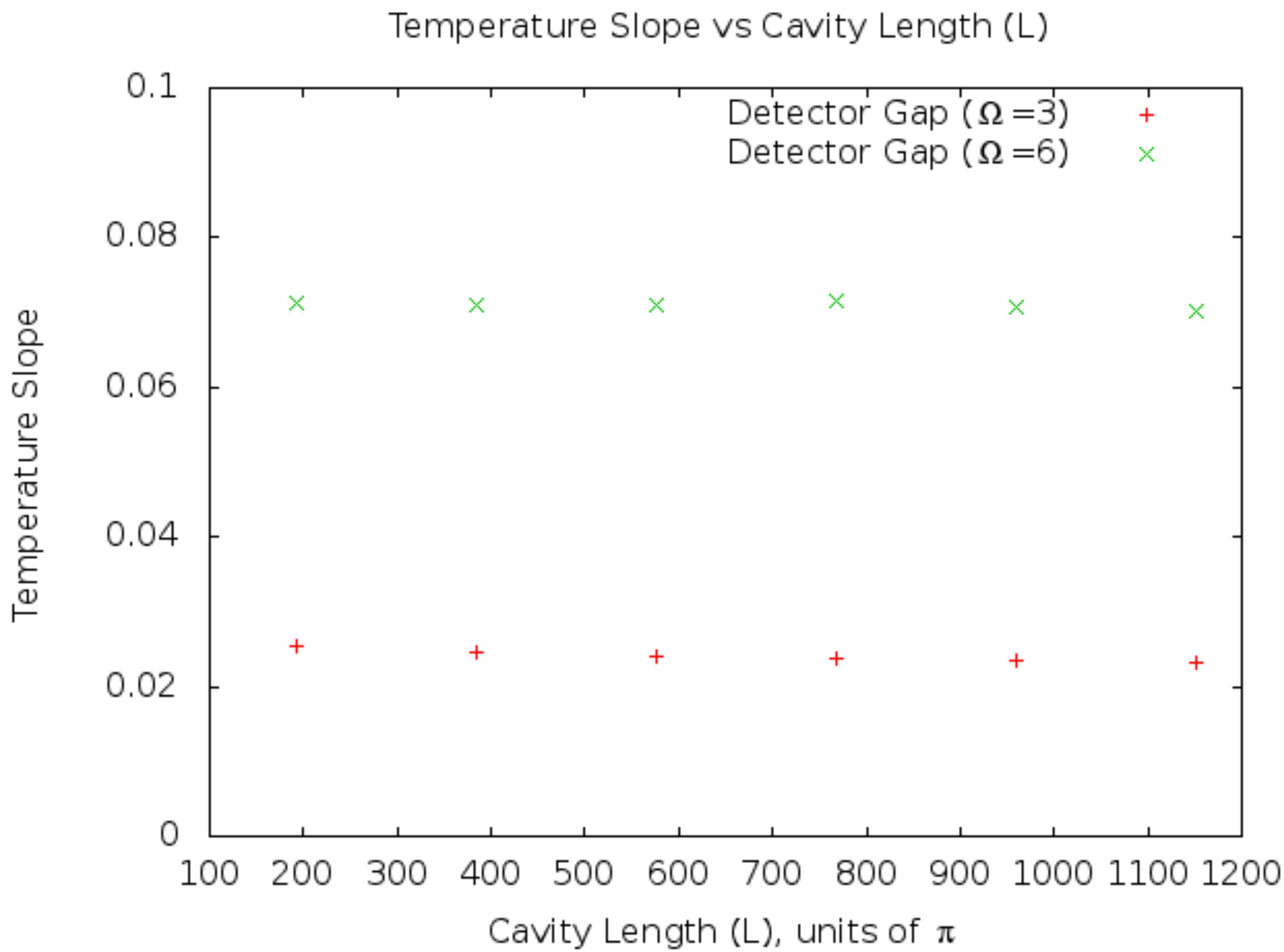

Thermality versus Cavity Length
n = 60
Thermality Ratio (r²/δ)
0.0002
0.00018
0.00016
0.00014
0.00012
0.0001
8e-05
6e-05
0
20
40
60
80
100
120
140
160
180
Cavity Length (L)

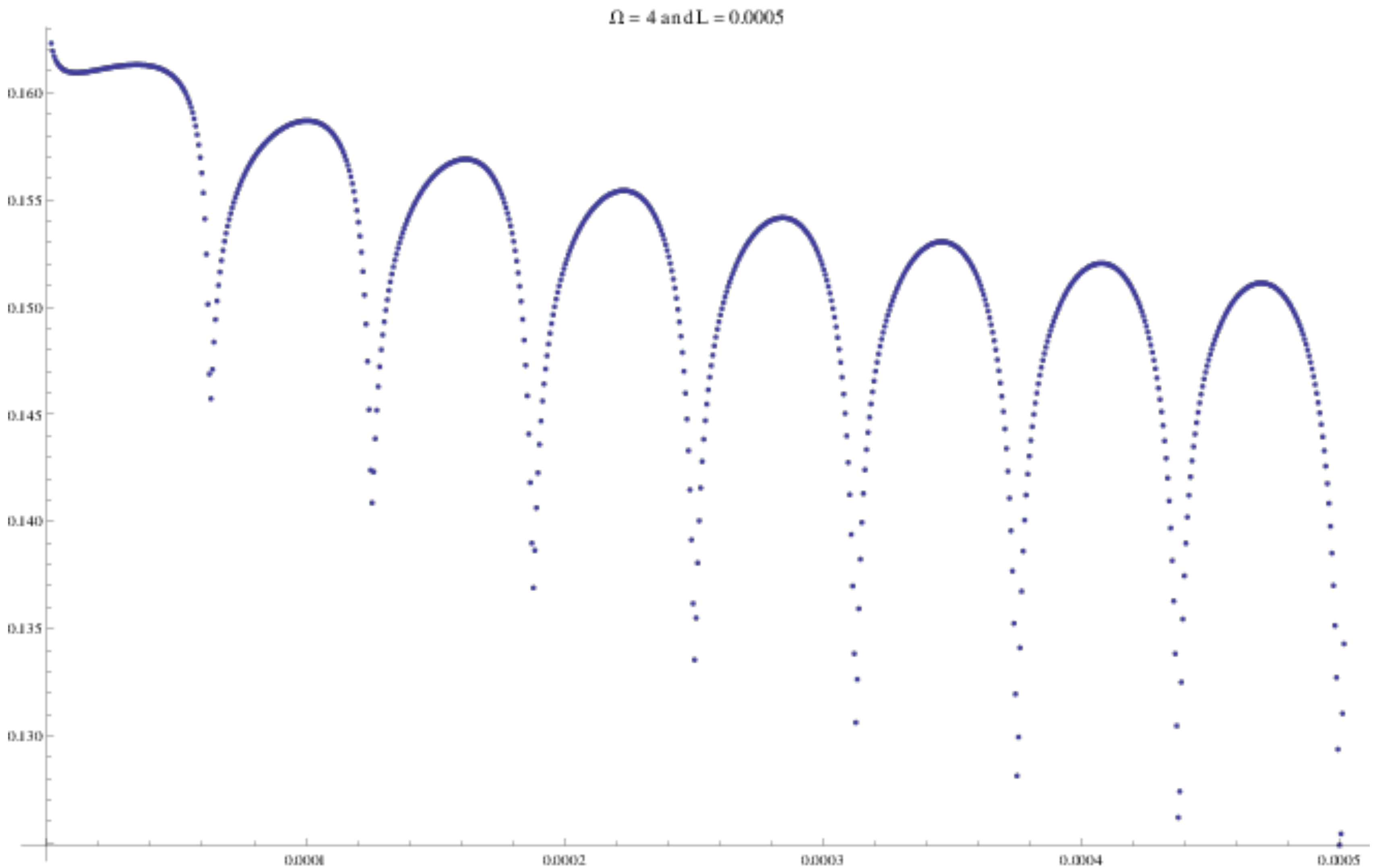


Thermalization (L = 96 pi)
slope
Slope of Temperature
Time (t)
0.025
0.02
0.015
0.01
0.005
0
-0.005
0
2
4
6
8
10
12

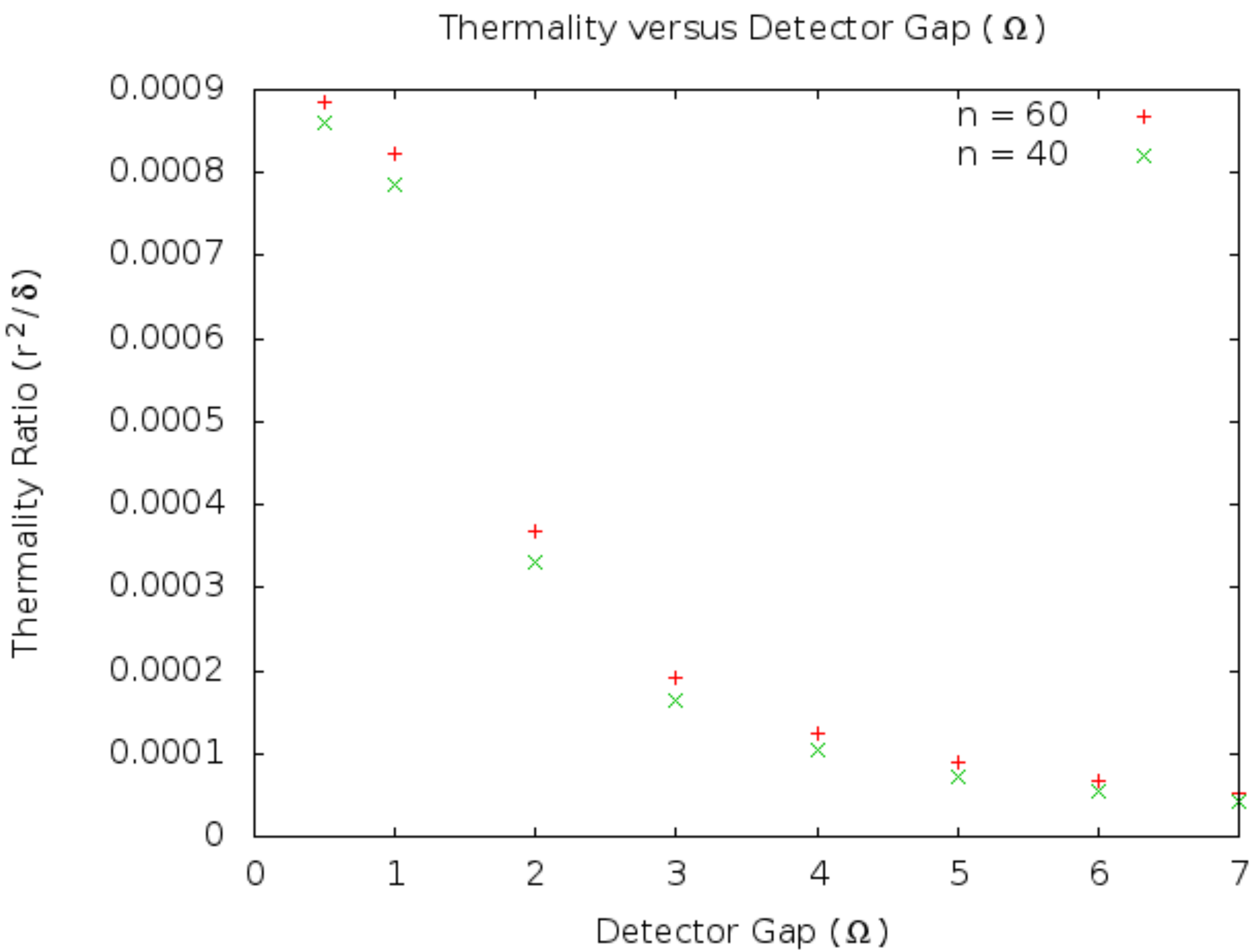


Thermality versus Number of Modes
Δt = 12
Δt = 16
Thermality Ratio (r²/δ)
Number of Modes (n)
0.00016
0.00014
0.00012
0.0001
8e-05
6e-05
4e-05
2e-05
0
0
10
20
30
40
50
60
70
80
90


Temperature Slope vs Cavity Length (L)
Detector Gap (Ω =3)
Detector Gap (Ω =6)
Temperature Slope
0.1
0.08
0.06
0.04
0.02
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
Cavity Length (L), units of π



Ω = 4 and L = 0.0005
0.160
0.155
0.150
0.145
0.140
0.135
0.130
0.0001
0.0002
0.0003
0.0004
0.0005



Thermality versus Detector Gap (Ω)
Thermality Ratio (r²/δ)
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0
0
1
2
3
4
5
6
7
Detector Gap (Ω)
n = 60
n = 40