Perturbative Non-Fermi Liquids from Dimensional Regularization

Denis Dalidovich
(Perimeter Institute)

Collaborator:
Sung-Sik Lee
(McMaster University and
Perimeter Institute)

References:

D. Dalidovich and Sung-Sik Lee, Phys. Rev. B 88, 245106 (2013).

Motivation

- Experimental:

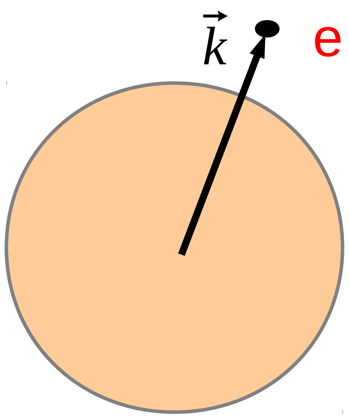
Numerous examples of non-Fermi liquid behaviour in various compounds (doped cuprates, pnictides, heavy-fermion compounds, ruthenades, organic materials).

- Theoretical:

– Development of controlled and reliable schemes that account for the interaction-induced non-Fermi liquid behaviour.

– Understanding universal properties of phases using low-energy effective field theories.

Landau Fermi liquid theory



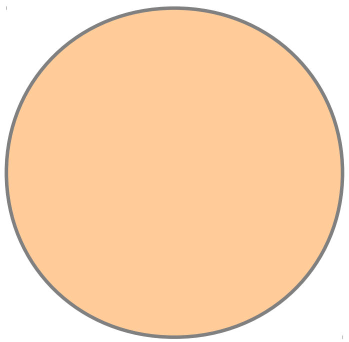
– low energy excitations have the same quantum numbers as for the gas of free fermions

$$G(\vec{k},\omega) = \frac{Z}{i\omega - v_F(k - k_F)}$$

– Quasiparticle residue Z and Fermi velocity v_F are renormalized by interactions (L.D. Landau, 1956)

States with critical Fermi surface

- These states have



– well defined Fermi surface
– but no Landau quasiparticles

$$G(\vec{k},\omega) \neq \frac{Z}{i\omega - v_F(k-k_F)}$$

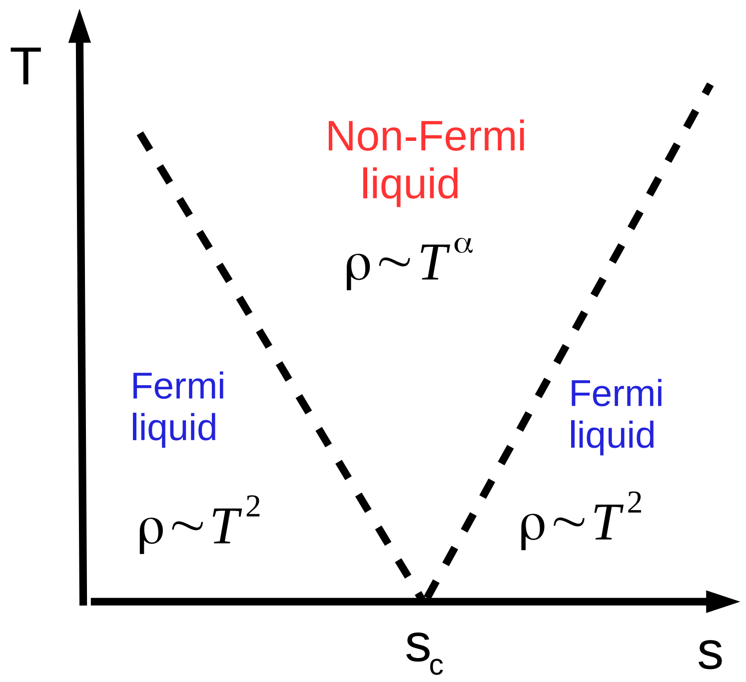
- An example includes $1D$ Luttinger liquids with

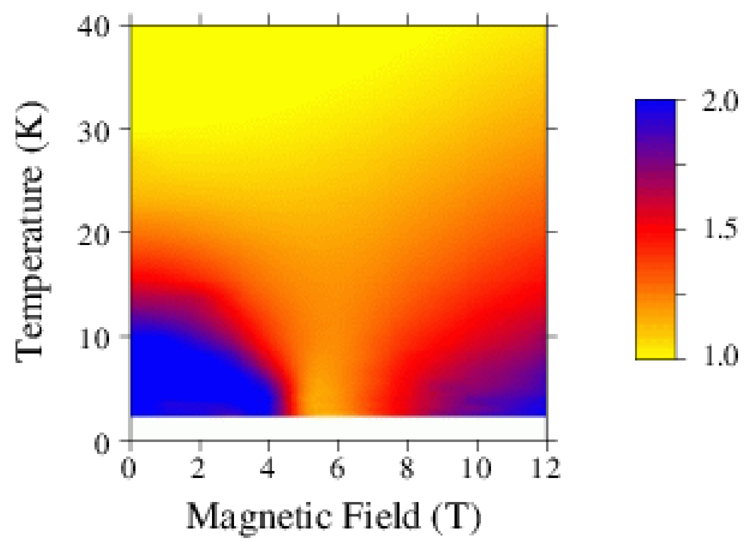
$$G(k,\omega) \sim (i\omega - k)^{\alpha-1}\,(i\omega + k)^{\alpha}$$

- States with critical Fermi surface in $D > 1$?

Critical Fermi surface and phase transitions in metals

- Order parameter ϕ becomes non-zero as s is tuned





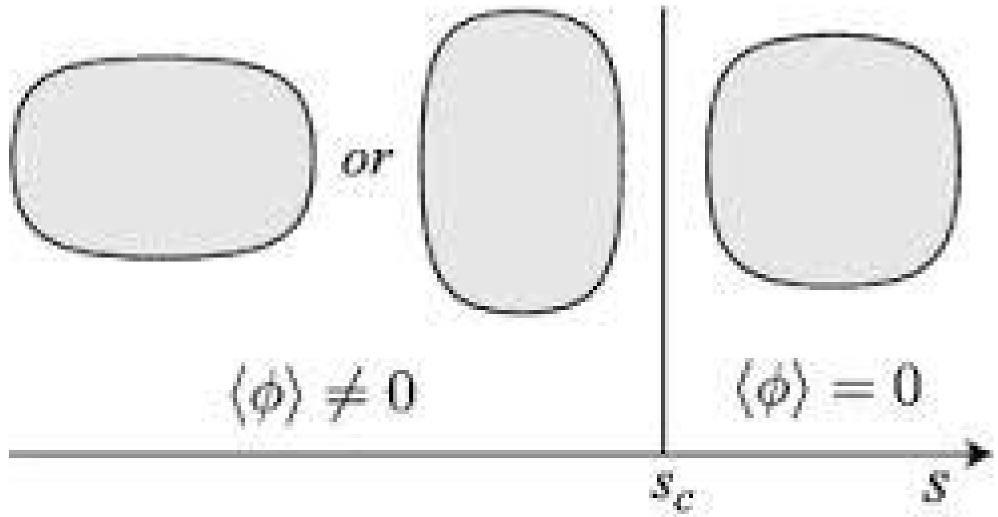
Fermi liquid resistivity $\rho \sim T^2$. In non-Fermi liquid state $\rho \sim T^{\alpha}$ with $1 < \alpha < 2$.

[From R. S. Perry, et.al., Phys. Rev. Lett. **86**, 2661, (2001)]

Ising Nematic Order

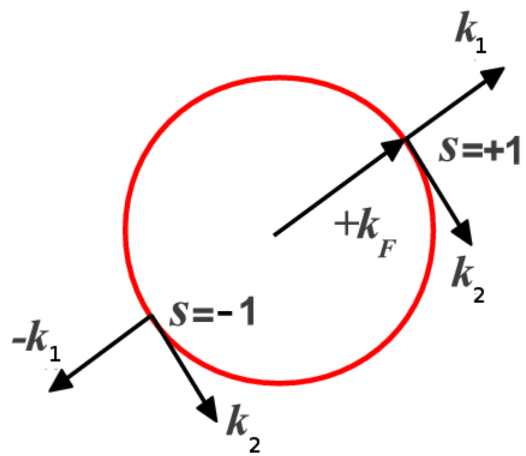
- Order breaks the rotational symmetry of the lattice; translational symmetry is preserved.

- Order parameter ϕ has the Ising character. Its quantum fluctuations are described by a scalar bosonic field $\phi(\vec{x}, \tau)$.



The model for the low-energy physics I

- Two patches of Fermi surface are coupled with one critical boson in (2+1) dimension



Two opposite patches of the Fermi surface with 2+1-dimensional left and right moving fermions.

$$
\begin{aligned}
S &= \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k}{(2\pi)^3}\psi^\dagger_{s,j}(k)\left[ik_0 + sk_1 + k_2^2\right]\psi_{s,j}(k) \\
&+ \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\left[k_0^2 + k_1^2 + k_2^2\right]\phi(-k)\phi(k) \\
&+ \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^3k d^3q}{(2\pi)^6}\lambda_s\,\phi(q)\,\psi^\dagger_{s,j}(k+q)\psi_{s,j}(k).
\end{aligned}
$$

The model for the low-energy physics II

- The right and left moving fermions are combined into one spinor

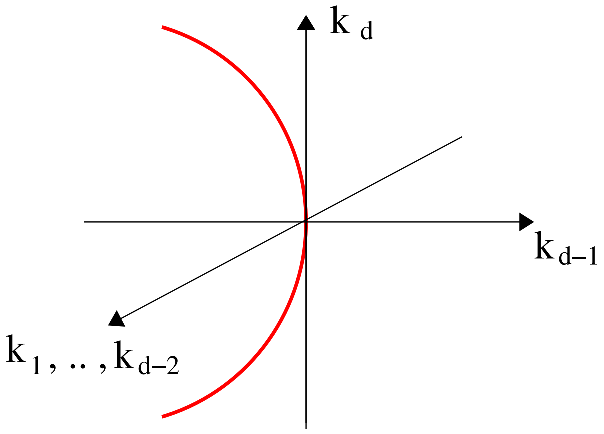
$$\Psi_j(k) = \begin{pmatrix} \psi_{+,j}(k) \\ \psi^\dagger_{-,j}(-k) \end{pmatrix}.$$

$$\begin{aligned} S &= \sum_j \int \frac{d^3k}{(2\pi)^3} \bar{\Psi}_j(k) \left[ik_0\gamma_0 + i(k_1 + k_2^2)\gamma_1 \right] \Psi_j(k) \\ &+ \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[k_0^2 + k_1^2 + k_2^2 \right] \phi(-k)\phi(k) \\ &+ \frac{e}{\sqrt{N}} \sum_j \int \frac{d^3k d^3q}{(2\pi)^6} \phi(q) \bar{\Psi}_j(k+q)\gamma_0\Psi_j(k), \end{aligned}$$

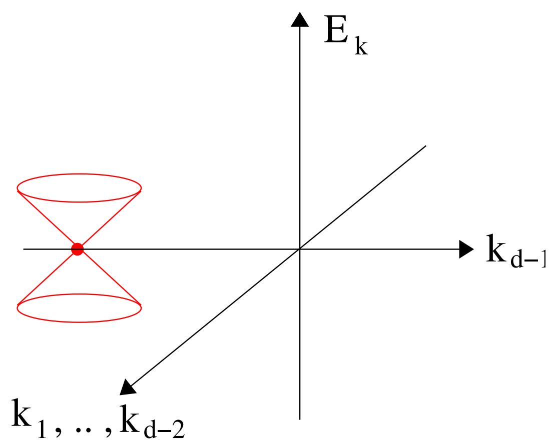
- $\gamma_0 = \sigma_y$, $\gamma_1 = \sigma_x$ are the gamma-matrices for the two-component spinor, $\bar{\Psi} \equiv \Psi^\dagger\gamma_0$.

Action including co-dimension of Fermi surface

$$S = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \left[i\vec{\Gamma}\cdot\vec{K} + i\gamma_{d-1}\delta_k \right] \Psi_j(k) + \frac{1}{2}\int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k_{d-1}^2 + k_d^2 \right] \phi(-k)\phi(k)$$
$$+ \frac{ie}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\bar{\Psi}_j(k+q)\gamma_{d-1}\Psi_j(k).$$



(a)

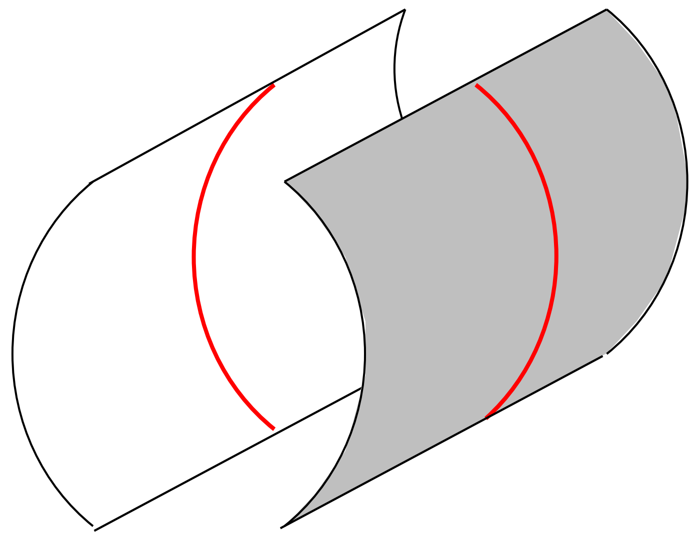


(b)

The 1D Fermi surface is embedded in the d-dimensional momentum space, $\vec{K} \equiv (k_0, k_1, \ldots, k_{d-2})$, $\vec{\Gamma} \equiv (\gamma_0, \gamma_1, \ldots, \gamma_{d-2})$. For each k_d, there is a Fermi point at $(k_1, k_2, ..., k_{d-2}, k_{d-1}) = (0, 0, .., 0, -\sqrt{d-1}k_d^2)$ (red dot). $\delta_k = k_{d-1} + \sqrt{d-1}k_d^2$ and the spinor has the energy dispersion with two bands.

$$E(k) = \pm\sqrt{\sum_{i=1}^{d-2} k_i^2 + \delta_k^2} = 0 \quad \Leftrightarrow \quad \begin{cases} k_i = 0 & i = 1, \ldots, d-2, \\ \delta_k = 0 & \Rightarrow k_{d-1} = -\sqrt{d-1}k_d^2. \end{cases}$$

Co-dimension of Fermi surface in $d = 3$



- If $d = 3$, the dimensionality of codimension is $d_{\text{codim}} = 1$. Fermi lines in three dimensional momentum space can be obtained by turning on p-wave superconducting order parameter that gaps out the cylindrical Fermi surface except for the line node (red line).

Scaling transformation

$$S = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \left[i\vec{\Gamma}\cdot\vec{K} + i\gamma_{d-1}\delta_k \right] \Psi_j(k) + \frac{1}{2}\int \frac{d^{d+1}k}{(2\pi)^{d+1}} \left[|\vec{K}|^2 + k_{d-1}^2 + k_d^2 \right] \phi(-k)\phi(k)$$
$$+ \frac{ie}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\bar{\Psi}_j(k+q)\gamma_{d-1}\Psi_j(k).$$

- Quadratic part is invariant under

$$\vec{K} = \frac{\vec{K}'}{b}, \qquad k_{d-1} = \frac{k'_{d-1}}{b}, \qquad k_d = \frac{k'_d}{\sqrt{b}}, \qquad \Psi(k) = b^{\frac{d}{2}+\frac{3}{4}}\Psi'(k'), \qquad \phi(k) = b^{\frac{d}{2}+\frac{3}{4}}\phi'(k')$$

$$\Downarrow$$

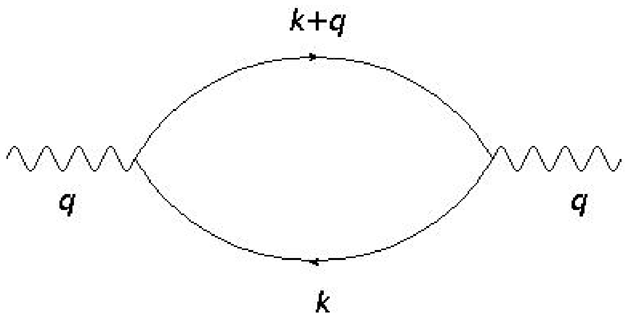
$$\boxed{e' = b^{\frac{1}{2}(\frac{5}{2}-d)} e}$$

- e is irrelevant for $d > d_{\text{cr}} = 5/2$, and is relevant for $d < 5/2$.
  Interacting non-Fermi liquid state is accessed perturbatively in $d = 5/2 - \epsilon$.
  Physical case corresponds to $\epsilon = 1/2$.

Minimal subtraction scheme

- Replace the bare boson propagator by the one-loop corrected one (retaining only the relevant terms). μ is the running energy scale

$$S = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k)\left[i\vec{\Gamma}\cdot\vec{K} + i\gamma_{d-1}\delta_k\right]\Psi_j(k) + \frac{1}{2}\int \frac{d^{d+1}k}{(2\pi)^{d+1}} D_1^{-1}(k)\phi(-k)\phi(k)$$
$$+ \frac{ie\mu^{\epsilon/2}}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k\, d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\bar{\Psi}_j(k+q)\gamma_{d-1}\Psi_j(k).$$



$$D_1(k) = \frac{1}{k_d^2 + Ce^2\mu^{\epsilon}\frac{|\vec{K}|^{d-1}}{|k_d|}}, \qquad C = \text{const}$$

- Add the part with the counter terms

$$S_{CT} = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k)\left[i(Z_1-1)\vec{\Gamma}\cdot\vec{K} + i(Z_2-1)\gamma_{d-1}\delta_k\right]\Psi_j(k) + \frac{(Z_3-1)}{2}\int \frac{d^{d+1}k}{(2\pi)^{d+1}}\; k_d^2\phi(-k)\phi(k)$$
$$+ (Z_4-1)\frac{ie\mu^{\epsilon/2}}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k\, d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\bar{\Psi}_j(k+q)\gamma_{d-1}\Psi_j(k)$$

Renormalization

- The renormalized action gives the finite quantum effective action.

$$
\begin{aligned}
S_{ren} &= \sum_j \int \frac{d^{d+1}k_B}{(2\pi)^{d+1}} \bar{\Psi}_{Bj}(k_B)\left[i\vec{\Gamma}\cdot\vec{K}_B + i\gamma_{d-1}\delta_{k_B}\right]\Psi_{Bj}(k_B) + \frac{1}{2}\int \frac{d^{d+1}k_B}{(2\pi)^{d+1}}\, k_{Bd}^2 \phi_B(-k_B)\phi_B(k_B) \\
&+ \frac{ie_B}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k_B d^{d+1}q_B}{(2\pi)^{2d+2}} \phi_B(q_B)\bar{\Psi}_{Bj}(k_B+q_B)\gamma_{d-1}\Psi_{Bj}(k_B)
\end{aligned}
$$

$$
\vec{K} = (Z_2/Z_1)\vec{K}_B, \qquad k_{d-1} = k_{B,d-1}, \qquad k_d = k_{B,d}, \qquad \Psi(k) = \Psi_B(k_B)/\left[Z_2(Z_2/Z_1)^{(d-1)}\right]^{1/2}
$$

$$
\phi(k) = \phi_B(k_B)/\left[Z_3(Z_2/Z_1)^{(d-1)}\right]^{1/2}, \qquad e_B = Z_3^{-1/2}(Z_2/Z_1)^{(d-1)/2}\mu^{\epsilon/2}e
$$

- The quantities of interest are

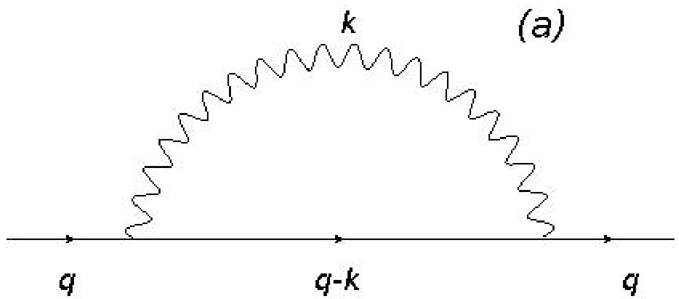
$$
\begin{aligned}
z(e) &= 1 - \frac{(\partial \ln(Z_2/Z_1)}{\partial \ln \mu} && \text{– dynamical critical exponent} \\
\beta(e) &= \frac{\partial e}{\partial \ln \mu} && \text{– beta-function for the coupling constant} \\
\eta_\psi(e) &= \frac{1}{2}\frac{\partial \ln Z_\Psi}{\partial \ln \mu} && \text{– anomalous dimension for fermions} \\
\eta_\phi(e) &= \frac{1}{2}\frac{\partial \ln Z_\phi}{\partial \ln \mu} && \text{– anomalous dimension for bosons}
\end{aligned}
$$

One-loop results

- Up to the two-loop level, the counter terms have only simple poles in ϵ,

$$Z_n - 1 = \frac{Z_{n,1}}{\epsilon}.$$

- The only diagram contributing to renormalization



$$Z_{1,1} = -\frac{e^{4/3}}{N} u_1, \qquad u_1 = 0.0875$$

- The beta-function $\boxed{\beta = -\frac{\epsilon}{2} e + 0.02920 \left(\frac{3}{2} - \epsilon\right) \frac{e^{7/3}}{N}}$ gives

Gaussian fixed-point (unstable)

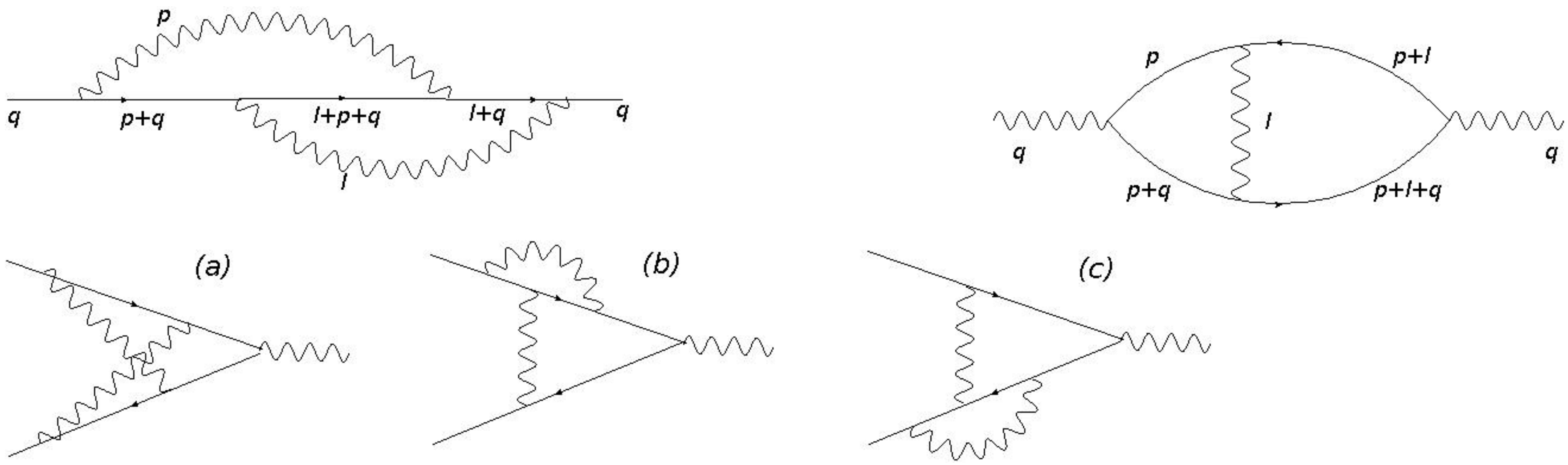
$$e = 0, \qquad z = 1,$$
$$\eta_\Psi = 0, \qquad \eta_\phi = 0$$

Interacting fixed-point (stable)

$$e^{*\frac{4}{3}}/N = 11.417\epsilon, \qquad z = 3/(3 - 2\epsilon),$$
$$\eta_\Psi = -\frac{\epsilon}{2}, \qquad \eta_\phi = -\frac{\epsilon}{2}.$$

Two-loop results I

- The diagrams that contribute at two-loops are



$$S_{CT} = \sum_j \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \bar{\Psi}_j(k) \Big[i(Z_1-1)\vec{\Gamma}\cdot\vec{K} + i(Z_2-1)\gamma_{d-1}\delta_k \Big] \Psi_j(k) + \frac{(Z_3-1)}{2} \int \frac{d^{d+1}k}{(2\pi)^{d+1}} \; k_d^2 \phi(-k)\phi(k)$$
$$+ \; (Z_4-1)\frac{ie\mu^{\epsilon/2}}{\sqrt{N}}\sqrt{d-1}\sum_j \int \frac{d^{d+1}k d^{d+1}q}{(2\pi)^{2d+2}} \phi(q)\bar{\Psi}_j(k+q)\gamma_{d-1}\Psi_j(k)$$

- As a result of renormalization

$$Z_{1,1} = -\left(e^{4/3}/N\right) u_1 - \left(e^{8/3}/N^2\right) u_2,$$
$$Z_{2,1} = -\left(e^{8/3}/N^2\right) v_2,$$
$$Z_{3,1} = 0$$

$$u_1 \approx 0.08758634,$$
$$u_2 \approx -0.01777769,$$
$$v_2 \approx 0.000867775$$

Two-loop results II

- The beta-function at two-loops

$$\beta = -\frac{\epsilon}{2}e + 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} - 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$

gives the fixed point at

$$\frac{e^{*4/3}}{N} = 11.417\epsilon + 55.498\epsilon^2.$$

- Dynamical critical exponent and anomalous dimensions:

$$z = \frac{3}{3-2\epsilon}, \qquad \eta_\psi = -\frac{\epsilon}{2} + 0.07541\epsilon^2, \qquad \eta_\phi = -\frac{\epsilon}{2}.$$

- The scaling of boson and fermion propagators

$$D(k) = \frac{1}{k_d^2}\; f\left(\frac{|\vec{K}|^{1/z}}{k_d^2}\right), \qquad G(k) = \frac{1}{|\delta_k|^{1-0.1508\epsilon^2}}\; g\left(\frac{|\vec{K}|^{1/z}}{\delta_k}\right).$$

Conclusions and Future Work

- A new dimensional regularization scheme has been developed with one-dimensional Fermi surface embedded in general dimensions

- The Ising-nematic quantum phase transition is described by a stable non-Fermi liquid fixed point just below the critical dimension $d_c = 5/2$, at which the fermion-boson coupling is marginal.

- Physical properties (thermodynamic, transport) can be computed using a controlled scheme order by order in ϵ.

- This dimensional regularization scheme can be generalized to include disorder, as well as consider more complicated types of interactions such as those between fermions and gauge bosons.