Detecting Particles near an AdS Black Hole

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arXiv:1406.2688

June 18, 2014

Hawking Radiation

- Black holes emit thermal radiation
- Black holes have a “temperature”
- Hartle-Hawking vacuum contains particles

Hawking Radiation

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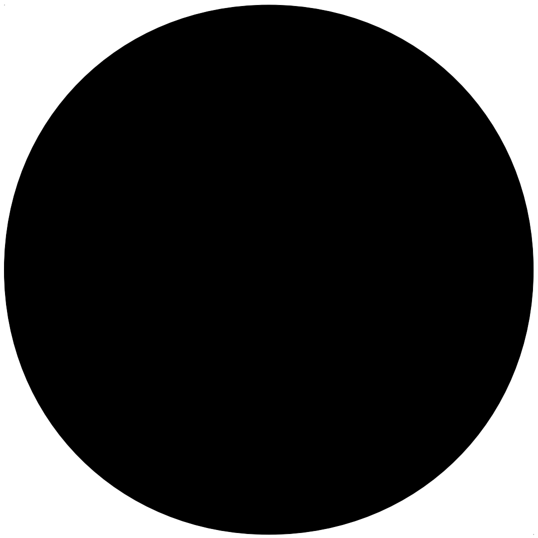
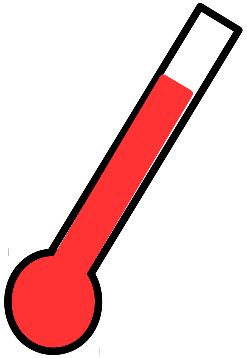
- Black holes have a “temperature”
- Hartle-Hawking vacuum contains particles
- How can we measure the temperature of a black hole?

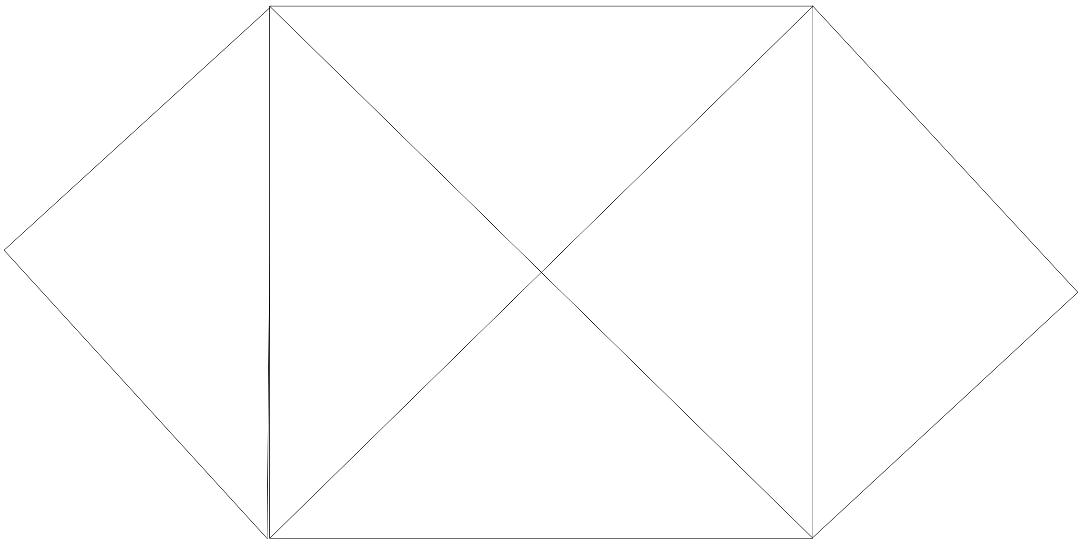

Measuring Temperature

- Idea: put a two-level system above a black hole
- Model Hawking radiation as a scalar
- Excitation = Detection
- Transition rate should obey thermality condition

$$\dot{F}(E) = e^{-E/T_{loc}}\dot{F}(-E)$$

Which Black Hole?

- Schwarzschild



Which Black Hole?

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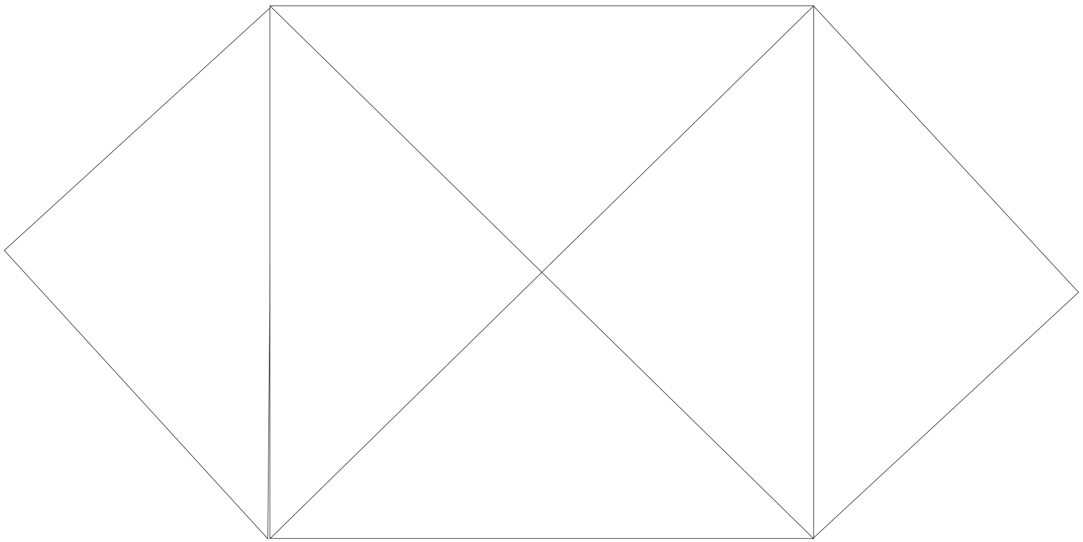
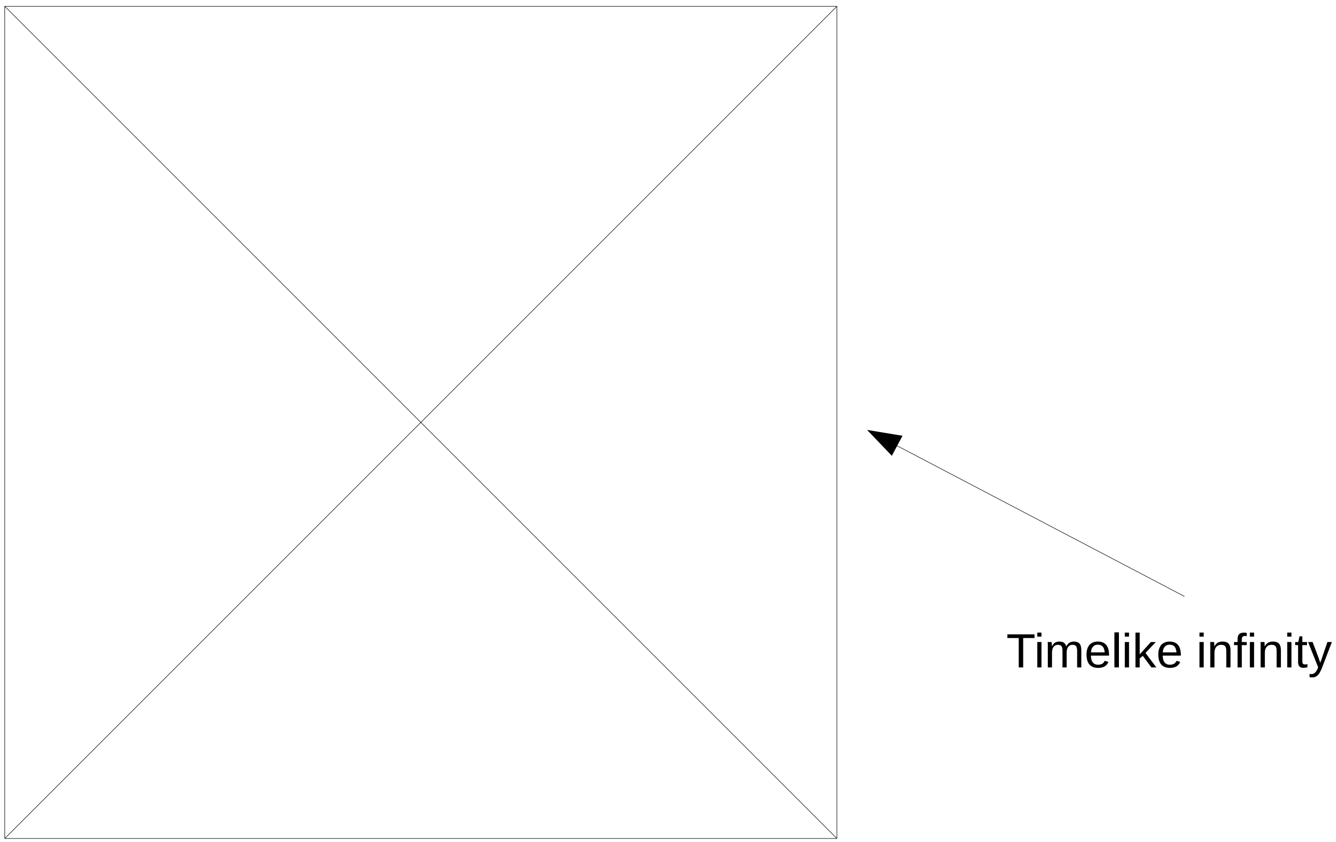


- Done by Hodgkinson, Louko, Ottewill (Phys. Rev. D 89, 104002 (2014))

Schwarzschild Anti-de Sitter Space



Schwarzschild Anti-de Sitter Space

- Schwarzschild (uncharged spinless) black hole
- Asymptotically AdS
- Timelike infinity
- Reflective boundary conditions

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2$$

$$f(r) = \frac{r^2}{L^2} + 1 - \frac{r_0}{r},\ f(r_+) = 0$$

$$T_H = \frac{3r_+^2 + 1}{4\pi r_+}$$

Why SAdS?

- Well-studied system
- Physical importance: AdS/CFT duality
- Conformal coupling similar to Schwarzschild case
- Detector response not done before

The Unruh-Dewitt Detector

$$H_{int}(\tau) = c\chi(\tau)\mu(\tau)\phi(x(\tau))$$

- A monopole detector with small coupling constant *c*, switching function $\chi(\tau)$
- Simplest detector has two energy states with gap E
- First-order transition probability over trajectory:

$$P(E) = c^2 |\langle 0_d| \mu(0) |E_d\rangle|^2 F(E)$$

The Response Function F(E)

- F(E) is independent of physical details of the detector besides energy gap
- For calculations, can be written in terms of the Wightman function $W(\mathsf{x},\mathsf{x}') = \langle\Psi|\,\phi^\dagger(\mathsf{x})\phi(\mathsf{x}')\,|\Psi\rangle$

$$F(E) = 2\lim_{\epsilon\to 0}\mathrm{Re}\int_{-\infty}^{\infty} du\,\chi(u)\int_0^{\infty} ds\,\chi(u-s)e^{-iEs}W_\epsilon(u,u-s)$$

$$W_\epsilon(\tau',\tau'') = W_\epsilon(x(\tau'),x(\tau''))$$

Special Case: Stationary Trajectory

- In this case, we can integrate over infinite time and take the average, and the regulator can just be taken to zero pointwise.
- Since $W_\varepsilon(u,u-s)=W_\varepsilon(s)$ depends only on s, we end up with

$$F(E) = \lim_{\epsilon\to 0} \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} ds\, e^{-iEs} W_\epsilon(s)$$

$$\dot{F}(E) = \lim_{\epsilon\to 0} \int_{-\infty}^{\infty} ds\, e^{-iEs} W_\epsilon(s)$$

- Works for static detectors and circular geodesics

Which Vacuum?

- Hartle-Hawking vacuum: black hole in thermal equilibrium with environment, radiation present
- Boulware vacuum: static observers see no particles, no radiation present

Conformal Radial Klein-Gordon Equation

$$r^* = -\int_r^\infty \frac{dr'}{f(r')}$$

$$w_{\omega lm} = (4\pi\omega)^{-1/2} e^{-i\omega t} Y_{lm}(\theta, \phi) r^{-1} \Phi_{\omega lm}(r)$$

$$[\partial_{r^*}^2 + \omega^2 - \tilde{V}(r^*)]\Phi = 0$$

$$\tilde{V}(r) = f(r)V(r)$$

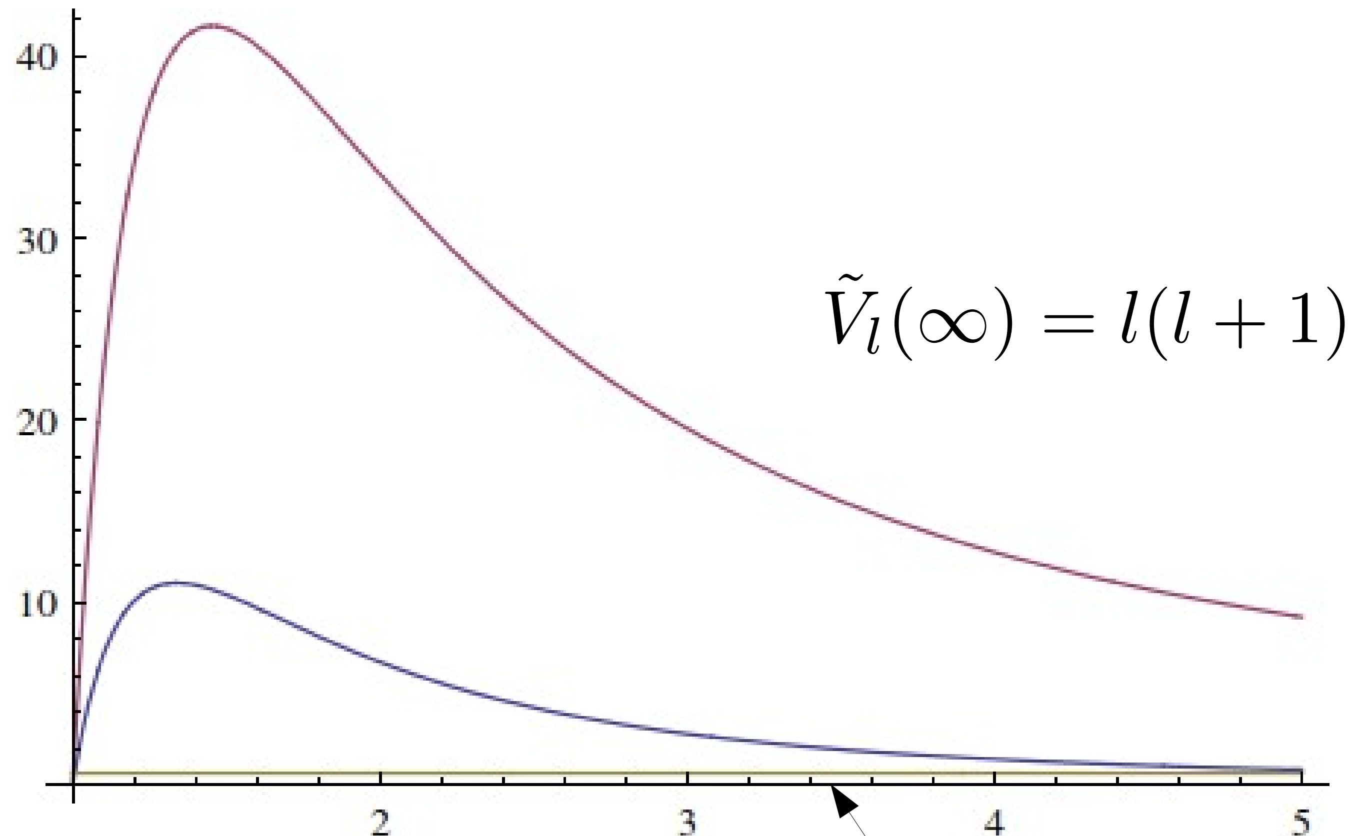
$$V(r) = \frac{l(l+1)}{r^2} + \frac{r_0}{r^3}$$

$$R_{\omega lm} = \Phi_{\omega lm}/r$$

Effective Potential

$\tilde{V}_l$

$$r_+/L = 0.1$$



$$\tilde{V}_l(\infty) = l(l+1)$$

$l = 1$

$l = 0$

r/r_+

$$E^2 = T_H^2$$

Static Transition Rate

- Dependence on one energy for each angular momentum

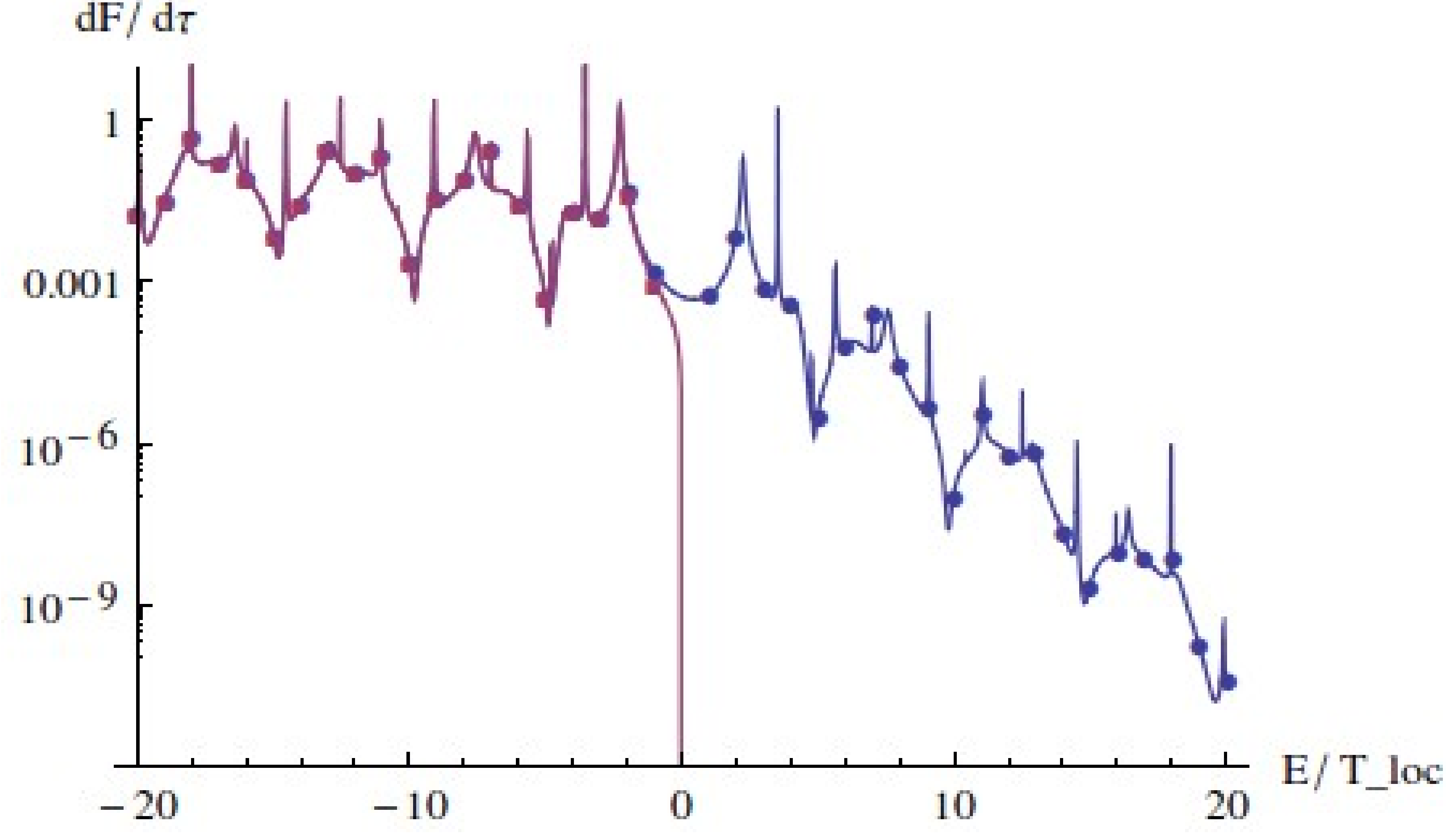
$$\dot{F}_H(E) = \frac{1}{2E}\frac{1}{e^{E/T_{loc}} - 1}\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}R^2_{\tilde{\omega}l}(r)$$

$$\dot{F}_B(E) = \Theta(-E)\frac{1}{2|E|}\sum_{l=0}^{\infty}\frac{2l+1}{4\pi}R^2_{\tilde{\omega}l}(r)$$

$$\tilde{\omega} = \sqrt{f(r)}E,\ T_{loc} = T_H/\sqrt{f(r)}$$

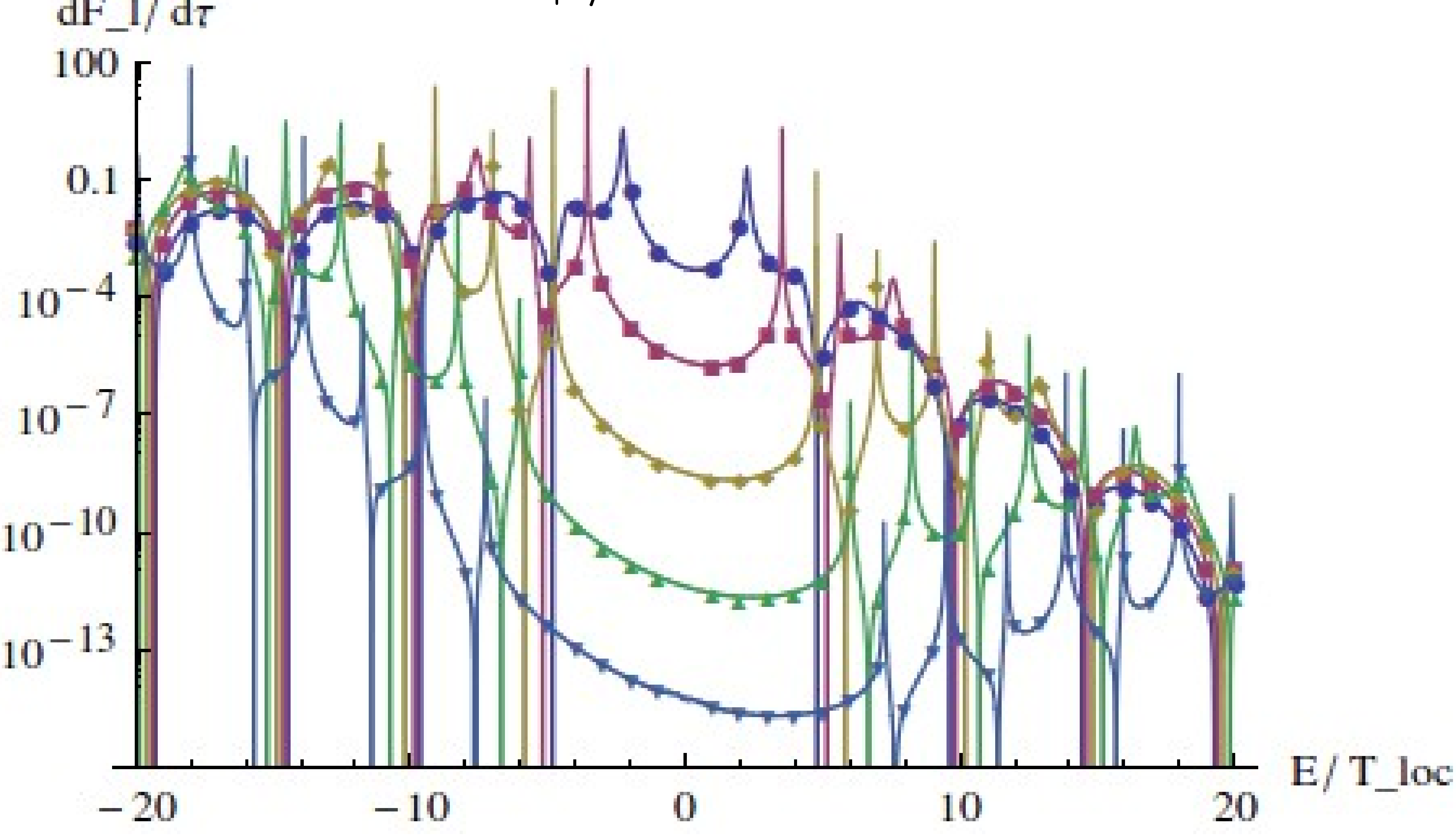
Hartle-Hawking vs. Boulware

$$r_+/L = 0.1, r/r_+ = 10$$



HH Angular Momentum Decomposition

$$r_+/L = 0.1$$

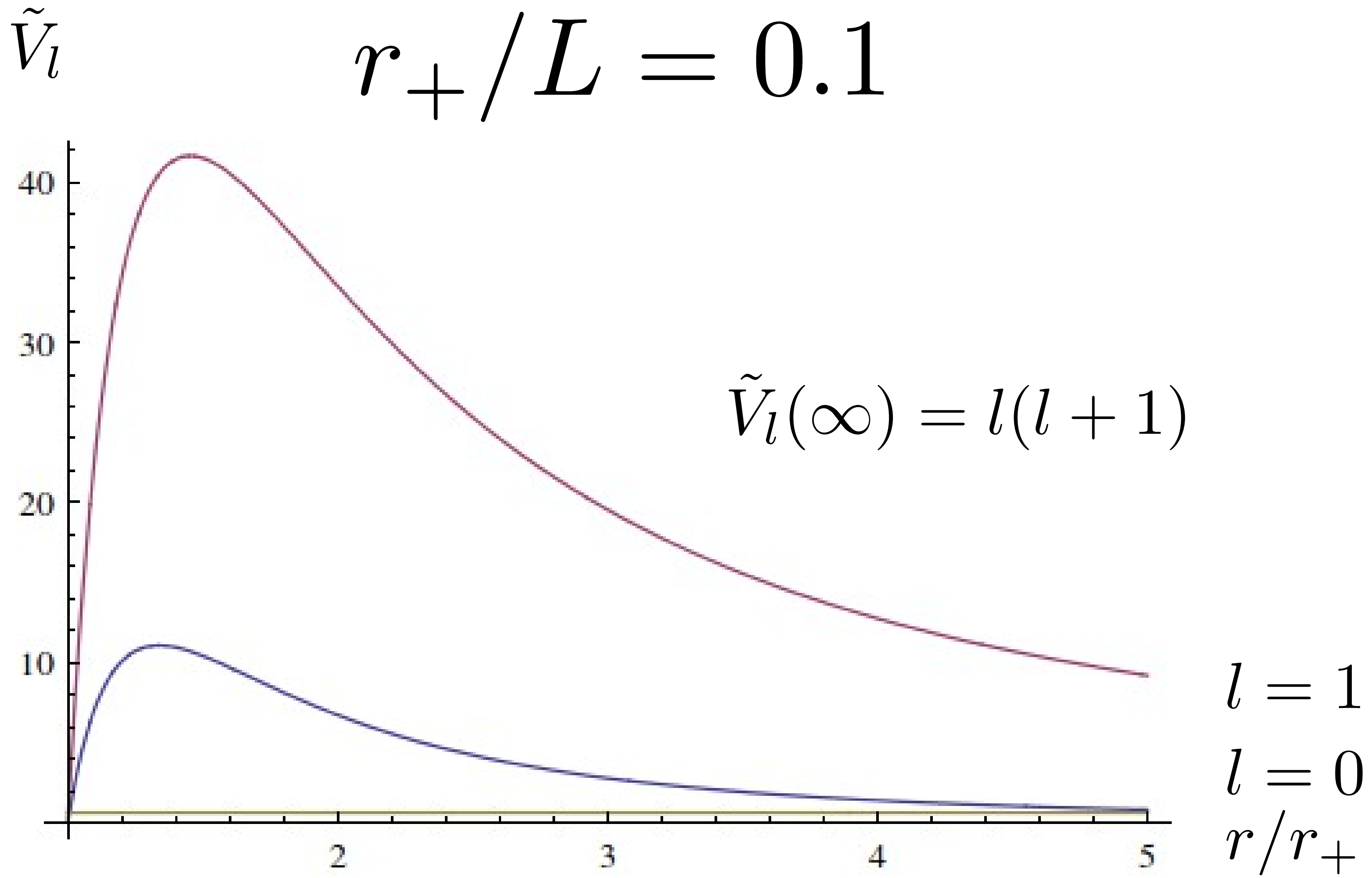


Features of Static Transition Rate

- Angular momentum contributions go to zero at certain points: mode at detector is zero
- Peaks: Quasinormal modes (“trapped modes”)
- Higher angular momentum contributions are nontrivial at higher detector energies
- Peaks get sharper with higher angular momentum, due to reflective boundary

Effective Potential

$$r_+/L = 0.1$$



Circular Geodesic Detector

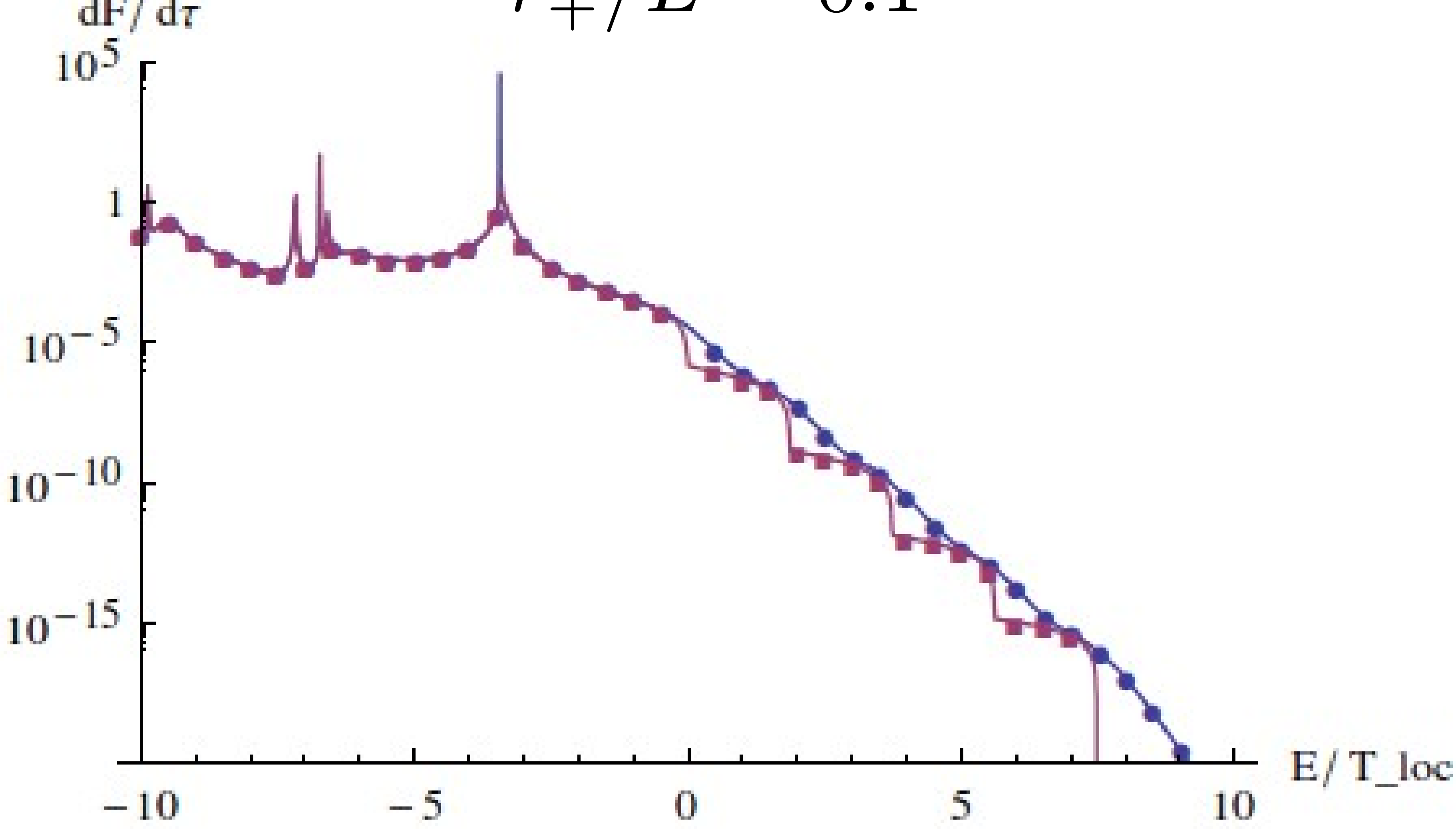
- Solving equations of motion for circular orbit yields

$$dt/d\tau = \sqrt{\frac{2r}{2r - 3r_0}}$$

$$d\phi/d\tau = \sqrt{\frac{r_0 + 2r^3}{r^2(2r - 3r_0)}}$$

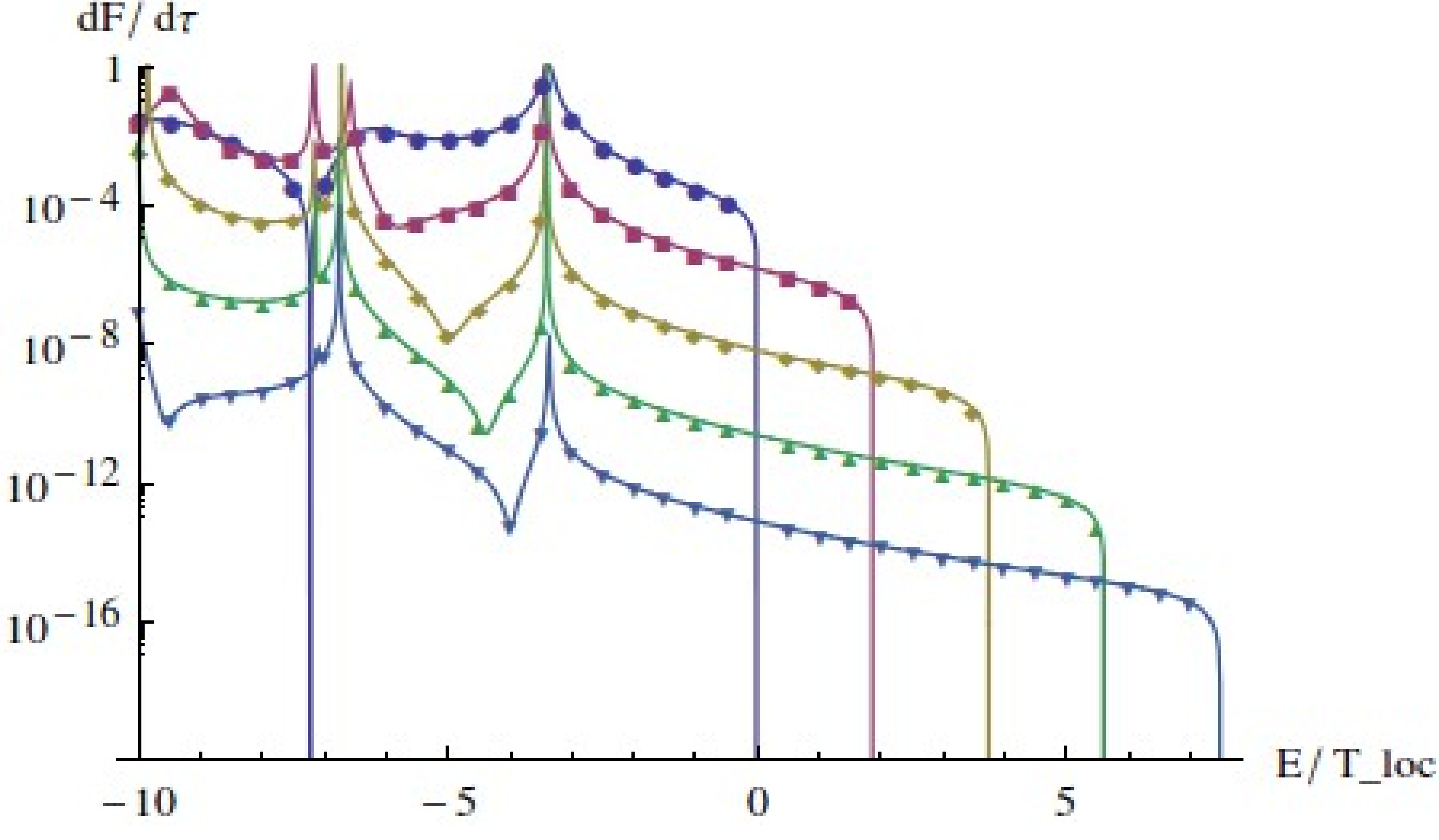
Hartle-Hawking vs. Boulware

$r_+/L = 0.1$



Boulware Angular Momentum Decomposition

$$r_+/L = 0.1$$

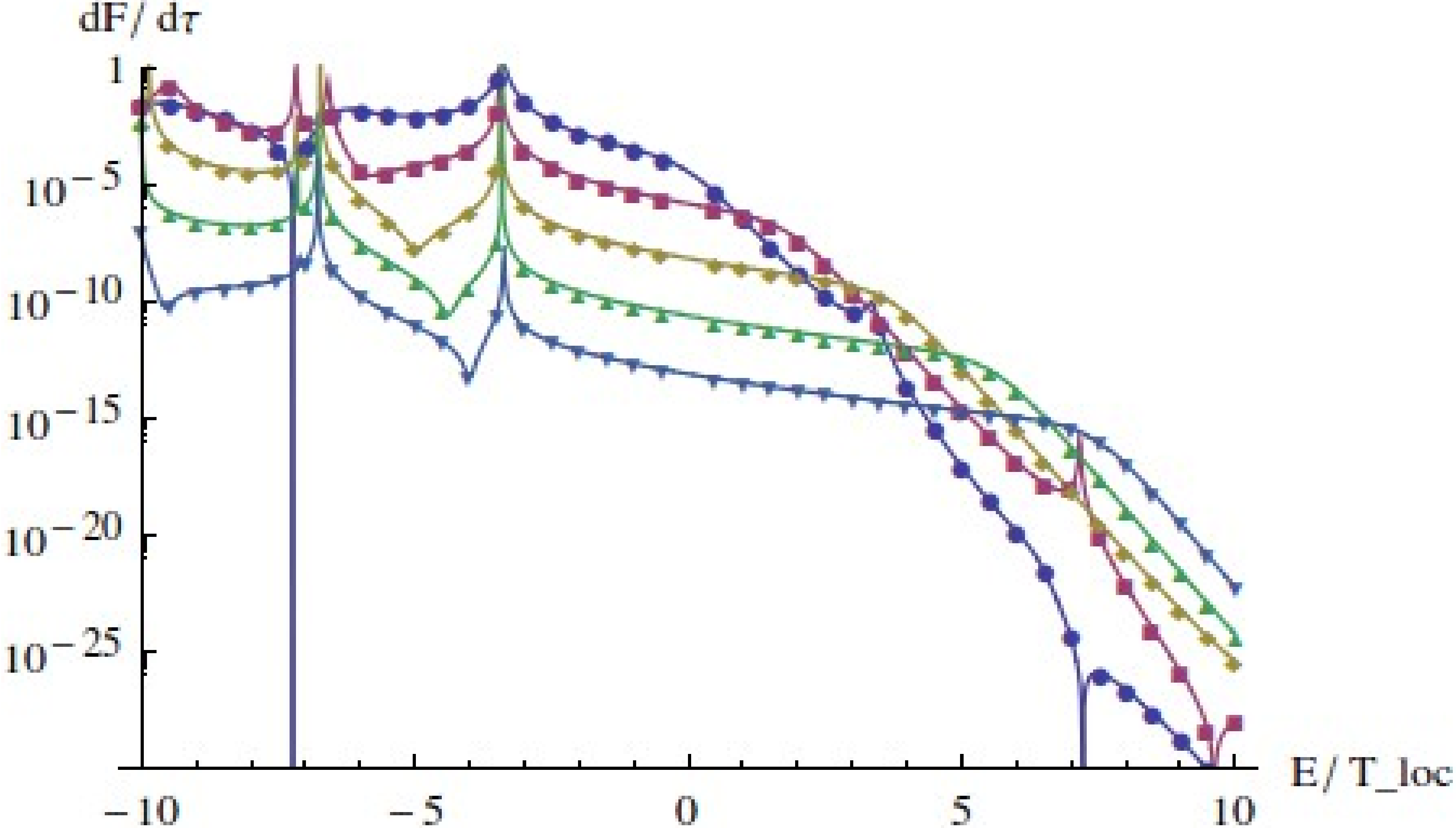


“Steps”

- Contribution nonzero for positive $\omega_- = (mb - E)/a$
- Dropoff is dependent on $l \geq |m|$
- Circular geodesic motion creates excitation regardless of field state

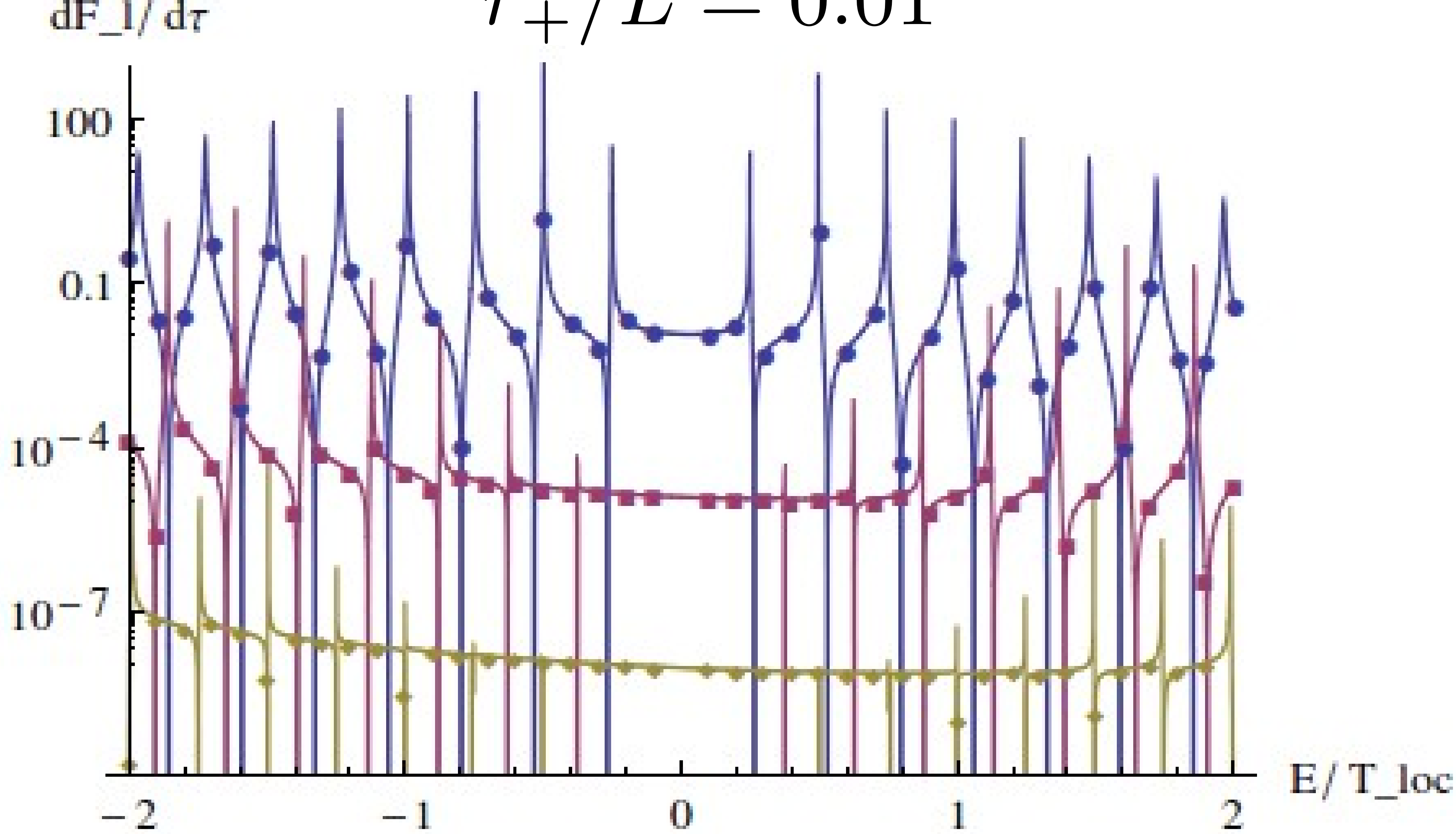
Hartle-Hawking Angular Momentum Decomposition

$$r_+/L = 0.1$$



HH Angular Momentum Decomposition

$$r_+/L = 0.01$$



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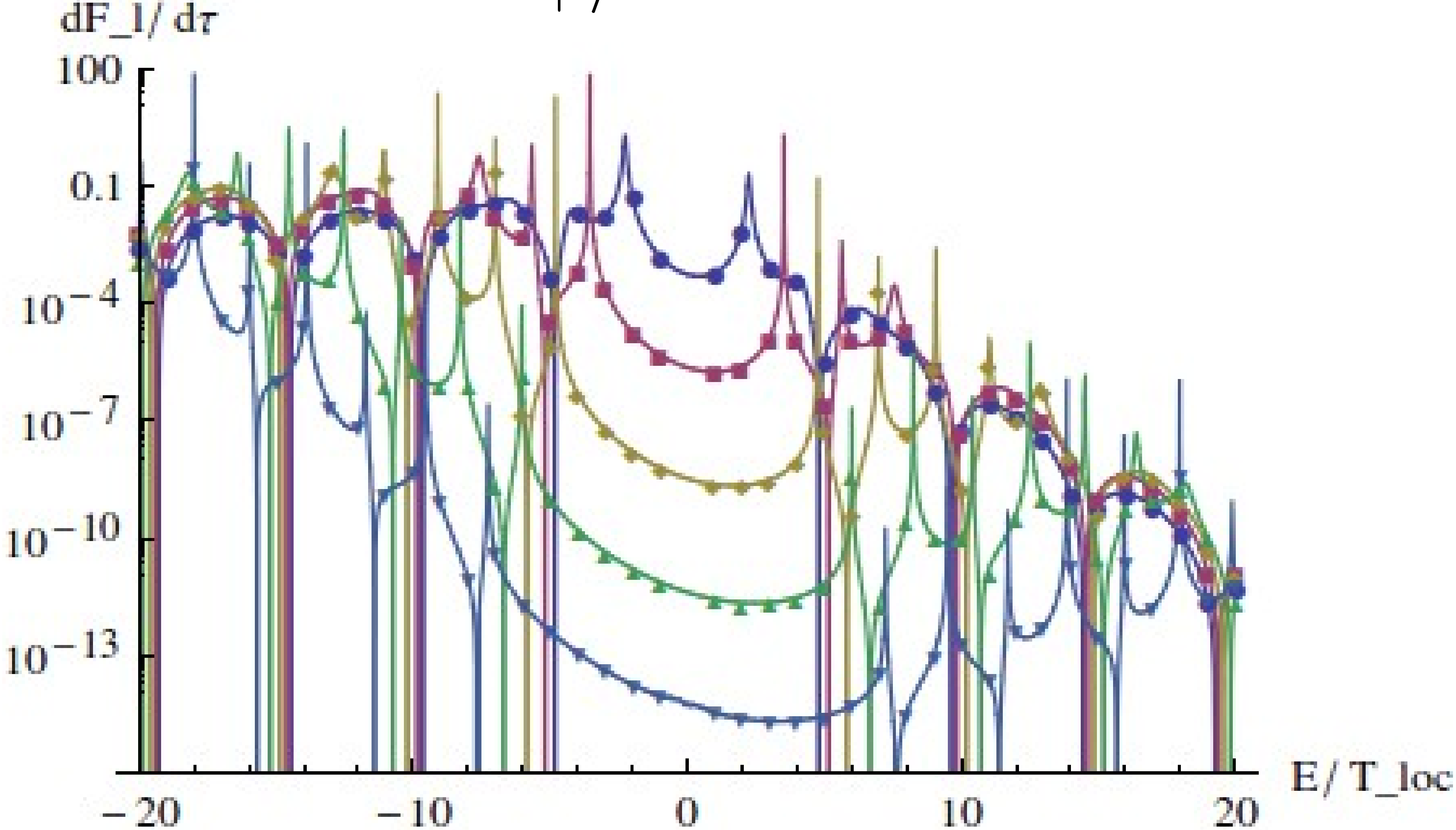
$$r_+/L = 0.1$$



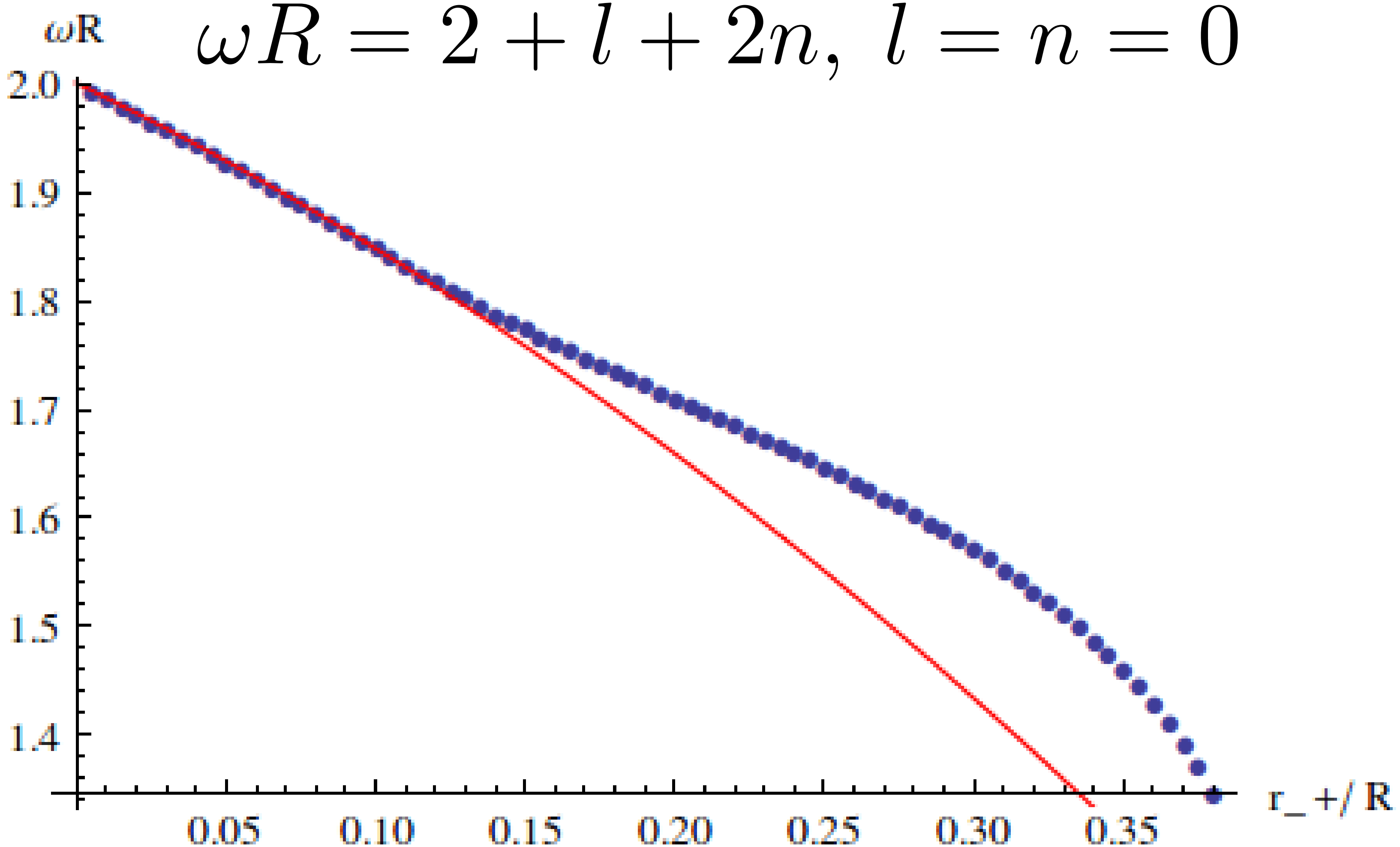
Small Black Hole Limit

- When the black hole is very small compared to the AdS length, the spacetime is “almost AdS”
- Higher angular momentum modes contribute less off-peak at energies shown
- Peaks become sharper, and approach AdS normal frequencies

$$\omega L = 2 + l + 2n$$

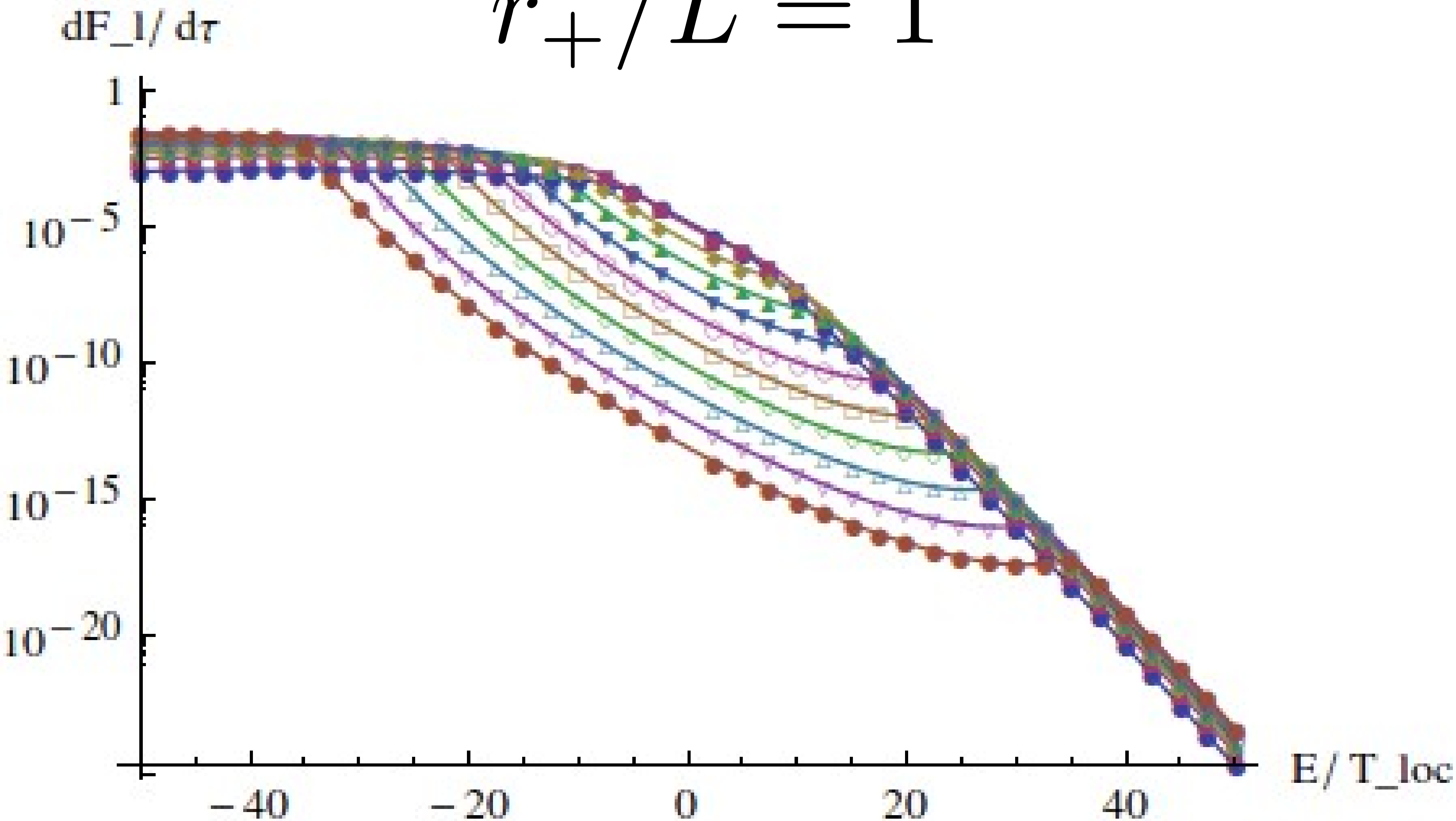
First Peak Frequency

$$\omega R = 2 + l + 2n,\ l = n = 0$$



HH Angular Momentum Decomposition

$$r_+/L = 1$$



Large(r) Black Hole Limit

- No transition between small and large black holes, since effective potential vanishes at infinity
- Peaks subsumed by greater exponential decay; not very interesting
- Boundary conditions mean this is *not* Schwarzschild-like

Conclusions

- Radiation is thermal, but not featureless
  - Peaks due to quasinormal resonances
  - Troughs due to zeroes of modes
- Small black holes converge to AdS sharp peaks
- Large black holes have no peaks
- Circular geodesic detectors are excited in either vacuum

Next Steps

- More general trajectories, e.g. radial infall
- More general spacetimes, e.g. SAdS geon
- Multiple detector scenarios

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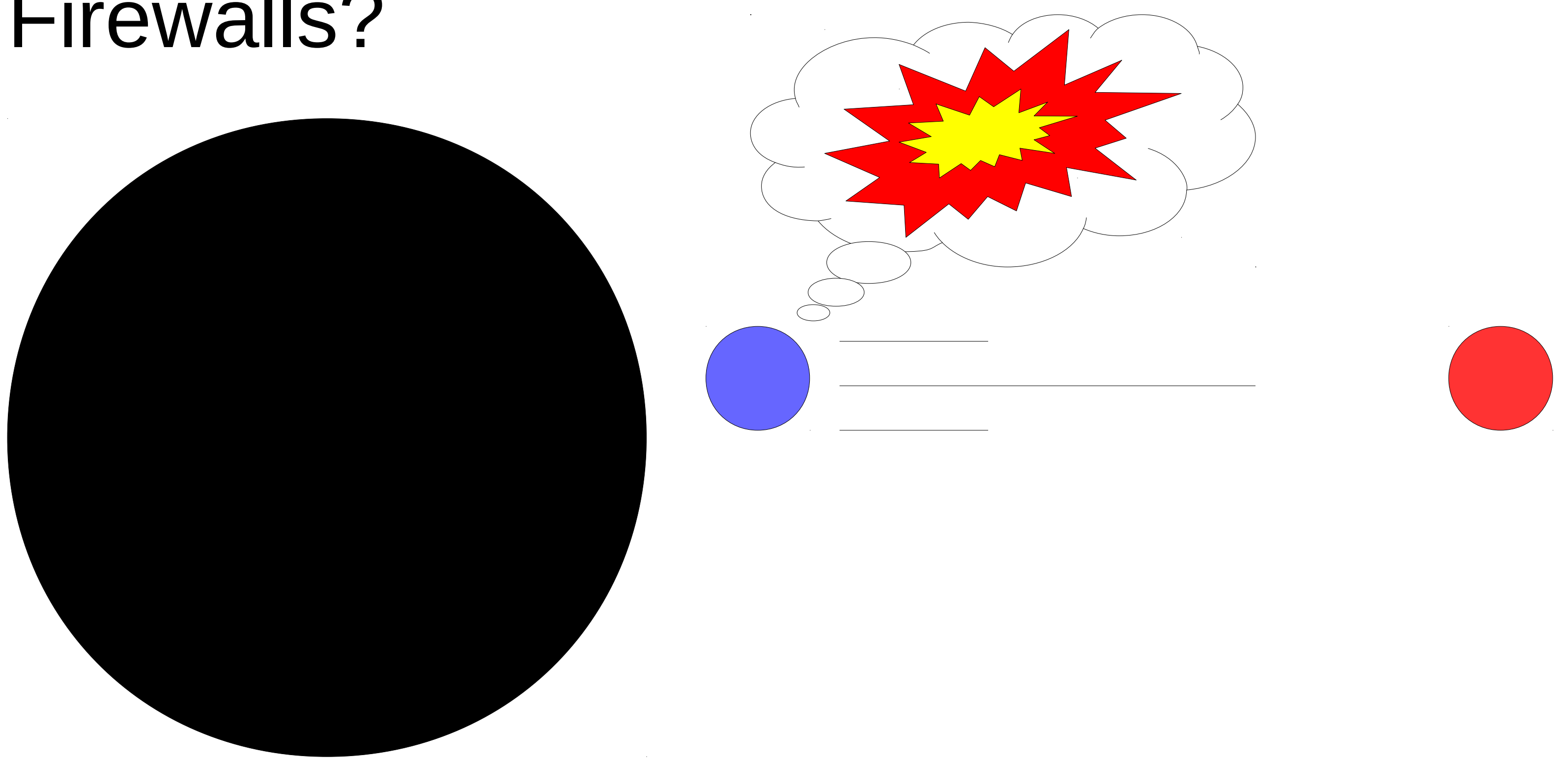
- More general spacetimes, e.g. SAdS geon
- Multiple detector scenarios
- Firewalls?



Acknowledgements

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- NSERC

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Thank You!