Trajectory Management of Self-Accelerating Beams Through Fourier-Space Phase Engineering

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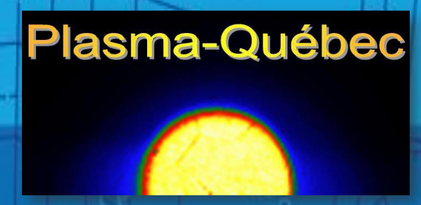
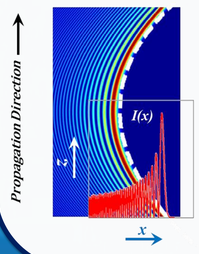


Introduction

Self-accelerating beams are optical beams featured by a transversely bending trajectory. Among them Airy beams have the unique characteristic of freely accelerating along parabolic trajectory without diffracting.

1D Airy Beam



Airy Beam' s Properties

- Non-diffractive propagation
- Bending trajectory
- Self-healing

References

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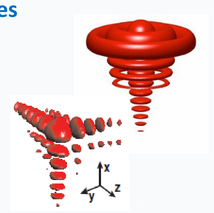
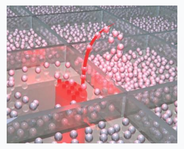
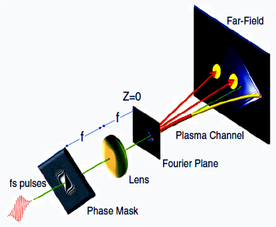


Optical Airy Bullets

Airy Beam's Applications



Optical Trapping



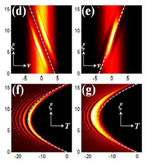
Curved-Plasma Channels

Motivation

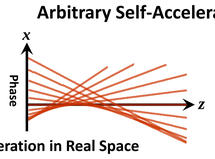
- Nowadays, self-accelerating beams can be designed to propagate along arbitrary convex trajectory. Most of them are produced by directly imposing a phase in the real space. A generalized method is lack to generate accelerating beam through engineering the spectrum.
- Our previous work showed that it is possible to map the spectrum into the propagation distance via suitable nonlinearities. So there may be a mapping relationship between spectrum and the trajectory of the self-accelerating beam even for linear regime.

Here, we propose a method to generate arbitrary self-accelerating beams through direct spectrum-to-distance mapping in linear regime.

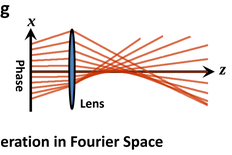
Mapping Via Nonlinearities



Arbitrary Self-Accelerating



Generation in Real Space



Generation in Fourier Space

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I. Kaminer, *et al*, PRL 108, 163901 (2012)
E. Greenfield, *et al*, PRL 106, 213902 (2011)
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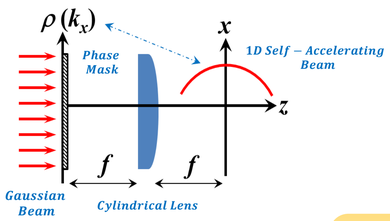
Theory

Setup



The spectral evolution is :

$$E(k_x, z) = exp[i\mu(k_x, z)]$$

where: $\mu(k_x, z) = -\frac{(k_x^2)}{2k}z + \rho(k_x)$

Diffraction Term — *Imposed Phase*

Mapping Spectrum to Space

Mapping I: $k_x \Leftrightarrow (x, z)$ — Phase Gradient $x = -\partial\mu(k_x, z)/\partial k_x$

In a small area Δx, spectral density is: $\frac{\Delta k_x}{\Delta x} = 1/\left|\frac{\partial^2\mu(k_x,z)}{\partial k_x^2}\right|$

Mapping II: $k_x \Leftrightarrow z$ — Spectral Density Singularity $\partial^2\mu(k_x, z)/\partial k_x^2 = 0$

Key Spatial Frequency

Phase Mask ⟺ Trajectory

Trajectory ⟺ Phase Mask

In our research, 1D self-accelerating beams are generated in the real space by phase-modulating a Gaussian beam in the spectral regime and computing the Fourier transformation through a cylindrical lens (see setup). We found the existence of a mapping relationship between spectrum and propagation distance in which different positions in the trajectory are mapped to different frequencies in the spectrum. From the knowledge of this spectrum-to-space mapping, we can predict the trajectory of a self-accelerating beam, given the spectral phase mask, or engineer the spectral phase mask for any desired convex trajectory.

Paraxial Beams

Phase Mask ⟺ Trajectory

Monotonic Key Spatial Frequency | *Non – Monotonic Key Spatial Frequency*

Spectral Phase Modulation

$$\rho(k_x) = \left(\alpha\frac{k_x}{k}\right)^3$$

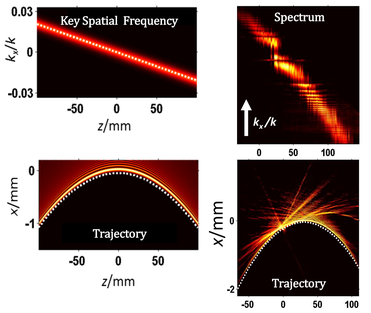
Mapping II: Key Spatial Frequencies

$$\frac{\partial^2\mu(k_x, z)}{\partial k_x^2} = 0 \quad \Rightarrow \quad k_{xc}(z) = \frac{k^2}{6\alpha^3}z$$

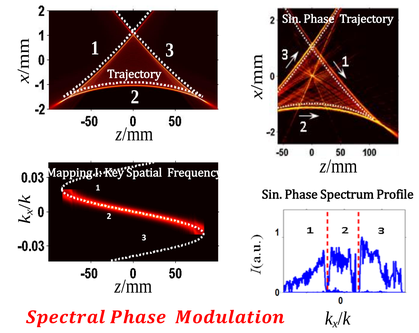
Mapping I + II: Trajectory Equations

$$x = -\frac{\partial\mu[k_{xc}(z), z]}{\partial k_x} \Rightarrow x(z) = \frac{k}{12\alpha^3}z^2$$

Simulations | Experiment Results



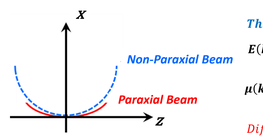
Simulations | Experiment Results



Spectral Phase Modulation

$$\rho(k_x) = \beta sin\left(\alpha\frac{k_x}{k}\right)$$

Non-Paraxial Beams



The spectral evolution is :

$E(k_x, z) = exp[i\mu(k_x, z)]$ where:

$\mu(k_x, z) = \sqrt{k^2 - k_x^2}z + \rho(k_x)$

Diffraction Term — *Imposed Phase*

Trajectory ⟺ Phase Mask

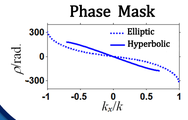
Elliptic Trajectory

$$x(z) = a\left(b^2 - z^2\right)^{1/2}/b$$

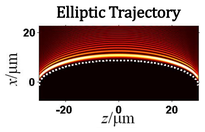
Hyperbolic Trajectory

$$x(z) = c\left(d^2 + z^2\right)^{1/2}/d$$

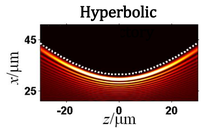
Phase Mask



Elliptic Trajectory



Hyperbolic

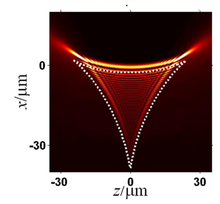


Phase Mask ⟺ Trajectory

Non – Paraxial Airy Beam

$$\rho(k_x) = (\alpha k_x/k)^3$$ *Phase Mask Modulation*

Cubic Phase Mask

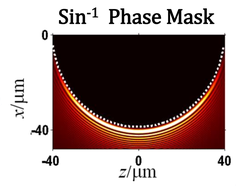


Circular Self – Accelerating Beam

$$\rho(k_x) = rsin^{-1}(k_x/k)$$ *Phase Mask Modulation*

$$x(z) = -\sqrt{r^2/k^2 - z^2}$$ *Trajectory Equation*

Sin^{-1} Phase Mask



Conclusion

We demonstrated a generalized scheme to generate and control single- or multi-path self-accelerating beams through spectrum-engineered position mapping. Furthermore, we showed that our method can be applied to paraxial and non paraxial self-accelerating beams

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