Coherent States and Particle Scattering *in* Loop Quantum Gravity

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Outline

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- Spinor helicity formalism

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- Implications

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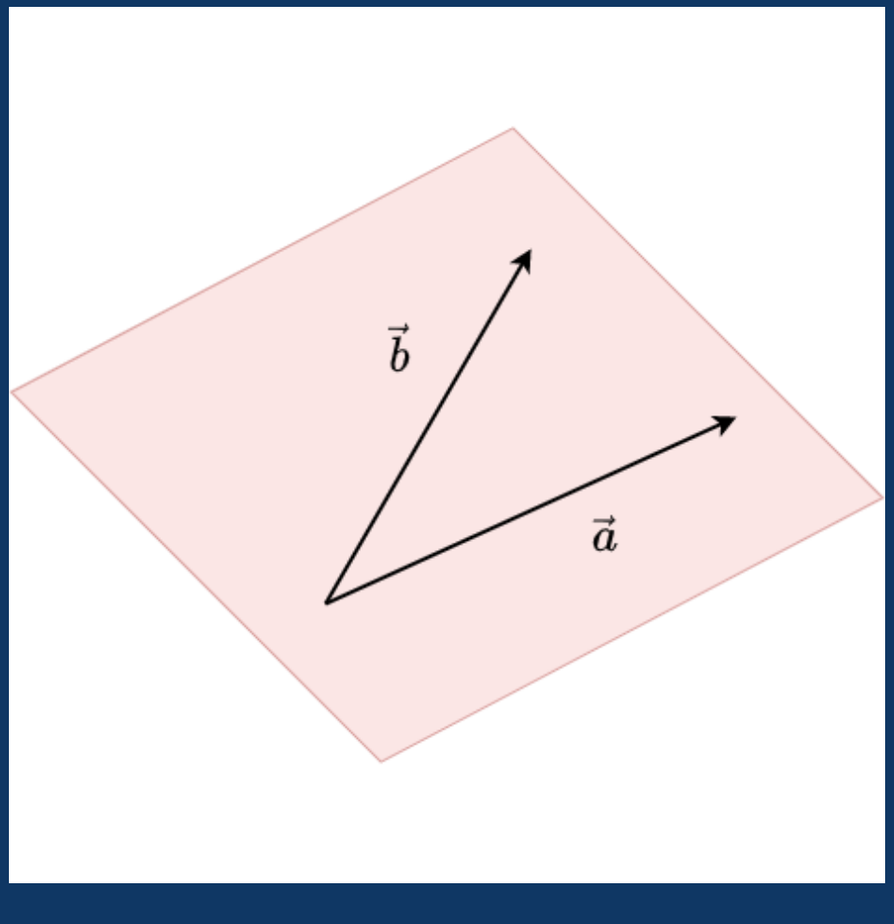
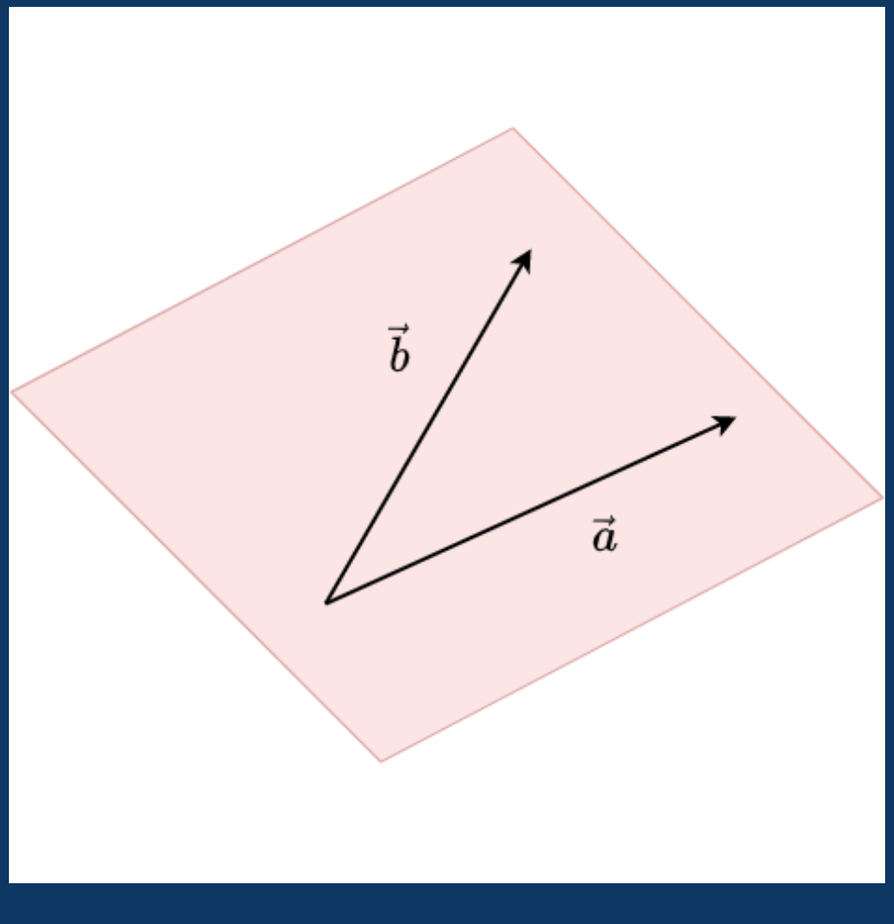
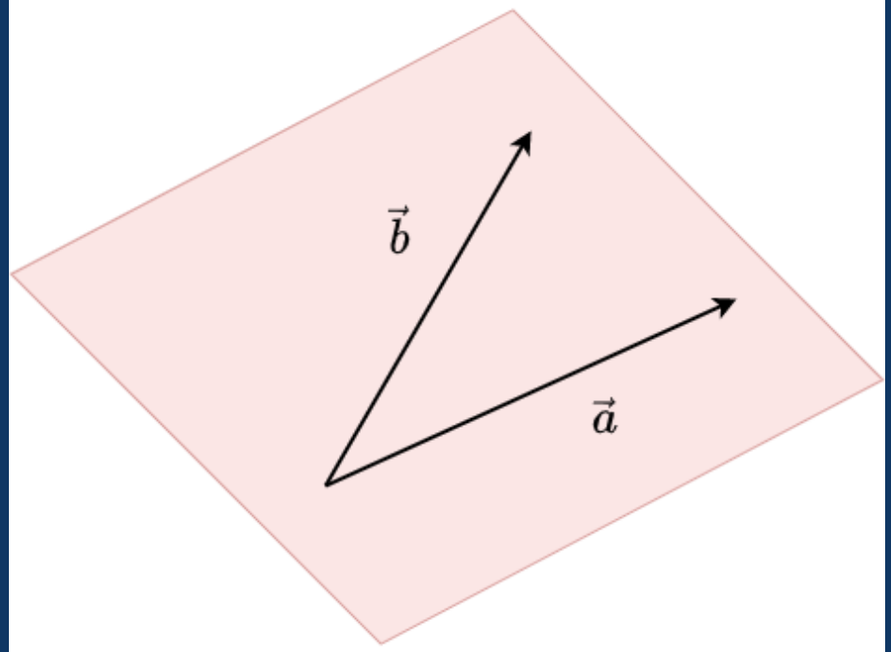
$$\overrightarrow{a}' = \alpha\,\overrightarrow{a} + \beta\,\overrightarrow{b}$$
$$\overrightarrow{b}' = \gamma\,\overrightarrow{a} + \delta\,\overrightarrow{b}$$





Lorentz Transformation



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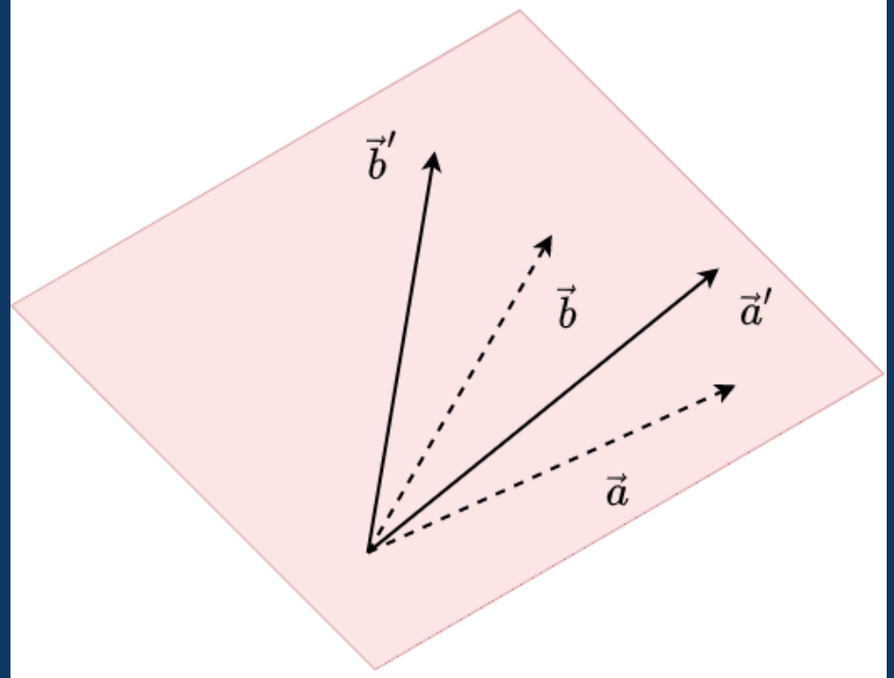

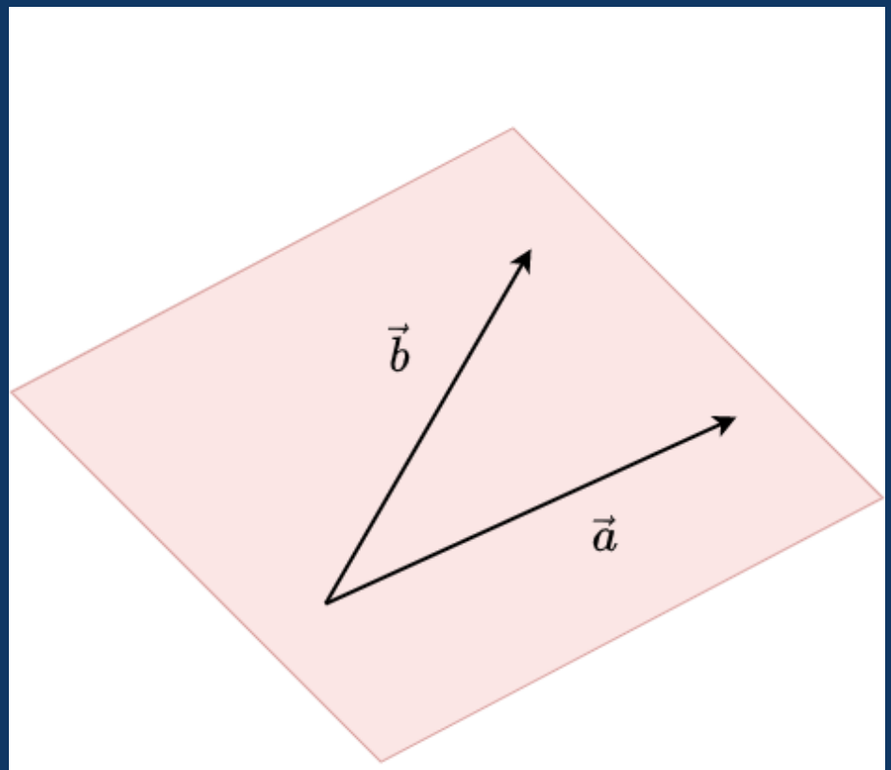

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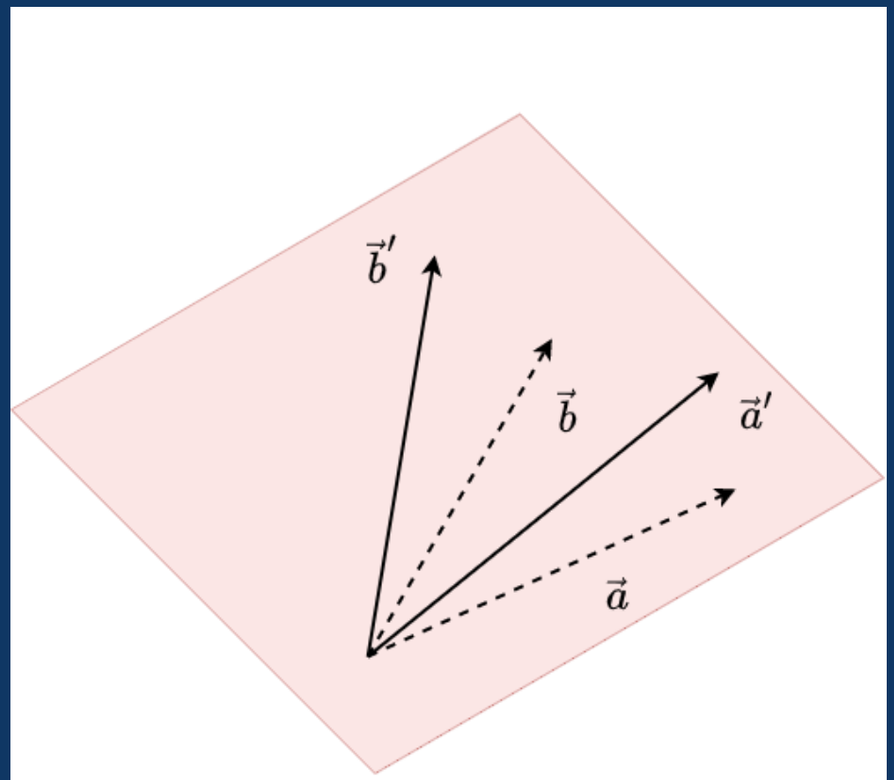

- the set of kinematic data corresponding to the scattering of N massless particles is labelled by a 2 dimensional plane in $\mathbb{C}^N$

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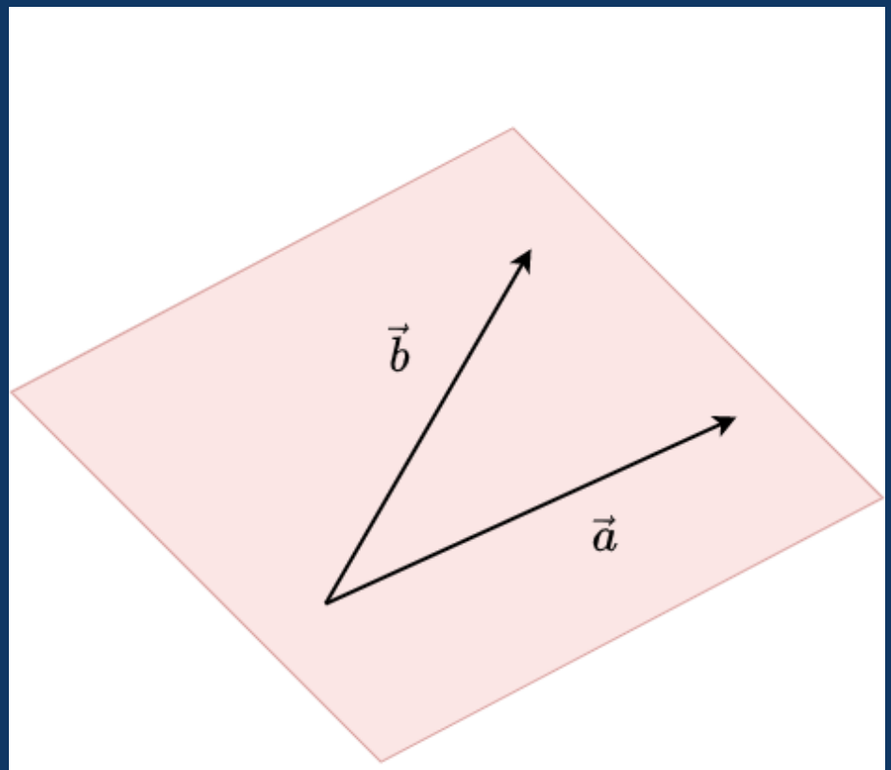

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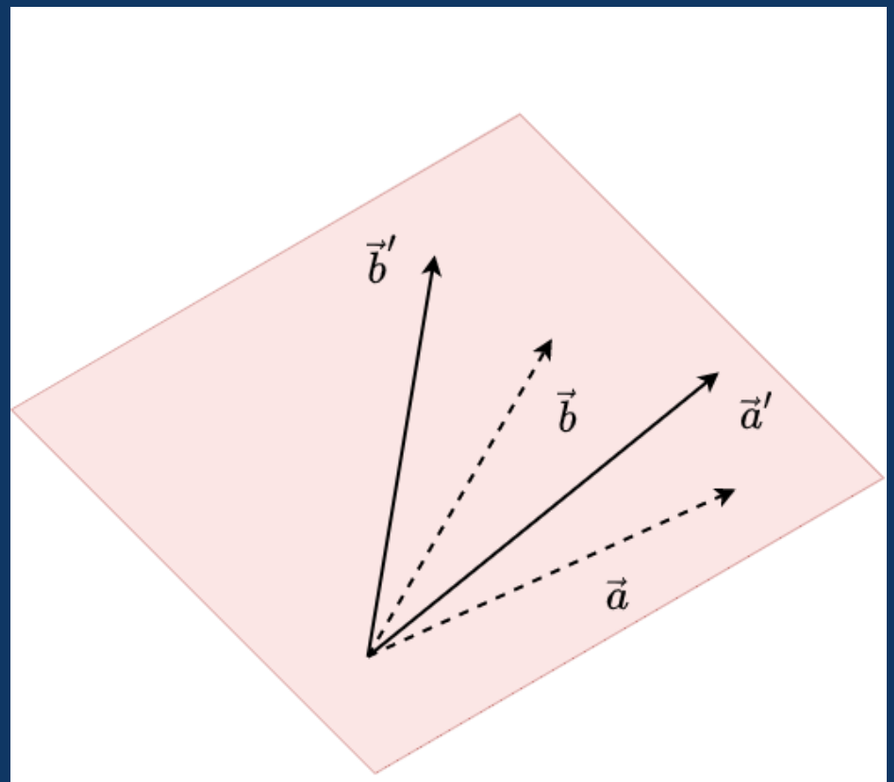

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The space of kinematic data for the scattering of N massless particles in Minkowski space consists of the set of all two-planes in $\mathbb{C}^N$, also known as the Grassmannian $Gr_{2,N}$.

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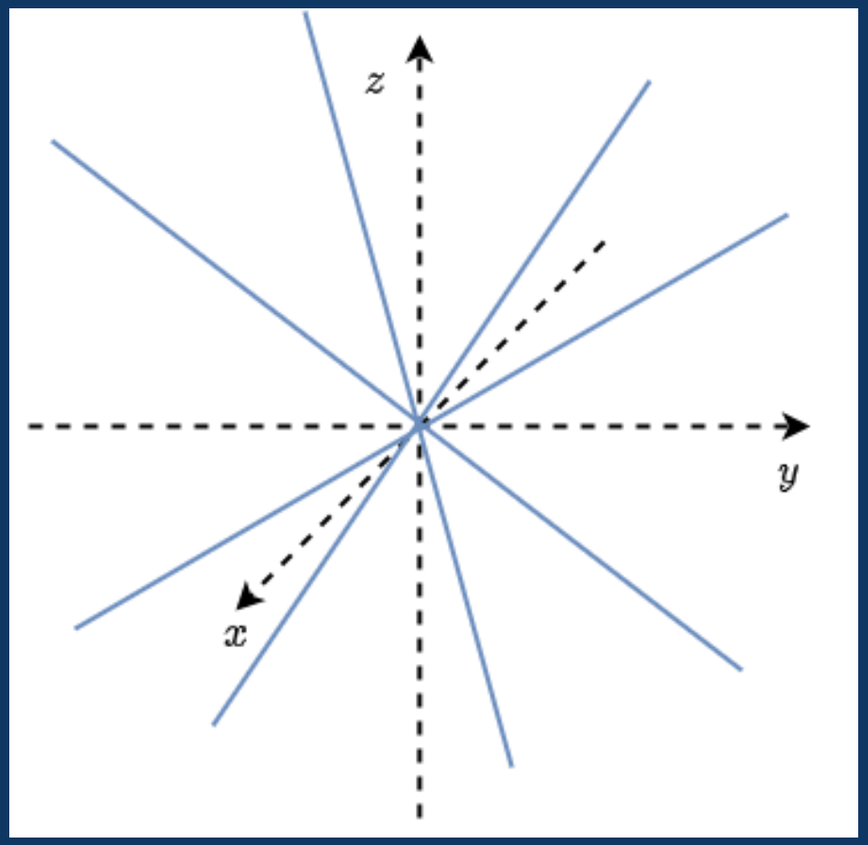
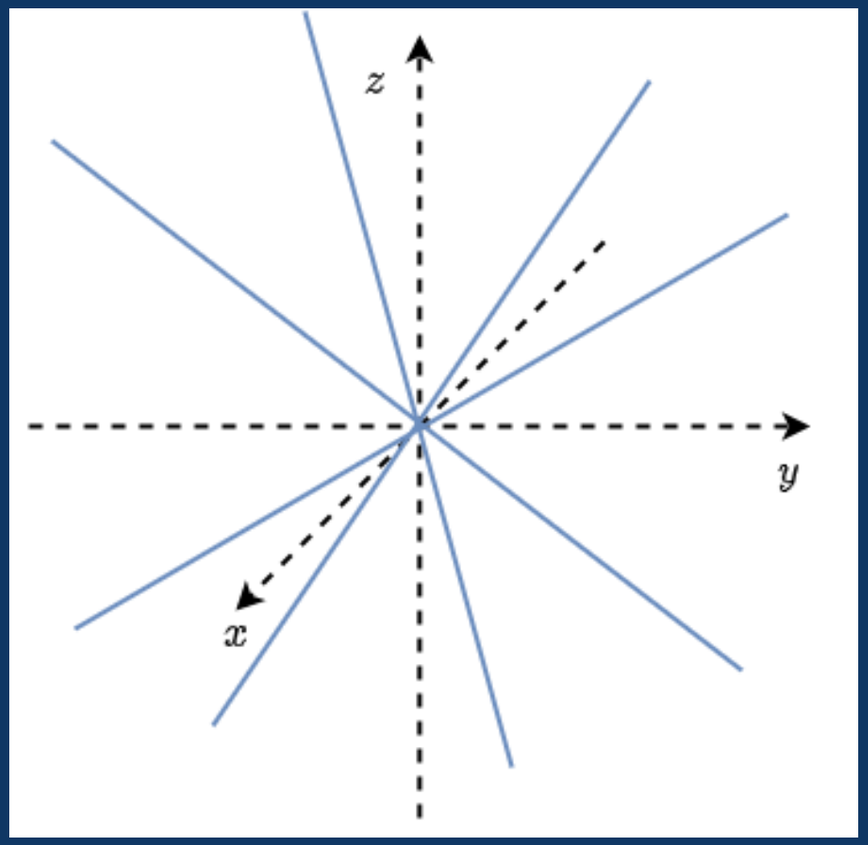


$Gr(3,1)$



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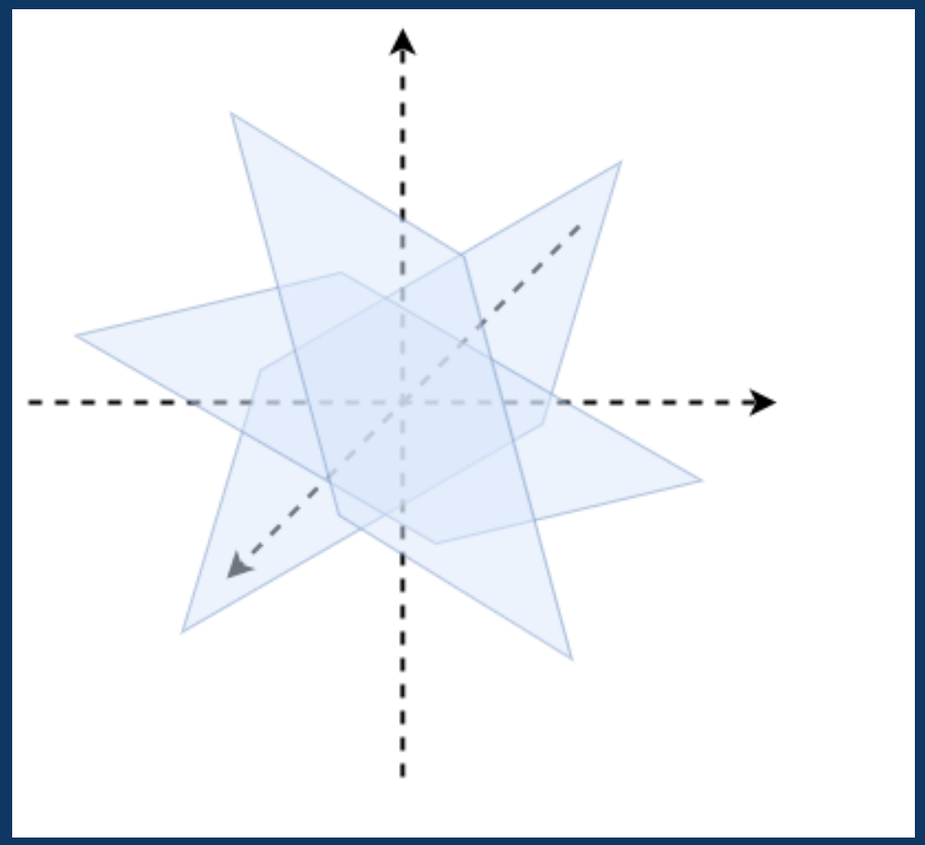

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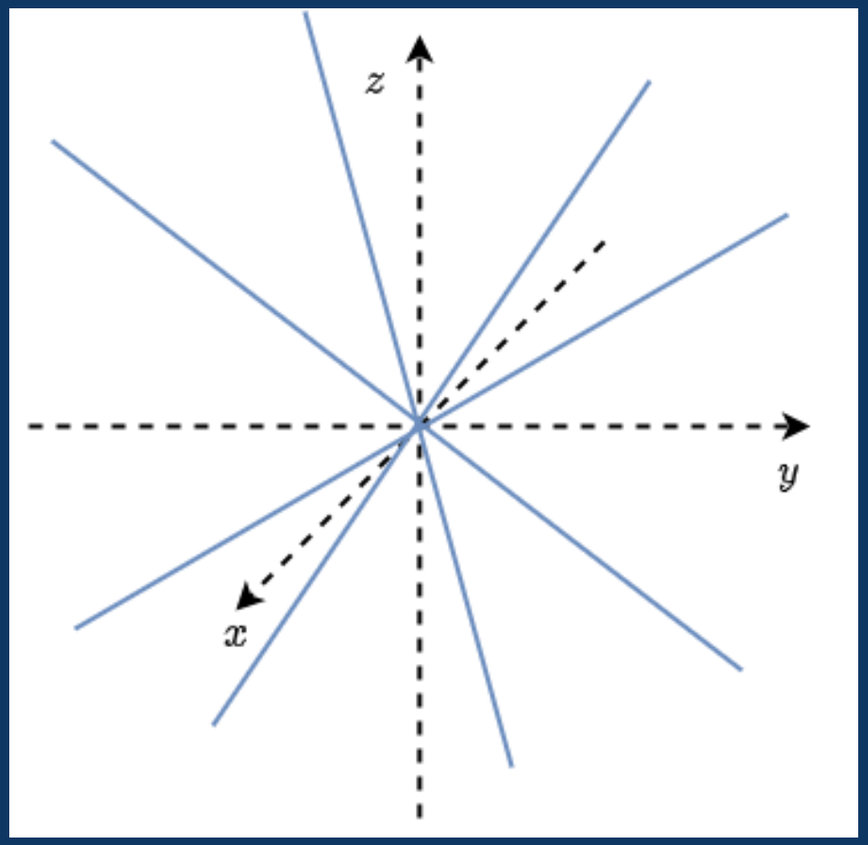

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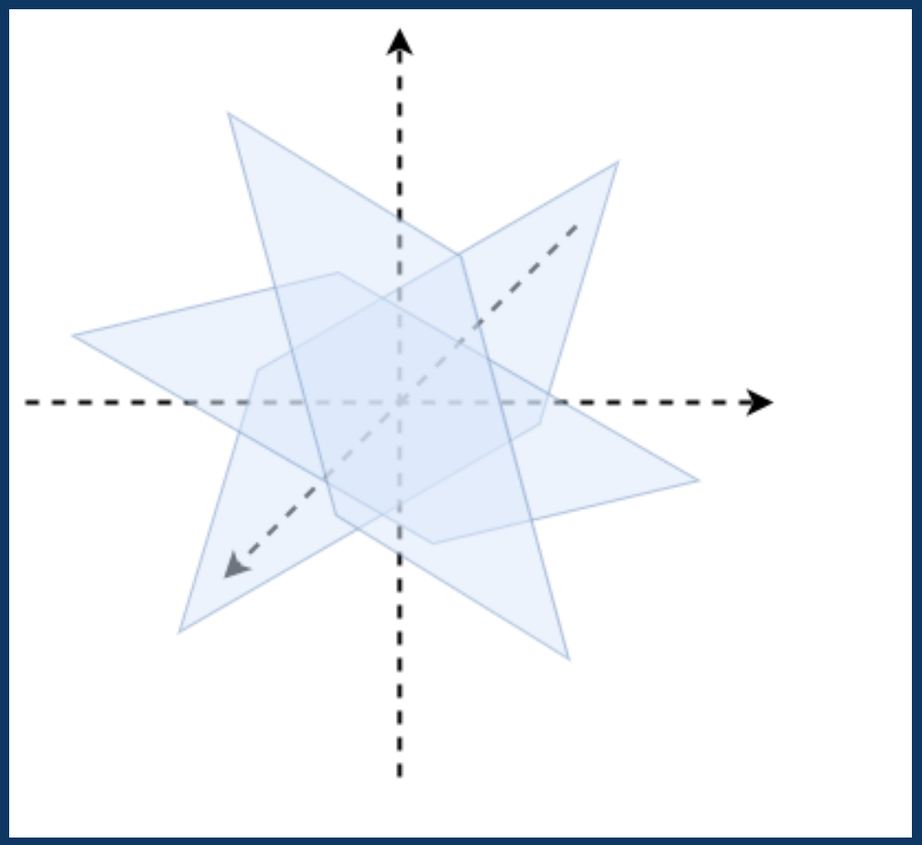

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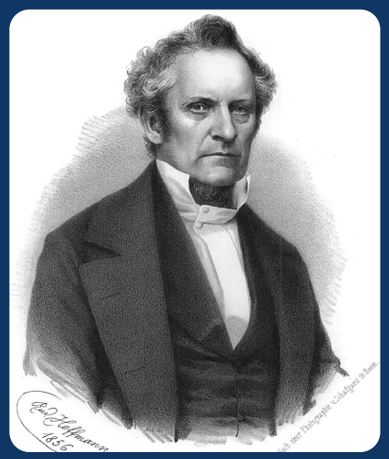
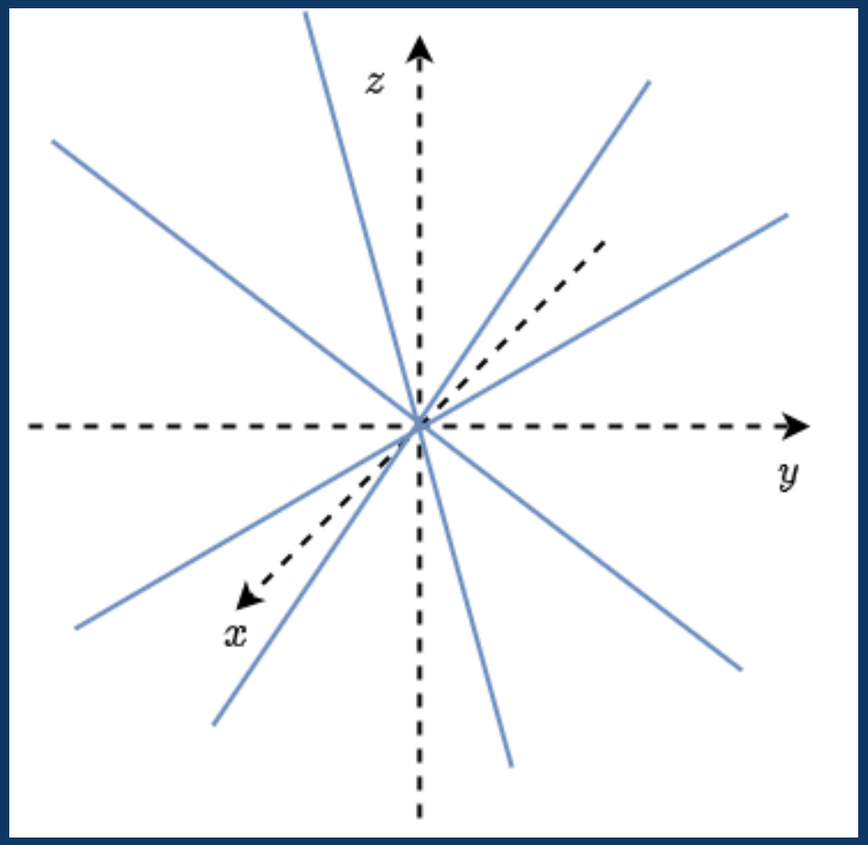


Julius Plücker



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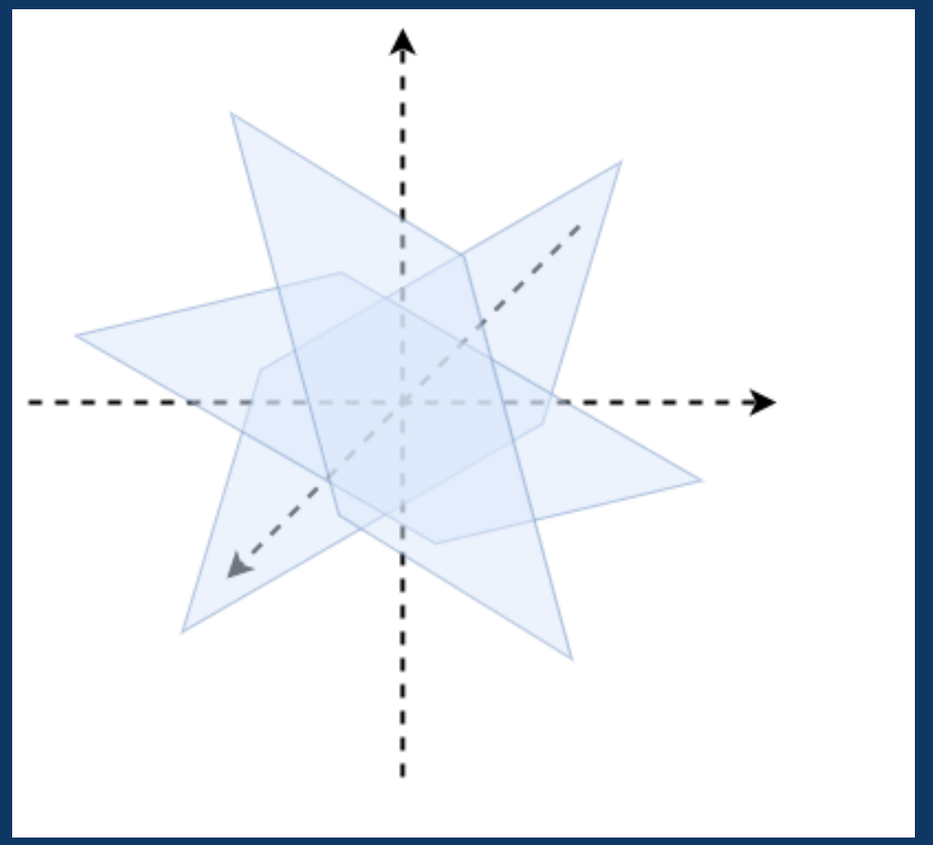

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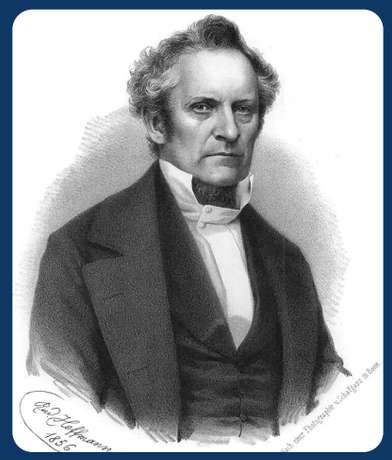

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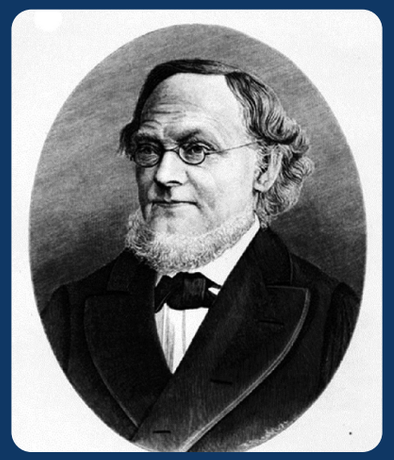
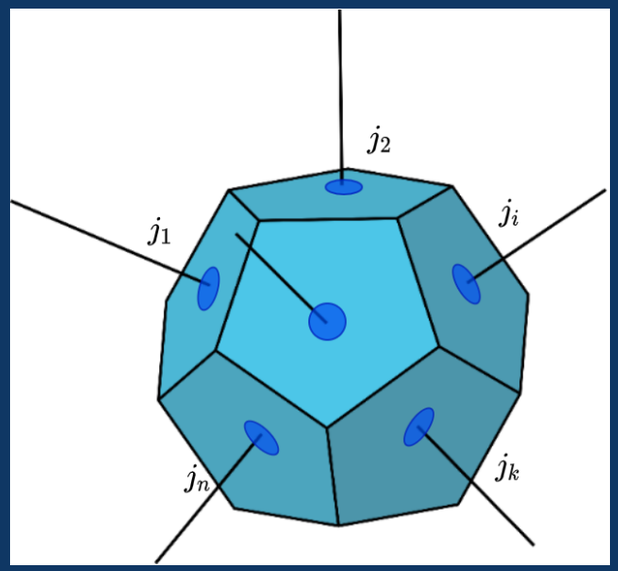


Hermann Grassmann

Intertwiners



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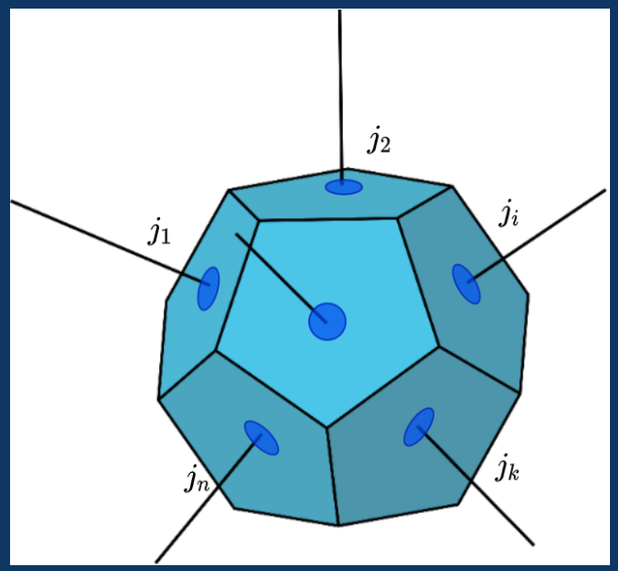

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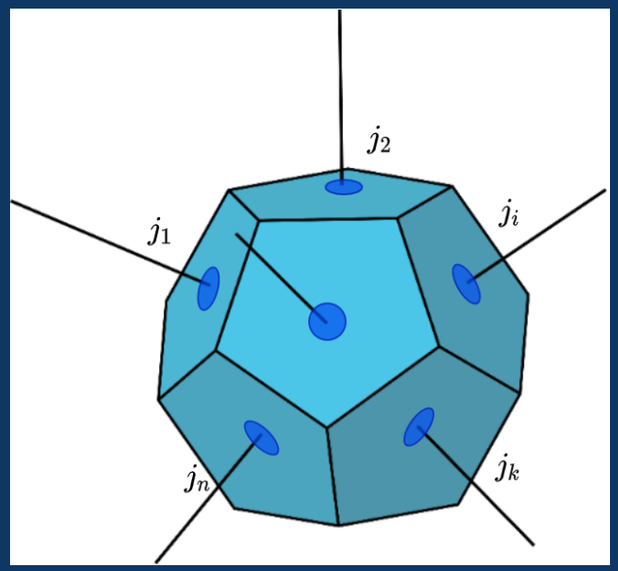

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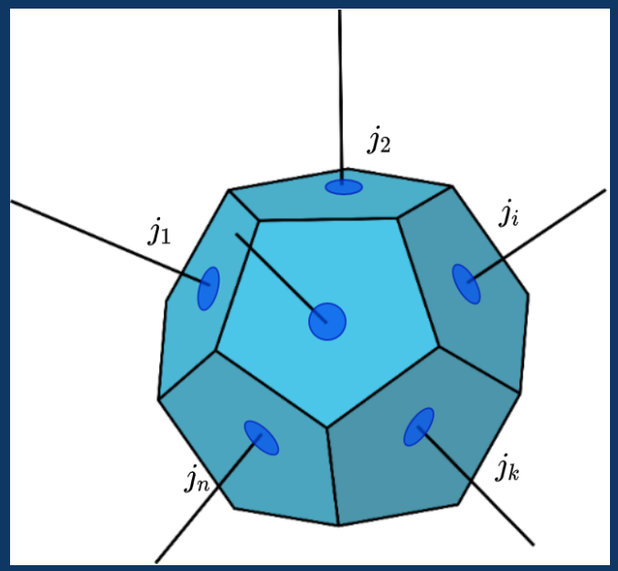

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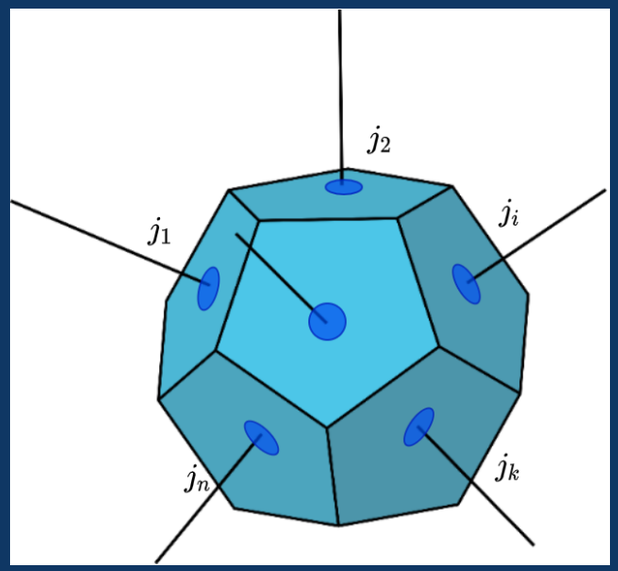

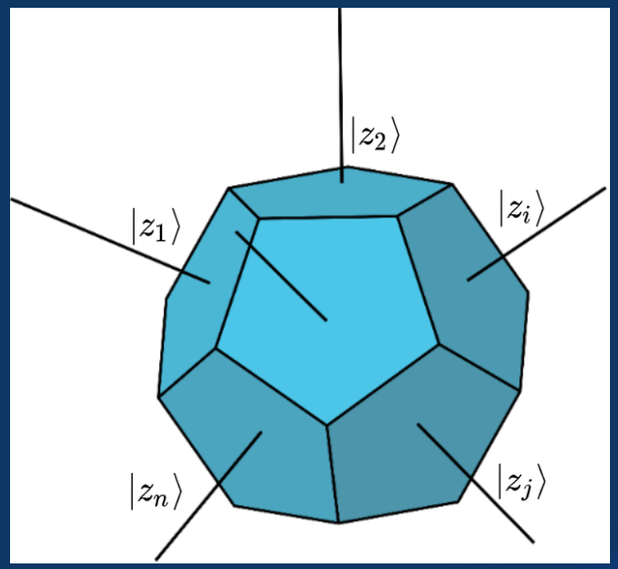

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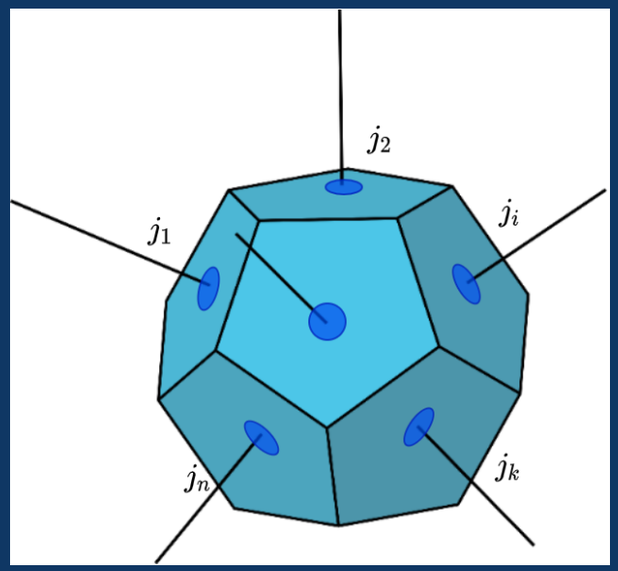

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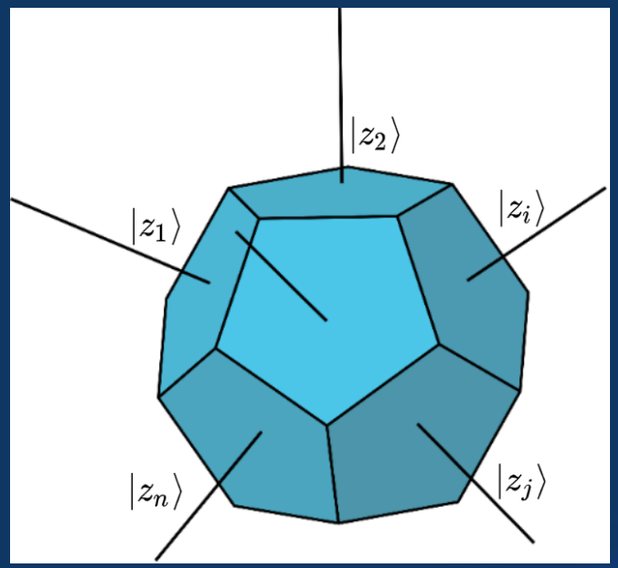

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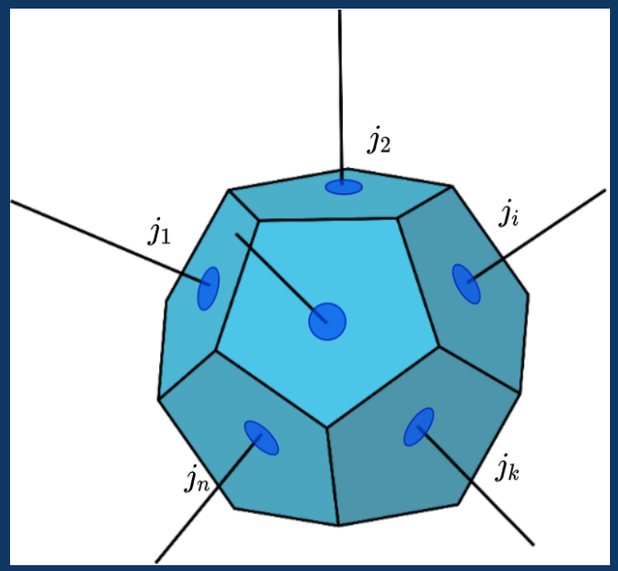

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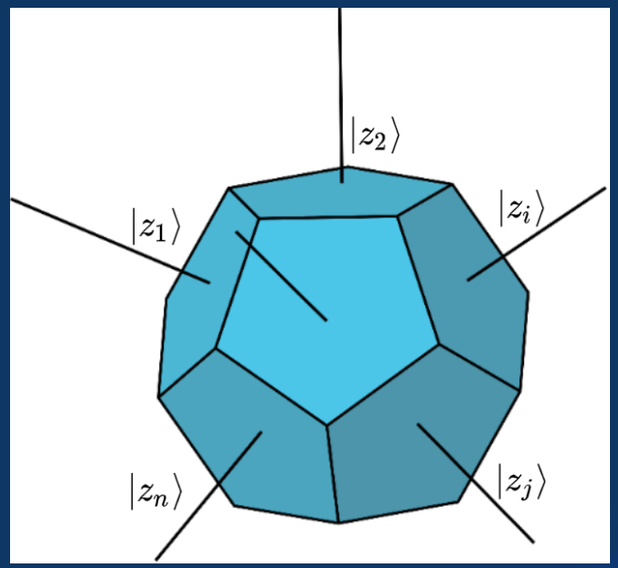

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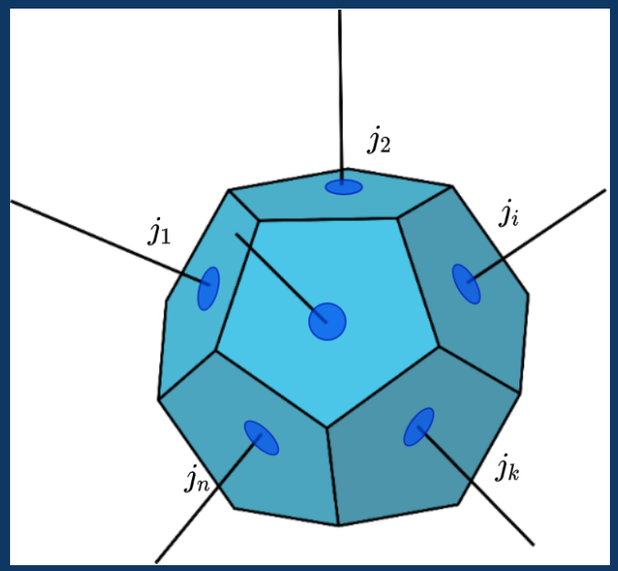

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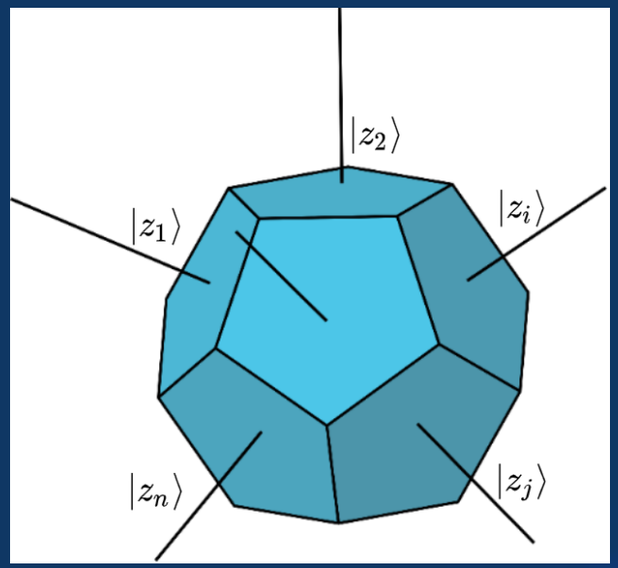
$$\|X_i\|^2 = \langle z_i \,|\, z_i\rangle = 2\,A_i$$



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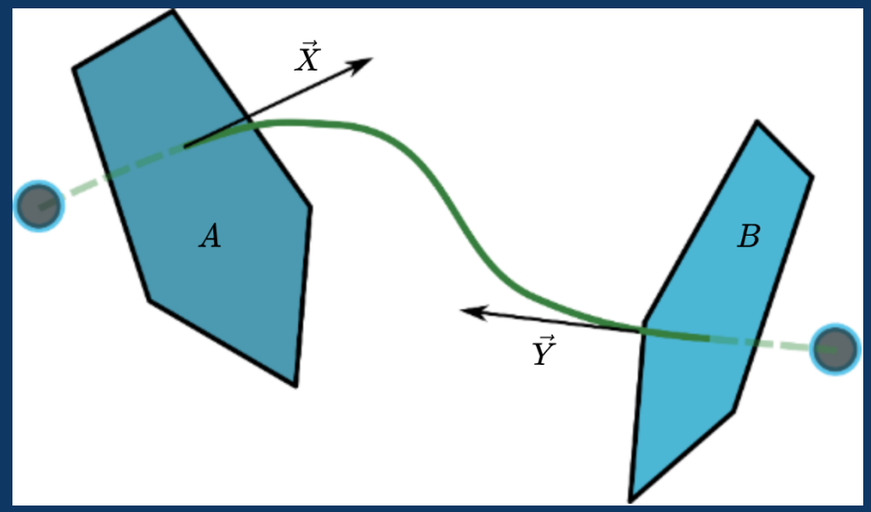

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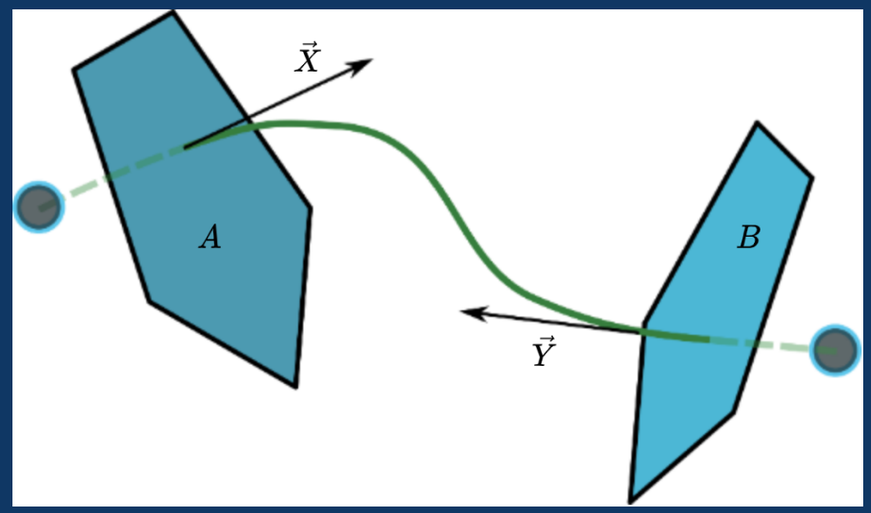

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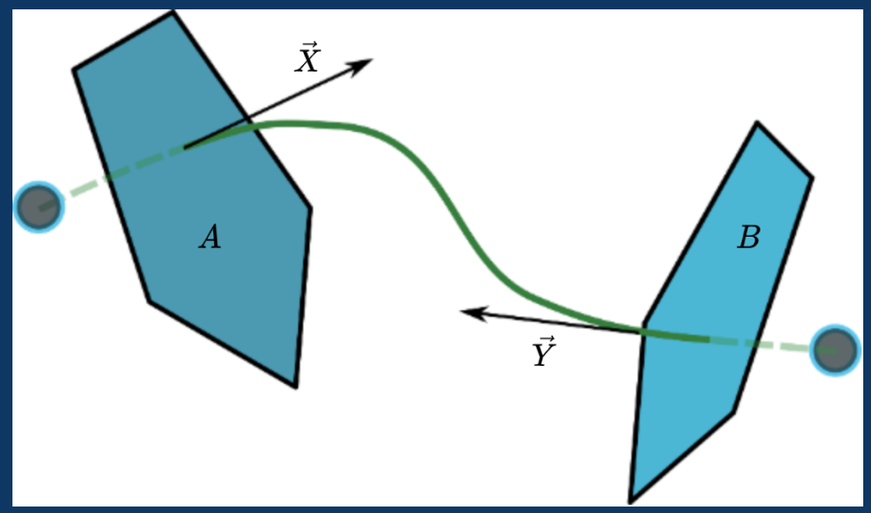

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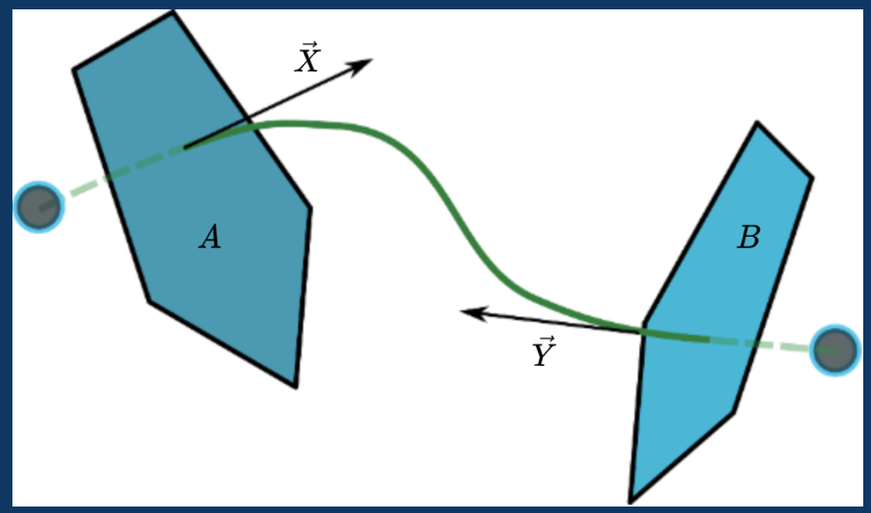

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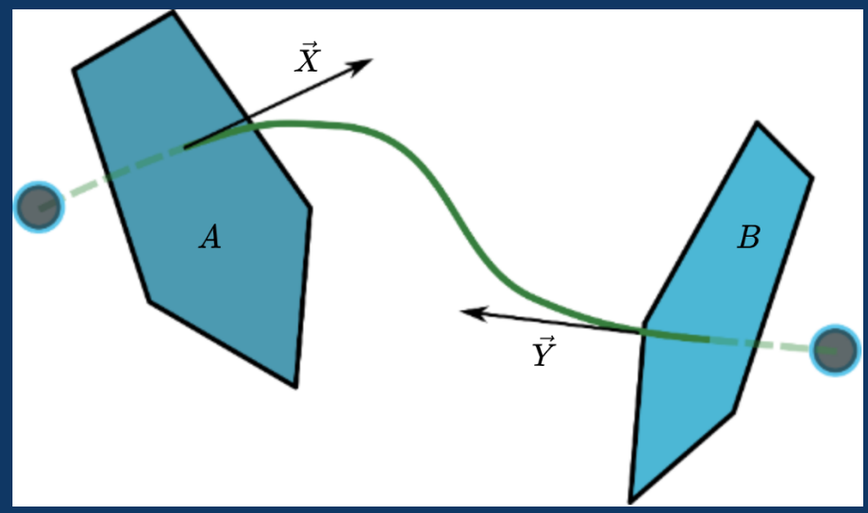

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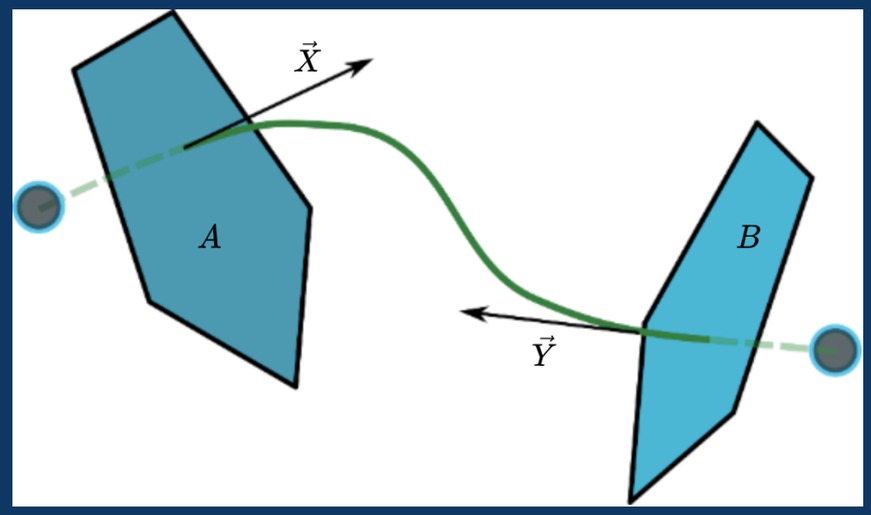

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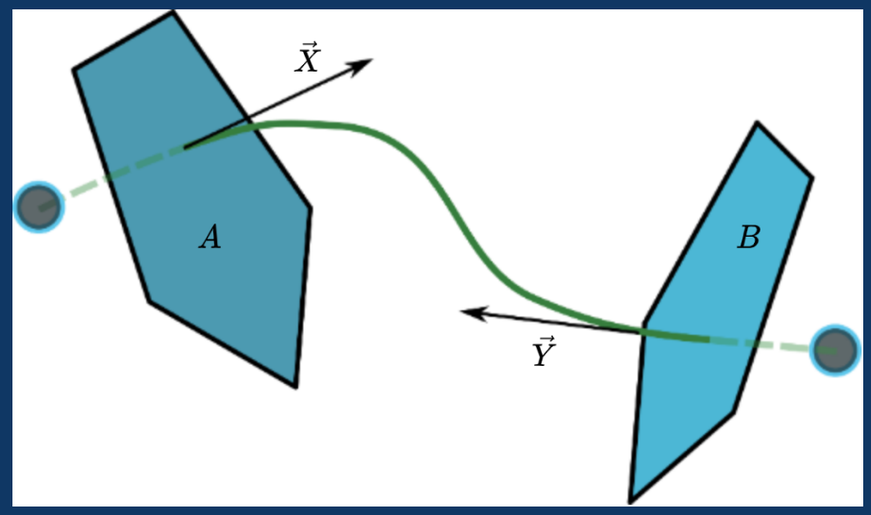

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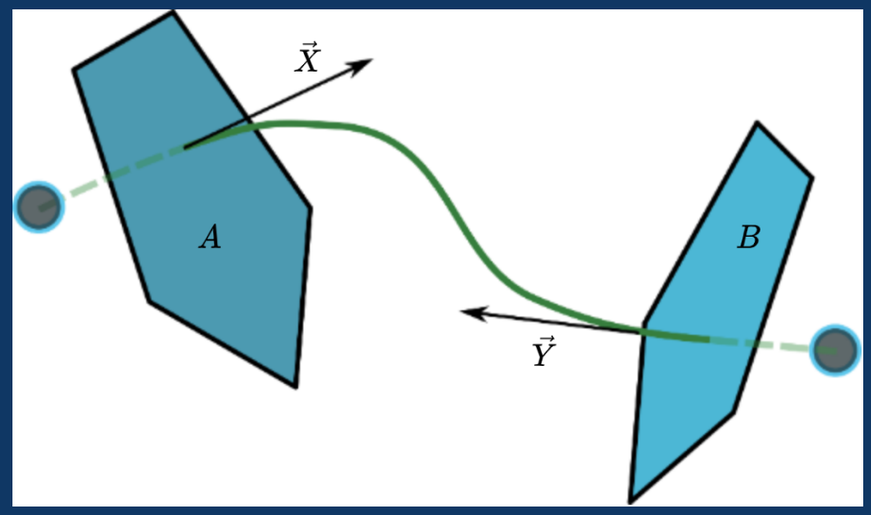

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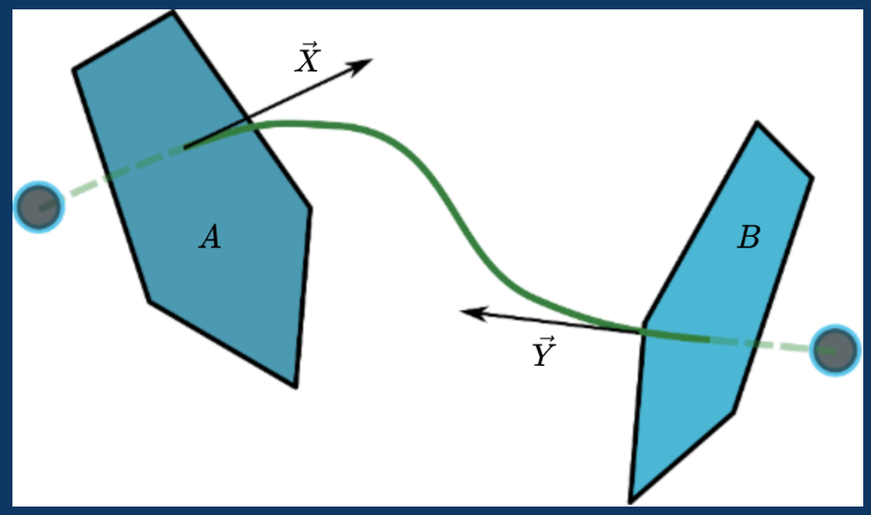

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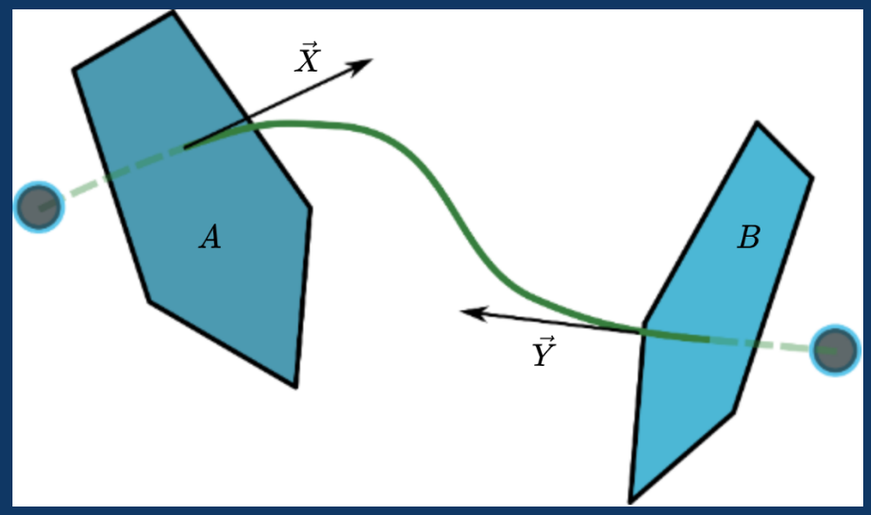

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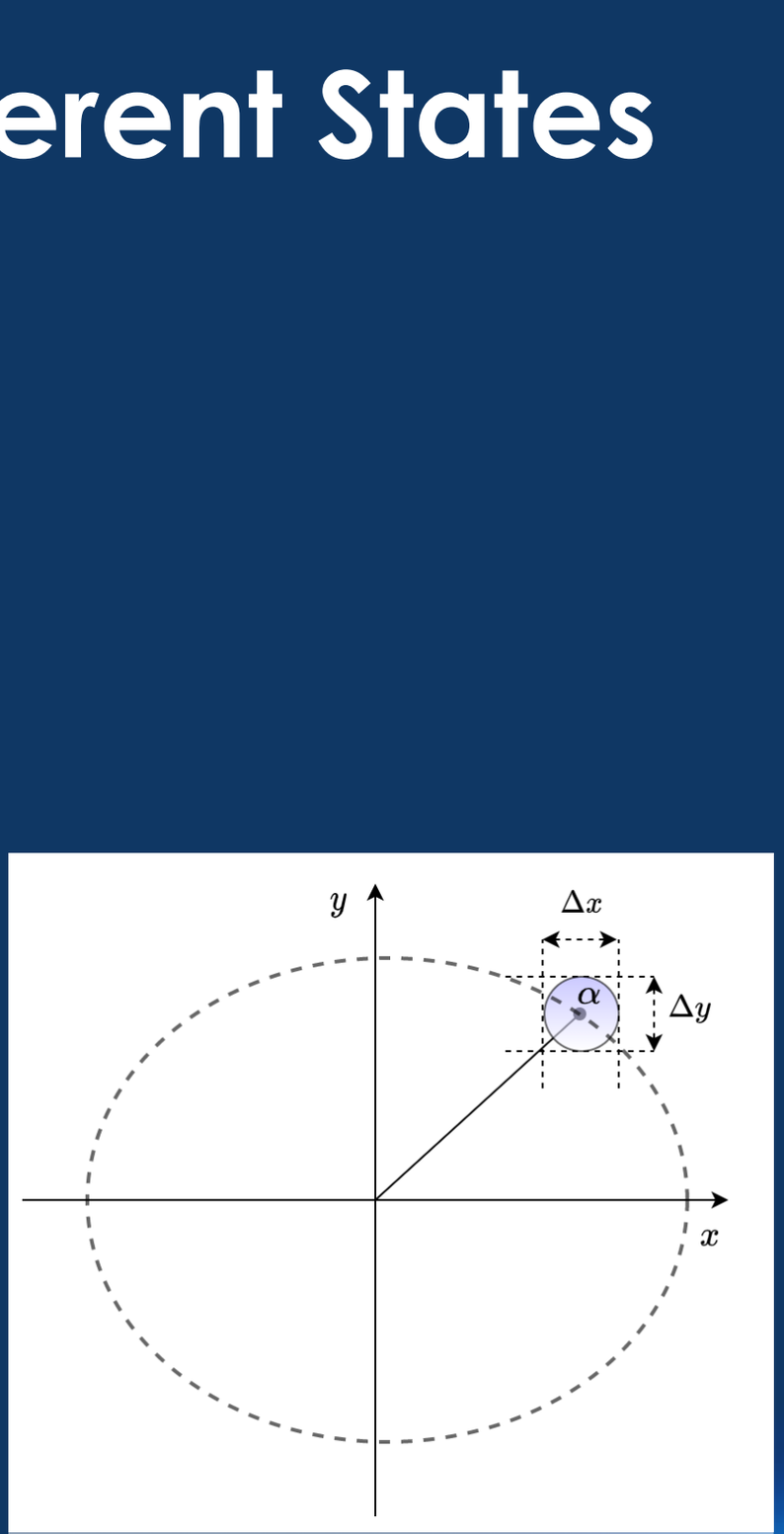

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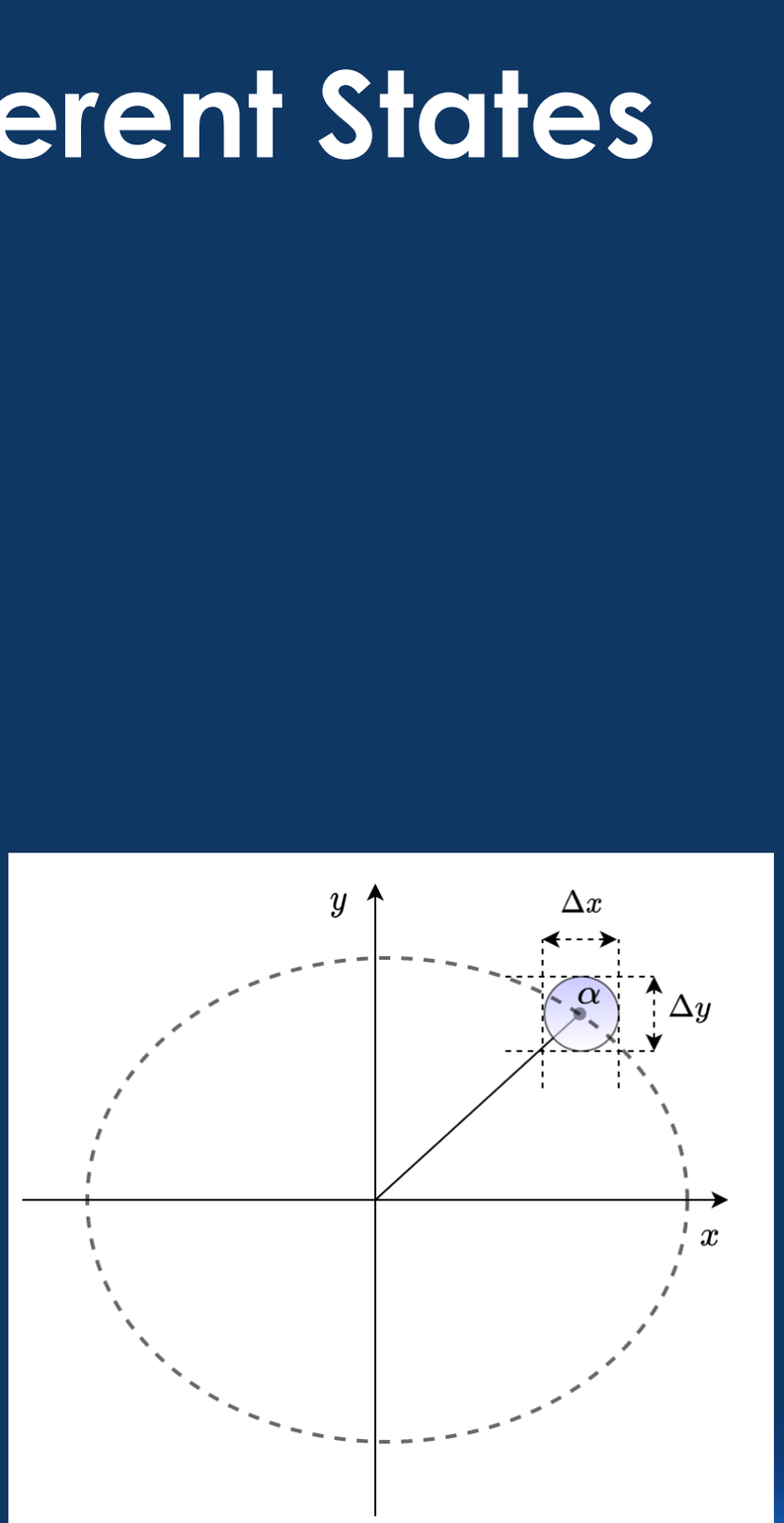

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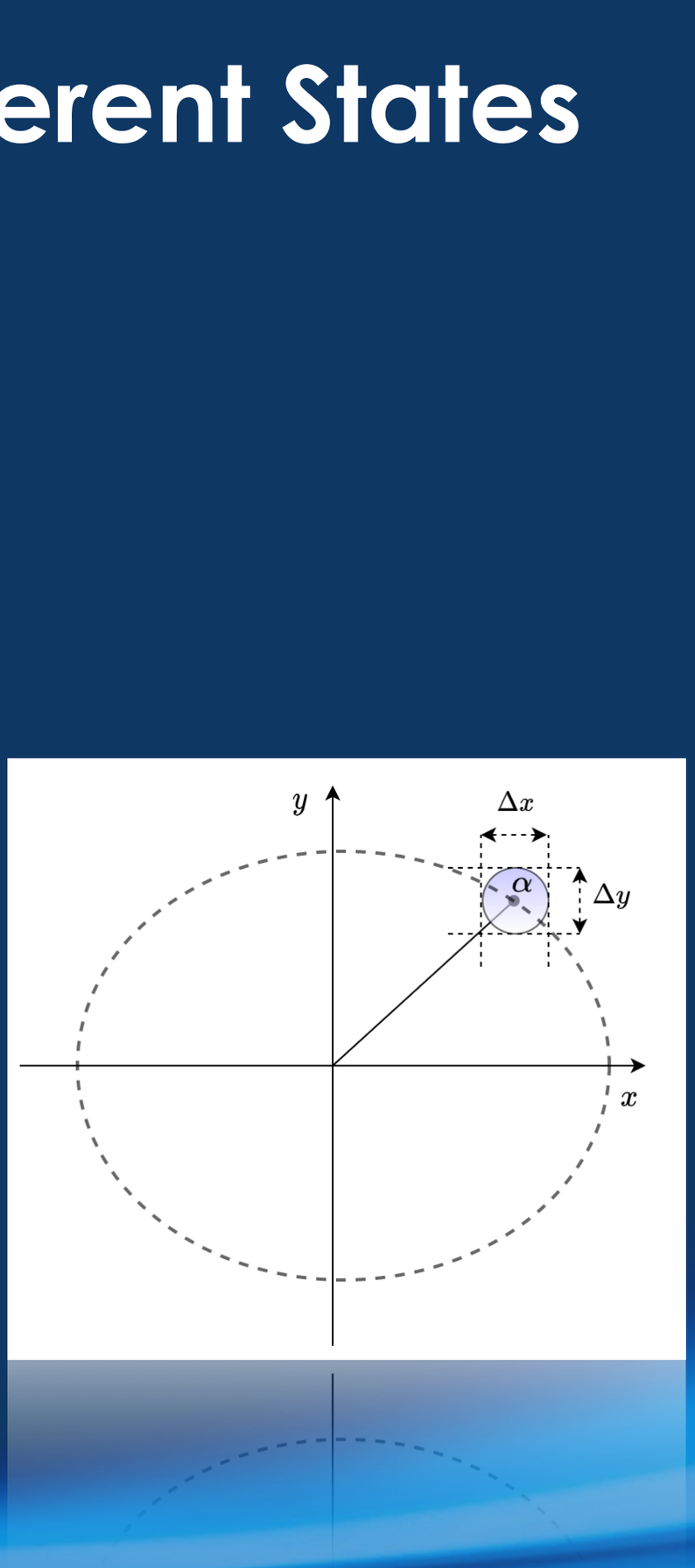

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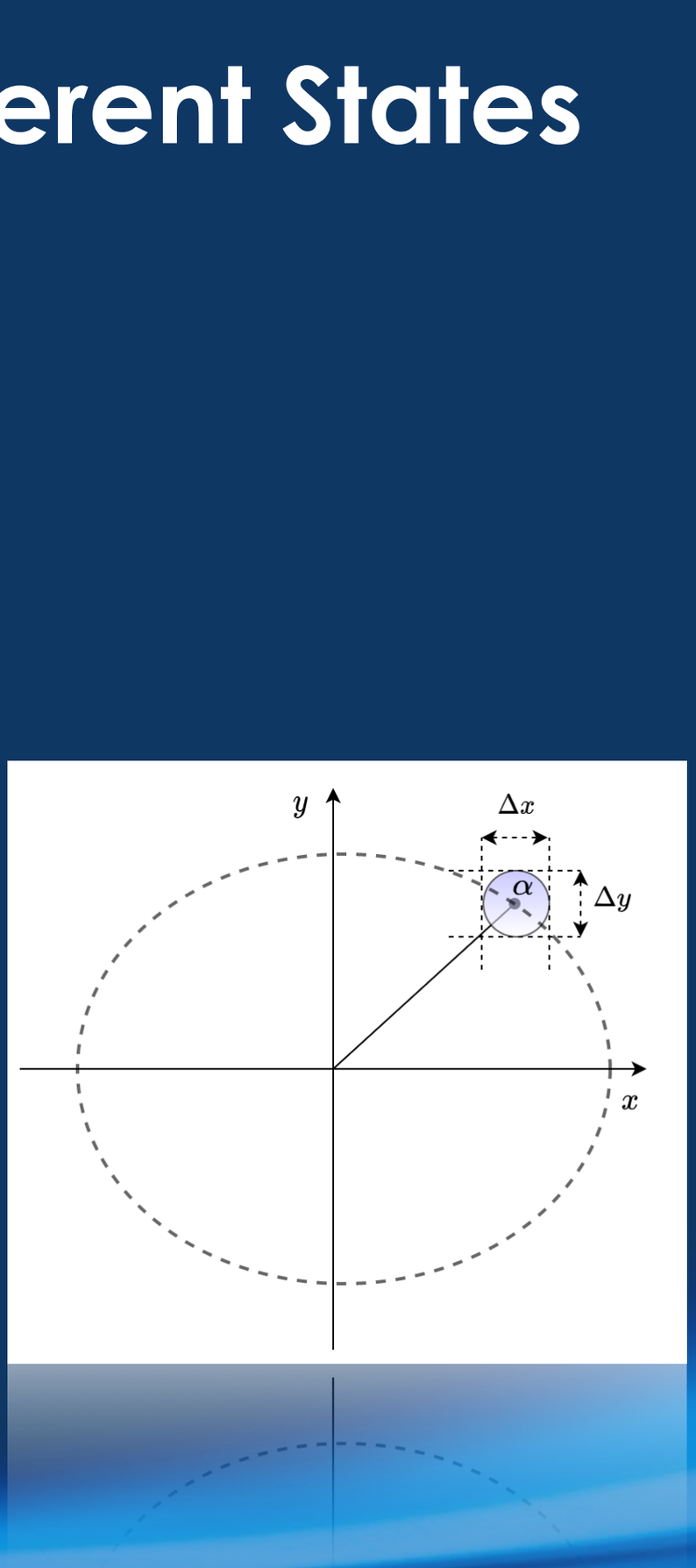

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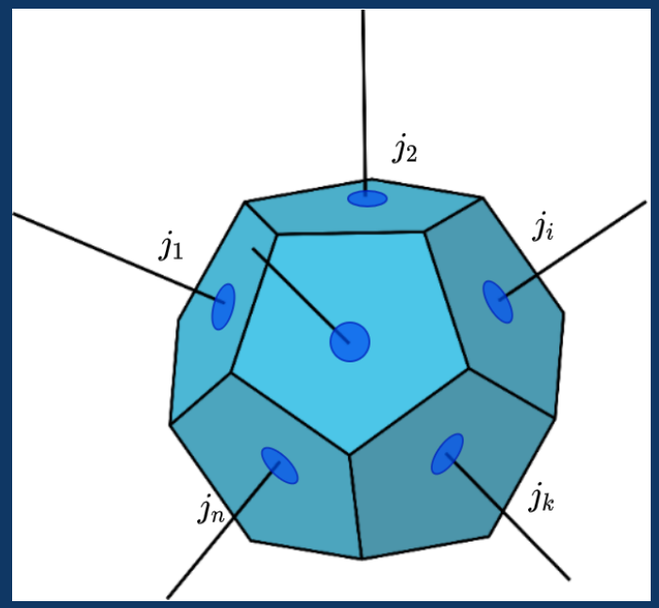

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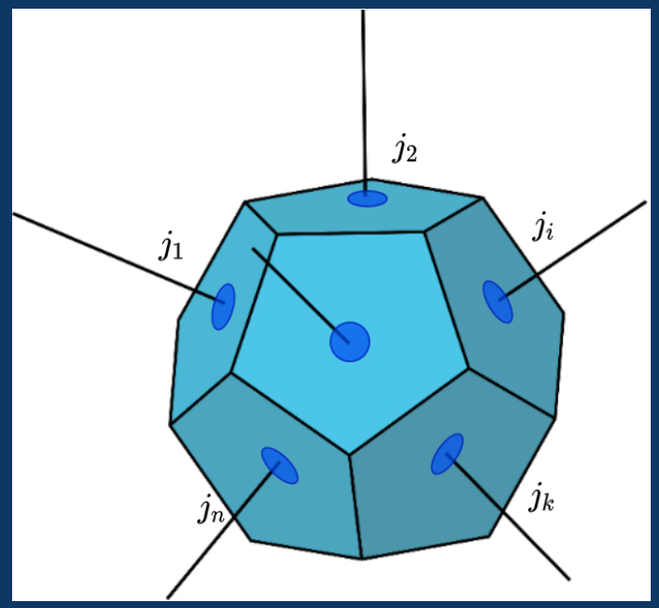

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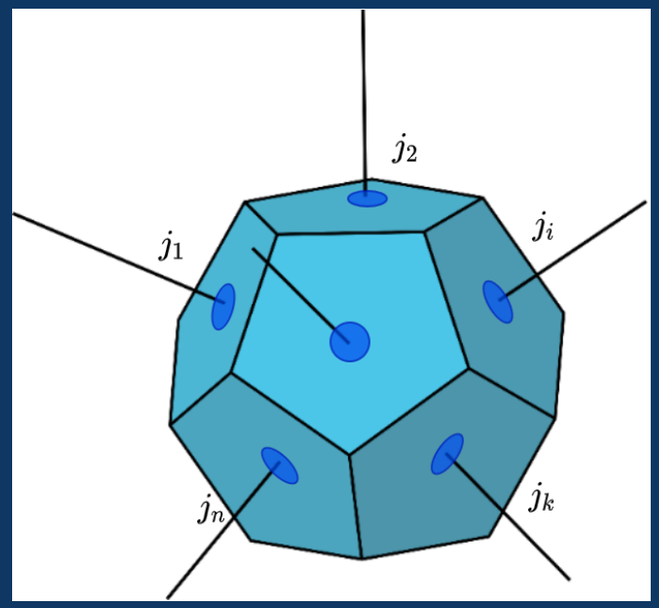

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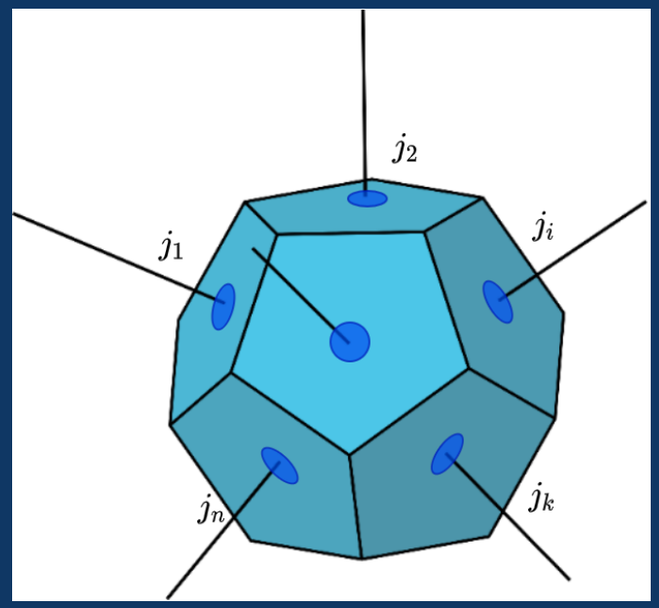

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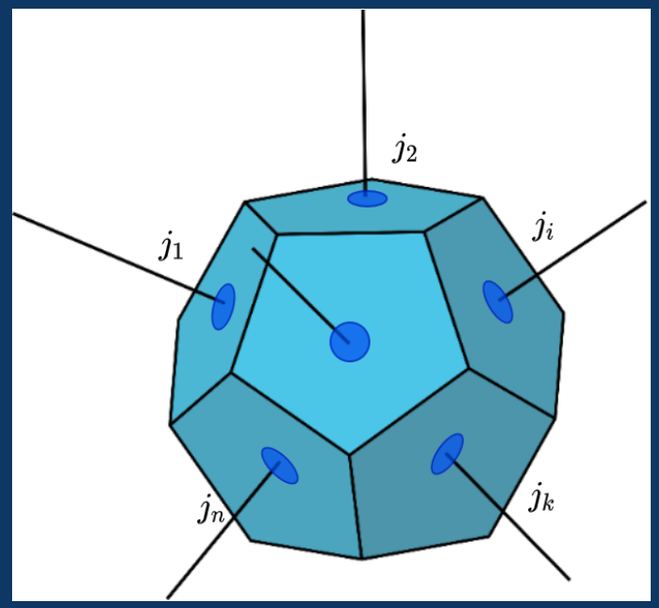

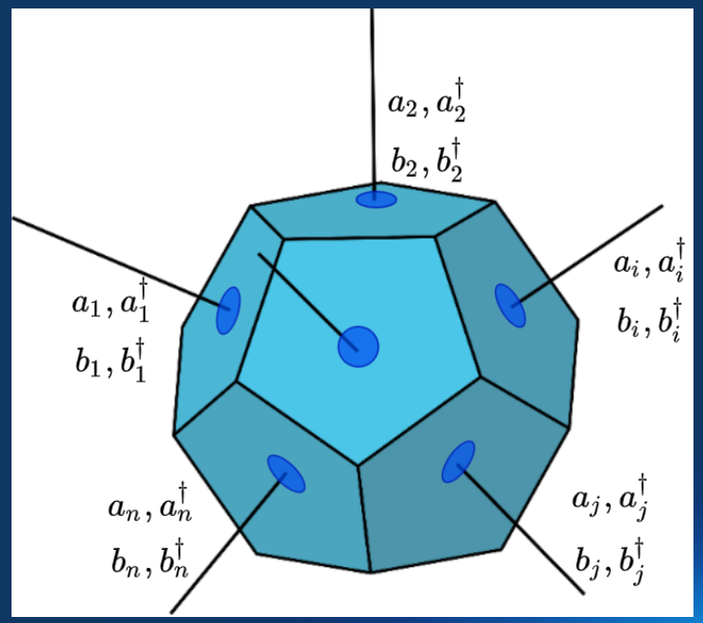

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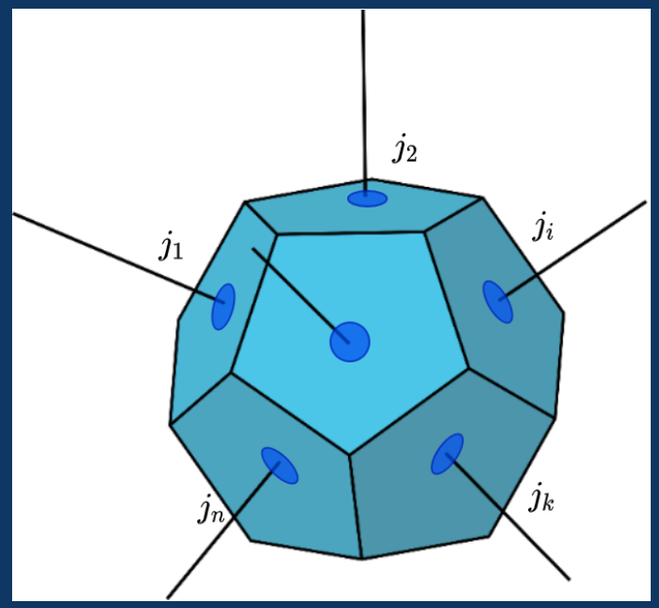

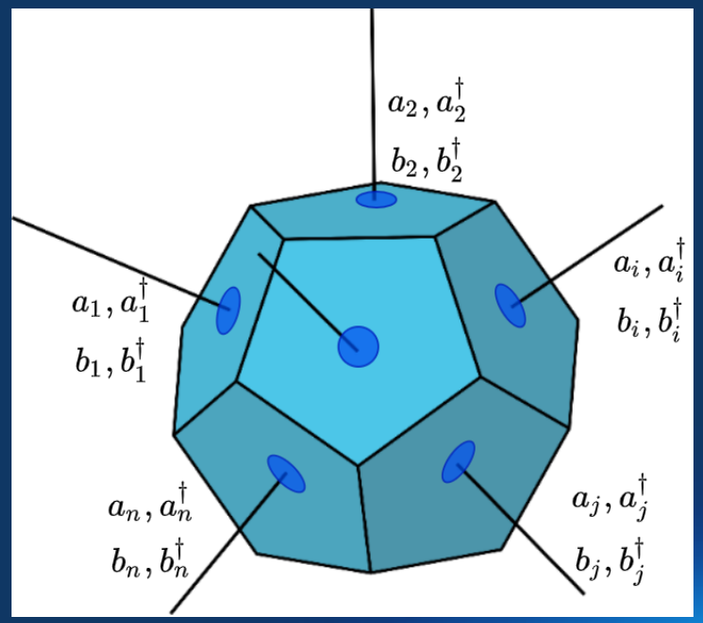

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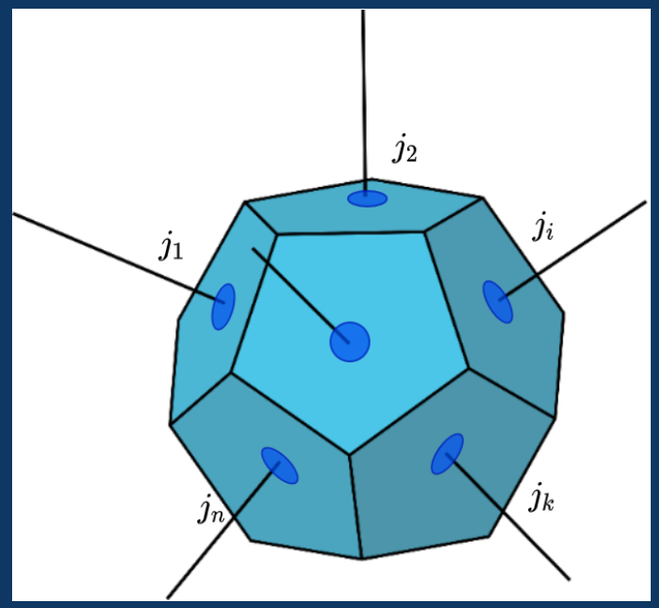

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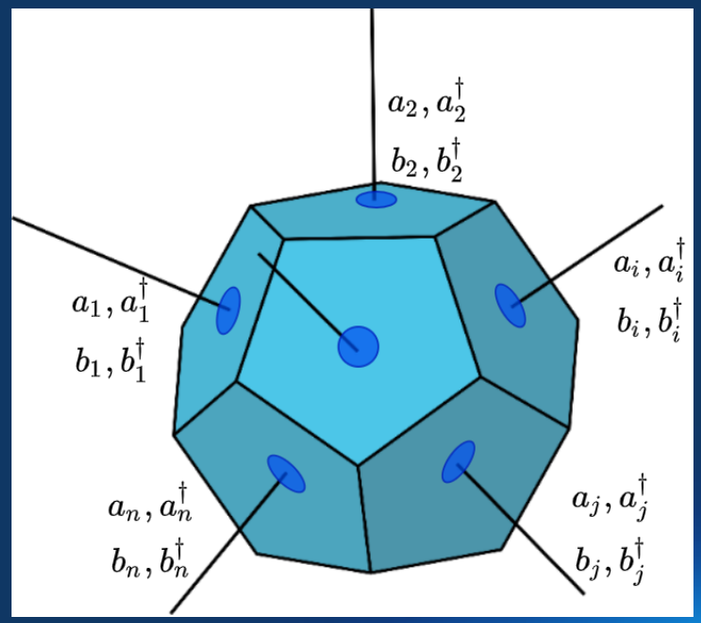

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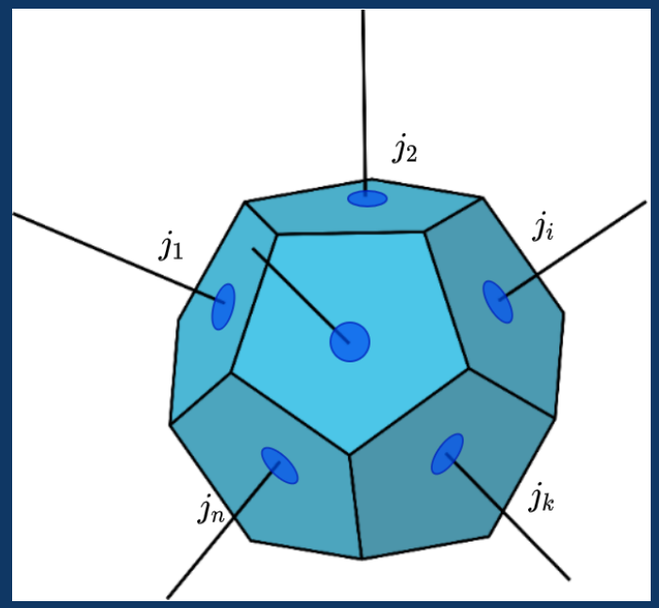

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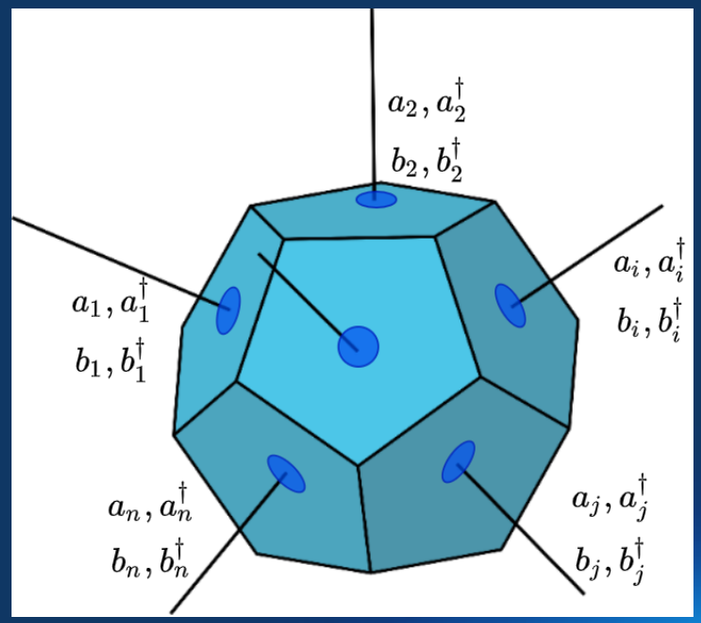

$$\left|j; m\right\rangle \equiv \left|n_a, n_b\right\rangle; \quad \text{where} \quad j = \frac{1}{2}\left(n_a + n_b\right); \quad m = \frac{1}{2}\left(n_a - n_b\right)$$

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$\mathfrak{u}(N)$ Intertwiner Algebra (cont.)

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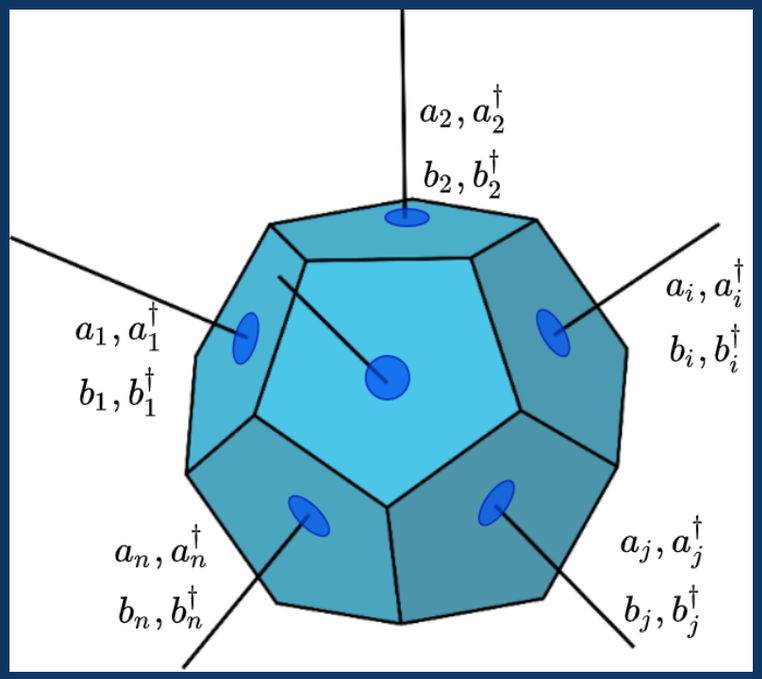

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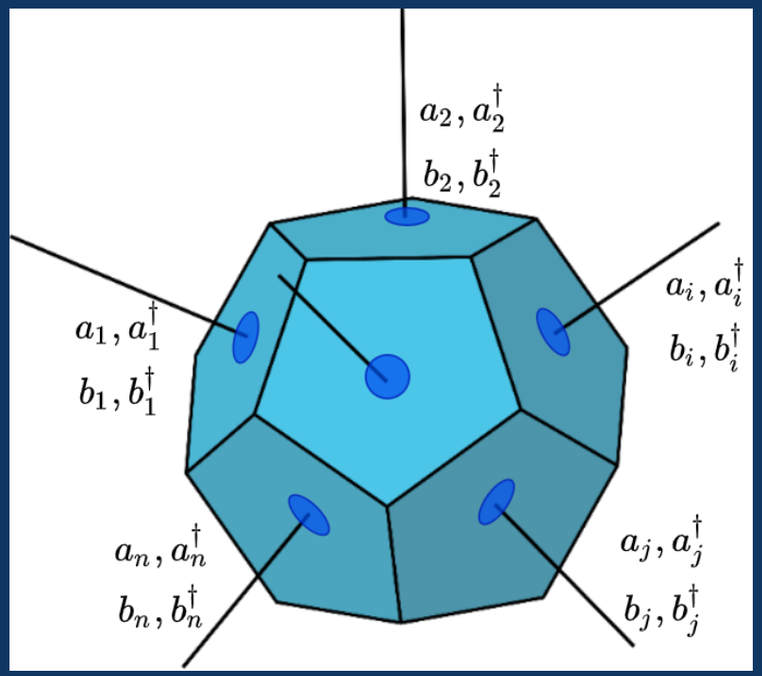

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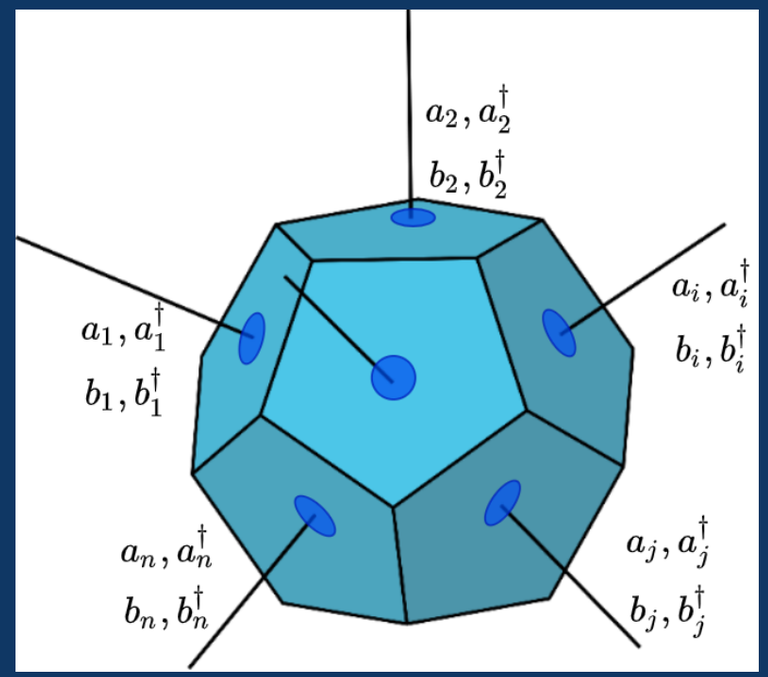

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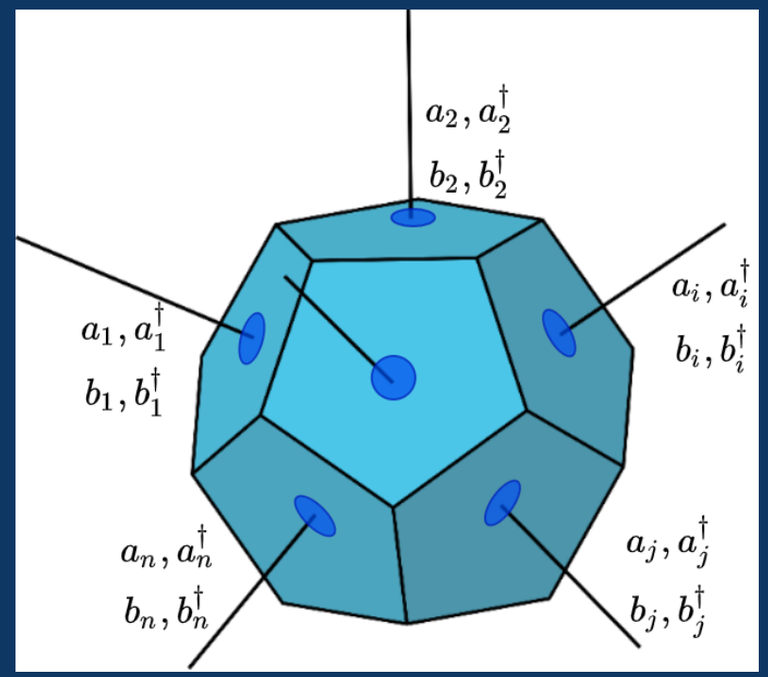

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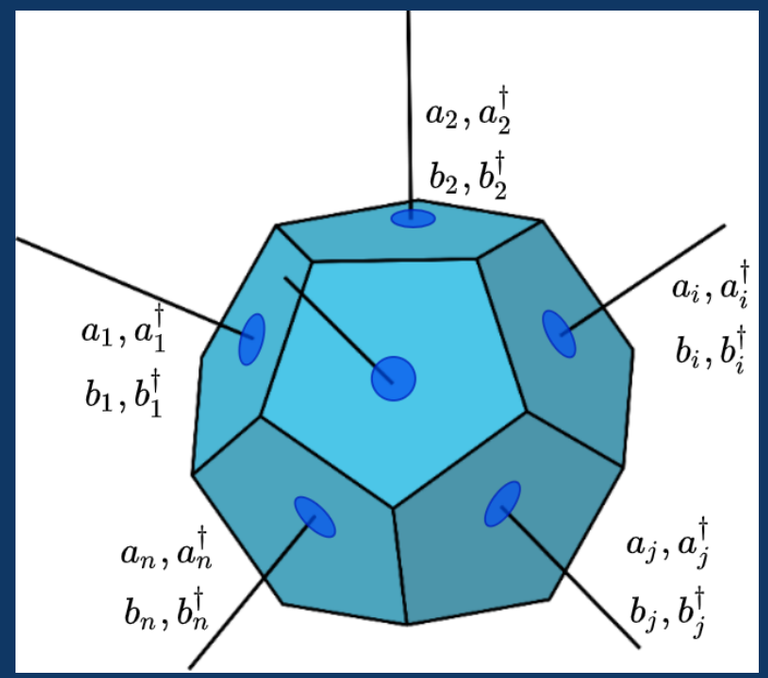

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- Minimum uncertainty states: $\Delta E_{ij}/\langle E_{ij}\rangle \sim 1/\sqrt{J}$; which vanishes in the limit of large J.

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The space of unique labels of the FL coherent states is given by $\mathbb{C}^{2n}/GL(2,\mathbb{C})$ which is precisely the Grassmannian $Gr_{2,n}$: the set of all 2 planes in $\mathbb{C}^n$.

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N massless particles

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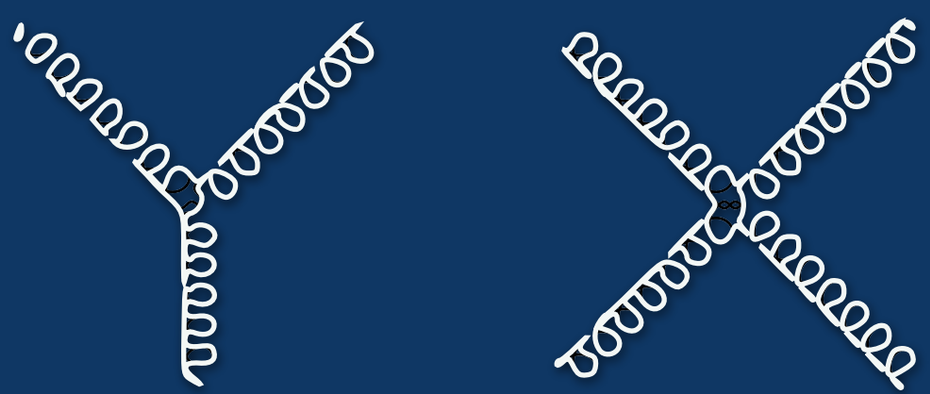

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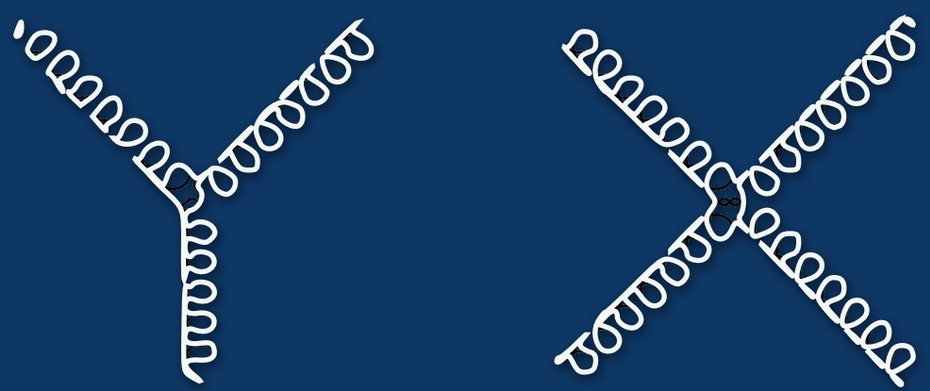

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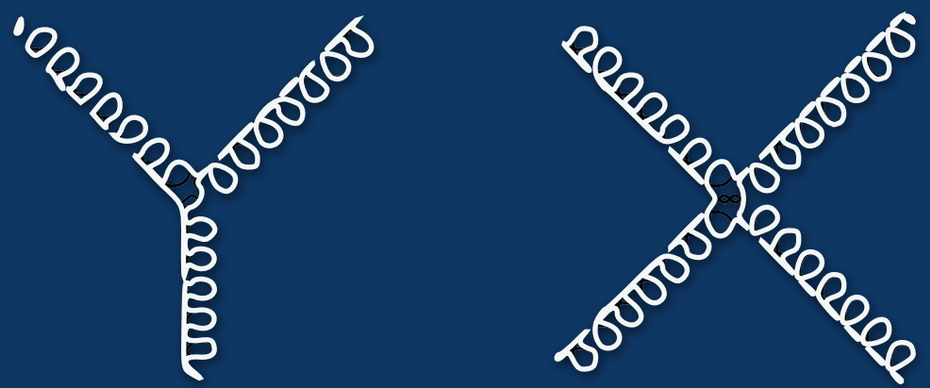

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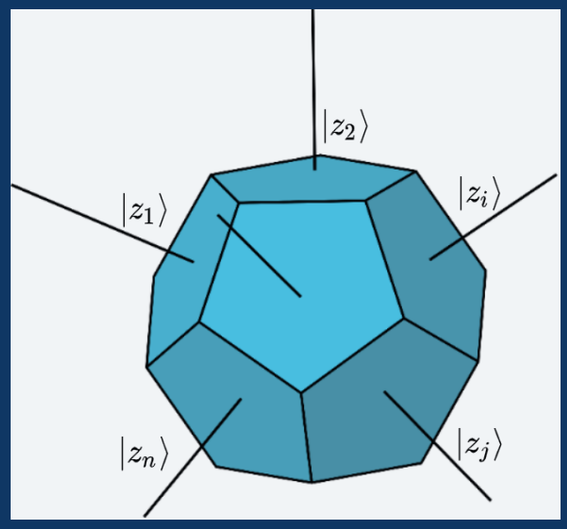

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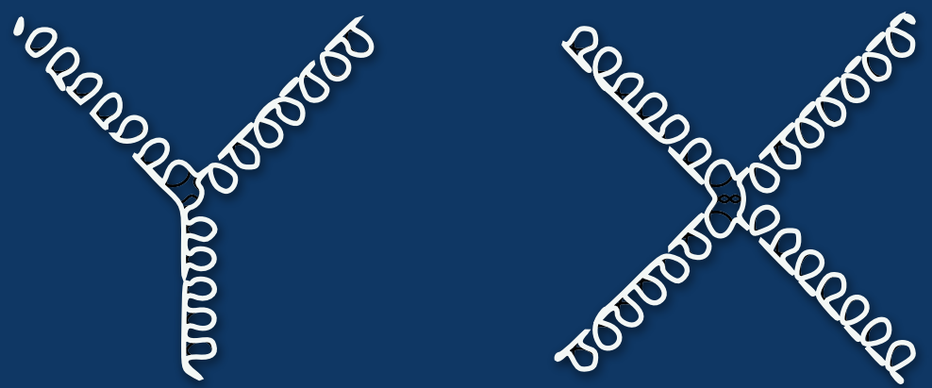

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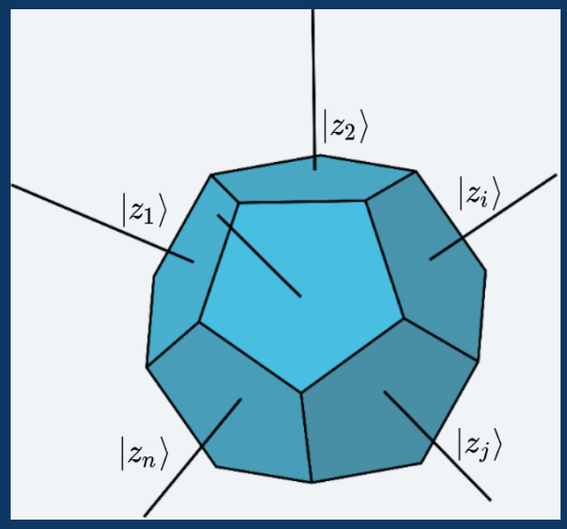

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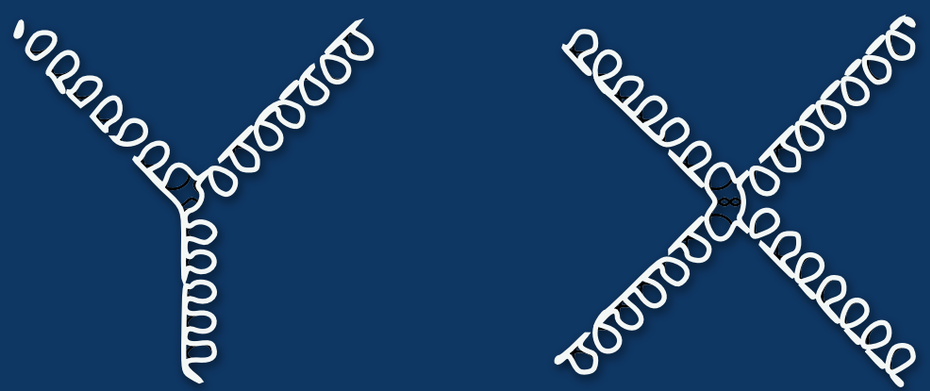

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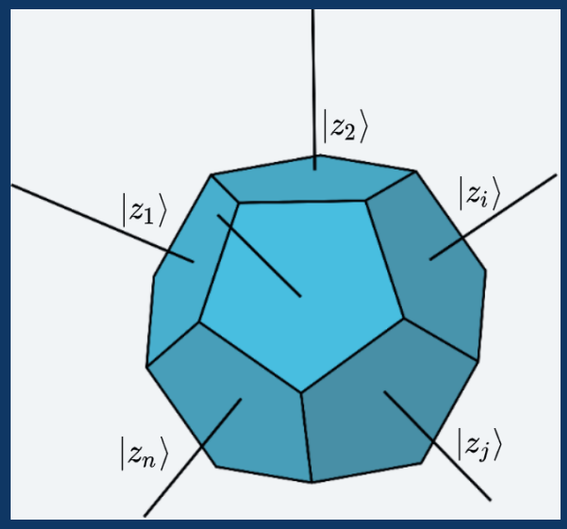

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hal-03438842, Vaid, Suresh, 2021

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$$\sum_{i=1}^{N} p_i^0 = 2E \Leftrightarrow \sum_{i=1}^{N} J_i^0 = 2A$$

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- Jose Barbón and collaborators have shown (1912.05996, 2006.06607) that there is a correspondence between the total momentum coming into or out of any region of spacetime and the complexity of the spacetime geometry within that region.

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That’s all folks

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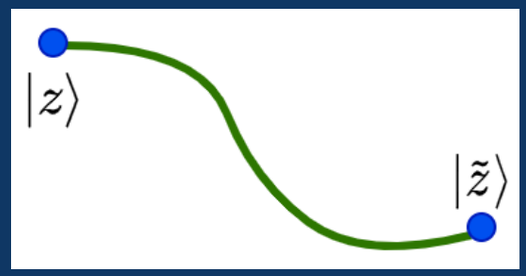

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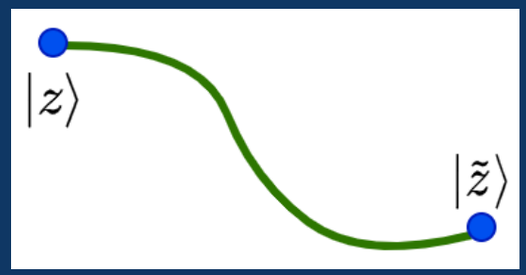

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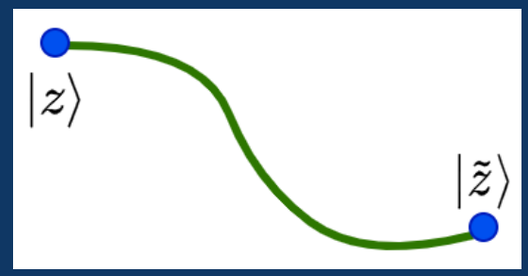

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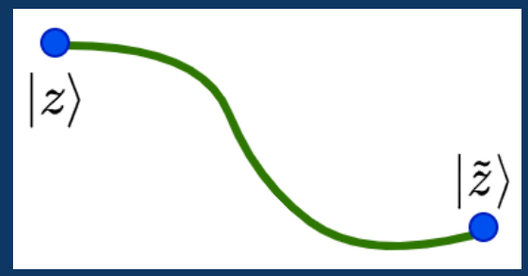

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