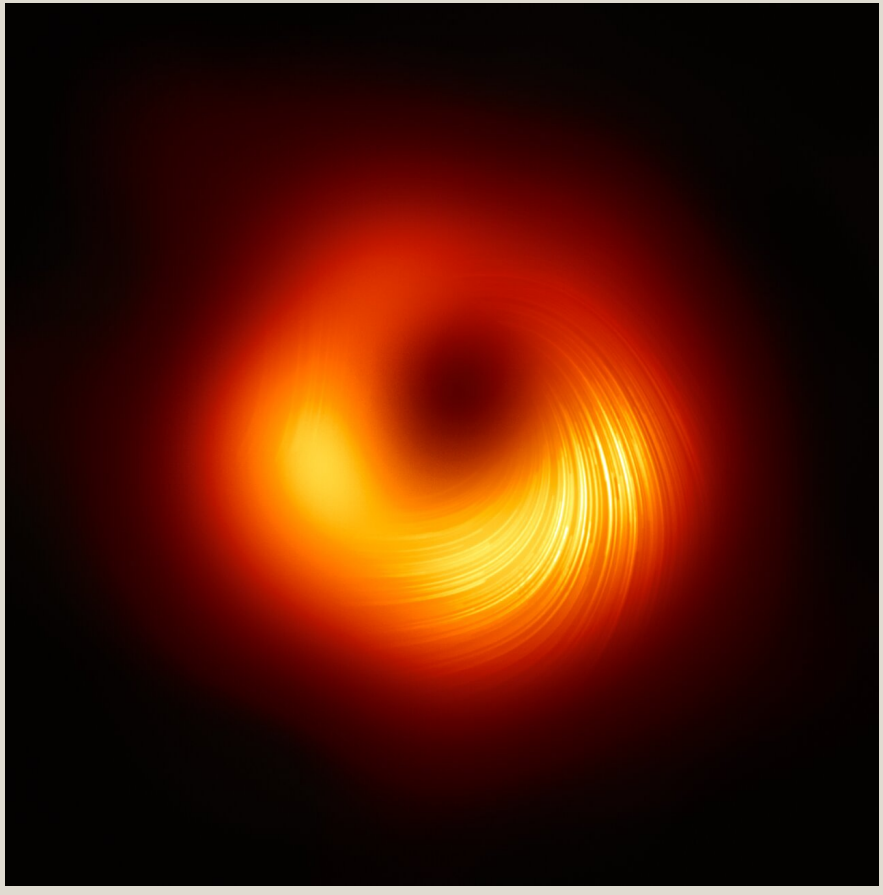
Kruskal black holes in LQG: Recent advances

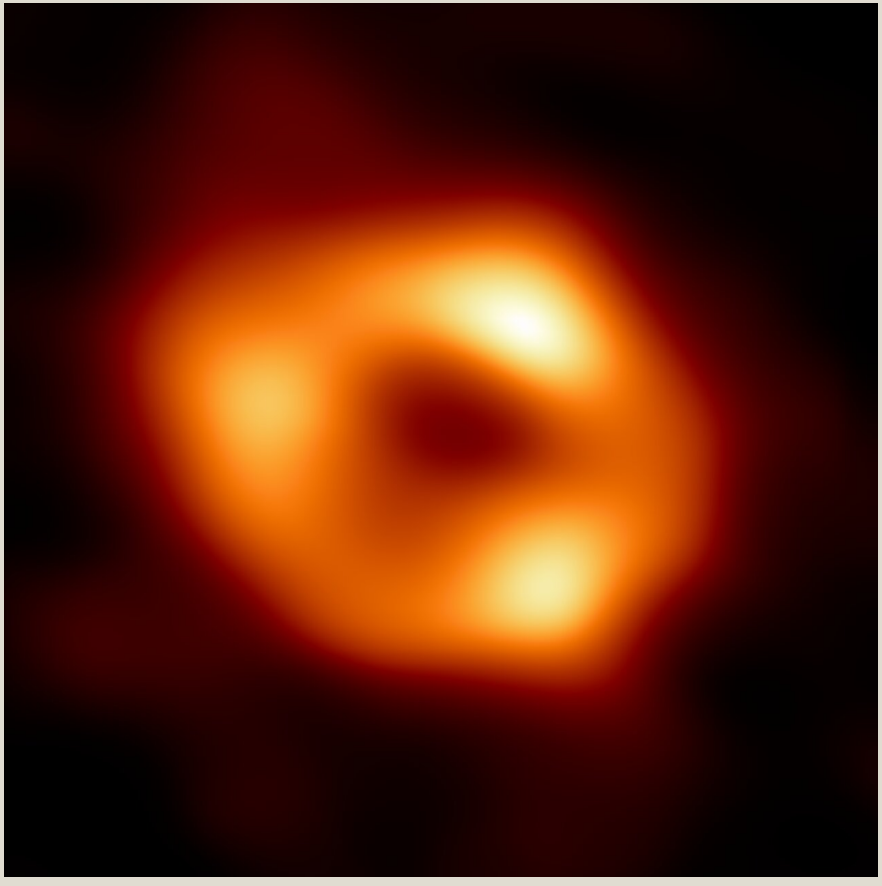
Javier Olmedo

Universidad de Granada

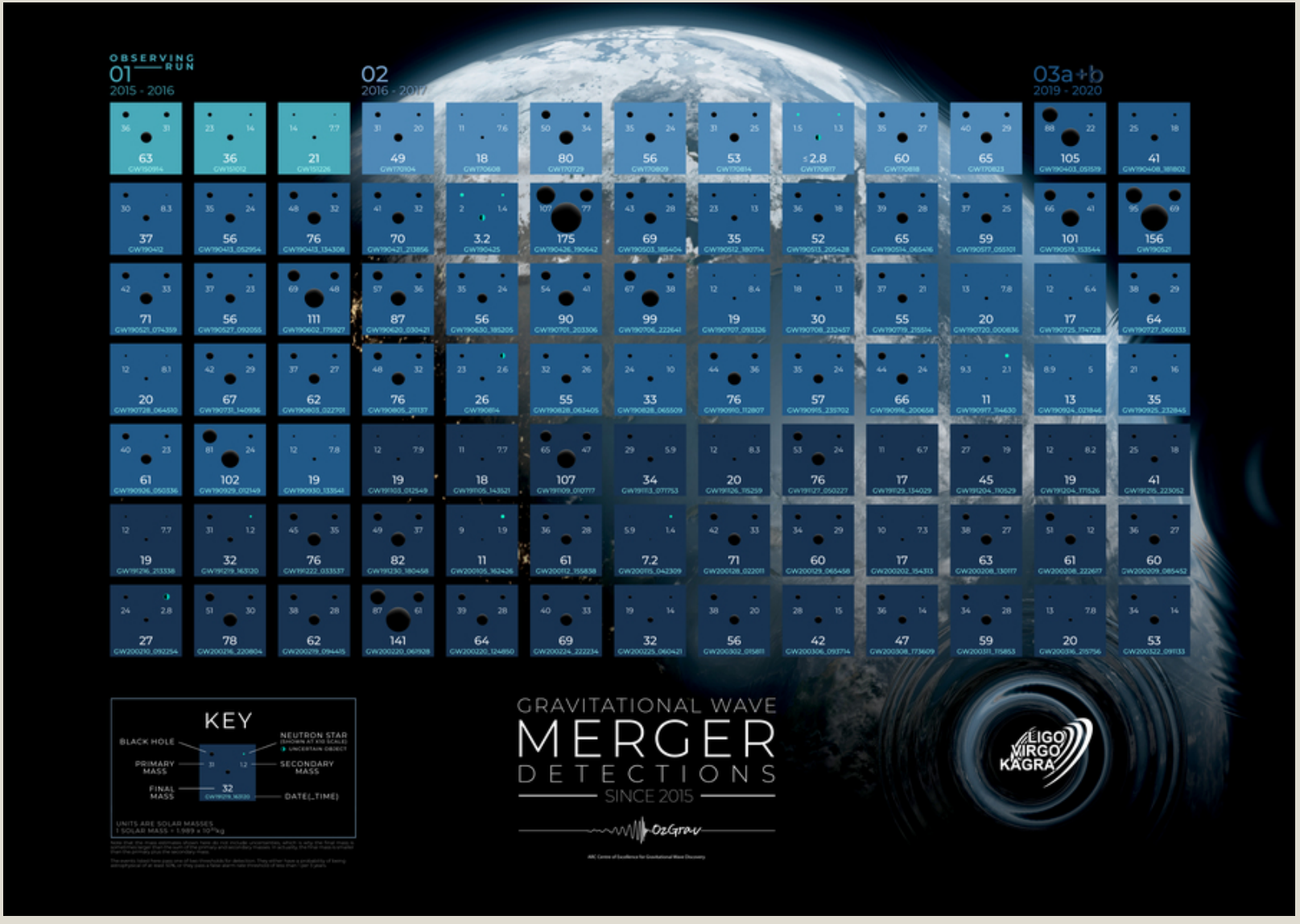
LOOPS 22

ENS - Lyon, 07/21/22





https://eventhorizontelescope.org/



https://www.ligo.caltech.edu/gallery

Introduction

- **Within LQG several strategies** allowed us to understand different aspects **about BH physics** and their potential consequences (BH entropy, spin foam models, QRLG, etc.)
- Here, I will focus on the quantization of **symmetry reduced** Kruskal spacetime. Despite its simplicity, it displays interesting physics and challenges.
- I will discuss two models: the so-called AOS and GOP models (though these are not the only ones ;)).
- Within the AOS model, I will mainly review its main properties (both interior and exterior regions).
- Regarding the GOP model, I will introduce an **improved dynamics** scheme and discuss several the physical aspects.
- Finally, I will also mention some new results about the spectrum of quasinormal modes of these effective geometries.

AOS model: main ideas

- We will follow the main lines of Kantowski-Sachs models: focus on space-times with **spherical symmetry in vacuum** and a **homogeneous slicing** (trivial constraint algebra), and adopt **LQC techniques**.
- We limit ourselves to **effective descriptions** (classical evolution equations modified to incorporate quantum geometry corrections).
- **Judicious choice of quantum parameters** entering in the definition of the Hamiltonian constraint that solves several limitations previous models.
- Extension to the **exterior region**.

AOS model

- The **spacetime metric** is

$$g_{ab}\mathrm{d}x^a\mathrm{d}x^b \equiv \mathrm{d}s^2 = -N_t^2\mathrm{d}t^2 + \frac{p_b^2}{p_c L_o^2}\mathrm{d}x^2 + p_c(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2).$$

- Here p_b and p_c are triad components, with b and c the components of the $su(2)$ connection.

- Besides, c/L_o and p_b/L_o, and b, p_c are invariant under a rescaling of the fiducial cell of length L_o.

- **Quantum corrections** to the classical theory.
  - i) $H[N]$ contains curvature F^i_{ab} of the gravitational connection A^i_a.
  - ii) F^i_{ab} can be obtained from $(h_\square/\mathcal{A}_\square)$, where $h_\square$ is a loop $\square$ of holonomies of A^i_a and $\mathcal{A}_\square$ the area enclosed by $\square$ (we choose it to be equal to the LQG area gap $\Delta = 4\sqrt{3}\pi\gamma\ell_{\mathrm{Pl}}^2$).
  - iii) curvature operators $\hat{F}_{\theta,\phi}$, $\hat{F}_{x,\theta}$, $\hat{F}_{x,\phi}$ are determined by the *fractional lengths* of the links in these plaquettes: δ_c for the x-directional link and δ_b for links in the 2-spheres.
- **LQC effective dynamics** dictated by

$$H_{\mathrm{eff}}[N] = -\frac{1}{2G\gamma}\Big[2\frac{\sin(\delta_c c)}{\delta_c}\,p_c + \Big(\frac{\sin(\delta_b b)}{\delta_b} + \frac{\gamma^2\delta_b}{\sin(\delta_b b)}\Big)\,p_b\Big], \quad N = \frac{\gamma\ p_c^{1/2}\ \delta_b}{\sin(\delta_b\, b)}.$$

The parameters δ_b and δ_c

- Our proposal: Choose the plaquettes to lie on the transition surface **where the space-time has largest curvature**. One then has:

$$2\pi\,\delta_c\delta_b\,|p_b|\big|_{\mathcal{T}} = \Delta, \qquad (1)$$

(Physical (fractional) area of the annulus around equators)

$$4\pi\,\delta_b^2\,p_c\big|_{\mathcal{T}} = \Delta. \qquad (2)$$

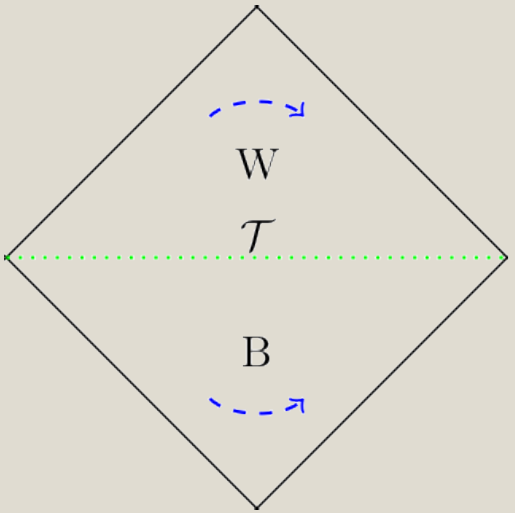
(Physical (fractional) area of the sphere)

- The solutions can be written in a closed form for macroscopic BHs ($m^2 \gg \Delta$):

$$\delta_b = \Big(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\Big)^{1/3},\ L_o\delta_c = \frac{1}{2}\Big(\frac{\gamma\Delta^2}{4\pi^2 m}\Big)^{1/3}.$$

where the mass is given by

$$m = \Big[\frac{\sin(\delta_c\, c)}{\gamma L_o \delta_c}\Big]\, p_c.$$



The exterior region

- We keep a homogeneous slicing, i.e. time-like hypersurfaces with $(-++)$ 3-metric.
- The spacetime metric is

$$\tilde{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b \equiv \mathrm{d}\tilde{s}^2 = -\frac{\tilde{p}_b^2}{\tilde{p}_c L_o^2}\mathrm{d}x^2 + \tilde{N}_\tau^2\mathrm{d}\tau^2 + \tilde{p}_c(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2).$$

- The connection and triad are now $su(1,1)$ valued (instead of $su(2)$). See for instance Liu & Noui (2017) for details. It amounts to replacing

$$b \to i\tilde{b},\ p_b \to i\tilde{p}_b; \qquad c \to \tilde{c},\ p_c \to \tilde{p}_c$$

- **The dynamics in the exterior** is dictated by

$$\tilde{H}_{\mathrm{eff}}[\tilde{N}] = -\frac{1}{2G\gamma}\Big[2\frac{\sin(\delta_{\tilde{c}}\,\tilde{c})}{\delta_{\tilde{c}}}\,\tilde{p}_c + \Big(-\frac{\sinh(\delta_{\tilde{b}}\,\tilde{b})}{\delta_{\tilde{b}}} + \frac{\gamma^2\delta_{\tilde{b}}}{\sinh(\delta_{\tilde{b}}\,\tilde{b})}\Big)\,\tilde{p}_b\Big], \quad \tilde{N} = \frac{\gamma\tilde{p}_c^{1/2}\,\delta_{\tilde{b}}}{\sinh(\delta_{\tilde{b}}\,\tilde{b})}.$$

- The mass m is given by the same explicit expression as for the interior, and, by continuity, the polymer parameters $\delta_{\tilde{b}} = \delta_b(m)$ and $\delta_{\tilde{c}} = \delta_c(m)$.

- The **exterior and interior** regions match **smoothly across the horizon**. One can explicitly introduce Eddington-Finkelstein-type coordinates.

Physical consequences in the interior

- Several curvature invariants are bounded above, reaching a maximum value at the transition surface that is mass independent, for instance

$$K\,|_{\mathcal{T}} = \frac{768\pi^2}{\gamma^4\Delta^2} + \mathcal{O}\Big(\big(\frac{\Delta}{m^2}\big)^{\frac{1}{3}}\,\ln\frac{m^2}{\Delta}\Big).$$

- Quantum corrections can be seen as an effective fluid with a stress-energy tensor $\mathfrak{T}_{ab}$ characterized by the energy density ρ and the radial and tangential pressures $\mathfrak{p}_x$ and $\mathfrak{p}_{||}$, respectively. One can easily see that, at the transition surface, the strong energy condition is violated

$$(\mathfrak{T}_{ab} - \frac{1}{2}g_{ab}\mathfrak{T})T^aT^b\Big|_{\mathcal{T}} < 0.$$

- Komar mass of the black and white hole horizons are equal in the large mass limit (unit mass amplification).

- This shows that there are highly non-trivial constraints that our effective geometry satisfies (besides it is free of limitations of previous proposals).

The exterior region

- Adopting Euclidean methods, the temperature associated with a Killing horizon has quantum corrections

$$T_{\rm H} = \frac{\hbar}{8\pi K m}\,\frac{1}{(1+\epsilon_m)}, \quad \epsilon_m = \frac{1}{256}\left(\frac{\gamma\Delta^{\frac{1}{2}}}{\sqrt{2\pi}m}\right)^{8/3},$$

  which are tiny for macroscopic black holes ($\epsilon_m \simeq 4\cdot 10^{-106}$ for solar mass black holes!).

- Asymptotic flatness of $\tilde{g}_{ab}$ is weaker than in textbook treatments. For instance, at spatial infinity, curvature falls-off sufficiently fast. However, some curvature invariants (like Kretschmann scalar) fall-off as r^{-4} (rather than r^{-6}).

- Note that, for a solar mass black hole, to find $\tilde{g}_{ab}$ differing from the "natural" flat metric η_{ab} by 10% we need to move away from the horizon a distance $\sim 10^{10^{24}}\, r_S$. Compared with the observable universe size $\sim 10^{22} r_S$. If the mass of the black hole is $10^6\, M_{\rm Pl}$, we need to move away by $\sim 10^{1158}\,\ell_{\rm Pl}$, while 5Gpc $\sim 10^{61}\,\ell_{Pl}$.

- Moreover, this non-standard fall-off is not an impediment for the ADM energy to be well-defined. Actually,

$$E_{\rm ADM} = \frac{m}{G}.$$

  Other definitions of mass, like the one involving the Ricci tensor or the horizon energy, all give values compatible with $E_{\rm ADM}$ (up to small corrections).

AOS model: criticism

- In arXiv:1902.07874 it was questioned the validity of the model since it has non-standard fall-off conditions:
  - we saw that the issue with asymptotics is subtle, but does not yields a physically inconsistent physical model.
- In arXiv:1902.04032 criticized the relation between Hamiltonian and Hamilton's equations.
  - In App A of arXiv:1806.02406 we showed that one can enlarge the phase space and introduce constraints that yield the correct Hamilton's equations out of the Hamiltonian.
  - Besides García-Quismondo & Mena-Marugán (2021) showed that a generalization of the dependence of polymer parameters alleviates this tension (for macroscopic BHs).
- Faraoni & Giusti (2021) claimed that quantum corrections in the innermost stable circular orbits diverge, and that there are important issues with the asymptotic behavior.
  - Our effective geometry has been shown to have an extremely good semiclassical behavior close to the horizon (nothing dramatic happens there).
  - Besides, we checked that for the range of masses of the black holes considered by us, important asymptotic corrections appear well beyond cosmological distances. No body should worry!

The GOP model: main ideas

- One considers spherically symmetric spacetimes in vacuum, without restrictions to homogeneous slicings.
- LQG-like quantization is known for the Abelian constraint.
- What is new:
  - **Novel choice of the polymer parameter** in the quantum Hamiltonian constraint in the spirit of the *improved dynamics*.
  - **Derivation of effective line elements** from semiclassical states.
  - We provide ideas towards the **quantum covariance** of the spacetime line element.

Classical system

- Phase space functions variables (K_x, E^x) and (K_φ, E^φ). The Hamiltonian is a linear combination of the constraints

$$H_T = H_{\text{old}}(N) + H_r(N_r)$$

fulfilling the algebra

$$\begin{aligned}
&\{H_r(N_r), H_r(\tilde{N}_r)\} = H_r(N_r\tilde{N}_r' - N_r'\tilde{N}_r),\\
&\{H_{\text{old}}(N), H_r(N_r)\} = H_{\text{old}}(N_r N'),\\
&\{H_{\text{old}}(N), H_{\text{old}}(\tilde{N})\} = H_r\left(\frac{E^x}{(E^\varphi)^2}\left[N\tilde{N}' - N'\tilde{N}\right]\right).
\end{aligned}$$

- The quantization of this theory is challenging (anomalous quantum algebra). Alonso-Bardaji Brizuela & Vera (2022) provided new ideas ideas in this direction (effective description).

New constraint algebra

- We Abelianize the scalar constraint

$$\tilde{H} := \frac{(E^x)'}{E^\varphi} H_{old} - 2\frac{\sqrt{E^x}}{E^\varphi} K_\varphi H_r$$

Now, smearing with the lapse, integrating by parts (with appropriate boundary conditions) and scaling with E^φ

$$H_{\text{new}}(N) = \int dx N \left(\sqrt{E^x} E^\varphi \left(1 + K_\varphi^2\right) - 2GME^\varphi - \frac{[(E^x)']^2 \sqrt{E^x}}{4E^\varphi} \right),$$

The phase space is extended $\{\tau, M\} = 1$ and the new constraint algebra is

$$\begin{aligned}
\{H_r(N_r), H_r(\tilde{N}_r)\} &= H_r(N_r \tilde{N}_r' - N_r' \tilde{N}_r), \\
\{H_{\text{new}}(N), H_r(N_r)\} &= H_{\text{new}}([N_r N]'), \\
\{H_{\text{new}}(N), H_{\text{new}}(\tilde{N})\} &= 0.
\end{aligned}$$

- This redefinition of the scalar constraint comes together with a redefinition of lapse and shift Lagrange multipliers.

Kinematical Hilbert space

- Spin networks

$$\langle K_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(i k_j \int_{e_j} dx\, K_x(x) \right) \prod_{v_j \in g} \exp\left(i \mu_j K_\varphi(v_j) \right),$$

  $k_j \in \mathbb{Z}$ is the valence associated with the edge e_j, and $\mu_j \in \mathbb{R}$ the valence associated with the vertex v_j.

- Kinematical Hilbert space

$$\mathcal{H}^g_{\text{kin}} = \mathcal{H}^m_{\text{kin}} \otimes \left[\bigotimes_{j=1}^{N} \ell^2_j \otimes L^2_j(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}}) \right],$$

  which is endowed with the inner product

$$\langle g, \vec{k}, \vec{\mu}, M | g', \vec{k}', \vec{\mu}', M' \rangle = \delta(M - M') \delta_{\vec{k},\vec{k}'} \delta_{\vec{\mu},\vec{\mu}'} \delta_{g,g'} \ .$$

- Kinematical operators have a well-defined action. For instance

$$\hat{M} | g, \vec{k}, \vec{\mu}, M \rangle = M | g, \vec{k}, \vec{\mu}, M \rangle,$$
$$\hat{E}^x(x) | g, \vec{k}, \vec{\mu}, M \rangle = \ell^2_{\text{Pl}} k_j | g, \vec{k}, \vec{\mu}, M \rangle.$$

Improved dynamics

- The scalar constraint is polymerized (point-holonomies) $K_\varphi \to \sin(\rho K_\varphi)/\rho$.
- The polymer parameter $\rho(x_j) = \rho_j$ is fixed by the *improved dynamics* condition

$$4\pi\ell_{\mathrm{Pl}}^2 k_j \rho_j^2 = \Delta.$$

- The action of the constraints on the kinematical Hilbert space is well defined. Solutions can be computed (explicitly).
- They impose, within this improve dynamics, the following restriction (minimum area of the spheres)

$$k_0 > \left(\frac{2GM\Delta}{4\pi\ell_{\mathrm{Pl}}^3}\right)^{2/3}$$

for macroscopic black holes. Hence, there is no singularity.

Parametrized observables: metric components

- Physical Hilbert space can be computed applying group averaging techniques.
- Metric components $\hat{g}_{\mu\nu}$ can be written in terms of parametrized observables (evolving constants of motion).
- For the lapse and shift, we have

$$\hat{N}^2(x_j) := \frac{1}{4}\frac{([\hat{E}^x(x_j)]')^2}{(\hat{E}^\varphi(x_j))^2},$$

$$\hat{N}^x_n(x_j) = \sqrt{\frac{\hat{E}^x(x_j)}{(\hat{E}^\varphi(x_j))^2}}\frac{\sin\left(\widehat{n\rho_j K_\varphi}(x_j)\right)}{n\rho_j}.$$

- Physics depends strongly on the value of n. But only $n = 1$ gives effective metrics independent of the choice of the gauge function K_φ. There are a covariance issues for $n \neq 1$.

Physical consequences

- There are semiclassical states for which x_j can be safely replaced by a continuous coordinate and fluctuations of the mass are negligible. In this continuous (semiclassical) limit, we have computed several curvature components.

- None of them blow up anywhere and depend on δx. For instance, for $\delta x = x_0 \left(\frac{\ell_{\rm Pl}}{x_0}\right)^s$ with $s = 0, 1, 2$, we see that

$$\begin{aligned}
\rho^{int}\big|_{\mathcal{T}} &= \frac{2\pi}{\Delta} \times \mathcal{O}\left(\left[\frac{\Delta}{2\pi r_S}\right]^{s/3+2/3}\right), \\
p_x^{int}\big|_{\mathcal{T}} &= -\frac{2\pi}{\Delta} \times \mathcal{O}\left(\left[\frac{\Delta}{2\pi r_S}\right]^{s/3}\right), \\
p_{||}^{int}\big|_{\mathcal{T}} &= -\frac{2\pi}{\Delta} \times \mathcal{O}\left(\left[\frac{\Delta}{2\pi r_S}\right]^{s/3}\right)
\end{aligned}$$

- The Einsten tensor tells us how effective corrections can be understood as an effective stress-energy tensor $T_{\mu\nu}$. It violates the dominant energy condition.

- The semiclassical states yield a black-to-white hole transition.

- Effective metrics have the standard asymptotic fall-off.

Quantum covariance

- Covariance is a subtle issue. We show (Gambini, Pullin and O. 2022) that for the length of a space-time curve $(t(x), x)$ in classical theory, we have an invariant length of the polygonal curve described by the collection $[...(\hat{t}_j, \hat{x}_j), (\hat{t}_{j+1}, \hat{x}_{j+1})...]$ where $\hat{x}_j|M, \hat{k}\rangle = \ell_{\text{Planck}}\sqrt{k_j}|M, \hat{k}\rangle$ and $\hat{t}_j|M, \hat{k}\rangle = t(x_j)|M, \hat{k}\rangle$, is invariant, at least for stationary slicings.

- At the quantum level, the 1+1 quantum line element $\widehat{\Delta s_j^2}$ is covariant if both time and radial coordinates are discrete and quantum, namely,

$$\widehat{\Delta s_j^2} = g_{\mu\nu}\widehat{(x_j^0, x_j^1)}\widehat{\Delta x_j^\mu}\,\widehat{\Delta x_j^\nu}, \quad x_j^0 = t_j, \quad x_j^1 = x_j.$$

- For example, let us consider the "tortoise" coordinate defined as

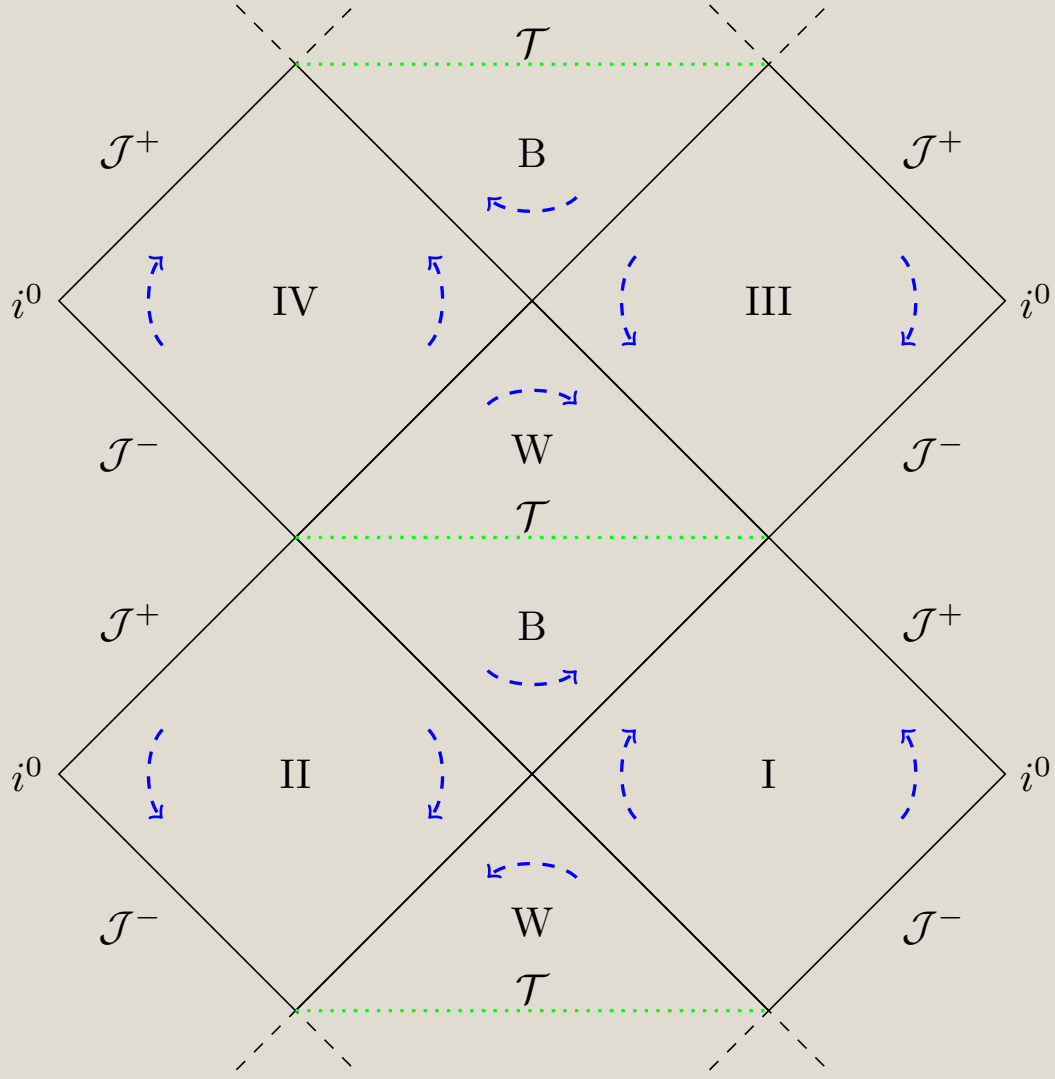
$$x_{j+1}^* - x_j^* = \left(1 + \frac{\delta x_j}{2x_j}\right)\left(1 - \frac{r_S}{x_j}\right)^{-1}\left(x_{j+1} - x_j\right).$$

This equation determines all x_j^* provided the value of, for instance, x_S^*. This change of radial coordinate leaves invariant the line element, by construction.

Limitations

- We provide a viewpoint towards quantum covariance for stationary slicings. For more general (time-dependent) coordinate transformation further work is needed.
- We computed curvature operators in the continuum limit. Extension to the discrete theory is still work in progress.
- Our model (neither AOS model) does not capture all the rich physics of collapsing (evaporating) scenarios. But many relevant calculations already use the Kruskal space-time (vacuum states of linear field theories, renormalized stress energy tensor, etc.).

Quantum extension of the Kruskal diagram



Quasinormal frequencies

- We have computed axial and polar quasinormal frequencies of both AOS (despite its unusual asymptotic behavior) and GOP effective geometries in del Corral and O. 2022 (see also arXiv:2012.13359 and arXiv:2109.02861).

- We adopted an effective description. For a metric of the form

$$\mathrm{d}s^2 = -G(r)\mathrm{d}t^2 + F(r)\mathrm{d}r^2 + H(r)\mathrm{d}\Omega^2,$$

  we provide effective EOM of perturbations inspired by LQC approaches (well-defined in the exterior, simplicity and agreement with classical theory).

- We showed numerically that departures from classical theory are a small percent ($\sim 0.01\%$) even for black holes as small as $r_S = 1000\,\ell_{\mathrm{Pl}}$. For bigger ones, departures essentially decay as $(r_S/\ell_{\mathrm{Pl}})^{-2/3}$.

- Besides, they both violate isospectrality (as opposed to the classical theory).

- However, these deviations are tiny for macroscopic black holes (they are suppressed again by the ratio $(r_S/\ell_{\mathrm{Pl}})^{-2/3}$ for black hole sizes $r_S \in [1000\,\ell_{\mathrm{Pl}}, 32000\,\ell_{\mathrm{Pl}}]$).

Outlook

- Full quantum dynamics of AOS model needs to be understood:
  - i) The quantum Hamiltonian constraint has a complicated action. Fortunately, we have a proposal worth to be explored.
  - ii) Classical and quantum Hamiltonian frameworks have been recently discussed in the literature for $SU(1,1)$ connections in 3+1 gravity. Interesting possibility: Application to our model.
- Extension to charged and rotating black holes (the poor's man strategy of using the Janis-Newman might not be enough).
- Include backreaction for the study of Cauchy horizons and black hole formation and evaporation.
- Shadows, superradiance, and black hole **spectroscopy**.

Outlook: pseudospectrum

Gravitational wave signatures of black hole quasi-normal mode instability

José Luis Jaramillo,[1] Rodrigo Panosso Macedo,[2,3] and Lamis Al Sheikh[1]

[1]*Institut de Mathématiques de Bourgogne (IMB), UMR 5584, CNRS, Université de Bourgogne Franche-Comté, F-21000 Dijon, France*
[2]*School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, United Kingdom*
[3]*CENTRA, Departamento de Física, Instituto Superior Técnico – IST, Universidade de Lisboa – UL, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*

Black hole (BH) spectroscopy has emerged as a powerful approach to extract spacetime information from gravitational wave (GW) observed signals. Yet, quasinormal mode (QNM) spectral instability under high wave-number perturbations has been recently shown to be a common classical general relativistic phenomenon [1]. This requires to assess its impact on the BH QNM spectrum, in particular on BH QNM overtone frequencies. We conclude: i) perturbed BH QNM overtones are indeed potentially observable in the GW waveform, providing information on small-scale environment BH physics, and ii) their detection poses a challenging data analysis problem of singular interest for LISA astrophysics. We adopt a two-fold approach, combining theoretical results from scattering theory with a fine-tuned data analysis on a highly-accurate numerical GW ringdown signal. The former introduces a set of effective parameters (partially lying on a BH Weyl law) to characterise QNM instability physics. The latter provides a proof-of-principle demonstrating that the QNM spectral instability is indeed accessible in the time-domain GW waveform, though certainly requiring large signal-to-noise ratios. Particular attention is devoted to discuss the patterns of isospectrality loss under QNM instability, since the disentanglement between axial and polar GW parities may already occur within the near-future detection range.

Summary

- We provide a quantum extension adopting (AOS) and deriving (GOP) an effective LQC description of eternal (macroscopic) black holes. They share the following properties.
  - i) Admit an infinite number of trapped, anti-trapped and asymptotic regions.
  - ii) High-curvature regions are regular (singularity resolution).
  - iii) Some of the simplest curvature scalars have mass-independent upper bounds (in GOP this is not always the case).
  - iv) Away from quantum regions curvature falls-off sufficiently fast for ADM mass to be well defined.
  - v) Consecutive asymptotic regions have same ADM mass.
- GOP model covariance reduces ambiguities in the quantum theory.
- Quasinormal frequencies agree extremely well with the classical theory, even for small black holes. But deviations exist, as well as the isospectrality is broken.

Thank you!!!