Effective spin foam models
and
Effective actions for their continuum limit

Bianca Dittrich,
Perimeter Institute and Radboud University

Loops 2022, Lyon

Work with
Seth Asante, Johanna Borissova, Hal Haggard,
Athanasios Kogios, Jose Padua-Arguelles

Invite you to a journey through spin foams
with an unusual view point,
but it will pay off!

See key mechanisms of spin foams at work
and recognize universal behaviour,
independent of specific models.

Invite you to a journey through spin foams
with an unusual view point,
but it will pay off!

See key mechanisms of spin foams at work
and recognize universal behaviour,
independent of specific models.

Resolution of ~~flatness~~ anomaly problem.

Defining a spin foam model from minimal input

Want to construct a path integral for triangulated space times, where:

- areas are independent variables [LQG, black hole entropy, geometry defined from entanglement, string bits, …]
- areas have (asymptotically equidistant) discrete spectrum [LQG: Rovelli-Smolin, Ashtekar-Lewandowski, Continuum Edge-Modes: Wieland, Freidel-Geiller-Pranzetti]

Defining a spin foam model from minimal input

Want to construct a path integral for triangulated space times, where:

- areas are independent variables

[LQG, black hole entropy, geometry defined from entanglement, string bits, …]

- areas have (asymptotically equidistant) discrete spectrum

[LQG: Rovelli-Smolin, Ashtekar-Lewandowski,
Continuum Edge-Modes: Wieland, Freidel-Geiller-Pranzetti]

Simplest action:

Area Regge action

$$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

[Regge, Rovelli, Barrett-Rocek-Williams,
Asante-BD-Haggard]

Gives Length Regge action = discrete GR action for
area configurations arising from length assignments.
But area configurations much more general than length configurations.

Defining a spin foam model from minimal input

Want to construct a path integral for triangulated space times, where:

- areas are independent variables

[LQG, black hole entropy, geometry defined from entanglement, string bits, …]

- areas have (asymptotically equidistant) discrete spectrum

[LQG: Rovelli-Smolin, Ashtekar-Lewandowski,
Continuum Edge-Modes: Wieland, Freidel-Geiller-Pranzetti]

Simplest action:

Area Regge action

$$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

[Regge, Rovelli, Barrett-Rocek-Williams,
Asante-BD-Haggard]

Gives Length Regge action = discrete GR action for area configurations arising from length assignments.
But area configurations much more general than length configurations.

State sum model:

$$Z = \sum_{A_t\text{-}values} e^{iS_{\mathrm{AR}}}$$

Does that lead to GR?

Defining a spin foam model from minimal input

Want to construct a path integral for triangulated space times, where:

- areas are independent variables

[LQG, black hole entropy, geometry defined from entanglement, string bits, …]

- areas have (asymptotically equidistant) discrete spectrum

[LQG: Rovelli-Smolin, Ashtekar-Lewandowski,
Continuum Edge-Modes: Wieland, Freidel-Geiller-Pranzetti]

Simplest action:

Area Regge action

$$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

[Regge, Rovelli, Barrett-Rocek-Williams, Asante-BD-Haggard]

Gives Length Regge action = discrete GR action for area configurations arising from length assignments.
But area configurations much more general than length configurations.

State sum model:

$$Z = \sum_{A_t\text{-}values} e^{iS_{\mathrm{AR}}}$$

Does that lead to GR?

Discrete level:

Need to impose constraints (weakly) to reproduce discretized GR.

➡ Effective spin foam for EPRL/FK.

Defining a spin foam model from minimal input

Want to construct a path integral for triangulated space times, where:

- areas are independent variables [LQG, black hole entropy, geometry defined from entanglement, string bits, …]
- areas have (asymptotically equidistant) discrete spectrum [LQG: Rovelli-Smolin, Ashtekar-Lewandowski, Continuum Edge-Modes: Wieland, Freidel-Geiller-Pranzetti]

Simplest action:

Area Regge action

$$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

[Regge, Rovelli, Barrett-Rocek-Williams, Asante-BD-Haggard]

Gives Length Regge action = discrete GR action for area configurations arising from length assignments.

But area configurations much more general than length configurations.

State sum model:

$$Z = \sum_{A_t\text{-}values} e^{iS_{\mathrm{AR}}}$$

Does that lead to GR?

Discrete level:

Need to impose constraints (weakly) to reproduce discretized GR.

→ Effective spin foam for EPRL/FK.

Continuum Limit:

Does lead to GR! [BD, BD-Kogios]

→ Even Barrett-Crane can lead to GR.

Reproducing discretized GR.

Constraining discrete areas

Area Regge action

$$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

Equation of motion:

$$0 = \epsilon_t(A_t)$$

Curvature + Shape Mismatch =0

Does not reproduce discrete GR.

Constraining discrete areas

Area Regge action $$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

Equation of motion: $$0 = \epsilon_t(A_t)$$

Curvature + Shape Mismatch =0

Does not reproduce discrete GR.

But we know: Constraining variations of areas to configurations arising from length assignments (aka simplicity constraints),
we reproduce discrete gravity equation of motion.

Area-Length constraints:

- Polynomial equations in areas

Constraining discrete areas

Area Regge action $$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

Equation of motion: $$0 = \epsilon_t(A_t)$$

Curvature + Shape Mismatch =0

Does not reproduce discrete GR.

But we know: Constraining variations of areas to configurations arising from length assignments (aka simplicity constraints), we reproduce discrete gravity equation of motion.

Area-Length constraints:

- Polynomial equations in areas
- Diophantine equations for discrete areas.

There are very few solutions, does not allow for semi-classical limit.
Can only impose constraints 'weakly', allowing for some deviations.

[Asante-BD- Haggard 2020]

Constraining discrete areas

Area Regge action $$S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$$

Equation of motion: $$0 = \epsilon_t(A_t)$$

Curvature + Shape Mismatch =0

Does not reproduce discrete GR.

But we know: Constraining variations of areas to configurations arising from length assignments (aka simplicity constraints), we reproduce discrete gravity equation of motion.

Area-Length constraints:

- Polynomial equations in areas
- Diophantine equations for discrete areas.

There are very few solutions, does not allow for semi-classical limit.
Can only impose constraints 'weakly', allowing for some deviations.

[Asante-BD- Haggard 2020]

$\boxed{\text{Discrete (independent) area spectra force enlargement of length configuration space.}}$

Weak implementation of Area-Length constraints

How sharply can we implement the constraints?

The answer is provided by the LQG phase space!

Weak implementation of Area-Length constraints

How sharply can we implement the constraints?

The answer is provided by the LQG phase space!

(Constrained) Area-Angle Regge action

[BD, Speziale 2007]

To localize Area-Length constraints on 4-simplices, it is best to introduce 3D dihedral angles as auxiliary variables. Matches quantum numbers in EPRL/FK type spin foams.

Area-Length constraints =

Shape matching constraints

τ shared by σ and σ'

$$\phi_i^\tau = \Phi_i^{\tau,\sigma}(A), \qquad \phi_i^\tau = \Phi_i^{\tau,\sigma'}(A)$$

'Integrate out' angles

Coupling of σ and σ'

$$\Phi_i^{\tau,\sigma}(A) = \Phi_i^{\tau,\sigma'}(A)$$

Weak implementation of Area-Length constraints

How sharply can we implement the constraints?

The answer is provided by the LQG phase space!

(Constrained) Area-Angle Regge action

[BD, Speziale 2007]

To localize Area-Length constraints on 4-simplices, it is best to introduce 3D dihedral angles as auxiliary variables. Matches quantum numbers in EPRL/FK type spin foams.

Area-Length constraints =

Shape matching constraints

τ shared by σ and σ'

$$\phi_i^\tau = \Phi_i^{\tau,\sigma}(A), \qquad \phi_i^\tau = \Phi_i^{\tau,\sigma'}(A)$$

'Integrate out' angles

Coupling of σ and σ'

$$\Phi_i^{\tau,\sigma}(A) = \Phi_i^{\tau,\sigma'}(A)$$

But constraints do not commute.

Are second class. [BD, Ryan 2008]

$$\{\phi_1^\tau - \Phi_1^{\tau,\sigma}(A), \phi_2^\tau - \Phi_2^{\tau,\sigma}(A)\} \sim \gamma$$

Weak implementation of Area-Length constraints

How sharply can we implement the constraints?

The answer is provided by the LQG phase space!

(Constrained) Area-Angle Regge action

[BD, Speziale 2007]

To localize Area-Length constraints on 4-simplices, it is best to introduce 3D dihedral angles as auxiliary variables. Matches quantum numbers in EPRL/FK type spin foams.

Area-Length constraints =
Shape matching constraints

τ shared by σ and σ'

$$\phi_i^\tau = \Phi_i^{\tau,\sigma}(A), \qquad \phi_i^\tau = \Phi_i^{\tau,\sigma'}(A)$$

'Integrate out' angles

Coupling of σ and σ'

$$\Phi_i^{\tau,\sigma}(A) = \Phi_i^{\tau,\sigma'}(A)$$

But constraints do not commute.
Are second class. [BD, Ryan 2008]

$$\{\phi_1^\tau - \Phi_1^{\tau,\sigma}(A), \phi_2^\tau - \Phi_2^{\tau,\sigma}(A)\} \sim \gamma$$

The Barbero-Immirzi parameter is an anomaly parameter and controls how sharply we can implement constraints.

[BD, Ryan 2010]

Weak implementation of Area-Length constraints

How sharply can we implement the constraints?

The answer is provided by the LQG phase space!

(Constrained) Area-Angle Regge action

[BD, Speziale 2007]

To localize Area-Length constraints on 4-simplices, it is best to introduce 3D dihedral angles as auxiliary variables. Matches quantum numbers in EPRL/FK type spin foams.

Area-Length constraints =
Shape matching constraints

τ shared by σ and σ'

$$\phi_i^\tau = \Phi_i^{\tau,\sigma}(A), \qquad \phi_i^\tau = \Phi_i^{\tau,\sigma'}(A)$$

'Integrate out' angles

Coupling of σ and σ'

$$\Phi_i^{\tau,\sigma}(A) = \Phi_i^{\tau,\sigma'}(A)$$

But constraints do not commute.
Are second class. [BD, Ryan 2008]

$$\{\phi_1^\tau - \Phi_1^{\tau,\sigma}(A), \phi_2^\tau - \Phi_2^{\tau,\sigma}(A)\} \sim \gamma$$

The Barbero-Immirzi parameter is an anomaly parameter and controls how sharply we can implement constraints.

[BD, Ryan 2010]

Also for bi-vector simplicity constraints: Becomes partially second class upon quantization. Anomaly controlled by γ.
Thus weak implementation of (part of the) simplicity constraints in EPRL/FK.

[EPRL/FK, Perez;
with edge modes: Freidel-Geiller-Pranzetti]

Completing the construction of effective spin foams

Attach intertwiner coherent state to each $\tau \subset \sigma$ pair:

$$|\mathscr{K}(\phi^\tau; \Phi^{\tau\sigma}(A))\rangle$$

[Livine-Speziale]

'Integrate out' angles

Coupling of σ and σ'

$$G^{\sigma,\sigma'}(A)$$

(Approx) Gaussian factors,

with spread controlled by γ.

Completing the construction of effective spin foams

Attach intertwiner coherent state to each $\tau \subset \sigma$ pair:

$$|\mathscr{K}(\phi^\tau; \Phi^{\tau\sigma}(A))\rangle$$

[Livine-Speziale]

'Integrate out' angles

Coupling of σ and σ'

$$G^{\sigma,\sigma'}(A)$$

(Approx) Gaussian factors,

with spread controlled by γ.

Effective spin foam model:

$$Z = \sum_{A_t\text{-values}} e^{iS_{\mathrm{AR}}} \prod_\tau G^{\sigma,\sigma'}$$

Oscillating factor
with Area Regge action

Gaussian factor peaked
on constraints.

Completing the construction of effective spin foams

Attach intertwiner coherent state to each $\tau \subset \sigma$ pair:

$$|\mathcal{K}(\phi^\tau; \Phi^{\tau\sigma}(A))\rangle$$

[Livine-Speziale]

'Integrate out' angles

Coupling of σ and σ'

$$G^{\sigma,\sigma'}(A)$$

(Approx) Gaussian factors,

with spread controlled by γ.

Effective spin foam model:

$$Z = \sum_{A_t\text{-}values} e^{iS_{\mathrm{AR}}} \prod_\tau G^{\sigma,\sigma'}$$

Oscillating factor with Area Regge action

Gaussian factor peaked on constraints.

Does that lead to discrete GR?

Yes, but γ needs to be small.

Due to general issue due to anomaly / 'weak constraint implementation'.

(There seems to be a gap in the literature.)

Relation to other spin foam models

- Huge computational advantage: minutes on laptop as compared to weeks on HPC for EPRL/FK
- One main reason: already integrated out 3D dihedral angles: Much, much fewer variables

- Does define transition amplitudes for LQG Hilbert space!

Relation to other spin foam models

- Huge computational advantage: minutes on laptop as compared to weeks on HPC for EPRL/FK
- One main reason: already integrated out 3D dihedral angles: Much, much fewer variables

- Does define transition amplitudes for LQG Hilbert space!

- Can easily construct Lorentzian model with space-like, time-like tetrahedra, triangles, edges [Asante, BD, Padua-Arguelles 2021]
- Allows much easier control on what configuration to include:
    - Orientations
    - Topology change and light cone irregularities [Asante, BD, Padua-Arguelles 2021 and 2022]

Relation to other spin foam models

- Huge computational advantage: minutes on laptop as compared to weeks on HPC for EPRL/FK
- One main reason: already integrated out 3D dihedral angles: Much, much fewer variables

- Does define transition amplitudes for LQG Hilbert space!

- Can easily construct Lorentzian model with space-like, time-like tetrahedra, triangles, edges [Asante, BD, Padua-Arguelles 2021]
- Allows much easier control on what configuration to include:
  - Orientations
  - Topology change and light cone irregularities [Asante, BD, Padua-Arguelles 2021 and 2022]

- Using $\exp(iS_{\text{Regge}})$: appears as 20-j symbol in spin foam model for Poincare (higher gauge) group
[Girelli-Pfeiffer-Popescu, Baratin-Freidel, Mikovic-Vojnovic, Asante-BD-Girelli-Riello-Tsimiklis]

Relation to other spin foam models

- Huge computational advantage: minutes on laptop as compared to weeks on HPC for EPRL/FK
- One main reason: already integrated out 3D dihedral angles: Much, much fewer variables

- Does define transition amplitudes for LQG Hilbert space!

- Can easily construct Lorentzian model with space-like, time-like tetrahedra, triangles, edges [Asante, BD, Padua-Arguelles 2021]
- Allows much easier control on what configuration to include:
  - Orientations
  - Topology change and light cone irregularities [Asante, BD, Padua-Arguelles 2021 and 2022]

- Using $\exp(iS_{\text{Regge}})$: appears as 20-j symbol in spin foam model for Poincare (higher gauge) group
  [Girelli-Pfeiffer-Popescu, Baratin-Freidel, Mikovic-Vojnovic, Asante-BD-Girelli-Riello-Tsimiklis]

- Barrett-Crane spin foam model: Semi-classical limit described by Area-Regge action $\Rightarrow$ Effective spin foams without constraints

- Conjecture: Integrating out intertwiners in EPRL/FK leads to same form as effective spin foams
  [First indications: Asante-Simao-Steinhaus]

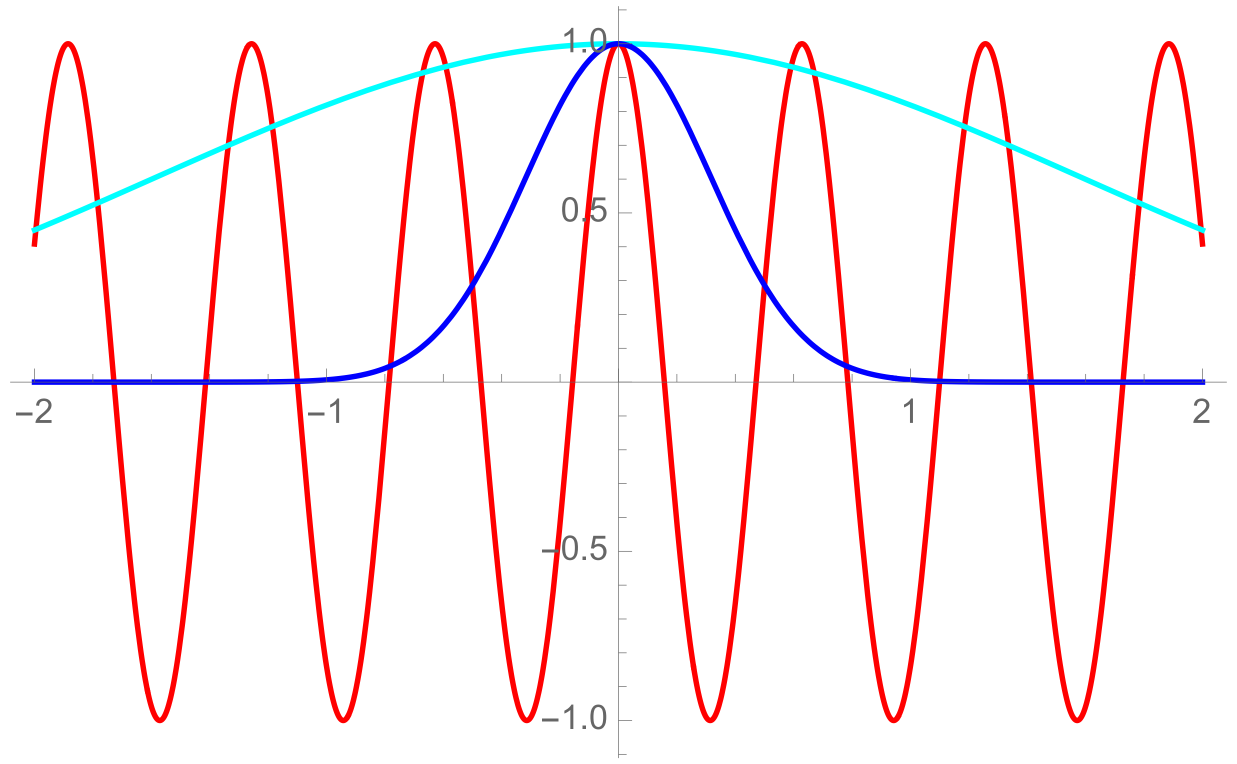
Do (effective) spin foams work?

Is 'weak constraint implementation' sufficient to obtain correct semi-classical limit?

$$Z = \sum_{A_t\text{-values}} e^{iS_{\mathrm{AR}}} \prod_{\tau} G^{\sigma,\sigma'}$$

Oscillating factor
with Area Regge action

Gaussian factor peaked
on constraints.



Oscillating and Gaussian factor for two different γ's.

(Standard) semi-classical limit:
Stronger and stronger oscillations wash out Gaussians/ Constraints.
Should allow only a few oscillations over Gaussian width:

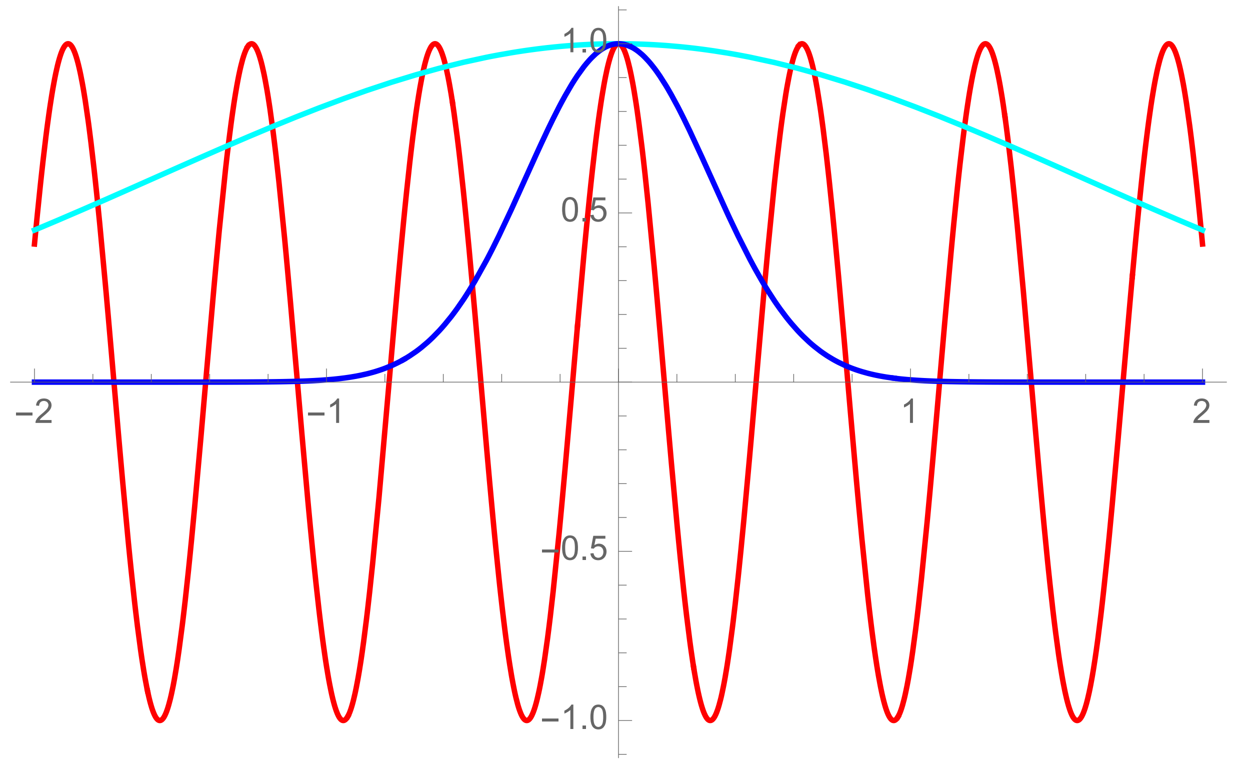
Do (effective) spin foams work?

Is ‘weak constraint implementation’ sufficient to obtain correct semi-classical limit?

$$Z = \sum_{A_t\text{-values}} e^{iS_{\mathrm{AR}}} \prod_{\tau} G^{\sigma,\sigma'}$$

Oscillating factor
with Area Regge action

Gaussian factor peaked
on constraints.



Oscillating and Gaussian factor for two different γ’s.

(Standard) semi-classical limit:
Stronger and stronger oscillations wash
out Gaussians/ Constraints.
Should allow only a few oscillations over
Gaussian width:

$$\sqrt{\gamma}\frac{\sqrt{\text{Area}}}{\ell_P}\frac{\delta S_{AR}}{\delta \text{Area}} \leq \mathcal{O}(1)$$

[Han 2013,
Asante-BD-Haggard 2020 PRL]

γ has to be small.

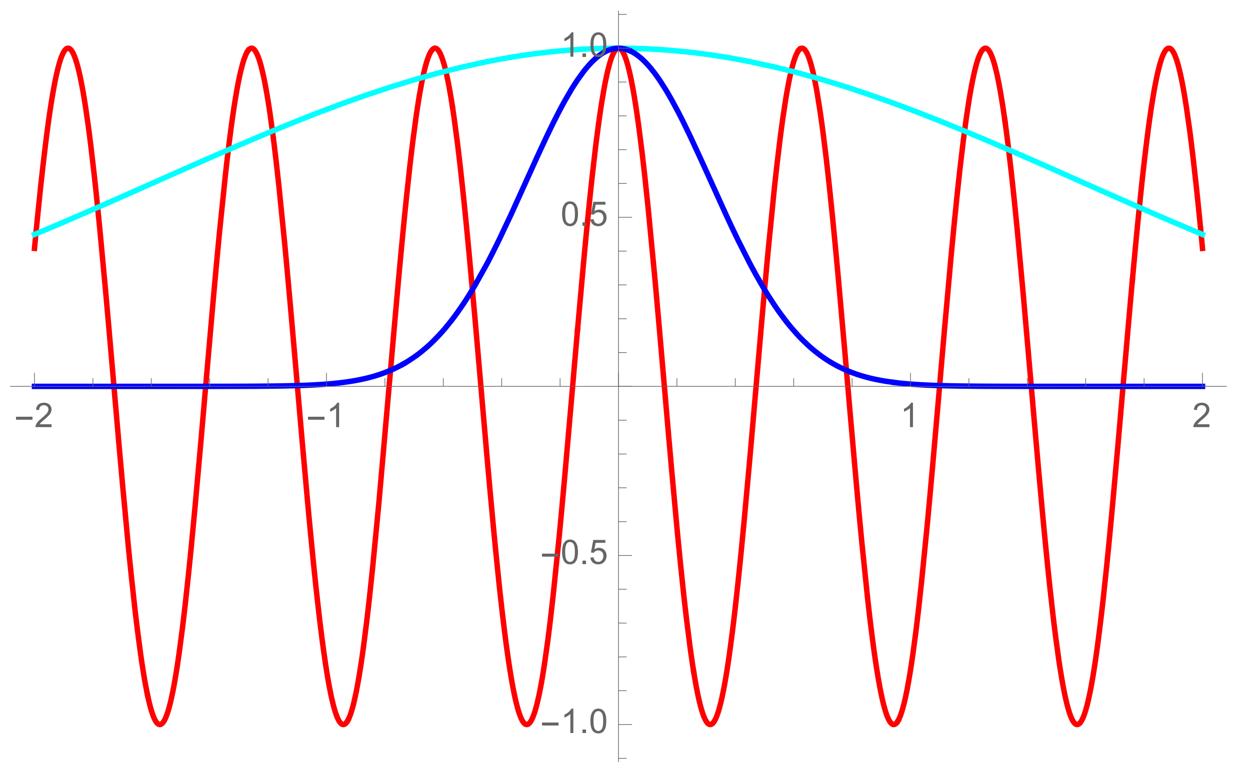
Do (effective) spin foams work?

Is 'weak constraint implementation' sufficient to obtain correct semi-classical limit?

$$Z = \sum_{A_t\text{-values}} e^{iS_{\mathrm{AR}}} \prod_{\tau} G^{\sigma,\sigma'}$$

Oscillating factor with Area Regge action

Gaussian factor peaked on constraints.



Oscillating and Gaussian factor for two different γ's.

(Standard) semi-classical limit:
Stronger and stronger oscillations wash out Gaussians/ Constraints.
Should allow only a few oscillations over Gaussian width:

$$\sqrt{\gamma}\,\frac{\sqrt{\mathrm{Area}}}{\ell_P}\,\frac{\delta S_{AR}}{\delta \mathrm{Area}} \leq \mathscr{O}(1)$$

[Han 2013,
Asante-BD-Haggard 2020 PRL]

γ has to be small.

Alternative view point:

Action is now complex. Thus saddle points in complex plain, imaginary part leading to suppression of amplitude. [Asante-BD-Haggard 2020 CQG]

Thus, the imaginary part of saddle point (controlled by γ) should be small. [For EPRL: Han, Huang, Liu, Qu 2021]

But how small?

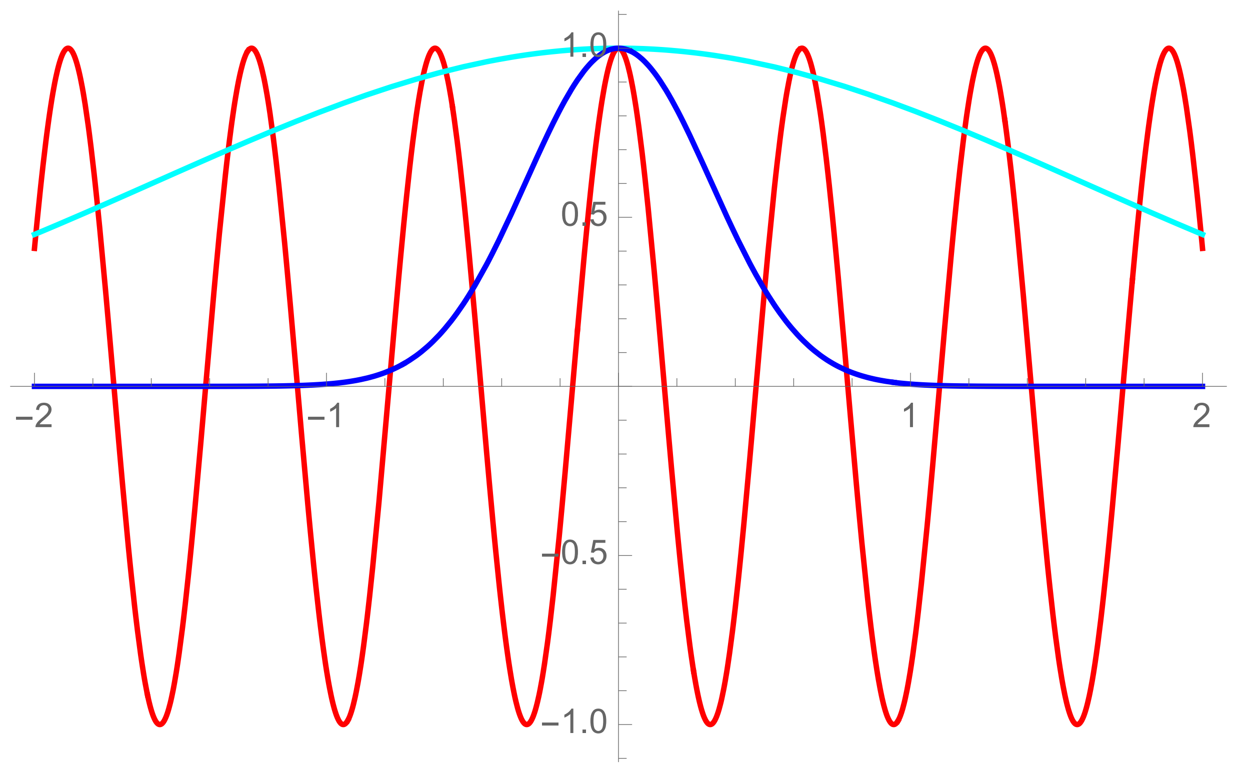
Do (effective) spin foams work?

Is 'weak constraint implementation' sufficient to obtain correct semi-classical limit?

$$Z = \sum_{A_t\text{-values}} e^{iS_{\mathrm{AR}}} \prod_{\tau} G^{\sigma,\sigma'}$$

Oscillating factor with Area Regge action

Gaussian factor peaked on constraints.



Oscillating and Gaussian factor for two different γ's.

(Standard) semi-classical limit:
Stronger and stronger oscillations wash out Gaussians/ Constraints.
Should allow only a few oscillations over Gaussian width:

$$\sqrt{\gamma}\,\frac{\sqrt{\mathrm{Area}}}{\ell_P}\,\frac{\delta S_{AR}}{\delta \mathrm{Area}} \leq \mathcal{O}(1)$$

[Han 2013, Asante-BD-Haggard 2020 PRL]

γ has to be small.

Alternative view point:

Action is now complex. Thus saddle points in complex plain, imaginary part leading to suppression of amplitude.

Thus, the imaginary part of saddle point (controlled by γ) should be small.

[Asante-BD-Haggard 2020 CQG]

[For EPRL: Han, Huang, Liu, Qu 2021]

But how small?

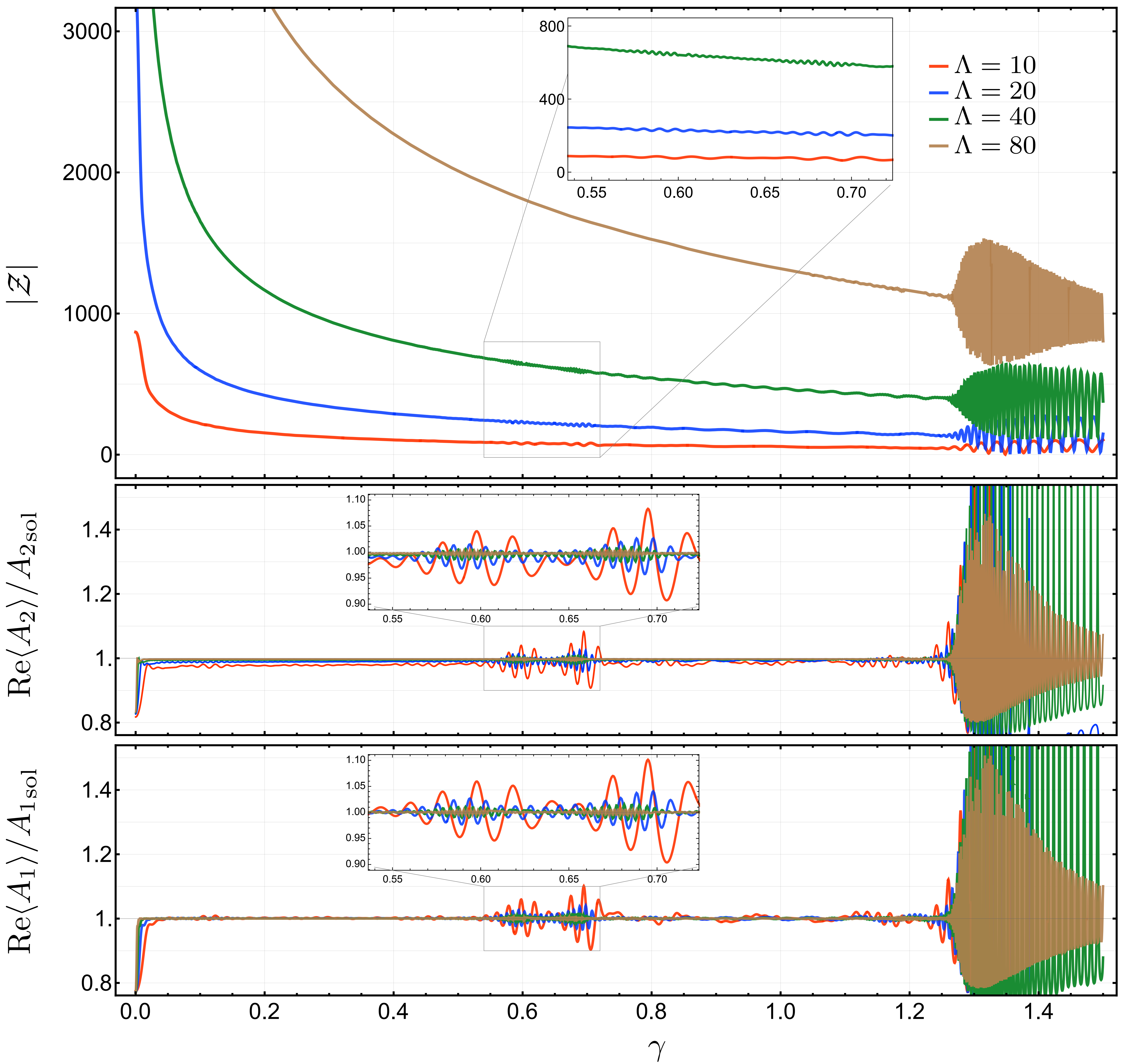
[Asante-BD-Haggard 2020 CQG] So far the only explicit path integral evaluation of expectation values, testing EOM.

Important effects beyond saddle point evaluation, but leading to a better than expected picture.

Testing effective spin foams

[Asante-BD-Haggard 2020]

Computation of area expectation values by full non-perturbative evaluation of path integral for triangulation with inner edge.



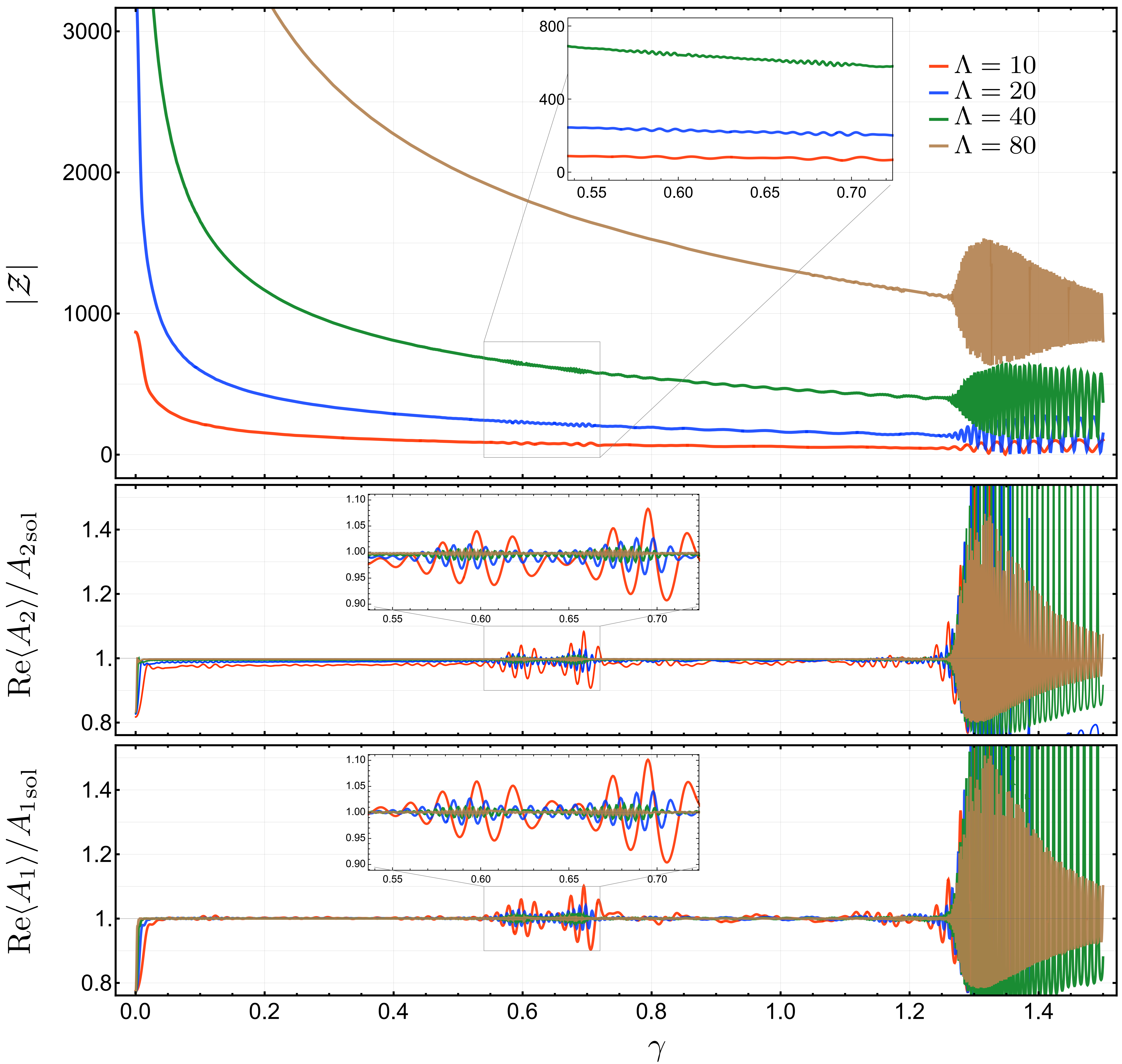
Low curvature

$$\epsilon_1(t_{\mathrm{sol}}) = 0\,, \qquad \epsilon_2(t_{\mathrm{sol}}) = \epsilon_3(t_{\mathrm{sol}}) = 0.034$$

Testing effective spin foams

[Asante-BD-Haggard 2020]

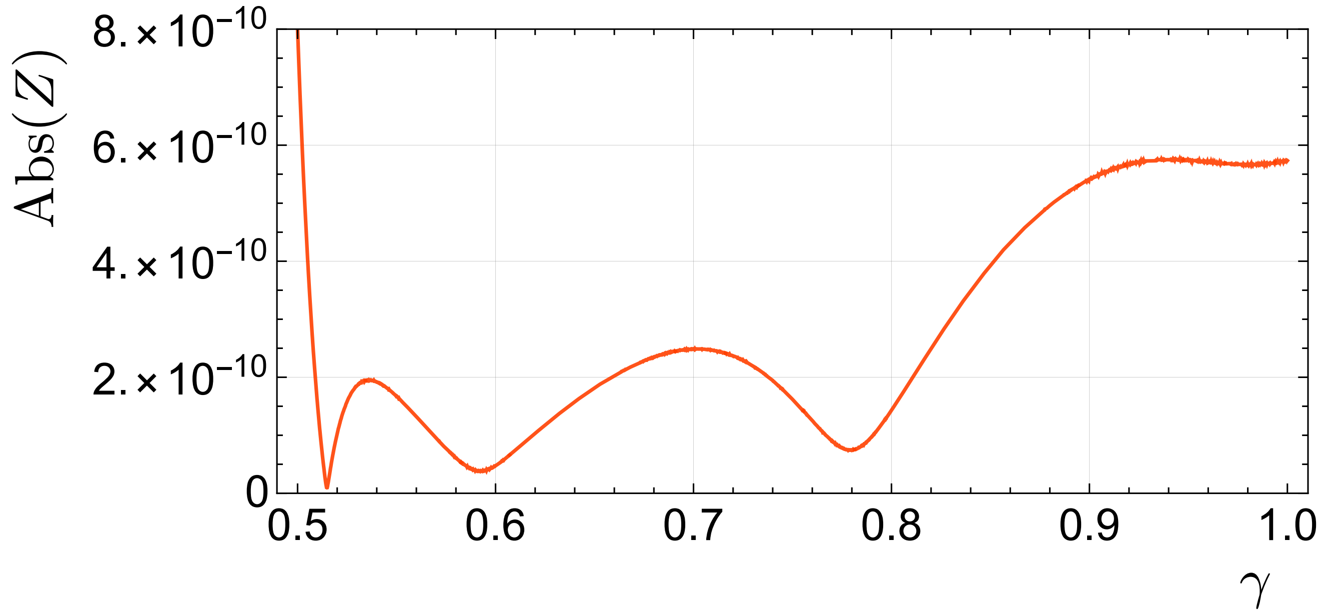
Computation of area expectation values by full non-perturbative evaluation of path integral for triangulation with inner edge.

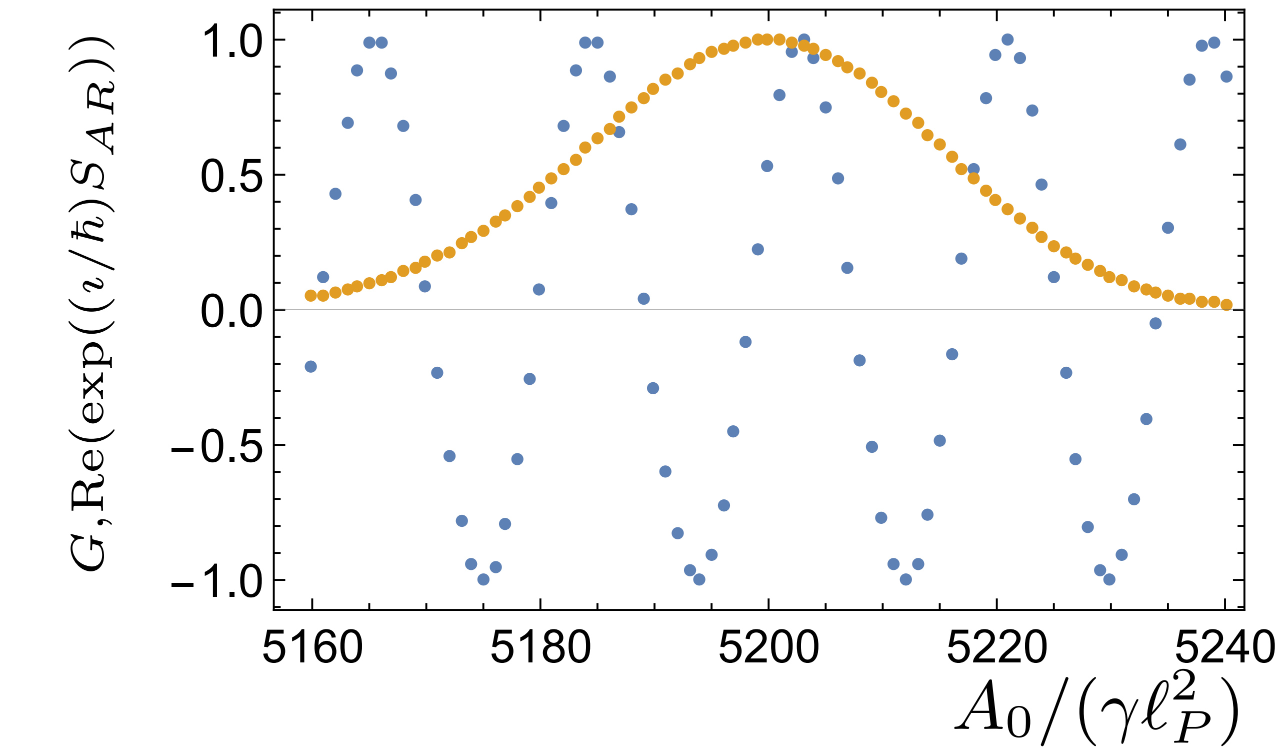


Low curvature

$$\epsilon_1(t_{\mathrm{sol}}) = 0\,, \qquad \epsilon_2(t_{\mathrm{sol}}) = \epsilon_3(t_{\mathrm{sol}}) = 0.034$$

Oscillations due to non-perturbative effects, resulting from interplay of discrete spectra and constraint implementation





Surprises:

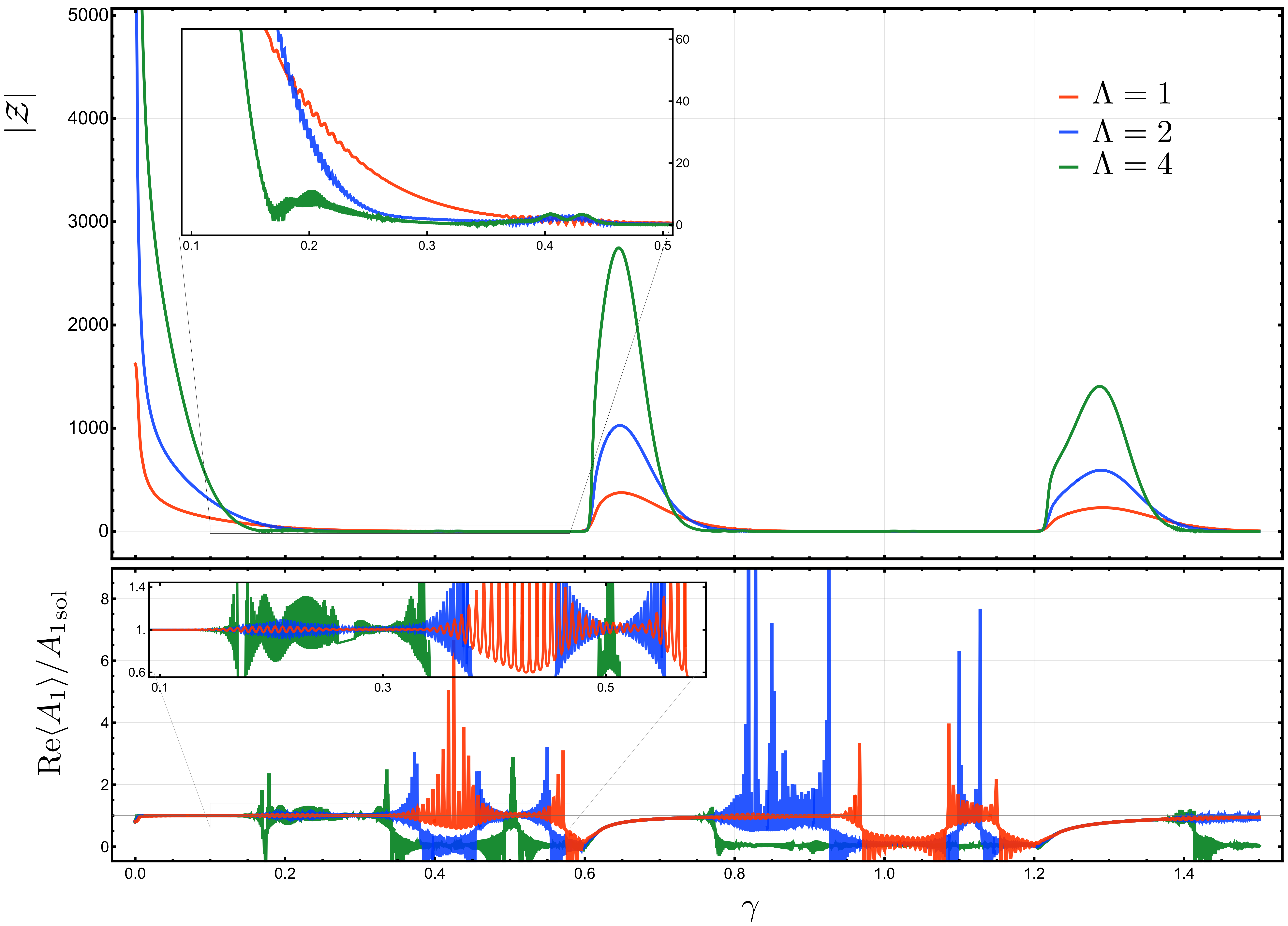
-threshold behaviour for oscillations

-threshold values independent of scale Lambda

Testing effective spin foams

[Asante-BD-Haggard 2020]

Computation of area expectation values by full non-perturbative evaluation of path integral for triangulation with inner edge.



Very large curvature

$$\epsilon_1(t_{\mathrm{sol}}) = 4.193\,, \qquad \epsilon_2(t_{\mathrm{sol}}) = -1.790\,, \qquad \epsilon_3(t_{\mathrm{sol}}) = -1.1432$$

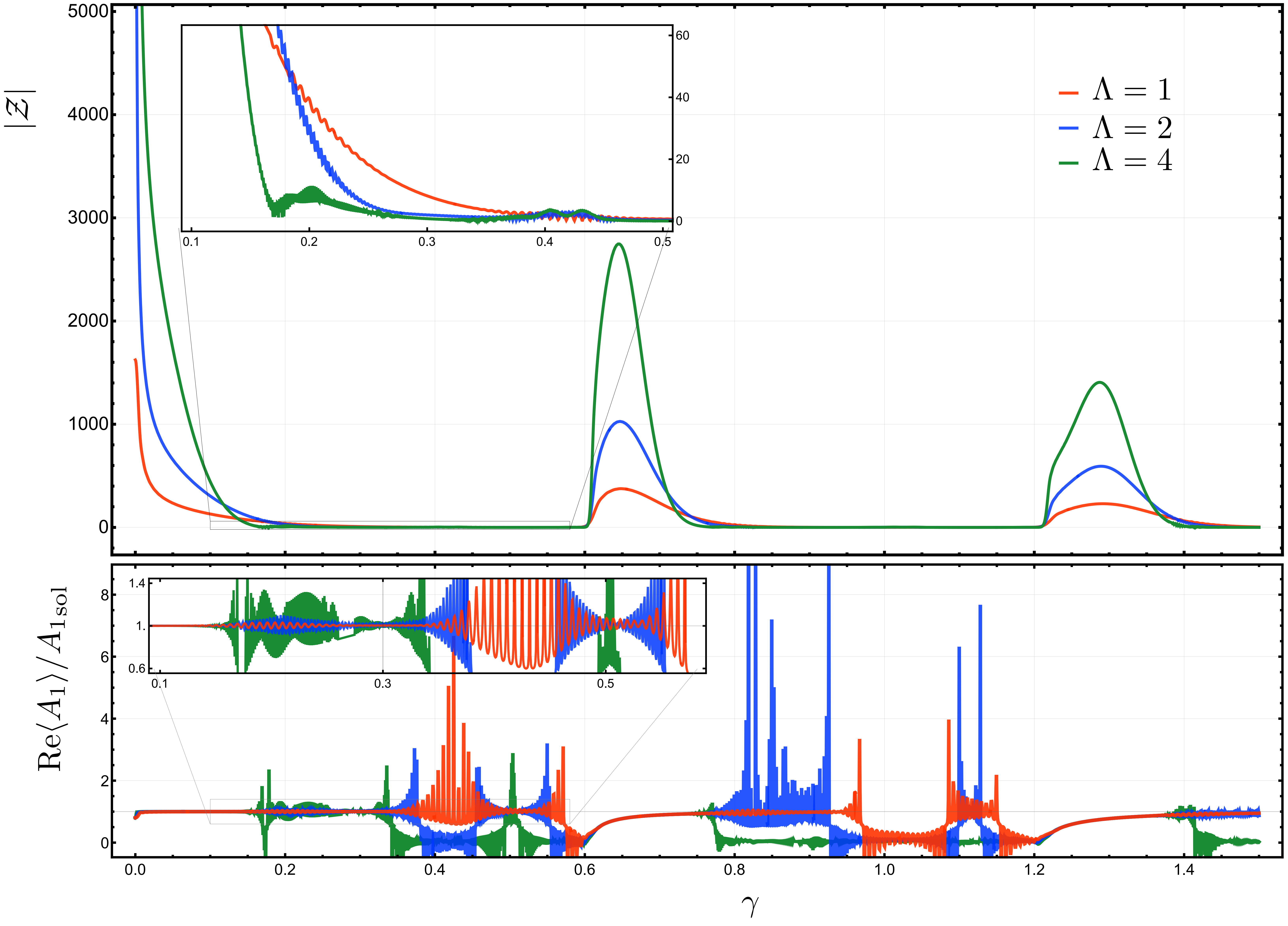
Reproduce classical EOM for

$\gamma < 0.15 \cdots 0.3$

Testing effective spin foams

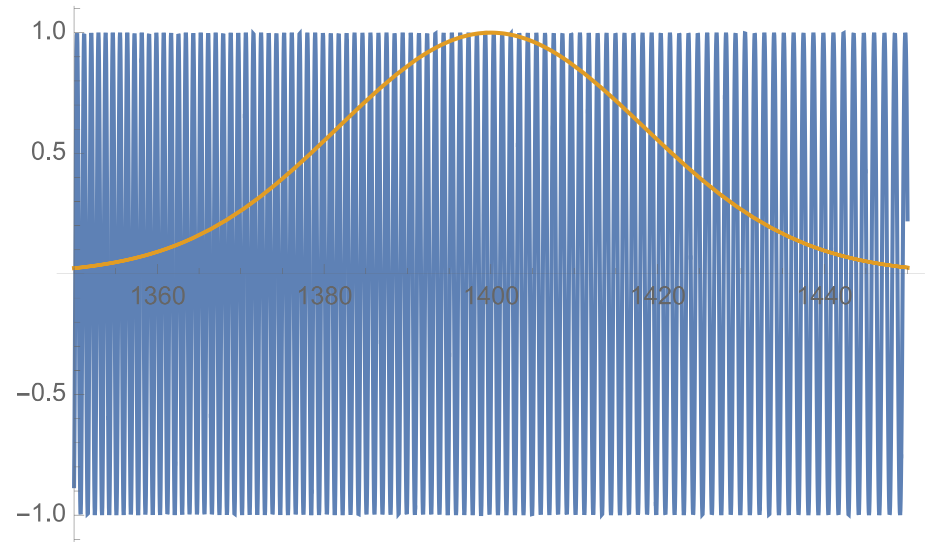
[Asante-BD-Haggard 2020]

Computation of area expectation values by full non-perturbative evaluation of path integral for triangulation with inner edge.

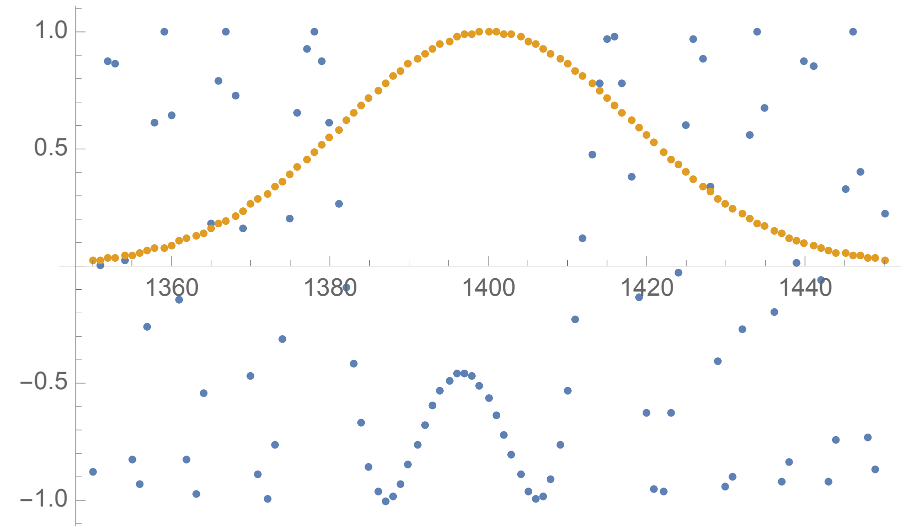


Very large curvature

$$\epsilon_1(t_{\rm sol}) = 4.193\,, \qquad \epsilon_2(t_{\rm sol}) = -1.790\,, \qquad \epsilon_3(t_{\rm sol}) = -1.1432$$



Maxima due to
Pseudo-Saddle Points.
(But not at $\epsilon\gamma = 2\pi \times N$)



Reproduce classical EOM for

$\gamma < 0.15 \cdots 0.3$

Discrete gravity from spin foams

- areas are independent variables
- have discrete (equi-distant) spectrum

$\Downarrow$

- Anomaly in Area-Length constraints
- Controlled by γ = Anomaly parameter
- Forces weak implementation

$\Downarrow$

- Anomaly parameter should be small for proper semi-classical limit

Expectation values recover discrete gravitational equations of motions - for a larger range of γ than expected.

[Asante-BD-Haggard 2020]

(Configurations with inner edge (three inner triangles) better behaved than configuration with just one triangle.)

Reproducing continuum GR (and corrections)

Surprise: The dynamics in the continuum limit provides a mechanism to suppress the Area-Length constraints, without any explicit constraint implementation.

Continuum limit for the Area Regge action

Despite its importance for spin foams, almost nothing was known about the continuum limit of the Area Regge Action.

Equation of motion: $0 = \epsilon_t(A_t)$ Curvature + Shape Mismatch =0

But discrete canonical analysis shows that it has much more propagating dof's than Length Regge Action. [Asante-BD-Haggard 2018]

Continuum limit for the Area Regge action

Despite its importance for spin foams, almost nothing was known about the continuum limit of the Area Regge Action.

Equation of motion: $0 = \epsilon_t(A_t)$ Curvature + Shape Mismatch =0

But discrete canonical analysis shows that it has much more propagating dof's than Length Regge Action. [Asante-BD-Haggard 2018]

[BD 2021]
[BD, Kogios 2022]

First analysis of (linearized) Area Regge action on hyper-cubic lattices and extraction of continuum limit.

Crucial (non-trivial) step: Split variables according to the order in which they contribute to the continuum limit.

Determined by scaling of Hessian blocks in derivatives.

Continuum limit for the Area Regge action

Despite its importance for spin foams, almost nothing was known about the continuum limit of the Area Regge Action.

Equation of motion: $0 = \epsilon_t(A_t)$ Curvature + Shape Mismatch =0

But discrete canonical analysis shows that it has much more propagating dof's than Length Regge Action. [Asante-BD-Haggard 2018]

[BD 2021]
[BD, Kogios 2022]

First analysis of (linearized) Area Regge action on hyper-cubic lattices and extraction of continuum limit.

Crucial (non-trivial) step: Split variables according to the order in which they contribute to the continuum limit.
Determined by scaling of Hessian blocks in derivatives.

[BD, Kogios 2022]

Finding: The two leading orders for the continuum limit result from variables forming an Area Metric.
The Area Metric has 20 components and can be parametrized by
-length metric h and (trace part)
-10 additional components χ (trace-free part)

Continuum limit for the Area Regge action

Hessian blocks in length metric: Gives discretized Einstein-Hilbert action. Second order in derivatives.

Crucial for continuum behaviour: Scaling of the Hessian blocks in derivatives.

$$H_{eff} = \underbrace{H_{hh}}_{\text{GR}} - \underbrace{H_{h\chi} H_{\chi\chi}^{-1} H_{\chi h}}_{\text{“Correction”}}$$

Continuum limit for the Area Regge action

Hessian blocks in length metric: Gives discretized Einstein-Hilbert action. Second order in derivatives.

Crucial for continuum behaviour: Scaling of the Hessian blocks in derivatives.

$$H_{eff} = \underbrace{H_{hh}}_{\text{GR}} - \underbrace{H_{h\chi} H^{-1}_{\chi\chi} H_{\chi h}}_{\text{“Correction”}}$$

Possibilities:

	h	χ
h	k^2	k^0
χ	k^0	k^2

Correction $\sim k^{-2}$

The “Correction” would dominate

In the continuum limit.

Even if we do implement the constraints.

	h	χ
h	k^2	k^2
χ	k^2	k^2

Correction $\sim k^{2}$

The “Correction” would is at the same

order as the GR part.

Constraints needed to make it subdominant.

	h	χ
h	k^2	k^2
χ	k^2	k^0

Correction $\sim k^{4}$

Higher order correction.

Constraints are not even needed.

Continuum limit for the Area Regge action

Hessian blocks in length metric: Gives discretized Einstein-Hilbert action. Second order in derivatives.

Crucial for continuum behaviour: Scaling of the Hessian blocks in derivatives.

$$H_{eff} = \underbrace{H_{hh}}_{\text{GR}} - \underbrace{H_{h\chi} H_{\chi\chi}^{-1} H_{\chi h}}_{\text{“Correction”}}$$

The dynamics of Area Regge leads to:

	h	χ
h	k^2	k^2
χ	k^2	k^0

Correction $\sim k^4$

Higher order correction.

Constraints are not even needed.

Area Regge = GR - Weyl^2

[BD, Kogios 2022]

	h	χ
h	k^2	k^2
χ	k^2	k^0

Correction $\sim k^4$

Area Regge = GR - Weyl^2

[BD, Kogios 2022]

	h	χ
h	k^2	k^2
χ	k^2	k^0

Correction $\sim k^4$

$$H \cdot h \ \sim \ \varepsilon\varepsilon\text{Riemann}$$

$$h \cdot H \cdot h \ \sim \ \text{Einstein-Hilbert}$$

$$\chi \cdot H \cdot h \ \sim -\chi \cdot \text{Weyl}$$

$\Rightarrow$ Correction $\sim -\,\text{Weyl}^2 + \mathcal{O}(k^6)$

Area Regge = GR+ Weyl^2

[BD, Kogios 2022]

	h	χ
h	k^2	k^2
χ	k^2	k^0

Correction $\sim k^4$

$$H \cdot h \;\sim\; \varepsilon\varepsilon\text{Riemann}$$

$$h \cdot H \cdot h \;\sim\; \text{Einstein-Hilbert}$$

$$\chi \cdot H \cdot h \;\sim -\chi \cdot \text{Weyl}$$

$\Rightarrow$ Correction $\sim -\,\text{Weyl}^2 + \mathcal{O}(k^6)$

- Dynamics of Area Regge calculus gives (Planck) mass to Area-Length constraint dof's.
- Do not need an explicit constraint implementation.
- Even if we do: Result does not change in an essential way. [to appear]
- Universality: Spin foam models differ in how/ whether constraints are implemented.

  But this does not seem to matter in the continuum limit.

Effective actions for the continuum limit of spin foams directly from continuum action. [Borissova, BD 2022]

Reviving the modified Plebanski action program. [Krasnov 2006]

Effective actions for the continuum limit of spin foams directly from continuum action.

[Borissova, BD 2022]

Reviving the modified Plebanski action program.

[Krasnov 2006]

Plebanski formulation for GR

Spinfoam quantization

Simplicity constraints partially imposed only weakly

Semi-classical limit:

Area-Regge action

Continuum limit:

Area-metrics

$S_{eff} = \text{GR} + \text{Weyl}^2 + \mathcal{O}(\partial^6)$

Effective actions for the continuum limit of spin foams directly from continuum action. Reviving the modified Plebanski action program.

[Borissova, BD 2022]

Plebanski formulation for GR

Spinfoam quantization
Simplicity constraints partially imposed only weakly

Semi-classical limit:
Area-Regge action

Continuum limit:
Area-metrics
$S_{eff} = \text{GR} + \text{Weyl}^2 + \mathcal{O}(\partial^6)$

For continuum action:
Simplicity constraints partially imposed only weakly

?

Modified Plebanski actions

[Krasnov 2006, …]

Replace simplicity constraints with potential for the corresponding degrees of freedom.
(Hope: Renormalization flow = flow of potential)

[Freidel 2008, Krasnov, …]

Surprise! For chiral theory modified Plebanski still leads to (a deformation of) GR.

[Alexandrov-Krasnov 2009, Speziale 2010]

For non-chiral theory general modified Plebanski leads to (messy) bi-metric theory.

Modified Plebanski actions

[Krasnov 2006, …]

Replace simplicity constraints with potential for the corresponding degrees of freedom.
(Hope: Renormalization flow = flow of potential)

[Freidel 2008, Krasnov, …]

Surprise! For chiral theory modified Plebanski still leads to (a deformation of) GR.

[Alexandrov-Krasnov 2009, Speziale 2010]

For non-chiral theory general modified Plebanski leads to (messy) bi-metric theory.

Strategy for determining dynamics:

- Integrate out connection and Lagrange multipliers [Bengtsson 1995]
- Leaves us with second order action for B-fields [Freidel 2008, Speziale 2010]
- Use clever parametrization of B-fields in terms of two length metrics and two uni-modular internal su(2) metrics: g^+, g^-, q^+, q^- [Freidel 2008]
- Simplicity constraints: $g^+ = g^-$ and $q^\pm = \text{Id}$

Modified Plebanski actions

[Krasnov 2006, …]

Replace simplicity constraints with potential for the corresponding degrees of freedom.
(Hope: Renormalization flow = flow of potential)

[Freidel 2008, Krasnov, …]

Surprise! For chiral theory modified Plebanski still leads to (a deformation of) GR.

[Alexandrov-Krasnov 2009, Speziale 2010]

For non-chiral theory general modified Plebanski leads to (messy) bi-metric theory.

Strategy for determining dynamics:

- Integrate out connection and Lagrange multipliers [Bengtsson 1995]
- Leaves us with second order action for B-fields [Freidel 2008, Speziale 2010]
- Use clever parametrization of B-fields in terms of two length metrics and two uni-modular internal su(2) metrics: g^+, g^-, q^+, q^- [Freidel 2008]
- Simplicity constraints: $g^+ = g^-$ and $q^\pm =$ Id

What we have to choose:

[Borissova, BD 2022]

- Which simplicity constraints to impose strongly and which only by potential term
- We want to reproduce: Area metric as free variable
- Effective action from Area Regge (as far as determined)

Effective actions for the continuum limit of spin foams

Choose:

[Borissova, BD 2022]

- $g^+ = g^-$ imposed strongly
- $q^\pm \approx \mathrm{Id}$ imposed weakly via Potential

- Leaves us with 20 degrees of freedom, can be packaged into Area Metric $A = A(g, q^+, q^-)$

Effective actions for the continuum limit of spin foams

Choose:

[Borissova, BD 2022]

- $g^+ = g^-$ imposed strongly
- $q^\pm \approx \mathrm{Id}$ imposed weakly via Potential

- Leaves us with 20 degrees of freedom, can be packaged into Area Metric $A = A(g, q^+, q^-)$

Linearized action:

$$S = \frac{1+\gamma^{-1}}{2}\int \mathscr{L}_{GR}(h_{\mu\nu} + \chi^+_{\mu\nu}) \;+\; \frac{1-\gamma^{-1}}{2}\int \mathscr{L}_{GR}(h_{\mu\nu} + \chi^-_{\mu\nu})$$
$$+\frac{1+\gamma^{-1}}{8}\int m_+^2\, \chi^+_{\mu\nu}\chi^{+\mu\nu} \;+\; \frac{1-\gamma^{-1}}{8}\int m_-^2\, \chi^-_{\mu\nu}\chi^{-\mu\nu}$$

Effective action:

$$S_{\mathrm{eff}} = \int \mathscr{L}_{GR}(h_{\mu\nu}) - \frac{1}{4}\left(\frac{1}{m_+^2 + \Delta} + \frac{1}{m_-^2 + \Delta}\right)\mathrm{Weyl}^2(h_{\mu\nu})$$

Ghost-free version of Einstein-Weyl Gravity.
Barbero-Immirzi-parameter only through mass terms.

Effective actions for the continuum limit of spin foams

Choose:

[Borissova, BD 2022]

- $g^+ = g^-$ imposed strongly
- $q^\pm \approx \mathrm{Id}$ imposed weakly via Potential

- Leaves us with 20 degrees of freedom, can be packaged into Area Metric $A = A(g, q^+, q^-)$

Linearized action:

$$S = \frac{1+\gamma^{-1}}{2} \int \mathscr{L}_{GR}(h_{\mu\nu} + \chi^+_{\mu\nu}) \; + \; \frac{1-\gamma^{-1}}{2} \int \mathscr{L}_{GR}(h_{\mu\nu} + \chi^-_{\mu\nu})$$

Area Regge action

$$+ \frac{1+\gamma^{-1}}{8} \int m_+^2 \, \chi^+_{\mu\nu} \chi^{+\mu\nu} \; + \; \frac{1-\gamma^{-1}}{8} \int m_-^2 \, \chi^-_{\mu\nu} \chi^{-\mu\nu}$$

Effective action:

$$S_{\text{eff}} = \int \mathscr{L}_{GR}(h_{\mu\nu}) - \frac{1}{4}\left(\frac{1}{m_+^2 + \Delta} + \frac{1}{m_-^2 + \Delta}\right) \mathrm{Weyl}^2(h_{\mu\nu})$$

Compatible with
Effective action found
From Area Regge action

Ghost-free version of Einstein-Weyl Gravity.
Barbero-Immirzi-parameter only through mass terms.

Effective actions for the continuum limit of spin foams

- Can model enlargement of configuration space also in the continuum: Area instead of Length metrics
- Modified Plebanski approach gives effective action consistent with Area Regge calculus result

- Puzzling sign difference for coupling between h and χ (can be easily removed by field redefinition)

  Future work: consider Area-Angle action.

Are there universality arguments for area metric actions?

Effective actions for the continuum limit of spin foams

- Can model enlargement of configuration space also in the continuum: Area instead of Length metrics
- Modified Plebanski approach gives effective action consistent with Area Regge calculus result

- Puzzling sign difference for coupling between h and χ (can be easily removed by field redefinition)

  Future work: consider Area-Angle action.

Are there universality arguments for area metric actions?

New opportunities for spin foam phenomenology!

- χ dof's are microscopic / Planck scale degrees of freedom [Perez]
- There is hair for black holes in Einstein-Weyl gravity

- We obtain a ghost-free version of Einstein-Weyl gravity
- Can extend to non-linear theory

Fully non-perturbative continuum limit

- Here we assumed that ‘perturbative continuum limit’ can be taken (background field method)
- Still lots of work to do to show that this is allowed

- Convergence of oscillatory path integral (instead of conformal factor problem) [Jose’s talk]

- Not being swamped by degenerate, too wild configurations, e.g. branching of baby universes [Jose’s talk]

Effective spin foam models will be helpful:

- Can fine-tune which configurations to allow and which not to allow
- Huge computational advantage

Conclusions

- Areas as more fundamental variables, discrete spectrum $\Rightarrow$ Anomaly problem

- Discrete level: Anomaly parameter γ has to be small for proper semi-classical limit
- Explicit test of equations of motions with effective spin foams: allowed range larger than expected

- Continuum limit: We are in luck! Area-Regge dynamics gives mass to degrees of freedom we want to suppress.

- Barrett-Crane model can lead to General Relativity!

- Mechanism can be modelled within Modified Plebanski approach - directly in the continuum.

Effective length metric action: ghost-free version of Einstein-Weyl gravity.

- Many opportunities for phenomenology! Area metric has twice as many degrees of freedom as length metric.

- Are there universality arguments for the type of corrections?

How does the anomaly arise?

$$B^{IJ} \to J^{IJ}$$

quantized as non-commuting SO(4) generators

(as in Lattice gauge theory)

without Immirzi parameter

$$B^{KL} \to \frac{\gamma^2}{\gamma^2+1}\left(J^{KL} - \frac{1}{\gamma} \star J^{KL}\right)$$

with Immirzi parameter

introduced through adding a quasi-topological term to the action [Holst]

"Non-diagonal" part of simplicity constraints becomes second class.

Non-commutativity parametrized by γ, which becomes an anomaly parameter.

[BD, Ryan 08; Perez '12; Asante, BD, Haggard '20]

(Naive) Semi-classical limit of spin foams

[Barrett-Williams, Barrrett-Dowdall-Fairbairn-Gomes-Hellmann, Conrady-Freidel, …]

Area Regge action $S_{\mathrm{AR}}(A_t) = \sum_t A_t \cdot \epsilon_t(A_t)$ Equation of motion: $0 = \epsilon_t(A_t)$

Seem to demand flatness

Length Regge action $S_{\mathrm{LR}}(L_e) = \sum_t A_t(L_e) \cdot \epsilon_t(L_e)$ Equation of motion: $0 = \sum_t \frac{\partial A_t}{\partial L_e} \epsilon_t(L_e)$

curvature angles

discretization of Einstein equations

Parametrization of (self-dual) two-form fields

$$B^{IJ}_{\mu\nu}$$

non-chiral Plebanski

$\star$ – eigenvectors :

$$B^i_{+\mu\nu} \qquad B^i_{-\mu\nu}$$

chiral Plebanski

Urbantke metric: extract length metric (conformal class) from chiral B-fields

$$\sqrt{g}\, g_{\mu\nu} = \epsilon^{ijk} B_{\pm\mu\alpha} B_{\pm\beta\gamma} B_{\pm\delta\nu}\, \epsilon^{\alpha\beta\gamma\delta}$$

$$b^j_{+i}\, \Sigma^i_{+\mu\nu}(e) \qquad b^j_{-i}\, \Sigma^i_{-\mu\nu}(\bar{e})$$

[Freidel '08]

two independent tetrads/length metric plus (internal gauge reduced) 5+5 additional fields

overall: 30 (internal gauge reduced) components

Simplicity constraints demand: $e = \bar{e}, \;\; b^j_{+i} = \delta^j_i, \;\; b^j_{-i} = \delta^j_i$ — 10 (internal gauge reduced) components

(generically) modified non-chiral Plebanski: mass terms for $(e - \bar{e}), b_+, b_-$ — 30 (internal gauge reduced) components

[Borissova-BD]: Impose $e = \bar{e}$ sharply, mass terms for b_+, b_- — 20 (internal gauge reduced) components

Indeed form area-metric: $A_{\mu\nu\rho\sigma} = \Pi_\epsilon \circ \; B^{IJ}_{\mu\nu} B^{I'J'}_{\rho\sigma} \delta_{IJI'J'}$

20 components