Scalar perturbations in cosmological scenarios with a modified Friedmann equation

Lisa Mickel

University of Sheffield

Supervisor: Steffen Gielen

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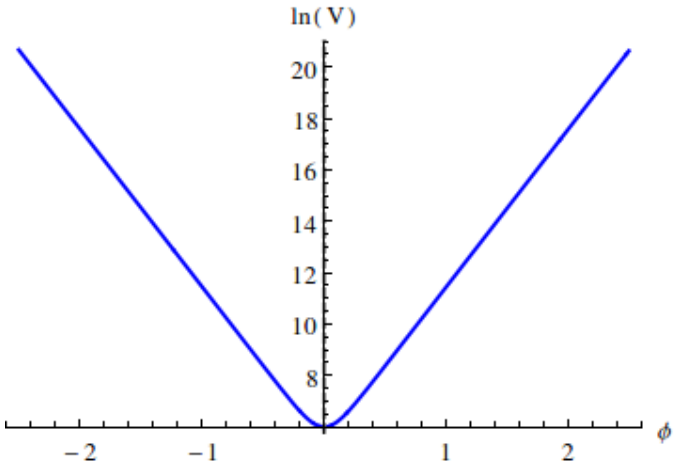
Motivation

- Effective Friedmann equations from quantum gravitational models

$$\frac{\frac{d}{d\phi}\langle\hat{V}\rangle}{\langle\hat{V}\rangle} = \frac{3H}{\phi'}$$



- Introduce general form of modified Friedmann equation

$$H^2 = \frac{\kappa}{3}N^2\rho\mathcal{F}$$

- Examples
  - Loop Quantum Cosmology (LQC): $\mathcal{F}_{\mathrm{LQC}} = 1 - \frac{\rho}{\rho_c}$
  - Group Field Theory (GFT): $\mathcal{F}_{\mathrm{GFT}} = 1 + \frac{v_0}{a^3} + \frac{\mathcal{V}}{a^6}$

- Statements about evolution of scalar perturbations from modified Friedmann equation?

Cosmological perturbation theory

Standard perturbation theory

- Homogeneous and isotropic universe: flat Friedmann-Lemaitre-Robertson-Walker metric

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 dx^i\, dx^j$$

- Linear perturbation theory

$$\begin{aligned} ds^2 = &- N^2(t)\left(1 + 2\tilde{\Phi}(t, x^i)\right) dt^2 + 2N(t)a(t)\,\partial_i B(t, x^i)\, dt\, dx^i \\ &+ a^2(t)\left[\left(1 - 2\psi(t, x^i)\right)\delta_{ij} + 2\partial_i\partial_j E(t, x^i)\right] dx^i\, dx^j \end{aligned}$$

- Matter content: single scalar field $\phi \to \phi + \delta\phi(t, x^i)$

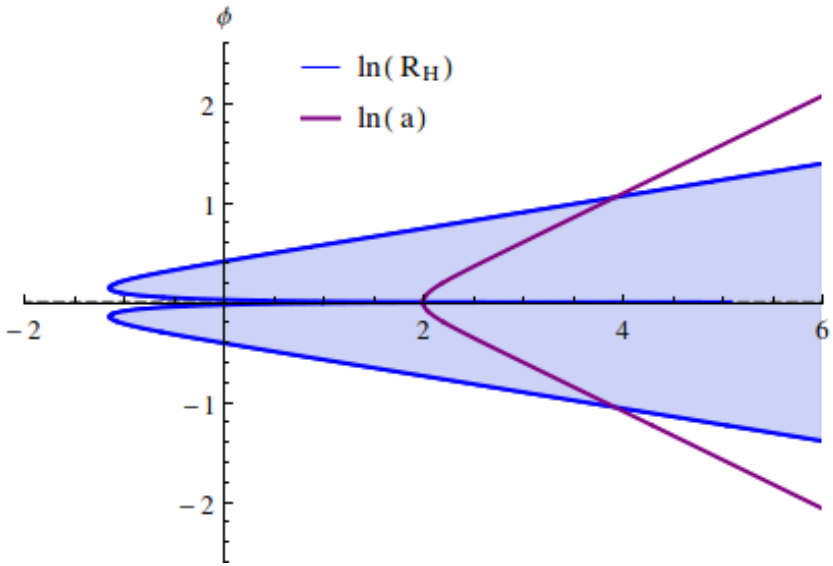
Gauge invariant perturbations

- Diffeomorphism invariance of perturbed coordinate system → gauge freedom
- Physical quantities = gauge invariant variables

Comoving curvature perturbation

$$\mathcal{R} = \psi + \frac{H}{\phi'}\delta\phi$$

- Physical wavelength of perturbations $\lambda \sim \frac{a}{k}$ → super- and sub-horizon modes

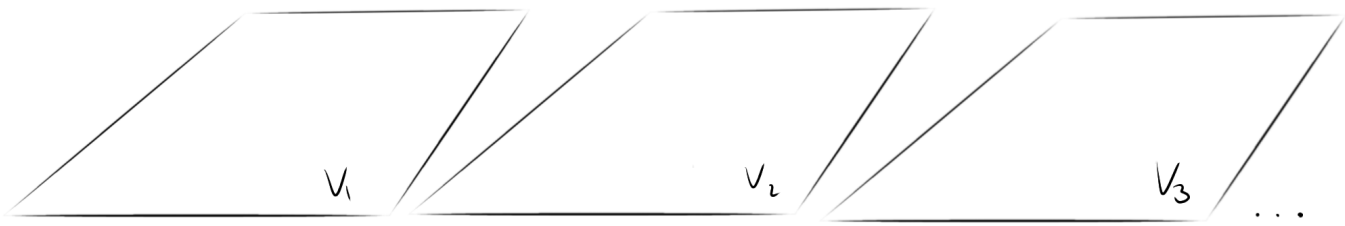


Dynamics from modified Friedmann equations

Separate universe framework

- Model universe as colletion of independent homogeneous patches that follow Friedmann dynamics - slightly different initial conditions
  [Alexander, Biswas, Brandenberger, '07], [Wands, Cardoso, '08], [Artigas, Grain, Venin, '21] ...

  [Wilson-Ewing, '16], [Gerhardt, Oriti, Wilson-Ewing, '18] ...



- Homogeneous perturbations in each patch with respect to background of the ensemble

$$\psi_p = \frac{1}{3}\left(1 - \frac{V_p}{V_{bg}}\right), \quad V_{bg} := \frac{1}{N_{\text{patches}}} \sum_p V_p$$

Dynamics of gauge invariant perturbation variables

- Recall $H^2 = \frac{\kappa}{3} N^2 \rho \mathcal{F}$
- Equations of motion from perturbing the Friedmann equation at linear order

$$H\psi' = -H^2\left(\tilde{\Phi} + \frac{\delta\rho}{2\rho} + \frac{\delta\mathcal{F}}{2\mathcal{F}}\right)$$

$$-\psi'' = \left(-\frac{N'}{N} - \frac{\mathcal{F}'}{2\mathcal{F}} + 3H\frac{\rho+P}{\rho}\right)\psi' + H\tilde{\Phi}' + \frac{H}{2}\left(\frac{\delta\mathcal{F}'}{\mathcal{F}} - \frac{\mathcal{F}'}{\mathcal{F}^2}\delta\mathcal{F}\right) + \frac{\kappa}{2}N^2\mathcal{F}(\rho+P)\left(\frac{\delta\rho}{\rho} - \frac{\delta\rho+\delta P}{\rho+P}\right)$$

Case A: $\mathcal{F} = \mathcal{F}(\rho)$

- Consider the case $\mathcal{F} \propto \rho$, $\delta\mathcal{F} \propto \delta\rho$
  - e.g. LQC [Wilson-Ewing, '16]
- Perturbation equations simplify

$$H\psi' = -H^2\tilde{\Phi} - \frac{\kappa}{6}N^2\left(\mathcal{F} + \frac{\mathrm{d}\mathcal{F}}{\mathrm{d}\rho}\rho\right)\delta\rho$$

- Can recover analogue to Diffeomorphism constraint, e.g. in comoving gauge ($\delta\phi = 0$)

$$\psi' + H\tilde{\Phi} = 0 \qquad \rightarrow \qquad \delta\rho = 0$$

- From $\delta\rho = -(\rho + P)\tilde{\Phi}$: conservation law for $\mathcal{R} = \psi$ as in GR

Case B: $\mathcal{F} \neq \mathcal{F}(\rho)$

- Equation of motion for $\mathcal{R}$ in comoving gauge ($\delta\phi = 0$)

$$\mathcal{R}' = -H\frac{\delta\mathcal{F}}{2\mathcal{F}}$$

- Example of a $\mathcal{F} \neq \mathcal{F}(\rho)$ case: GFT with a massless scalar field

$$\mathcal{F} = 1 + \frac{v_0}{a^3} + \frac{\mathcal{Y}}{a^6}\,, \quad \delta\mathcal{F} = 3\frac{v_0}{a^3}\psi + 6\frac{\mathcal{Y}}{a^6}\psi + \frac{\delta\mathcal{Y}}{a^6}\,.$$

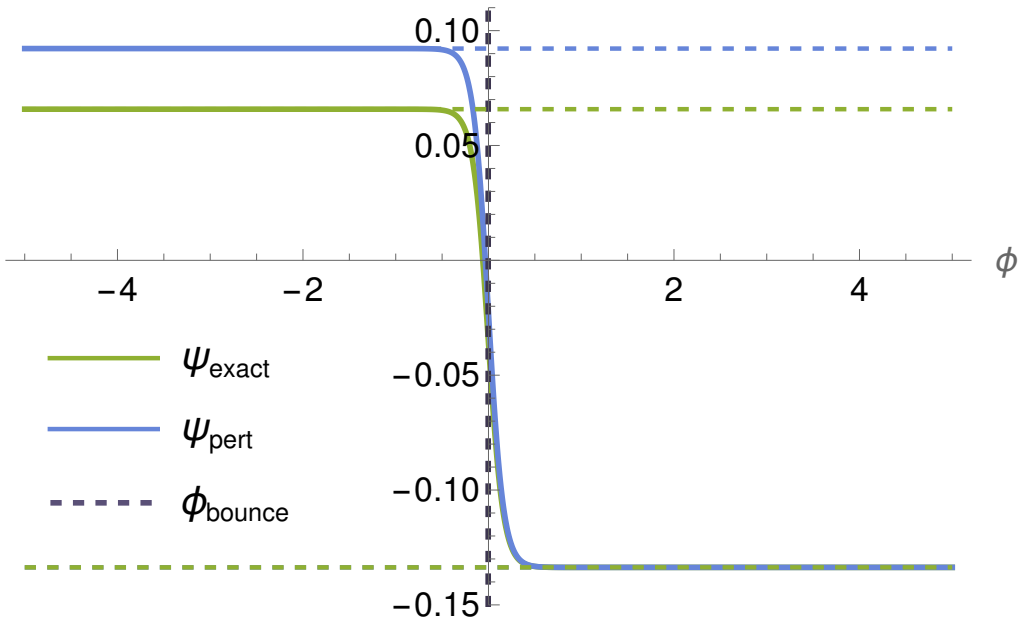


Figure: Evolution of $\mathcal{R}$ (in relational time ϕ)

Conclusion

Summary and Outlook

- Separate universe framework to obtain perturbation equations from general modified Friedmann equation
- Solutions to the first order equation of motion for the comoving curvature perturbation $\mathcal{R}$
  - recover conservation law of general relativity in $\mathcal{F}(\rho)$ case
  - non-trivial dynamics in general case
- Outlook: Consider also second order equation

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Thank you!