Classical dynamics for a simple model in Loop Quantum Gravity

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In collaboration with Eneko Aranguren and Etera R. Livine

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Outline

1. Introduction

2. Spinorial formalism and twisted geometries

3. 2-vertex model
   - Dynamics for the 2-vertex model
   - Homogeneous and isotropic sector
   - General case

4. Conclusions

Introduction

- Simple models within LQG to gain intuition about the open problems of the theory.
- Study dynamics in a truncation: **2-vertex model**.
- Spinorial formalism for LQG and twisted geometries.

- Symmetry reduced sector already studied.
  - Homogeneous and isotropic sector.
  - Interesting cosmological analogies and results.

- General case, out of the reduced sector.
  - Evolution of random initial configurations of spinors.
  - **Universal dependence** of the evolution regimes.
  - Reconstructing the associated polyhedra we compute the evolution of their volume and shape.

SPINORS AND NOTATIONS

- Spinors $z \in \mathbb{C}^2$: $\quad |z\rangle = \begin{pmatrix} z^0 \\ z^1 \end{pmatrix}, \qquad \langle z| = \begin{pmatrix} \bar{z}^0 & \bar{z}^1 \end{pmatrix}.$
- Geometric vector in 3d $\vec{X}(z)$, $\quad |z\rangle\langle z| = \frac{1}{2}\left(\langle z|z\rangle \mathbb{I} + \vec{X}(z)\cdot\vec{\sigma}\right).$
- Poisson brackets: $\quad \{z^A, \bar{z}^B\} = -i\delta^{AB} \qquad \{X^a, X^b\} = \epsilon^{abc} X^c.$
- Dual spinors: $\quad |z] \equiv -i\sigma_y |\bar{z}\rangle = \begin{pmatrix} -\bar{z}^1 \\ \bar{z}^0 \end{pmatrix}, \qquad [z|w] = \overline{\langle z|w\rangle}.$
- N spinors (one intertwiner with N legs): closure contraint

$$\vec{C} = \sum_i \vec{X}_i(z_i) = 0 \Rightarrow \begin{cases} \sum_i z_i^0 \bar{z}_i^1 = 0, \\ \sum_i |z_i^0|^2 = \sum_i |z_i^1|^2 = \frac{1}{2}\sum_i \langle z_i|z_i\rangle. \end{cases}$$

- Matching constraint along each edge,

$$C_k = \langle z_k^s | z_k^s \rangle - \langle z_k^t | z_k^t \rangle = 0.$$

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TWISTED GEOMETRIES AND QUANTIZATION

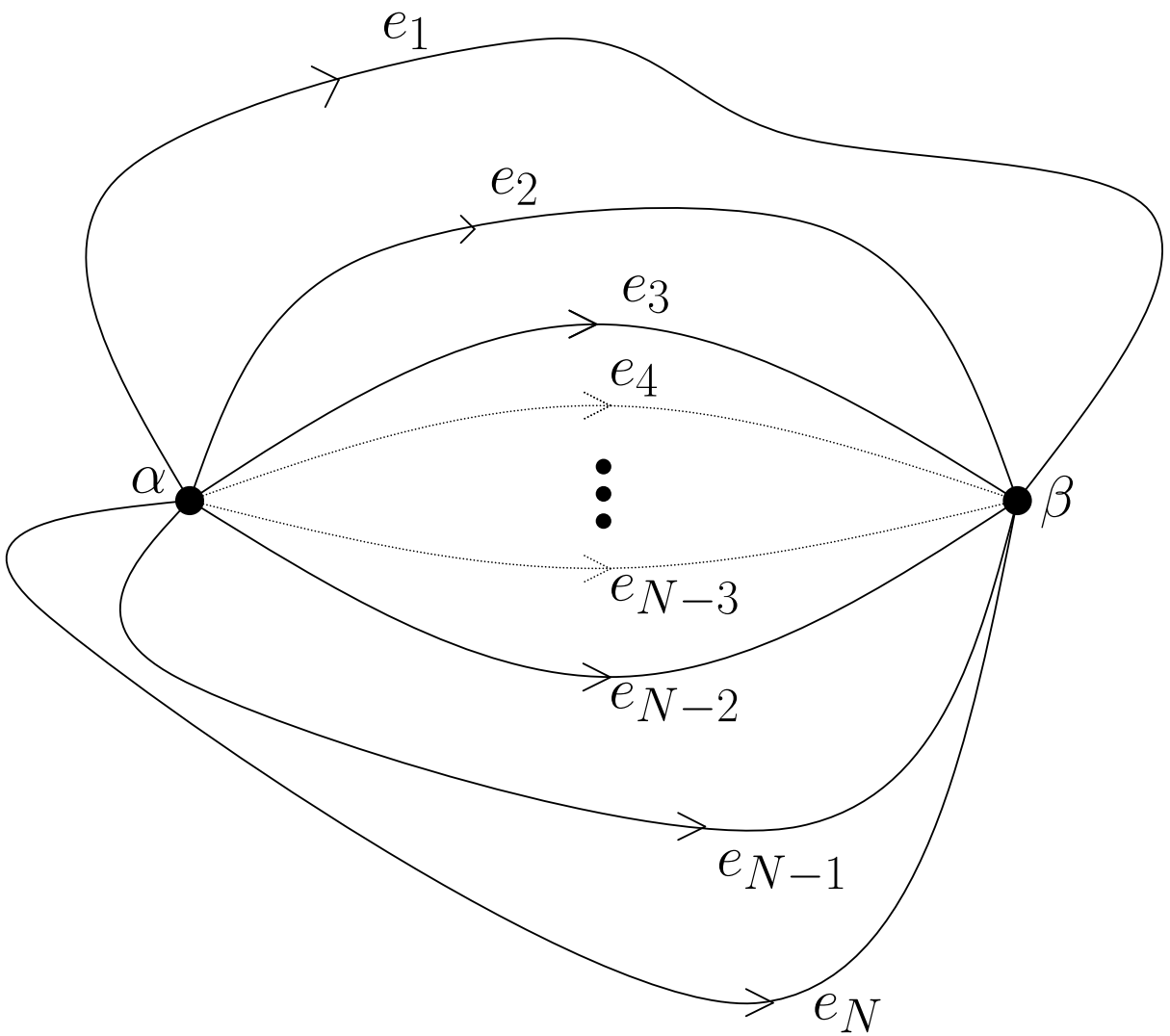
Minkowski theorem

If $\sum_{i=1}^{N} \vec{X}_i = 0$, there exists a unique convex polyhedron (up to translations) whose i-th face have orthogonal vector $\vec{X}_i$ and area $|\vec{X}_i|$.

Collection of spinors satisfying closure constraint	$=$	Collection of vectors satisfying Minkowski theorem

- Associate a polyhedron to each configuration of spinors. They are glued by faces of equal are $\rightarrow$ **Twisted geometries**.
  [Freidel, Speziale, 2010]
- Quantization of the spinors $\longrightarrow$ LQG kinematical Hilbert space.
  [Livine, Tambornino, 2012]
- Algorithm to reconstruct the polyhedron.
  [Bianchi, Dona, Speziale, 2011; Sellaroli, 2017]

THE 2-VERTEX GRAPH



DYNAMICS FOR THE 2-VERTEX GRAPH

- SU(2) invariant observables around each of the two vertices

$$E^{\alpha}_{kl} = \langle z^{\alpha}_k | z^{\alpha}_l \rangle \,, \qquad F^{\alpha}_{kl} = [z^{\alpha}_k | z^{\alpha}_l \rangle \,, \qquad \bar{F}^{\alpha}_{kl} = \langle z^{\alpha}_l | z^{\alpha}_k] = -\langle z^{\alpha}_k | z^{\alpha}_l] \,.$$

- Operators E_{ij} form a U(N) algebra.
- **Hamiltonian**: the lowest order polynomial ansatz for the spinors.

$$H = \sum_{k,l} \lambda E^{\alpha}_{kl} E^{\beta}_{kl} + \gamma F^{\alpha}_{kl} F^{\beta}_{kl} + \bar{\gamma} \bar{F}^{\alpha}_{kl} \bar{F}^{\beta}_{kl} \,.$$

- H commutes with the closure and matching constraints.
- **Equations of motion** for $|z^{\alpha}_i\rangle$:

$$\mathrm{d}_t |z_i\rangle = -\{H, |z_i\rangle\} = \sum_j \left(2i\bar{\gamma} \bar{F}^{\beta}_{ij} |z_j] - i\lambda E^{\beta}_{ij} |z_j\rangle \right) .$$

SYMMETRY REDUCED SECTOR

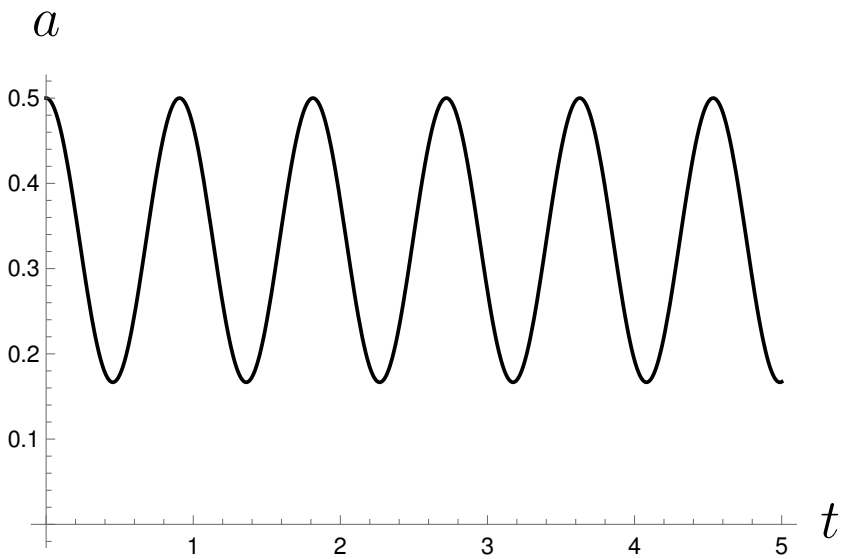
- We introduce the operators: $\mathcal{E}_{ij} \equiv E^{(\alpha)}_{ij} - E^{(\beta)}_{ji}$.
- They form a global $\mathfrak{u}(N)$ algebra: $[\mathcal{E}_{ij}, \mathcal{E}_{kl}] = \delta_{jk}\mathcal{E}_{il} - \delta_{il}\mathcal{E}_{kj}$.
- We look for a $\mathrm{U}(N)$ invariant subspace

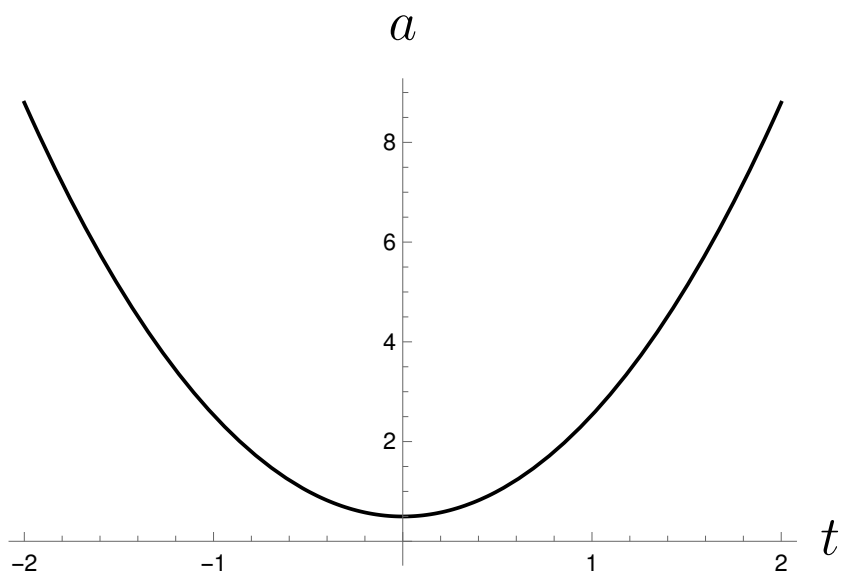
$$\boxed{{}^2\mathcal{H}_{inv} = \bigoplus_{J\in\mathbb{N}} \mathbb{C}\,|J\rangle}$$

- Classical Hamiltonian for the reduced sector depends only on two conjugated variables: (a, ϕ).

$\lambda^2 > 4\lvert\gamma\rvert^2$	$\rightarrow$	The evolution oscillates
$\lambda^2 \leq 4\lvert\gamma\rvert^2$	$\rightarrow$	The evolution diverges

OSCILLATORY AND DIVERGENT REGIMES



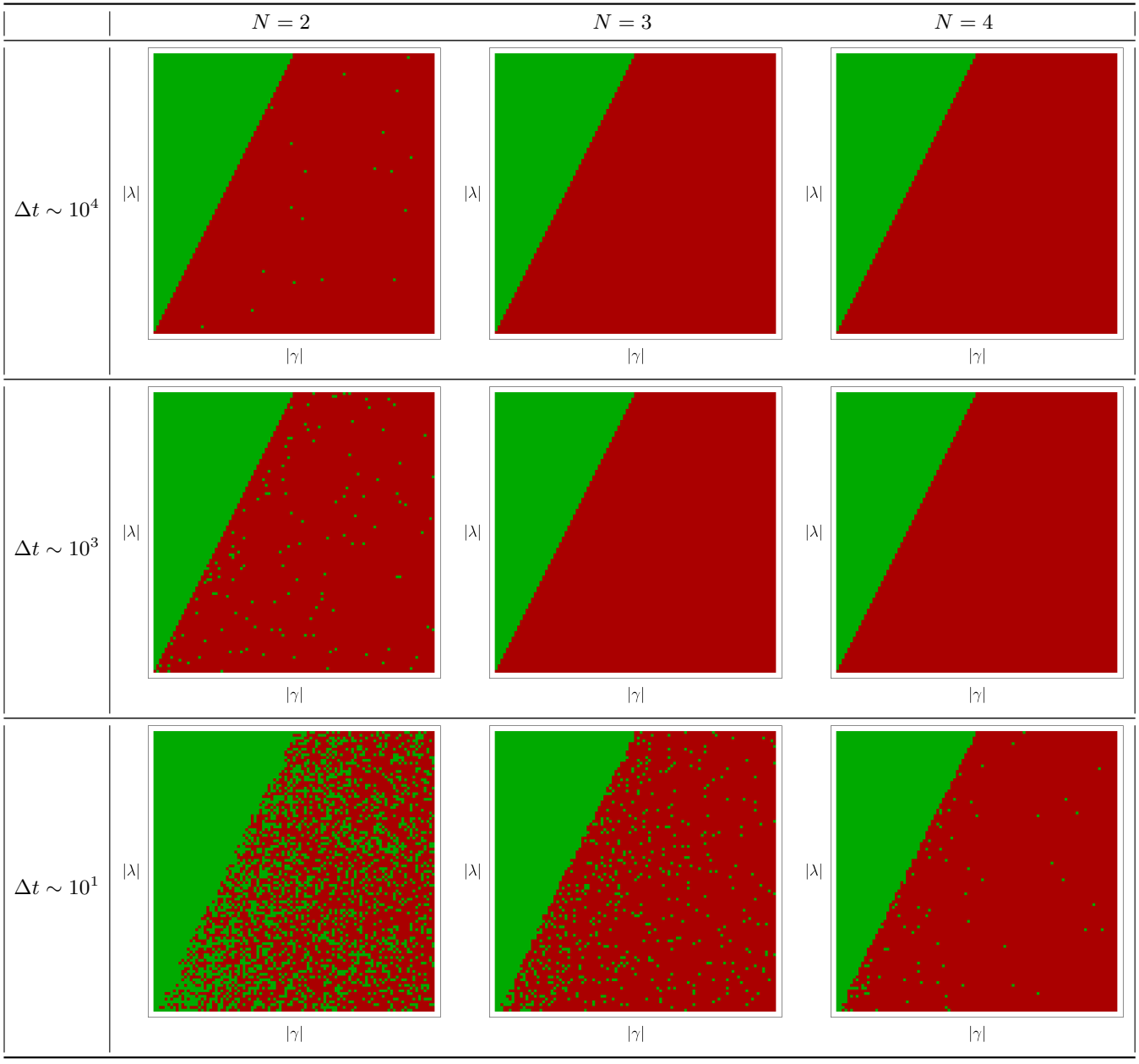


RANDOM CONFIGURATIONS OF CLOSED SPINORS

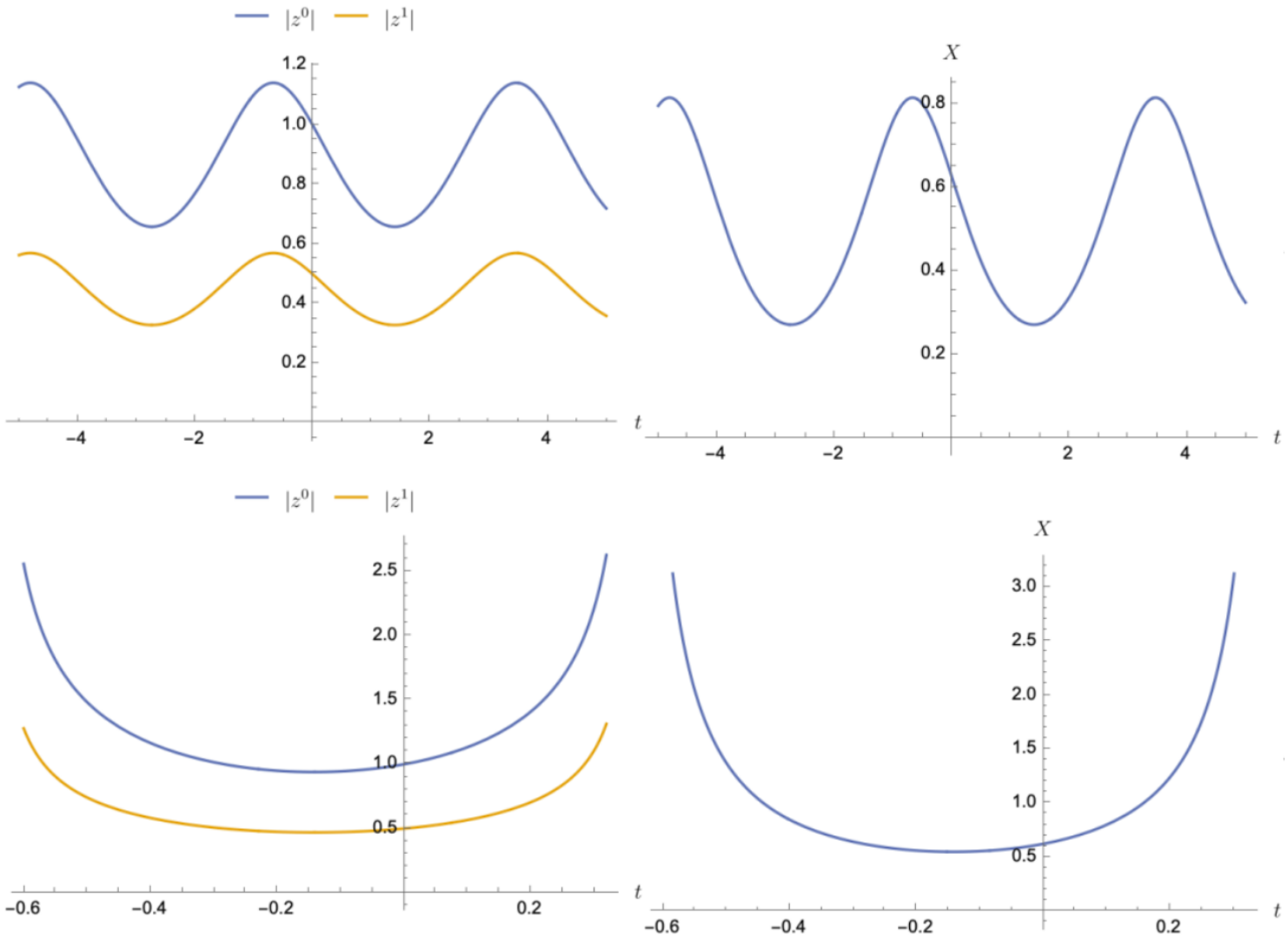
- Algorithm to generate random configurations of closed and matching spinors.
- Evolution of any initial configuration of spinors.
- Reconstruction of the associated polyhedra.
- Computation of the volume (using Sellarolli's code).
- Same regimes as the reduced case, independent of the number of edges and initial configurations.

$\lambda^2 > 4\|\gamma\|^2$	$\rightarrow$	The evolution oscillates
$\lambda^2 \leq 4\|\gamma\|^2$	$\rightarrow$	The evolution diverges

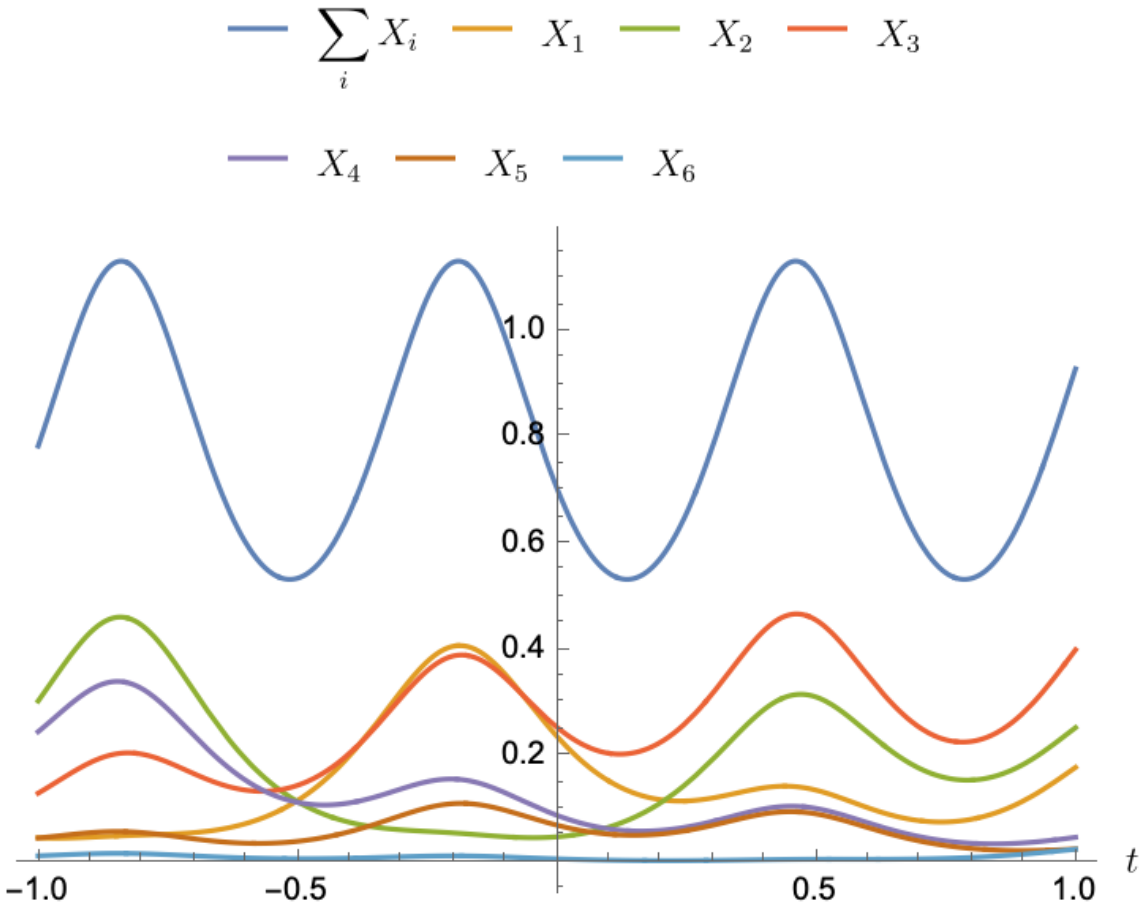
REGIMES DEPENDING ON THE COUPLING CONSTANTS



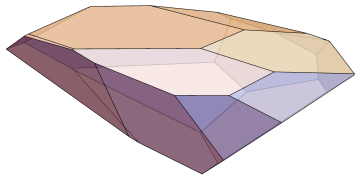
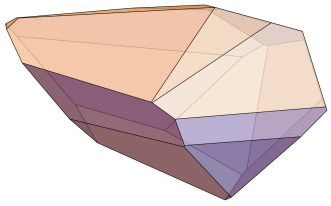
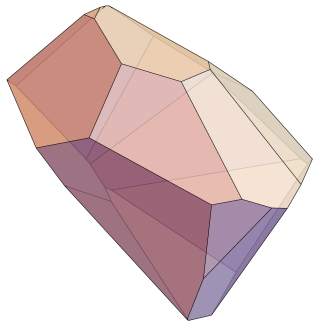
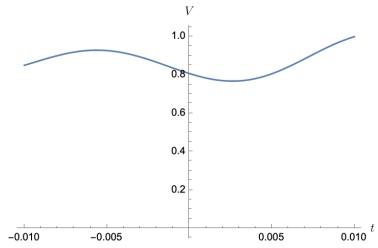
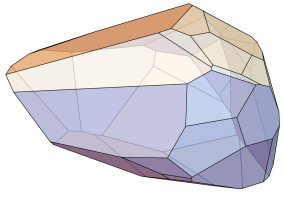
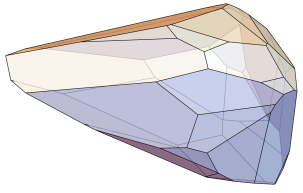
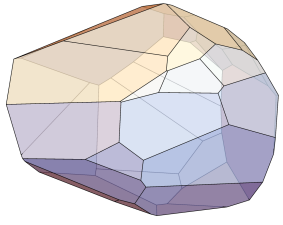
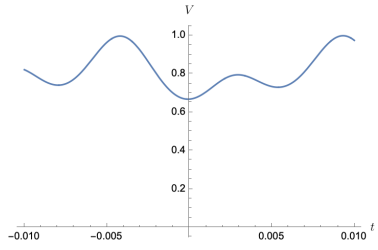
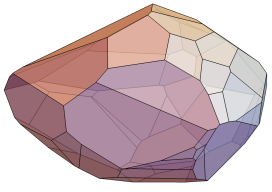
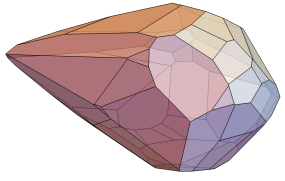
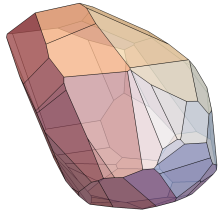
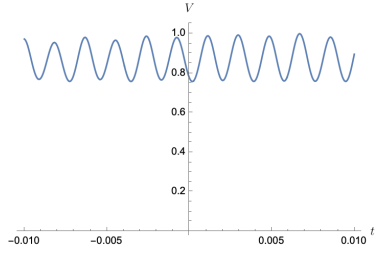
TWO EDGES


$|z^0|$ $|z^1|$
1.2
1.0
0.8
0.6
0.4
0.2
−4 −2 2 4
X
0.8
0.6
0.4
0.2
t
−4 −2 2 4 t
$|z^0|$ $|z^1|$
2.5
2.0
1.5
1.0
0.5
−0.6 −0.4 −0.2 0.2
t
X
3.0
2.5
2.0
1.5
1.0
0.5
−0.6 −0.4 −0.2 0.2 t

EVOLVING A RANDOM CONFIGURATION



Evolving a random configuration

	$t = -10^{-2}$	$t = 0$	$t = 10^{-2}$	Volume
$N = 20$				
$N = 40$				
$N = 60$				

CONCLUSIONS

- Dynamics for a truncation (2-vertex model) using the spinorial formalism and twisted geometries.
- Reduced sector with interesting **cosmological interpretation**.
- General case: evolution of random initial configurations (out of the homogeneous and isotropic sector).
- Oscillatory and divergent regimes with a universal dependence **only** on the coupling constants of the Hamiltonian.
- We studied also different Hamiltonians (different normalizations and edge dependent coupling constants). No regimes found.
- Possible path to explore more general cosmological models within the LQG theory.

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