

Dynamical reference frames: gauge-invariant observables, general covariance and locality

Christophe Goeller

based on 2206.01193, in collaboration with Philipp Höhn and Josh Kirklin

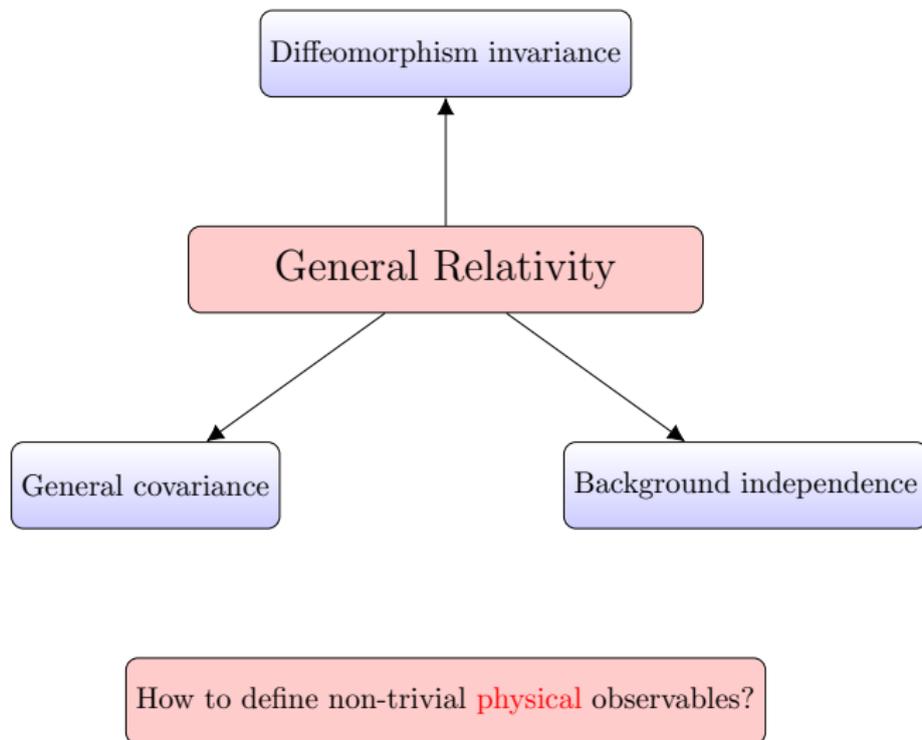
19, July 2022

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The tensions: locality, causality and dynamics

Observables must be gauge-invariant

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$$\psi(x) \rightarrow (f_*\psi)(x) = \psi(f^{-1}(x)) \neq \psi(x)$$

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Physical observables are non-local, have no bulk evolution and break causality?

From static to dynamical coordinate systems

We need to promote the fixed parametrisation of space-time into a dynamical one

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- From fixed $x \in \mathcal{M}$ to dynamical/dressed $x[\phi]$, where ϕ is a field configuration, such that it transforms covariantly

$$\forall f \text{ gauge diffeomorphism } x[f_*\phi] = f \circ x[\phi]$$

so that

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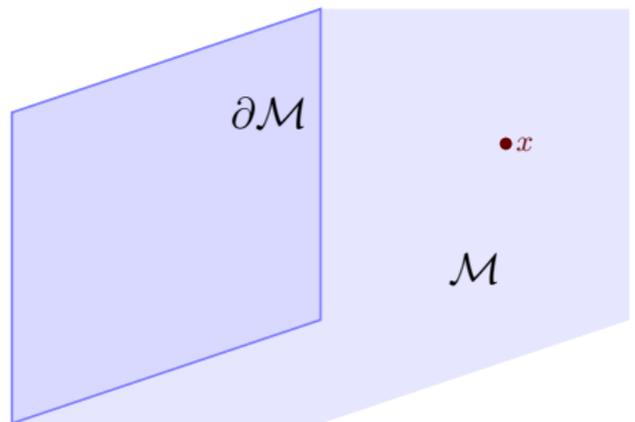
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$$\psi(x[\phi]) \rightarrow (f_*\psi)x[f_*\psi] = \psi \circ f^{-1} \circ f(x) = \psi(x)$$

- Can be generalized for whole space-time region, obtaining the reference frame $R[\phi]$.

An example: Boundary anchored geodesic frames

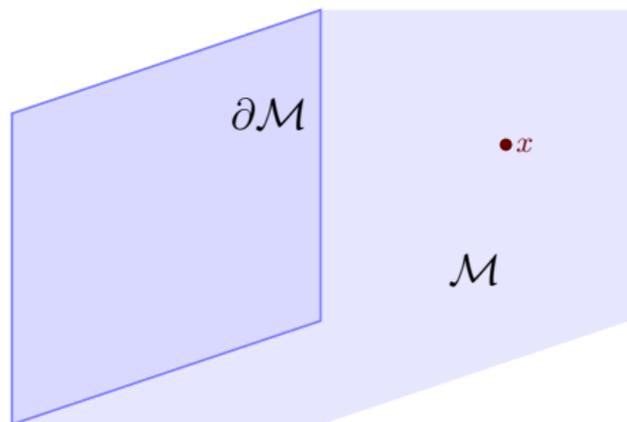
Let us consider (pure) gravity



- space-time \mathcal{M} with boundary $\partial\mathcal{M}$ and $x \in \mathcal{M}$
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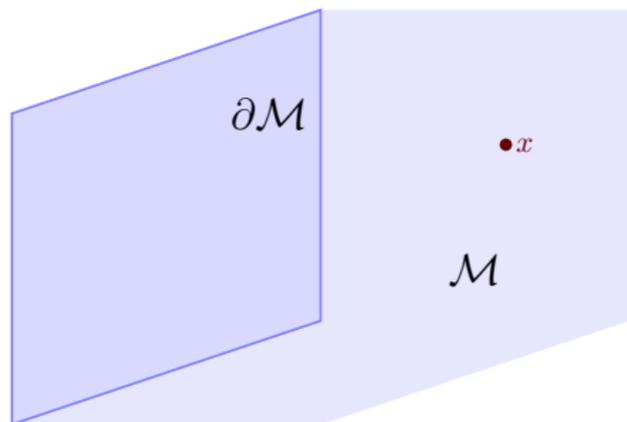
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Goal: dynamical parametrisation of x

→ Let's “shoot” a geodesic $\gamma[g]$ from the boundary

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- Under a small diffeomorphism f , the whole geodesic changes following

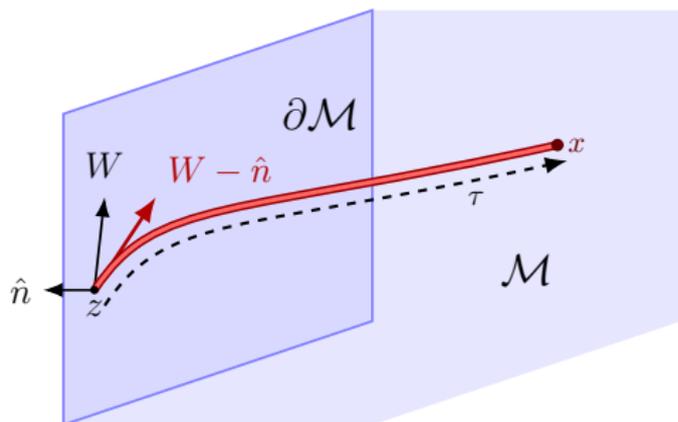
$$\gamma[f_*g] = f(\gamma[g])$$

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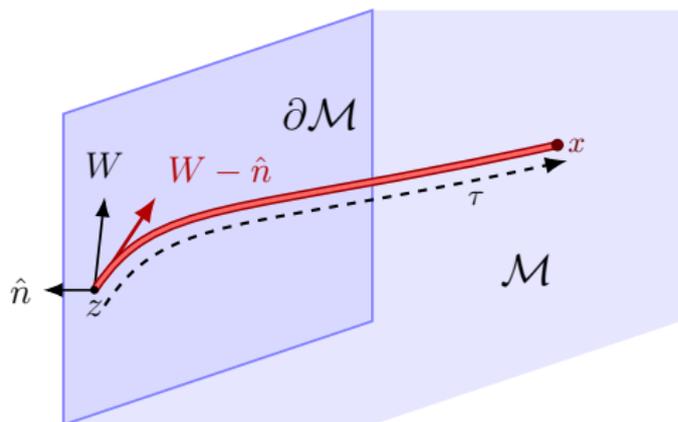
Let us define $x \in \mathcal{M}$ by the event being along the geodesic

- starting at $z \in \partial\mathcal{M}$
- with tangent vector $W - \hat{n}$
- at distance/proper time τ

$$x = x[g](z, \tau, W)$$

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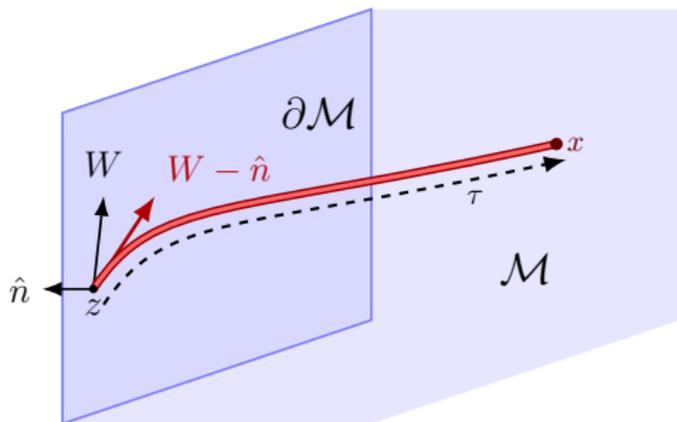
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- ψ a scalar field on space-time. We can define the gauge-invariant observable

$$O_{\psi, x} = \psi(x[g](z, \tau, W))$$

→ scalar field dressed with metric degrees of freedom.

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There is a whole set of admissibles (z, τ, W) for the geodesic construction

$$\mathcal{O} = \{(z, \tau, W) \mid z \in \partial\mathcal{M}, \tau \in \mathbb{R}_+, W \in T_z\partial\mathcal{M}\}$$

→ natural map between \mathcal{O} and space-time: the reference frame

$$R[g] : \mathcal{O} \rightarrow \mathcal{M}, (z, \tau, W) \mapsto x[g](z, \tau, W)$$

By construction, under a small diffeomorphism f , it transforms covariantly

$$R[f_*g] = f(R[g])$$

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- Dressed scalar field as a function on \mathcal{O}

$$\mathcal{O}_{\psi, R[g]} : \mathcal{O} \rightarrow \mathbb{R}, (z, \tau, W) \mapsto (R[g]^*\psi)(z, \tau, W) = \psi(R[g](z, \tau, W))$$

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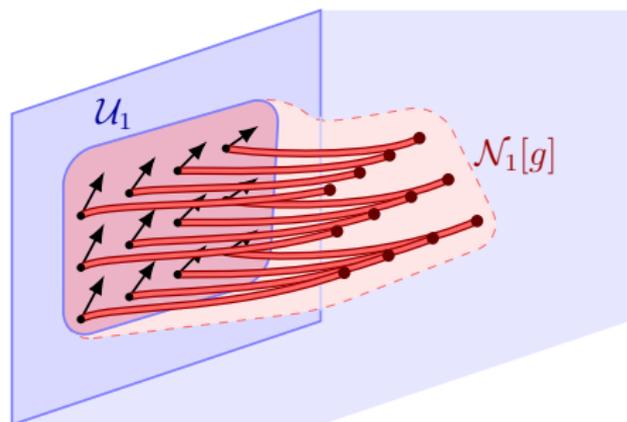
- “Dressed metric” as a function on \mathcal{O}

$$O_{g,R[g]} = R[g]^*g \quad \rightarrow \quad R[f_*g]^*f_*g = (f \circ R[g])^*f_*g = R[g]^*f^*f_*g = R[g]^*g$$

→ measures (the component of) g in a diffeo-invariant fashion

An example: Boundary anchored geodesic frames

Dressed observables as relational observables



Consider the subset \mathcal{O}_1 of \mathcal{O} constructed via

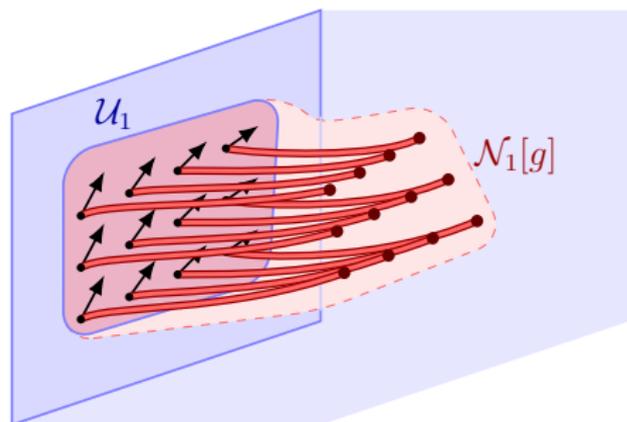
- $\mathcal{U}_1 \subset \partial\mathcal{M}$ and a boundary vector field W_1 on \mathcal{U}_1
- $\tau_1 \in \mathbb{R}_+$

$\rightarrow \mathcal{O}_1 = \{(z_1, \tau, W_1) | \tau < \tau_1, z_1 \in \mathcal{U}_1\}$

$$\mathcal{N}_1 = \{x[g](z, \tau, W) | (z, \tau, W) \in \mathcal{O}_1\}$$

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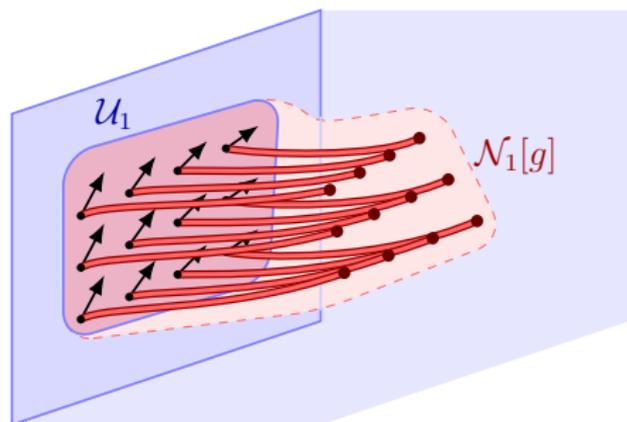
Assumption: \mathcal{O}_1 such that

$$R_1[g] : \mathcal{O}_1 \rightarrow \mathcal{N}_1 \quad \text{is invertible}$$

- Dynamical frame field: $(R_1[g])^{-1} : \mathcal{N}_1 \rightarrow \mathcal{O}_1$, $x \mapsto (T_1(x), Z_1(x)) = (\tau, z)$

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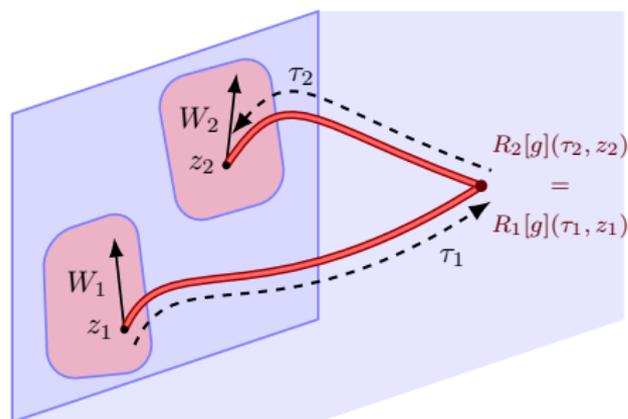
- Dynamical frame field: $(R_1[g])^{-1} : \mathcal{N}_1 \rightarrow \mathcal{O}_1$, $x \mapsto (T_1(x), Z_1(x)) = (\tau, z)$
- Any (covariant) tensor space-time quantity B on \mathcal{N}_1 can be pushforwarded to \mathcal{O}_1

$$O_{B, R_1[g]}(\tau, z) = (R_1[g]^{-1})_* B(\tau, z)$$

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Changes of frame as transition functions between coordinate systems

Consider \mathcal{O}_1 and \mathcal{O}_2 such that $\mathcal{N}_1[g] \cap \mathcal{N}_2[g] \neq \emptyset$



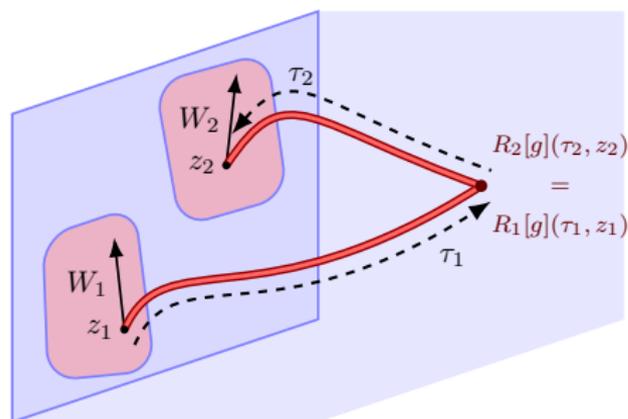
- Relation between

$$O_{B,R_1[g]}(\tau_1, z_1) \quad \text{and} \quad O_{B,R_2[g]}(\tau_2, z_2) \quad ?$$

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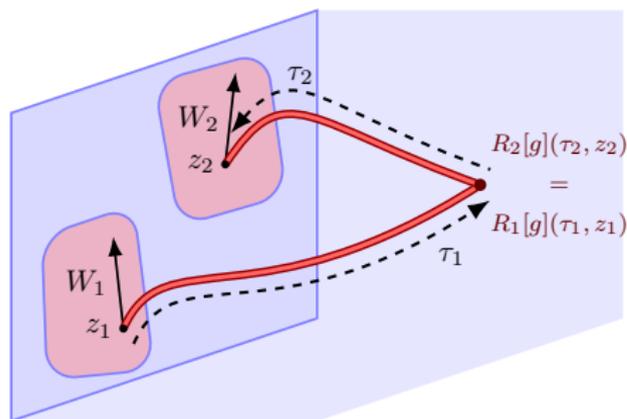
$$R_{1 \rightarrow 2} = (R_2[g])^{-1} \circ R_1[g] = R_1[g]^* R_2[g]$$

Analogue of transition function between two standard coordinate systems

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Analogue of transition function between two standard coordinate systems

$$O_{B,R_2}[\phi] = (R_{1 \rightarrow 2}[g])^* O_{B,R_1}[\phi] = (R_2[g])^* B[\phi],$$

→ dynamical but gauge-invariant change between two distinct gauge-invariant descriptions of B

We can construct relational atlases

Generalisation to more complicated dressings

We can go even further and look at more complicated dressings, even quasi-local ones

→ Instead of local dressings, we can look at dressings valued in any space \mathcal{K} that carries an action of the diffeomorphism group of \mathcal{M} .

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- \mathcal{K}^d : space of all d -dimensional submanifolds of \mathcal{M} . Under any diffeomorphism f

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→ Fix $\mathcal{A} \subset \partial\mathcal{M}$, and consider $\mathcal{K}_{\mathcal{A}}^d$ subset of \mathcal{K}^d homologous to \mathcal{A} .

→ Pick $b : \mathcal{K}^d \rightarrow \mathbb{R}$ covariant admitting a minimum and let $\Upsilon_{b,\mathcal{A}}[\phi] \in \mathcal{K}_{\mathcal{A}}^d$ be one.

Assuming small diffeomorphisms leave the boundary invariant

$$\Upsilon_{b,\mathcal{A}}[f_*\phi] = f(\Upsilon_{b,\mathcal{A}}[\phi]): \text{transforms like a dressing!}$$

We can construct a reference frame

$$R_{\Upsilon}[\phi] : \mathcal{O}^d \rightarrow \mathcal{K}^d, \mathcal{A} \mapsto \Upsilon_{b,\mathcal{A}}[\phi]$$

Conclusion

