LOOPS'22

Spherically-symmetric Geometries in a Matter Reference Frame as Quantum Gravity Condensates

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OUTLINE

01

Set Up

02

GFT Black Hole Quantum States

03

Recovery of Schwarzschild Spacetime

04

Outlooks

ONE

Set Up

SET UP: Overview

01

Define a GFT quantum black hole state.

02

Compare quantum observables with classical ones.

03

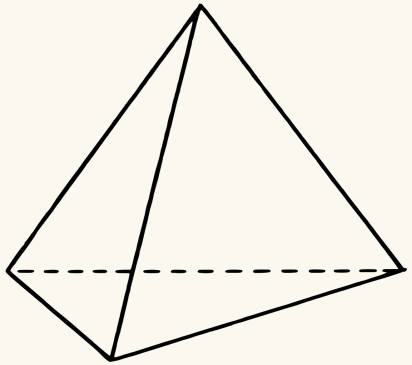
Reproduce a classical black hole spacetime.

1. Which quantum gravity state?

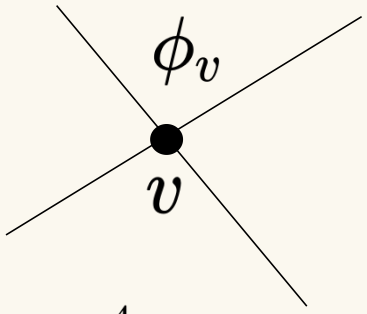
【 Chanllenges 】

2. Which matter reference frame?

Set Up: GFT Building Blocks



$$\varphi : SU(2)^4 \times \mathbb{R}^4 \to \mathbb{C}$$



$$[\hat{\varphi}_t(g_v, \phi_v), \hat{\varphi}^\dagger_{t'}(g_w, \phi_w)] = \delta_{t,t'}\delta^4(\phi_v - \phi_w) \int_{SU(2)} d\gamma \prod_{i=1}^{4} \delta(g_{v_i}\gamma g_{w_i}^{-1})$$

Matter Reference Frame

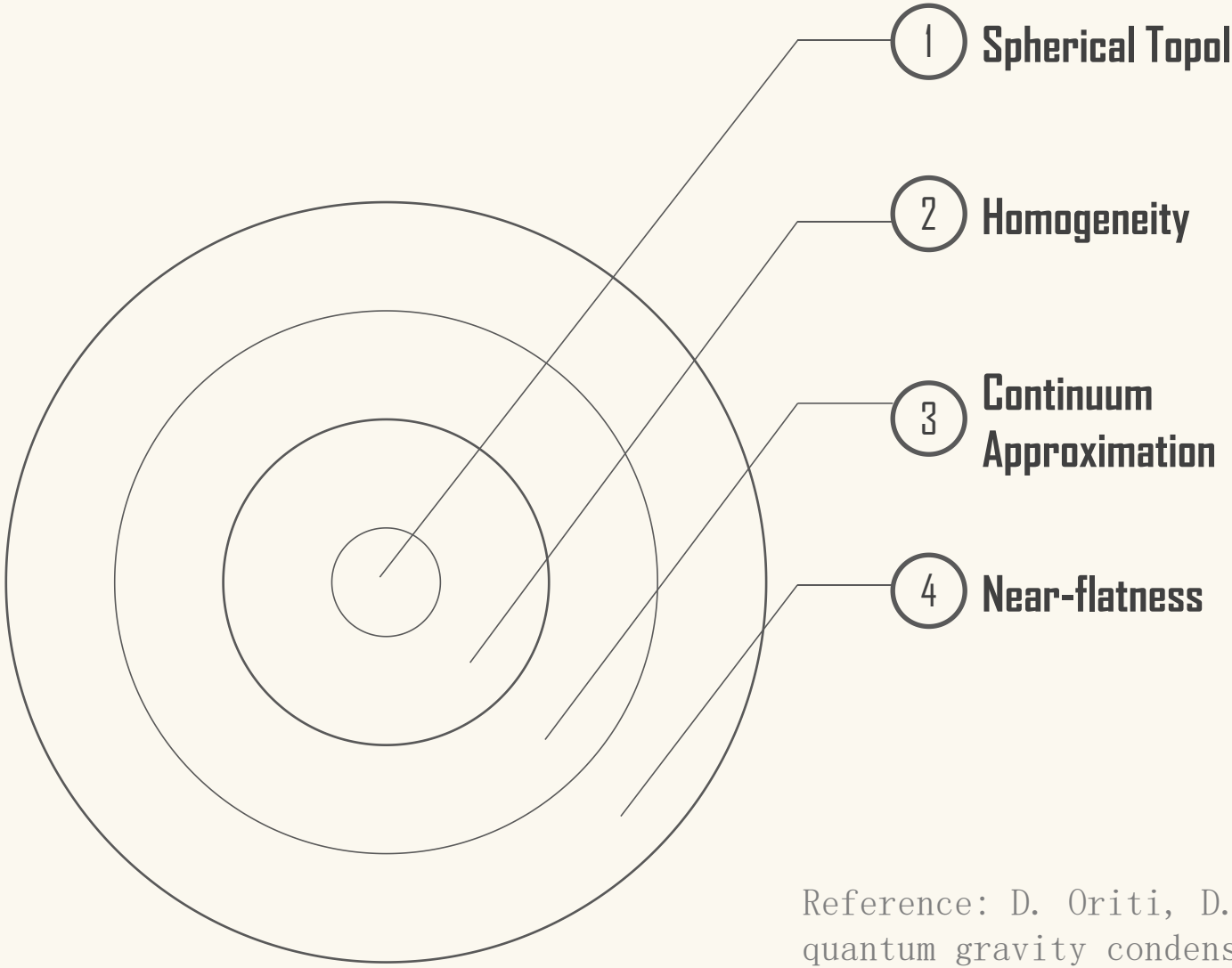
The Simplest Choice: 4 minimally coupled real scalar fields.

SU(2) Group Manifold

Second quantisation of LQG.

Reference: D. Oriti, Group field theory as the second quantization of loop quantum gravity, Classical and Quantum Gravity 33 (2016) 085005.

SET UP: Main Features



Space Foliation

A spherically symmetric spatial geometry is obtained through a foliation into homogeneous shells.

New Features

5. Reference frame peakedness
6. Semi-classicality

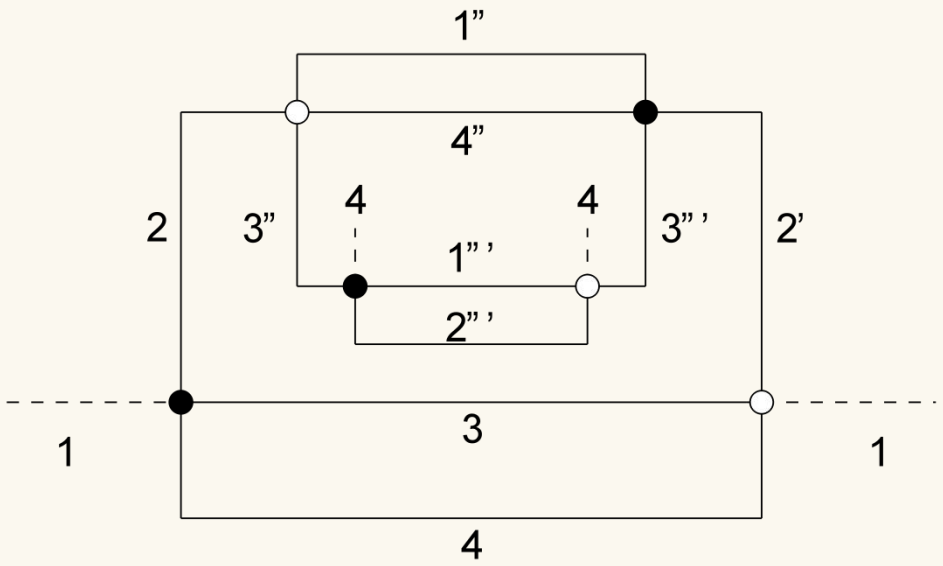
Reference: D. Oriti, D. Pranzetti and L. Sindoni, Black holes as quantum gravity condensates, Physical Review D 97 (2018) 066017.

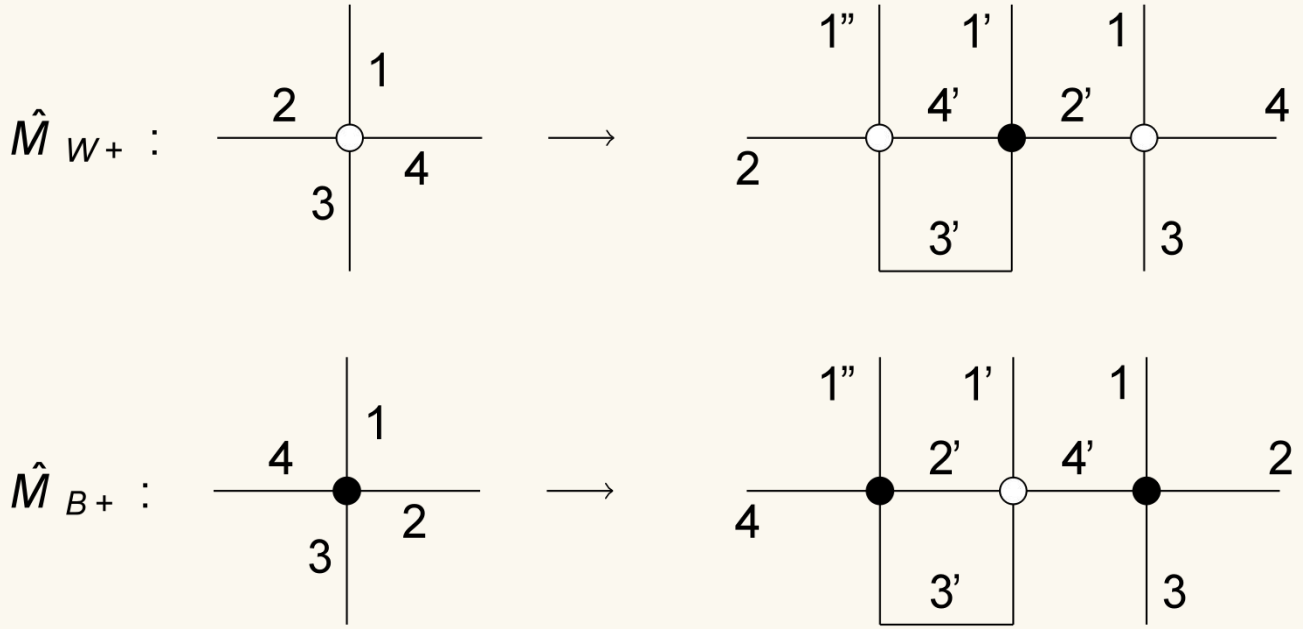
TWO

Black Hole

Consensate States

GFT STATE: Seed State and Refinements





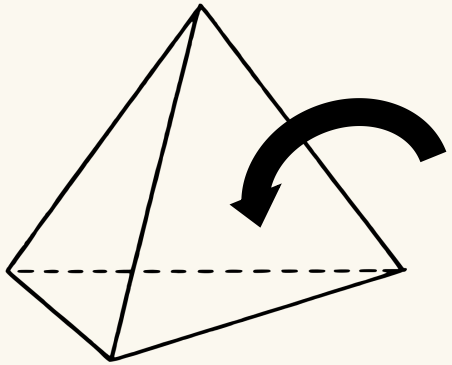
Seed state:

triangulation of a spherical shell with three layers.

Refinements:

add tetrahedral without changing the topology.

GFT STATE: Quantum Gravity Condensates



wavefunction

quantum geometrical information
(homogeneity)

$$\hat{\sigma}_t^{\dagger}(h;\phi_u) \equiv \int dg d\phi \sigma_u(hg,\phi)\hat{\varphi}_t^{\dagger}(g,\phi)$$

$$\hat{\sigma}_t(h;\phi_u) \equiv \int dg d\phi \sigma_u(hg,\phi)\hat{\varphi}_t(g,\phi)$$

$$\sigma_{\tau,u,\theta,\chi}(g;\phi) \equiv \eta_{\epsilon_0}(\phi_0-\phi_\tau,\pi_\tau)\eta_{\epsilon_1}(\phi_1-\phi_u,\pi_u)\eta_{\epsilon_2}(\phi_2-\phi_\theta,\pi_\theta)\eta_{\epsilon_3}(\phi_3-\phi_\chi,\pi_\chi)\varsigma(g;\phi)$$

$$\eta_{\epsilon_x} \equiv \mathcal{N}_{\epsilon_x}\exp\left(-\frac{(\phi_x-x)^2}{2\epsilon_x}\right)\exp(i\pi_x(\phi_x-x)) \qquad \mathcal{N}_{\epsilon_x}^2 = (\pi\epsilon_x)^{-1/2}$$

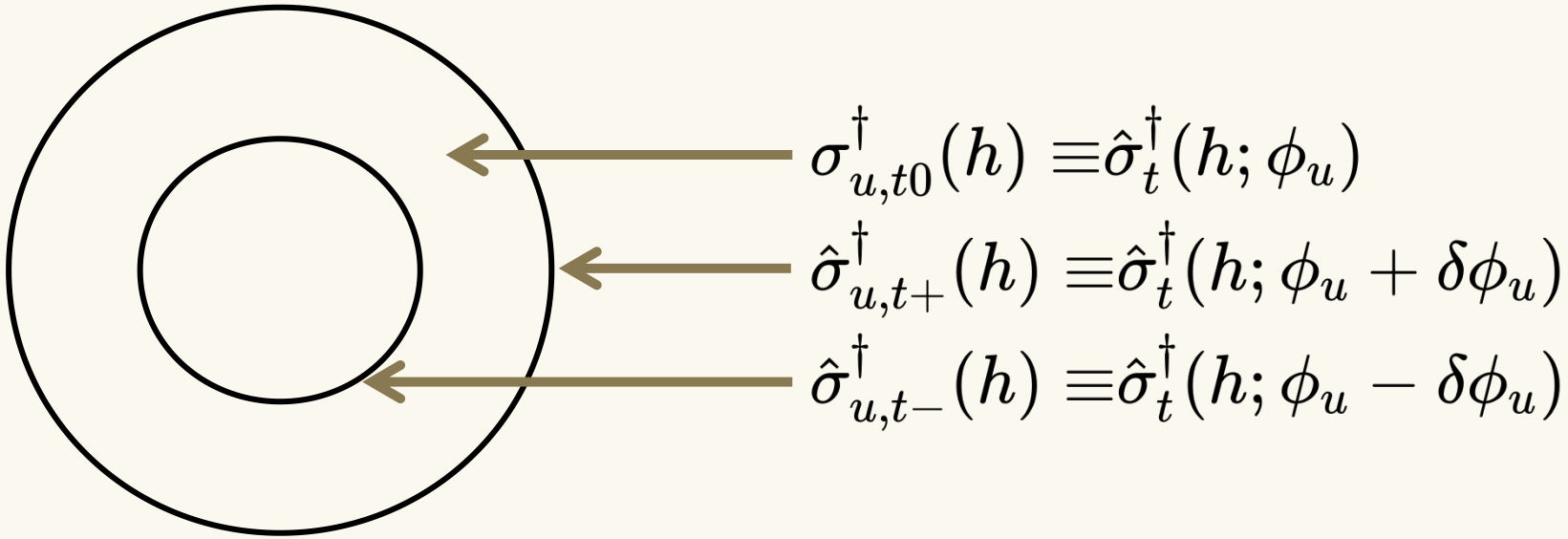
Reference: L. Marchetti and D. Oriti, Effective relational cosmological dynamics from quantum gravity, Journal of High Energy Physics 2021 (2021) 1.

GFT STATE: Shell State

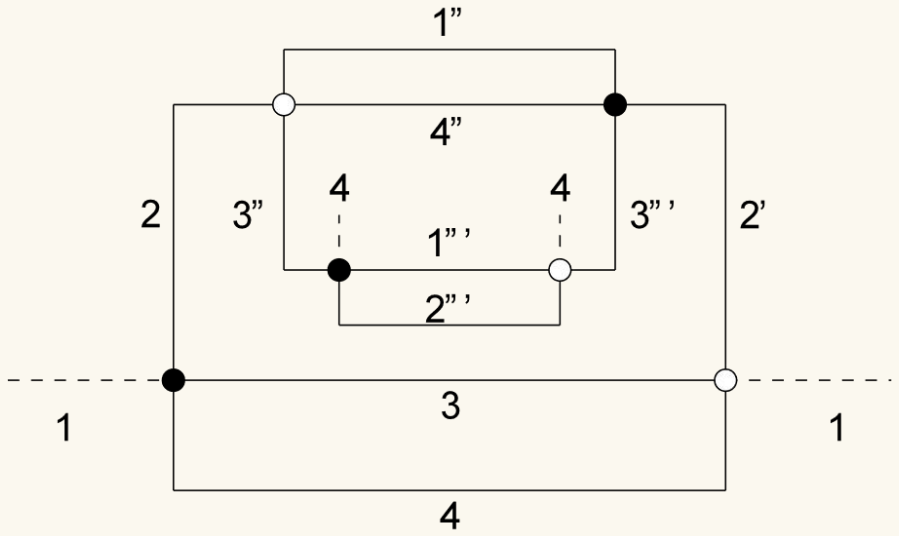
homogeneity → assumption

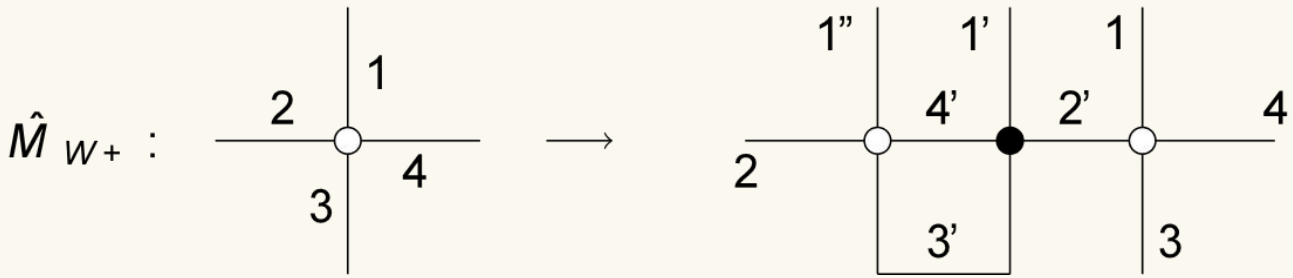
$$[\hat{\sigma}_t(h_v;\phi_u),\hat{\sigma}^\dagger_{t'}(h_w;\phi_{u'})] = \delta_{u,u'}\delta_{t,t'}\frac{1}{\tilde{\varsigma}(\phi_u)}\int_{SU(2)} d\gamma \prod_{i=1}^{4}\delta(\gamma h_{v_i} h_{w_i}^{-1})$$

to be found



GFT STATE: Shell State





$$|\tau_u\rangle \equiv \int dg^{10} \hat{\sigma}^{\dagger}_{u,B+}(e,g_2,g_3,g_4)\hat{\sigma}^{\dagger}_{u,W+}(e,g_2',g_3,g_4)\hat{\sigma}^{\dagger}_{u,B0}(g_1'',g_2',g_3''',g_4'')$$
$$\hat{\sigma}^{\dagger}_{u,W0}(g_1'',g_2,g_3'',g_4'')\hat{\sigma}^{\dagger}_{u,B-}(g_1'''',g_2'''',g_3'',e)\hat{\sigma}^{\dagger}_{u,W-}(g_1'''',g_2'''',g_3''',e)|0\rangle$$

$$\hat{\mathcal{M}}_{u,W+} \equiv \tilde{\varsigma}^2_{us}\int dk_2 dk_3 dk_4 dh_{4'} dh_{3'} dh_{2'} \hat{\sigma}^{\dagger}_{u,W+}(e,k_2,h_{3'},h_{4'})\hat{\sigma}^{\dagger}_{u,B+}(e,h_{2'},h_{3'},h_{4'})$$
$$\hat{\sigma}^{\dagger}_{u,W+}(e,h_{2'},k_3,k_4)\hat{\sigma}_{u,W+}(e,k_2,k_3,k_4)$$

GFT STATE: States and Operators

A shell with 6n vertices from the seed state through (n-1) refinements on each layer:

$$|\Gamma_n(\phi_u)\rangle \equiv \frac{1}{(n!)^3} \prod_{s=\{+,0,-\}} \tilde{\varsigma}_{us}^{n-1} \int \prod_{i=1}^{6n} dh_i J_\Gamma(h_1,\dots,h_{6n}) \hat{\sigma}^\dagger_{u,B+}(h_1)\dots\hat{\sigma}^\dagger_{u,W+}(h_{n+1})\dots\hat{\sigma}^\dagger_{u,B0}(h_{2n+1})$$
$$\dots\hat{\sigma}^\dagger_{u,W0}(h_{3n+1})\dots\hat{\sigma}^\dagger_{u,B-}(h_{4n+1})\dots\hat{\sigma}^\dagger_{u,W-}(h_{5n+1})\dots\hat{\sigma}^\dagger_{u,W-}(h_{6n})|0\rangle$$

Extended operators measure quantities on a specific layer:

$$\hat{\mathcal{O}}_t \equiv \int dg_v dg_w d\phi_v d\phi_w O(g_v,g_w;\phi_v,\phi_w)\hat{\varphi}^\dagger_t(g_v,\phi_v)\hat{\varphi}_t(g_w,\phi_w)$$

$$\hat{\mathbb{O}}_{u,ts} \equiv \int dh_v dh_w O_{us}(h_v,h_w)\hat{\sigma}^\dagger_{u,ts}(h_v)\hat{\sigma}_{u,ts}(h_w)$$

$$O_{us}(h,h') = \tilde{\varsigma}^3_{us}\prod_{s'\neq s}\tilde{\varsigma}^2_{us'} \int dg_v dg_w d\phi_v d\phi_w \sigma_{u,s}(h_v g_v,\phi_v) O(g_v,g_w;\phi_v,\phi_w)\sigma_{u,s}(h_w g_w,\phi_w)$$

Schwarzschild: GFT Observables

1

Volume

2

Area

$$\langle V_3(\phi_u)\rangle \propto 3\tilde{\varsigma}(\phi)^{-1}\delta\phi|_{\phi=\phi_u}$$

$$\langle \mathbb{A}_{sphere}(\phi_u)\rangle \propto \tilde{\varsigma}^{-1}(\phi_u)$$

$$\hat{\mathbb{V}}_{u,ts} = \int dh_v dh_w \hat{\sigma}^{\dagger}_{u,ts}(h_v) V(h_v,h_w) \hat{\sigma}_{u,ts}(h_w)$$

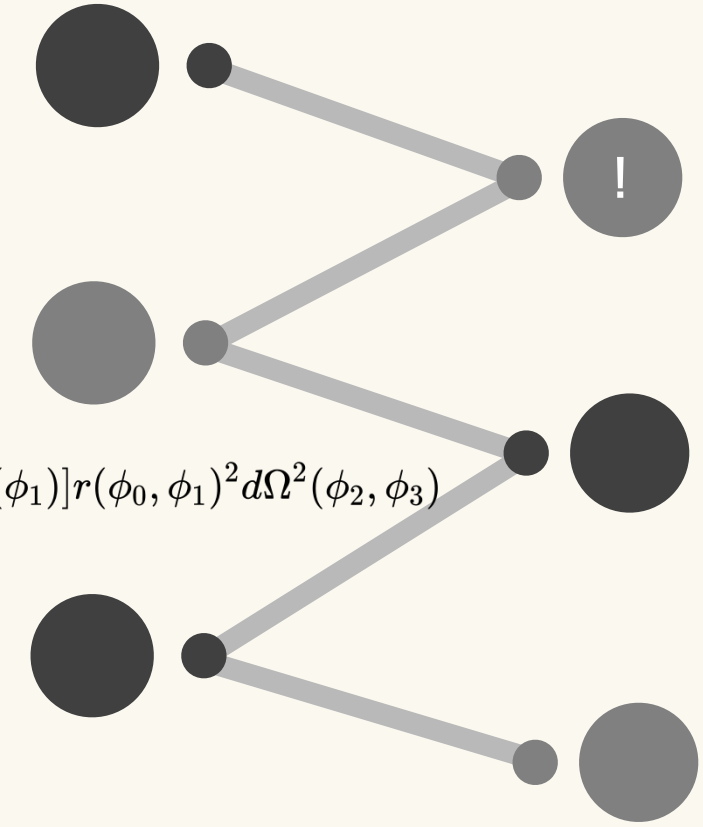
$$\hat{\mathbb{A}}_1(\phi_u) = \prod_{s=\{+,0,-\}} \tilde{\varsigma}^2_{us}\kappa \sum_{t=B,W} \int dh^v_I \hat{\sigma}^{\dagger}_{r,t+}(h^v_I) \sqrt{E^i_1 E^j_1 \delta_{ij}} \triangleright \hat{\sigma}_{r,t+}(h^v_I)$$

THREE

Recovery of Schwarzschild Spacetime

Schwarzshild: Usual strategy

GFT geometrical observables.



Classical Klein-Gordon equations of minimally coupled scalar fields.

'Relational' metric.

$$dh^2_{relational} = \frac{f_2[\tau(\phi_0), u(\phi_1)]}{\phi_1'(u)^2} d\phi_1^2 + f_4[\tau(\phi_0), u(\phi_1)] r(\phi_0, \phi_1)^2 d\Omega^2(\phi_2, \phi_3)$$

The corresponding 'relational' observables.

GFT wavefunction.

$$A_{rel}(\phi_u) \propto \langle \mathbb{A}_{sphere}(\phi_u) \rangle_{GFT}$$

$$V_{3rel}(\phi_u) \propto \langle V_3(\phi_u) \rangle_{GFT}$$

A Schwarzschild spacetime at continuum limit.

Schwarzschild: Relational Metric

1 $V_{rel}(\phi) \propto A_{rel}(\phi)\delta\phi$

2 GFT wavefunction depending on radial rod only.

Relational Metric

$$dh^2_{\text{rel}} = \frac{1}{\beta_1^2} d\phi_1^2 + r^2(\phi_1) d\Omega^2$$

Clock

Free and massless

$$\phi_0(t) = \beta_0 t$$

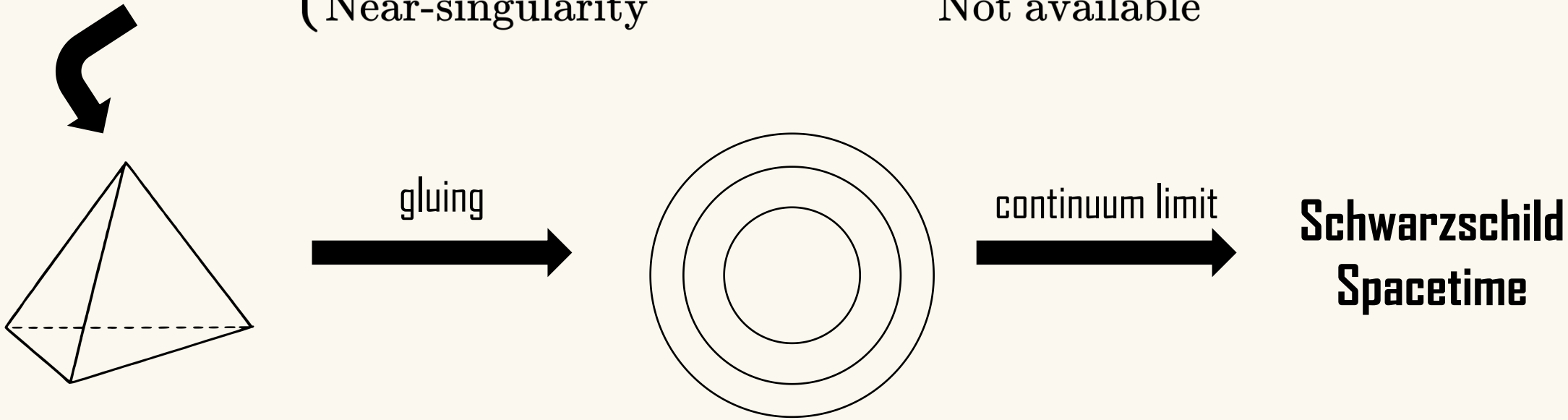
Radial Rod

Non-vaninshing potential

$$\phi_1(r) = \beta_1 r \sqrt{1 - \frac{2GM}{r}} + 2\beta_1 GM \operatorname{arctanh}\left(\sqrt{1 - \frac{2M}{r}}\right)$$

Schwarzschild: Recovery

$$\text{Schwarzschild}\begin{cases}\text{Asymptotic infinity}(\phi \to \infty) & \tilde{\varsigma}(\phi)^{-1} \propto \phi^2 \\ \text{Near-horizon}(\phi \sim 0) & \tilde{\varsigma}(\phi)^{-1} \propto (\phi^2 + 16\beta_1^2(GM)^2)^2 \\ \text{Near-singularity} & \text{Not available}\end{cases}$$



FOUR

Outlooks

Outlooks

Horizon Identification

States Generalisation

Quantum Dynamics

THANKS FOR ATTENTION