FRIEDRICH-SCHILLER-
UNIVERSITÄT
JENA

Discrete gravity dynamics from

Effective Spin foam models

Seth Kurankyi Asante, FSU Jena

[*w/ B Dittrich, H Haggard, J Padua-Argüelles*]

arXiv: 2004.07013 (PRL), arXiv: 2011.14468 (CQG), arXiv: 2104.00485 (CQG)

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What are effective spin foams ?

Effective spin foam models are discrete geometrical path integrals for quantum gravity.

$$\mathcal{Z} = \int_{M/\mathrm{Diff}(M)} [\mathcal{D}\mathrm{geom}]\, e^{-\mathrm{i}\, S[\mathrm{geom}]}$$

Geometry based on area variables

✅ allows different ‘gluing constraints’ - general family of models.

They need not be approximate.

✅ most efficient spin foam and fast computations to date.

Geometry from Areas

[*Schuller, Wohlfahrt '06*]

Area variables describe generalized geometries — more than Lorentzian geometry

Quantum geometries from LQG, Spin foams

Entanglement entropy

Holography

Black hole micro state counting

minimal area metric in String theory

Boundary edge modes

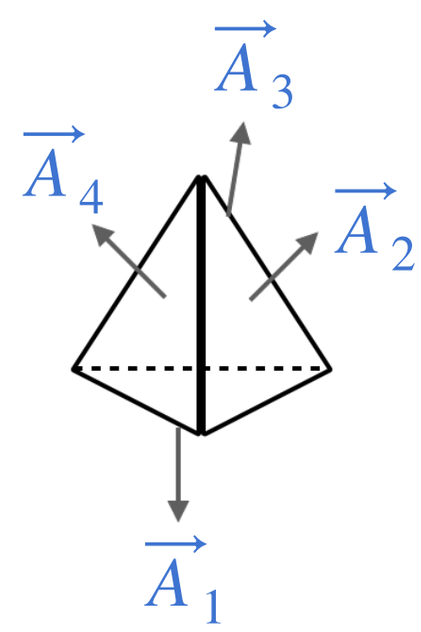
natural sympletic structure discrete gravity

Minkowski theorem: [*Minkowski*]

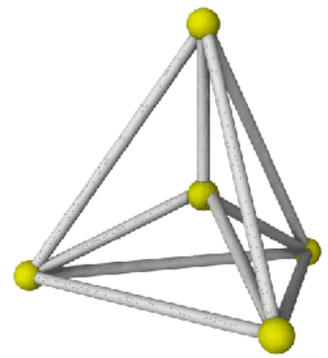
Area normal vectors capture geometry of tetrahedron

$$\overrightarrow{A}_i = A_i \hat{n}_i$$

constraint satisfied for classical tetrahedron $\sum_i^4 \overrightarrow{A}_i = 0$



Discrete gravity - Area Regge calculus



Building blocks are 4-simplices: each simplex has 10 triangles and 10 edges

Locally invert areas and lengths

[*Hero of Alexandria AD 60*]

Heron's formula $A_t(l_e) = a_t$

$$A_t^2(l_1, l_2, l_3) = \frac{1}{16}(l_1 + l_2 + l_3)(l_1 + l_2 - l_3)(l_1 - l_2 + l_3)(-l_1 + l_2 + l_3)$$

$$l_e = L_e^{\sigma}(a_t)$$

$$S_{\text{Regge}}[a_t] = \sum_h a_t \, \epsilon_t(a_{t'})$$

[*Bonzom*]

[*Barrett, Roček, Williams*] $\delta S_{\text{Regge}} \implies \epsilon_t(a_{t'}) = 0$ flatness

Flatness problem

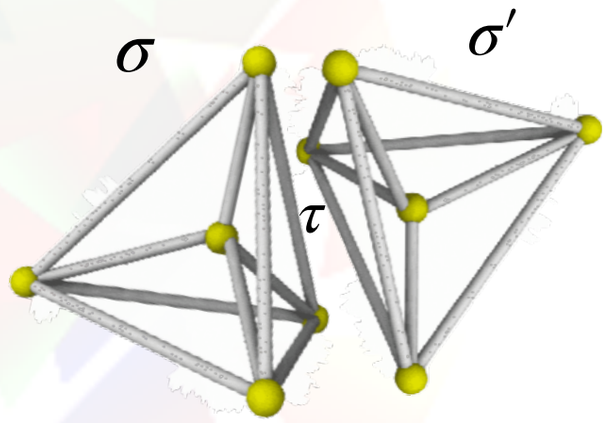
Flatness due to more degrees of freedom

[*SKA, Dittrich, Haggard*]

Constraints

General triangulation has a mismatch between shared tetrahedra

Gluing simplices



16 area variables 14 length variables

τ 4 areas vs. 6 lengths

Require that geometry of shared tetrahedron should match

match two 3d dihedral angles $\Phi^{\tau,\sigma}_{e_i}(a_t) = \Phi^{\tau,\sigma'}_{e_i}(a_t)$

(localized constraints) $\mathscr{C}^{\tau}_{i} := \phi^{\tau}_{e_i} - \Phi^{\tau,\sigma}_{e_i}(a_t) = 0$

$i = 1,2$ (non-opposite edges)

$\{\mathscr{C}^{\tau}_{i}, \mathscr{C}^{\tau}_{j}\} = (2\gamma \sin\alpha_{12})/a_t$

(second-class constraints)

Features of Quantum Geometry

Input from LQG, Spin foams

Discrete areas

- Discrete eigenvalues for area operators (almost equidistant) [*Rovelli, Smolin*]

$$a_t^2 \simeq \gamma^2 \ell_{\mathrm{P}}^2 j(j+1) \sim \gamma^2 \ell_{\mathrm{P}}^2 j^2 \qquad j \in \mathbb{N}/2$$

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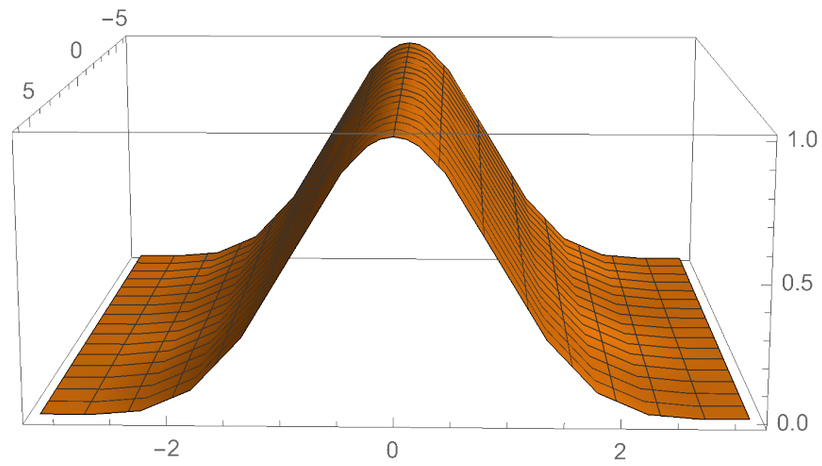
Constraints:

[*Engle, Perriera, Rovelli, Livine*]

- Weak imposition of constraints $\mathscr{C}_i^\tau := \phi_{e_i}^\tau - \Phi_{e_i}^{\tau,\sigma}(a_t)$ $\{\mathscr{C}_i^\tau, \mathscr{C}_j^\tau\} = (2\gamma \sin\alpha_{12})/a_t$

$$G_\tau = \langle K_{\Phi_{e_i}^{\tau,\sigma}} | K_{\Phi_{e_i}^{\tau,\sigma'}} \rangle$$

$$\text{ansatz} \quad \sim \mathscr{N}_k \exp\left(-\frac{\sum_i \mathscr{C}_i^2}{4\sigma^2(j)}\right)$$



See Simão's talk
Friday

Effective spin foam model

$$\mathcal{Z}_{\mathrm{ESF}} = \sum_{\{a\}} \mu(j) \prod_{t,\sigma} \exp\left(\frac{\mathrm{i}}{\hbar} S[a]\right) \prod_{\tau} G_\tau(a)$$

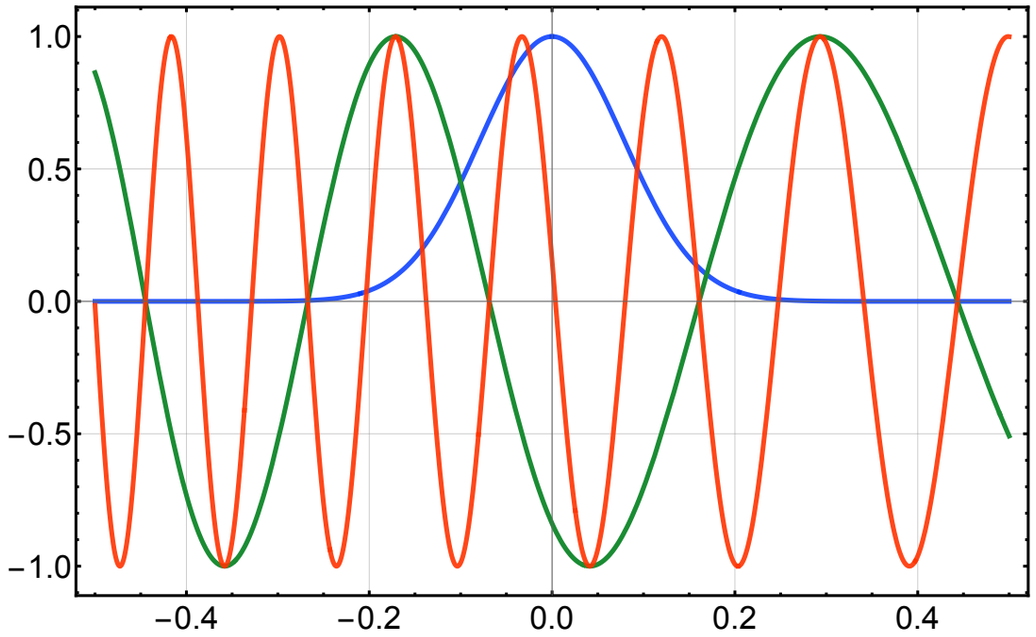
discrete area spectrum

$$S[a_t] = \sum_t a_t \, \epsilon_t \qquad a_t \sim \gamma \ell_{\mathrm{P}} j_t$$

Constraints:

$$G_\tau \sim \mathcal{N}_k \exp\left(-\frac{\sum_i \mathscr{C}_i^2}{4\sigma^2(j)}\right)$$

$$\sigma^2 \sim \{\phi^\tau_{e_1}, \phi^\tau_{e_2}\} = (2\gamma \sin\alpha_{12})/j_t$$

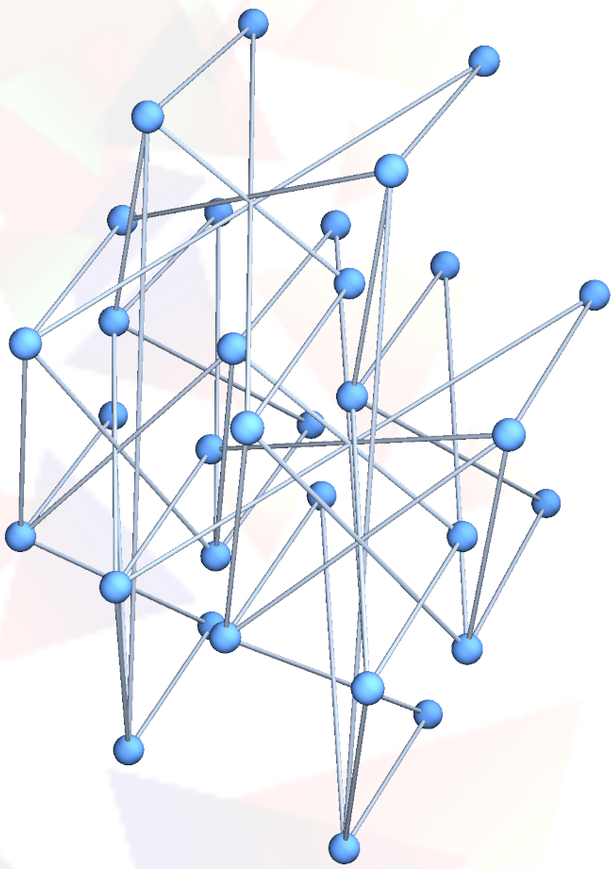


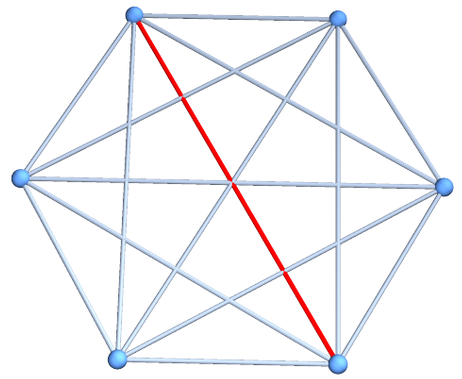
Asymptotic condition: $\gamma\sqrt{j_t}\,\mathrm{curv_t} \lesssim \mathcal{O}(1)$

[*SKA, Dittrich, Haggard*]

Testing the model

$$\langle \mathcal{O}(a) \rangle = \frac{1}{\mathcal{Z}_{\mathrm{ESF}}} \sum_{\{j\}} \mu(j) \prod_{t,\sigma} \exp\left(\frac{\mathrm{i}}{\hbar} S[\gamma, j]\right) \prod_{\tau} G_{\tau}(j)\, \mathcal{O}(a)$$





Triangulation with bulk edge

6 four-simplices

21 tetrahedra

29 triangles

Parameters of model :

γ	$\Lambda \sim j$	ϵ
area gap anisotropy	boundary data	curvature

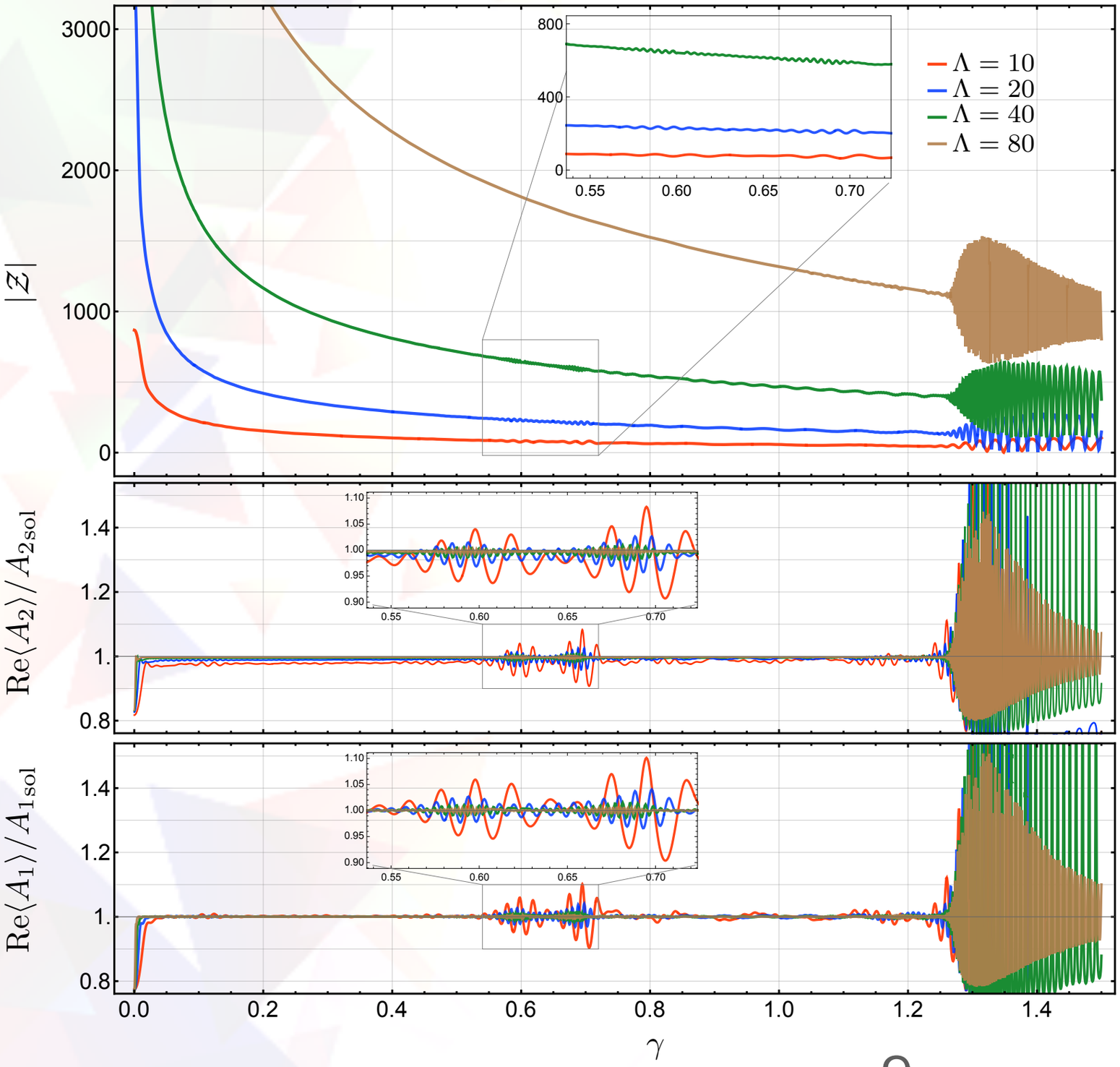
Numerical results

Bulk-Edge

Small curvature

$$\epsilon_{1_{\rm cl}} = 0, \qquad \epsilon_{2_{\rm cl}} = \epsilon_{3_{\rm cl}} = 0.034$$

symmetry reduced : $5 \rightarrow 3$ bulk areas



Surprises:

-threshold behaviour for oscillations

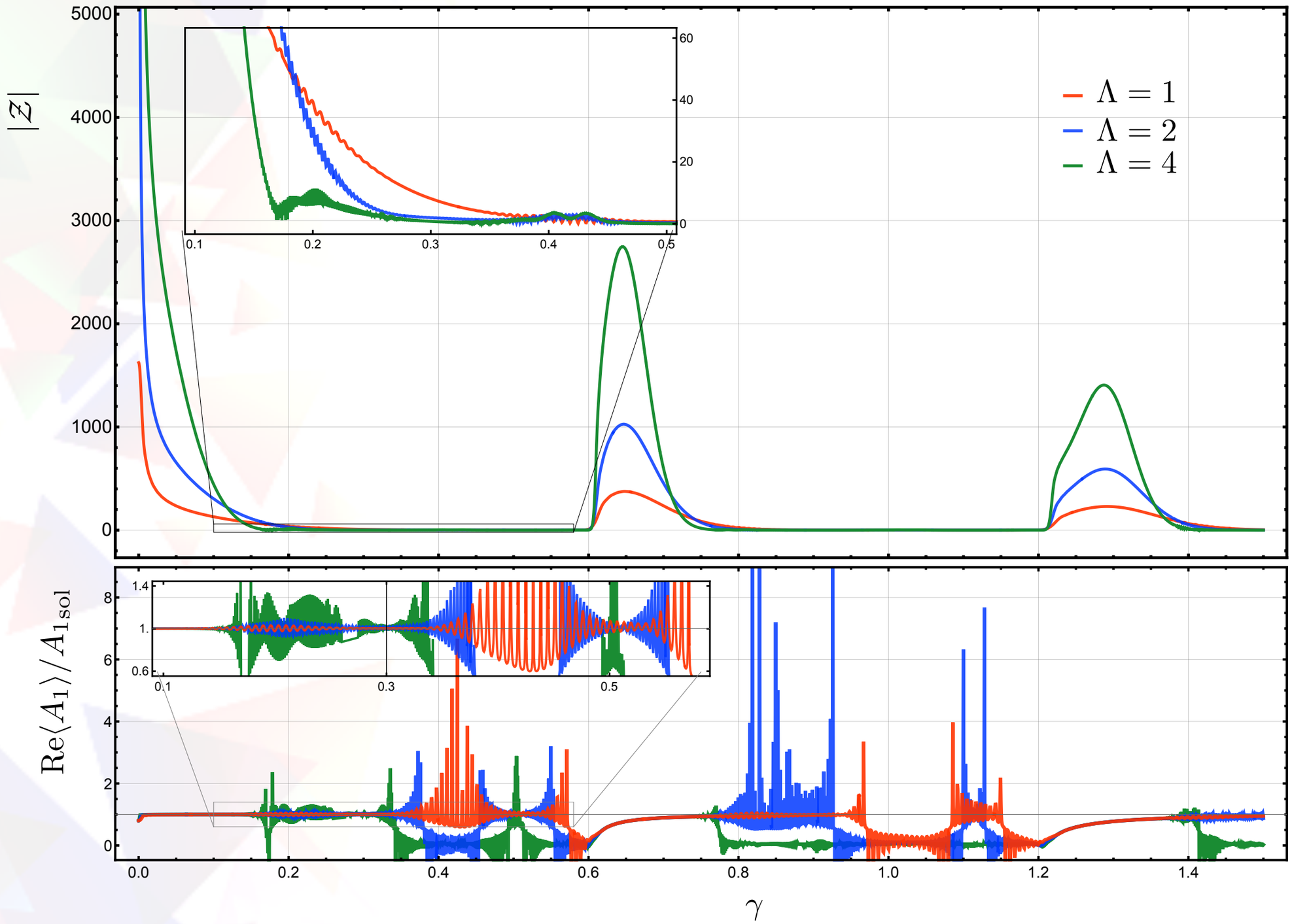
-threshold values independent of scale Λ

Improvement in semi-classical expectation values for large Λ

Bulk-Edge

$$\epsilon_{1_{\rm cl}} = 4.193, \qquad \epsilon_{2_{\rm cl}} = -1.790, \qquad \epsilon_{3_{\rm cl}} = -1.1432$$

Large curvature



agree with

$$\gamma\sqrt{j_t}\,\mathrm{curv_t} \lesssim \mathcal{O}(1)$$

Concluding Remarks

- First test of spin foam implementing equations of motion for gravity

  Resolves flatness problem - configurations with curvature appear

- Some surprising structures: sudden onset of oscillations likely related to resonances - due to scaling property of Regge action
- Can easily check stability of these features, if we change certain details of model

Good news:

- Get much larger acceptable γ range of values for small ϵ. For the continuum limit we expect to have small curvature (per building block).

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Outlook: **Coarse graining and test diffeomorphism invariance**

- Results support hope to restore diff. symmetry in continuum limit. [*Dittrich, Kogios, Borissova*]
  - inhomogeneously scaling.
  - require curvature to be sufficiently small.

THANK YOU !