The Complete Barrett-Crane Model and its Causal Structure

based on 2206.15442 and 2112.00091 JCAP 01(2022)050

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Munich Center of Quantum Science and Technology

Causal Structure in Quantum Gravity

Role of Causal Structure in GR

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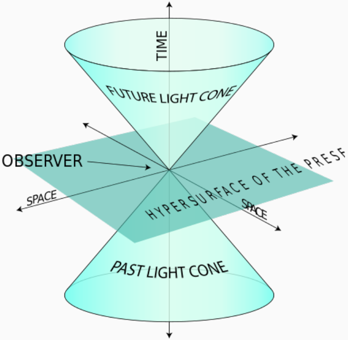

Figure: Wikipedia

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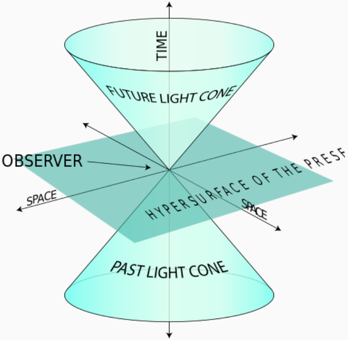

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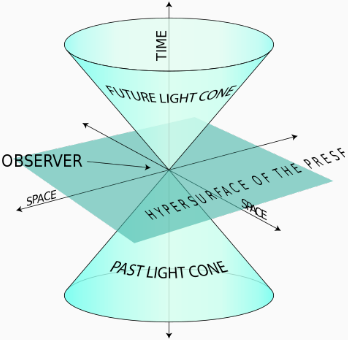

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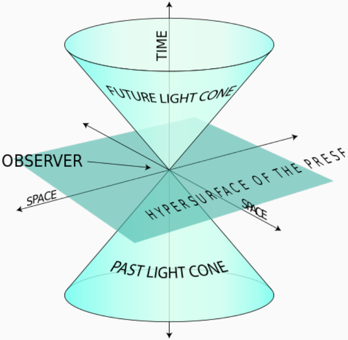

Causal structure = time orientation + bare causality

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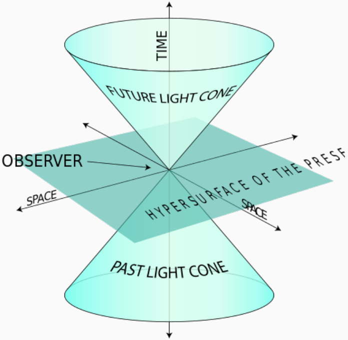

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Task

Construct a model that includes spacelike, lightlike and timelike tetrahedra and all possible interactions thereof

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Completing the Barrett-Crane Model

The BC GFT Model in the Extended Formulation

Barrett, Crane gr-qc/9904025; Baratin, Oriti 1108.1178; Oriti, Pithis, AJ 2112.00091

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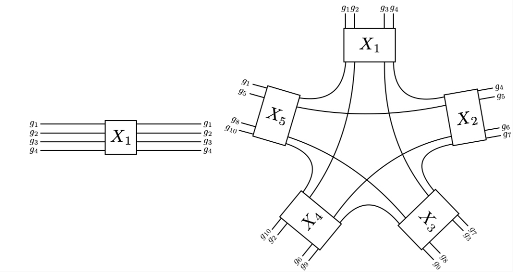

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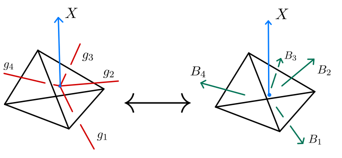

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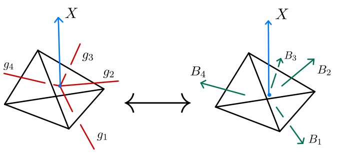

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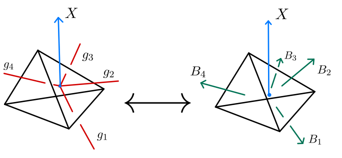

$X \in \mathrm{H}^3$ is assumed to be timelike $\Rightarrow$ Restriction to spacelike tetrahedra

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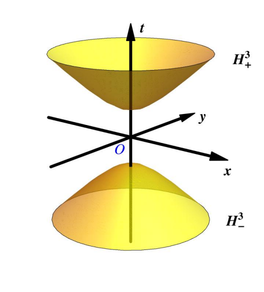

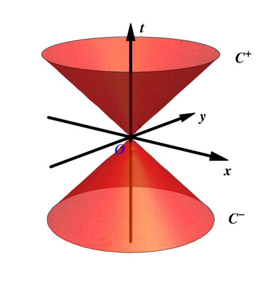

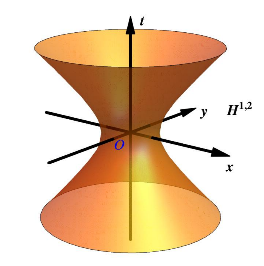

Oriti, Pithis, AJ 2206.15442

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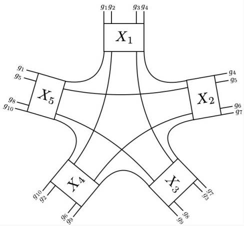

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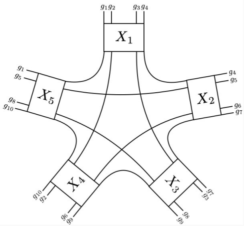

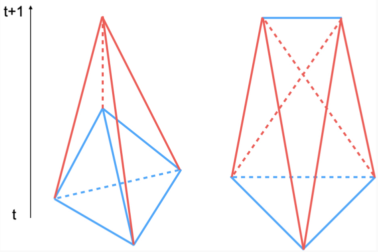

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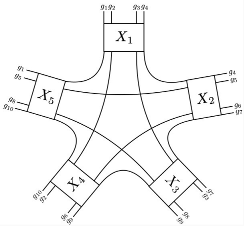

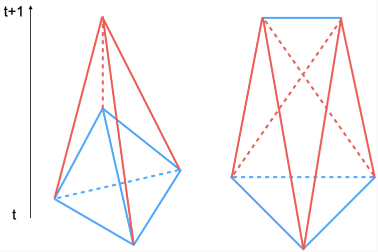

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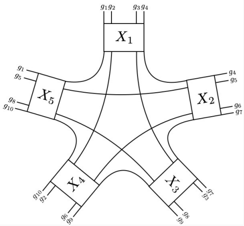

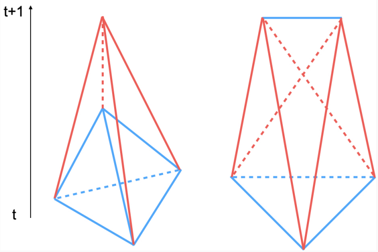

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- ✓ Quantum geometric results (already anticipated by Barrett, Crane gr-qc/9904025):
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- ✓ Determine behavior under spacetime reversal
  - Larger symmetry group $\mathrm{O}(1,3)$ or $\mathrm{Pin}(1,3)$
- ✓ Construct CDT-like GFT model, resulting in causal tensor model

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Back-Up

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6. Boundary states are closer to canonical LQG, incorporating the Barbero-Immirzi parameter γ

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- The criticisms are yet inconclusive

Action of the complete BC Model

Kinetic action

$$K[\varphi,\bar{\varphi}] = \sum_{\alpha} \int_{\mathrm{SL}(2,\mathbb{C})^4} [\mathrm{d}g]^4 \int_{\mathrm{SL}(2,\mathbb{C})/\mathrm{U}^{(\alpha)}} \mathrm{d}X_\alpha \, \bar{\varphi}(g_v; X_\alpha)\varphi(g_v; X_\alpha)$$

Vertex action

$$V[\varphi,\bar{\varphi}] = \sum_{\substack{n_+,n_0,n_- \geq 0 \\ n_+ + n_0 + n_- = 5}} \frac{\lambda^{(n_+,n_0,n_-)}}{5} V(n_+,n_0,n_-),$$

where the interaction $V(n_+,n_0,n_-)$ for a choice of normal signatures is given by

$$V(n_+,n_0,n_-) =$$

$$= \int [\mathrm{d}g]^{10} \int [\mathrm{d}X_\alpha]^5 \, \varphi_{1234}(X_{\alpha_1})\varphi_{4567}(X_{\alpha_2})\varphi_{7389}(X_{\alpha_3})\varphi_{962(10)}(X_{\alpha_4})\varphi_{(10)851}(X_{\alpha_5}) +$$

$$+ c.c.$$

Amplitudes of the complete BC Model

GFT partition function

$$Z = \int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} e^{-S[\varphi,\bar{\varphi}]}$$

$$= \sum_{\Gamma} \frac{1}{\text{sym}(\Gamma)} \prod_{(n_+,n_0,n_-)} \left(\lambda_{(n_+,n_0,n_-)}\right)^{\mathcal{V}_{(n_+,n_0,n_-)}(\Gamma)} \mathcal{A}_{\Gamma}$$

Spin foam amplitudes

$$\mathcal{A}_{\Gamma} = \prod_f \sum_{\nu_f} \int \mathrm{d}\rho_f \, \mathcal{A}_f(\rho_f,\nu_f) \prod_e \mathcal{A}_e^{\alpha_e}(\rho_{i_e},\nu_{i_e}) \prod_v \mathcal{A}_v^{\alpha_1,\dots,\alpha_5}(\rho_{v_a},\nu_{v_a})$$

with face, edge and vertex amplitudes

$$\mathcal{A}_f(\rho_f,\nu_f) = 4\left(\rho_f^2 + \nu_f^2\right)$$

$$\mathcal{A}_e^{\alpha_e}(\rho_{i_e},\nu_{i_e}) = 1$$

$$\mathcal{A}_v^{\alpha_1,\dots,\alpha_5}(\rho_{v_a},\nu_{v_a}) = \{10(\rho_{v_a},\nu_{v_a})\}_{(\alpha_1,\dots,\alpha_5)}$$

The vertex amplitude is defined through kernels

$$\{10(\rho,\nu)\}_{(\alpha_1,\dots,\alpha_5)} = \int [\mathrm{d}X_\alpha]^5 \prod_{a<b} K_{\alpha_a\alpha_b}^{(\rho_{ab},\nu_{ab})}(X_a,X_b)$$

Kernels

Non-mixed kernels

$$\delta(X,Y) = \sum_\nu \int \mathrm{d}\rho\, 4\left(\rho^2+\nu^2\right) \overline{K^{(\rho,\nu)}_{\alpha\alpha}(X,Y)}$$

Example: Timelike normal vectors $X = (1,0,0,0), Y = (\cosh(\eta), 0, 0, \sinh(\eta))$

$$K^{(\rho,\nu)}_{++}(X,Y) = \frac{\delta_{\nu,0}}{2} \int \mathrm{d}\Omega\, (X^\mu \xi_\mu)^{i\rho-1} (Y^\mu \xi_\mu)^{-i\rho-1} = ... = \frac{\sin(\rho\eta)}{\rho\sinh(\eta)}$$

Mixed kernels acting similar to δ-function

$$f(g_1) = \sum_\nu \int \mathrm{d}\rho\, 4(\rho^2+\nu^2) \int \mathrm{d}g_2\, \overline{D^{(\rho,\nu)}_{\alpha_1\alpha_2}(g_1^{-1}g_2)} f(g_2)$$

Example: Timelike and Spacelike normal vectors $X_+ = (1,0,0,0)$, $Y_- = (\sinh(\eta), 0, 0, \pm\cosh(\eta))$

$$K^{\rho}_{+-}(\eta,\pm 1) == \frac{\delta_{\nu,0}}{2} \int \mathrm{d}\Omega\, (X^\mu_+ \xi_\mu)^{i\rho-1} \left|Y^\mu_- \xi_\mu\right|^{-i\rho-1} = ... = \frac{i\cos(\rho\eta)}{\rho\cosh(\eta)}$$

CDT-like GFT

Main steps:

- Restrict to spacelike and timelike tetrahedra
- Restrict to two simplices $(1,0,4)$ and $(0,0,5)$
- Fix signature of faces $\Leftrightarrow$ choose irreps. $(\rho,0)$ or $(\nu,0)$
- Fix irreps $\rho_{ab}=\rho^*$, $\nu_{ab}=\nu^*$ and relate via α

$$\rho^{*2}=3\frac{1-\nu^{*2}}{1+4\alpha}-1,$$

Causal tensor model

Upon introducing a cutoff in the $\mathrm{SL}(2,\mathbb{C})$-magnetic indices, the resulting model is a causal tensor model

GFT partition function

$$Z=\sum_{\Gamma}\frac{1}{\mathrm{sym}(\Gamma)}\left(4\rho^{*2}N_{\rho^*}\right)^{F_s}\left(4\nu^{*2}N_{\nu^*}\right)^{F_t}\left(\lambda_{(4,1)}\mathcal{A}^{(4,1)}\right)^{V_{(4,1)}}\left(\lambda_{(3,2)}\mathcal{A}^{(3,2)}\right)^{V_{(3,2)}},$$

$$Z=\sum_{\Gamma}\frac{1}{\mathrm{sym}(\Gamma)}e^{-S_{\mathrm{eff}}(\Gamma)},$$