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Timelike faces and lightlike vectors within the CH-extended EPRL model asymptotics

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July 17, 2022

Motivation

- CH-extension [Conrady and Hnybida, 2010] of the EPRL model [Engle, Livine, Pereira and Rovelli] provides a spin foam model including timelike tetrahedra and timelike triangles
- Asymptotic analysis of timelike tetrahedra featuring spacelike [Kamiński, Kisielowski and Sahlmann, 2018] and timelike triangles [Liu and Han, 2019, Simão and Steinhaus, 2021] revealed: There are critical points corresponding to Lorentzian 4-simplices.
- Besides the expected spacelike vectors, there are also lightlike vectors associated to timelike triangles!

Asymptotic analysis

rewrite vertex amplitude of a single 4-simplex

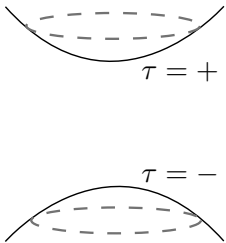
$$A_v = \int_{\mathrm{SL}(2,\mathbb{C})} \prod_{a=1}^{5} \mathrm{d}g_a \delta(g_5) \prod_{b<a} \langle D^{(n,\rho)}(g_a)\psi_{ab}, D^{(n,\rho)}(g_b)\psi_{ba}\rangle \tag{1}$$

such that dominant contributions in the large spin limit are given by stationary points of an action S_{ab} for each triangle interface ab,

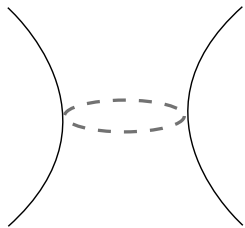
$$\delta_{\mathbf{z}_{ab}} S_{ab}(\mathbf{z}_{ab}, g_a, g_b) = 0, \quad \forall a, b \tag{2}$$

$$\sum_{a<b} \delta_{g_a} S_{ab}(\mathbf{z}_{ab}, g_a, g_b) = 0, \quad \forall a \tag{3}$$

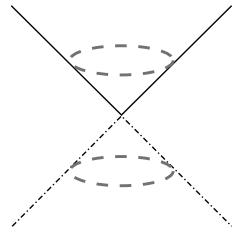
Coherent state boundary data



$\vec{n}^{\tau} \in H^{\tau}$



$\vec{m} \in H^{\mathrm{sp}}$



$\widetilde{m} \in C^{+}_{\mathrm{null}}$

Closure condition

for timelike tetrahedra a [Liu and Han, 2019, Simão and Steinhaus, 2021]:

$$\sum_{b:\, ab \text{ s.l.}} j_{ab} \vec{n}^{\tau}_{ab} + \sum_{b:\, ab \text{ t.l.}} s_{ab}(\vec{m}_{ab} + \mathcal{I}\beta_{ab}\widetilde{m}_{ab}) = 0 \tag{4}$$

$$\sum_{b:\, ab \text{ t.l.}} s_{ab}\mathcal{R}\beta_{ab}\widetilde{m}_{ab} = 0 \tag{5}$$

- $\vec{m}_{ab} + \mathcal{I}\beta_{ab}\widetilde{m}_{ab} \in \mathcal{H}^{\mathrm{sp}}$ is a unit-norm spacelike vector
- $(\mathcal{I}\beta_{ab})_{b\in\{1,2,3,4\}}$ not uniquely determined for tetrahedra with four timelike faces

Closure: imaginary part

$$\sum_{b:\, ab \text{ t.l.}} s_{ab} \mathcal{R}\beta_{ab} \widetilde{m}_{ab} = 0 \tag{6}$$

Different proposals [Liu and Han, 2019, Simão and Steinhaus, 2021] to restrict the model such that the closure equation implies $\mathcal{R}\beta_{ab} = 0$ for all timelike ab.

Bivector constraints

for timelike interfaces [Liu and Han, 2019, Simão and Steinhaus, 2021]:

$$\pi(\overline{g}_a)^{\wedge 2}\left(M_{ab}+\mathcal{I}\beta_{ab}\widetilde{M}_{ab}\right)=\pi(\overline{g}_b)^{\wedge 2}\left(M_{ba}+\mathcal{I}\beta_{ba}\widetilde{M}_{ba}\right) \tag{7}$$

$$\pi(\overline{g}_a)^{\wedge 2}\mathcal{R}\beta_{ab}\widetilde{M}_{ab}=\pi(\overline{g}_b)^{\wedge 2}\mathcal{R}\beta_{ba}\widetilde{M}_{ba} \tag{8}$$

bivector $M_{ab}+\mathcal{I}\beta_{ab}\widetilde{M}_{ab}$ describes timelike triangle ab in 4d Minkowski space

Action at critical points

For configurations satisfying $\mathcal{R}\beta_{ab} = 0 = \mathcal{R}\beta_{ba}$ one obtains the result already found for the case of $\beta_{ab} = 0 = \beta_{ba}$ [Liu and Han, 2019, Simão and Steinhaus, 2021]:

$$
\begin{aligned}
S_v^{\text{crit}}\big|_{\text{timelike}} &= i\gamma \sum_{ab}^{\text{timelike}} s_{ab}\, 2\arg\left(\frac{\alpha_{ba}}{\alpha_{ab}}\right) + i \sum_{ab}^{\text{timelike}} s_{ab} \ln\left|\frac{\alpha_{ba}}{\alpha_{ab}}\right|^2 \\
&= i\gamma \sum_{ab}^{\text{timelike}} s_{ab}\theta_{N_a^{(\vec{0},1)},N_b^{(\vec{0},1)}} + i \sum_{ab}^{\text{timelike}} s_{ab} \ln\left|\frac{\alpha_{ba}}{\alpha_{ab}}\right|^2
\end{aligned}
\tag{9}
$$

Summary and outlook

What we learned:

- $\vec{m}_{ab} + \mathcal{I}\beta_{ab}\widetilde{m}_{ab} \in \mathcal{H}^{\mathrm{sp}}$ is a unit-norm spacelike vector
- even for non-vanishing lightlike vectors, we can recover Lorentzian 4-simplices as well as the Regge action
- we can obtain the same geometric solutions for non-vanishing lightlike vectors

Open questions:

- asymptotic expansion of the vertex amplitude
- closure does not uniquely determine $\mathcal{I}\beta$ for tetrahedra with four timelike faces
- how to relate α_{ab} and β_{ab}

References


Engle, Jonathan and Livine, Etera and Pereira, Roberto and Rovelli, Carlo (2008)


Conrady, Florian and Hnybida, Jeff (2010)


Kamiński, Wojciech and Kisielowski, Marcin and Sahlmann, Hanno (2018)


Liu, Hongguang and Han, Muxin (2019)


Simão, José Diogo and Steinhaus, Sebastian (2021)

The End