Towards effective actions for the continuum limit of spin foams

Johanna Borissova

work with
Bianca Dittrich

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18/7/22 LOOPS 2022 Lyon jborissova@pitp.ca

Motivation

Extended configuration space for gravity (inspired by LQG & spin foams; string theory; entanglement geometry;...)

$$\{\text{length metrics } g\} \longrightarrow \{\text{area metrics } A\}$$

$g_{\mu\nu}$: **10 d.o.f.** – measures lengths of tangent vectors & angles

$A_{\mu\nu\rho\sigma}$ (cyclic) : **20 d.o.f.** – measures areas of parallelograms & dihedral angles

[Schuller & Wohlfarth, 2005; Punzi, Schuller & Wohlfarth, 2006]

Hints for extended gravitational configuration space in spin foams

anomaly in quantum constraint algebra

classical constraints ← imposed sharply → quantum constraints ← part imposed weakly

path integral extended over non-length-metric d.o.f. parametrizing fluctuating part of simplicity constraints

continuum limit of Area-Regge action

[Rovelli, 1993; Barrett, Rocek & Williams, 1999]

dynamics
[Dittrich, 2021; Dittrich & Kogios, 2022]

$$S_{\mathrm{AR}}[A_t] = \frac{1}{\kappa}\sum_{t\in\triangle} A_t \epsilon_t^A(A_t) \quad \longrightarrow \quad S_{\mathrm{eff}}[A] \longrightarrow S_{\mathrm{eff}}[g] = S_{\mathrm{GR}}[g] + \mathrm{Weyl}[g]^2 + \mathcal{O}(\lambda^4)$$

Goal: start from continuum $[\cdots] \longrightarrow S[A] \longrightarrow S_{\mathrm{eff}}[g] = S_{\mathrm{GR}}[g] + \text{corrections } [\cdots]$

Non-Chiral Plebanski Action

Non-chiral Plebanski theory = so(4) BF-theory (+ Holst term) **+ constraints**

[Plebanski, 1977; Capovilla, Jacobson, Dell & Mason, 1991; Reisenberger, 1995-98; Pietri & Freidel, 1999; Speziale 2010]

$$S_{\text{Pleb.}}[B,\omega,\phi] = \int \delta_{IJKL} B^{IJ} \wedge F^{KL}(\omega) + \frac{1}{2\gamma}\epsilon_{IJKL} B^{IJ} \wedge F^{KL}(\omega) - \frac{1}{2}\phi_{IJKL} B^{IJ} \wedge B^{KL}$$

$B^{IJ}_{\mu\nu}$: **36 d.o.f.** – so(4)-valued 2-form (4d) incl. 6 so(4) gauge d.o.f; can be parametrized using Urbantke theorem & isomorphism $so(4) \cong su(2)_+ \oplus su(2)_-$ [Urbantke, 1984; Freidel, 2008; Speziale, 2010]

$$B^{IJ}_{\mu\nu}(36) \quad \Leftrightarrow \quad (g^+_{\mu\nu}(10), g^-_{\mu\nu}(10), b^i_{+a}(8), b^i_{-a}(8))$$

2 x length metrics
2 x unimodular 3x3 matrices

$\delta\phi$: 20 simplicity constraints

full set of constraints imposed sharply at the classical level – two sectors of solutions

gravitational sector (leads to Palatini-Holst action for GR) & **topological sector** (no propagating d.o.f.)

Modified Non-Chiral Plebanski Action

Modified non-chiral Plebanski theory = so(4) BF-theory (+ Holst term) **+ potential**

[Krasnov, 2006-07; Bengtsson, 2007; Speziale, 2010; Beke, Palmisano & Speziale, 2012]

Motivation: in spin foam quantization only part of constraints imposed sharply – rest imposed weakly

Idea: impose only part of constraints sharply at the classical level – **suppress rest via potential**
(integrate out connection and relevant Lagrange multiplier d.o.f. [Freidel, 2008; Speziale, 2010])

$$S_{\text{mod. Pleb. eff.}}[e_\mu^{+I}, e_\mu^{-I}, q_{ab}^+, q_{ab}^-] = S^+_{\text{su(2)-BF}}[e_\mu^{+I}, q_{ab}^+] + S^-_{\text{su(2)-BF}}[e_\mu^{-I}, q_{ab}^-] + \int V[e_\mu^{+I}, e_\mu^{-I}, q_{ab}^+, q_{ab}^-]$$

2 x tetrads associated to length metrics $g^\pm_{\mu\nu} = e_\mu^{\pm I} e_\nu^{\pm J} \delta_{IJ}$

2 x unimodular internal metrics $q^\pm_{ab} = b^i_{\pm a} b^j_{\pm b} \delta_{ij}$

From Modified Non-Chiral Plebanski to Action for Area Metric

Area metric from so(4) B-fields

introduce cyclic area metrics for each su(2)-sector [Reisenberger, 1998]: (use parametrization of B-fields)

$$A^{\pm}_{\mu\nu\rho\sigma} \equiv \frac{1}{2}\Big((B^i_{\pm} \otimes B_{\pm i})_{\mu\nu\rho\sigma} - \frac{1}{4!}(B^i_{\pm} \otimes B_{\pm i})_{\alpha\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma}\Big) = A^{\pm}(g^{\pm}, q^{\pm})$$

note: $A^{+}_{\mu\nu\rho\sigma} \overset{!}{=} A^{-}_{\mu\nu\rho\sigma} \quad \leftrightarrow \quad 20 \text{ simplicity constraints}$ (for non-degenerate tetrads, [Reisenberger, 1998])

here: want action for **area metric** → from so(4) B-fields extract the required number of d.o.f. for a cyclic area metric by **imposing only the length-metric induced part of the simplicity constraints**

$$B^{IJ}_{\mu\nu}(36) - \text{so}(4)\ \text{gauge}(6) \quad \longrightarrow \quad \boxed{g^{+}_{\mu\nu} \overset{!}{=} g^{-}_{\mu\nu}} \quad \longrightarrow \quad 20\ \text{d.o.f. for area metric}$$

10 equations (~ subset of simplicity constraints imposed sharply)

remaining d.o.f. can be organized into a cyclic area metric given by the sum of the area metrics for each sector:

$$A_{\mu\nu\rho\sigma}(g, q^{+}, q^{-}) \equiv A^{+}_{\mu\nu\rho\sigma}(g, q^{+}) + A^{-}_{\mu\nu\rho\sigma}(g, q^{-})$$

$$A_{\mu\nu\rho\sigma}(20) \quad \Leftrightarrow \quad (g_{\mu\nu}(10), q^{+}_{ab}(5), q^{-}_{ab}(5))$$

1 x length metric $g_{\mu\nu} = e^I_\mu e^J_\nu \delta_{IJ}$

2 x unimodular internal metrics $q^{\pm}_{ab} = b^i_{\pm a} b^j_{\pm b}\delta_{ij}$

From Modified Non-Chiral Plebanski to Action for Area Metric

$$(g_{\mu\nu}, q^+_{ab}, q^-_{ab}) \quad \Leftrightarrow \quad A_{\mu\nu\rho\sigma} \qquad \longrightarrow \qquad S_{\text{mod. Pleb. eff.}}[g_{\mu\nu}, q^+_{ab}, q^-_{ab}] \quad \Leftrightarrow \quad S[A]$$

Length- and internal metrics from Area metric

consider inversion $A(g, q^+, q^-)$ to $g(A), q^+(A), q^-(A)$ perturbatively

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}, \quad q^\pm_{ab} = \delta_{ab} + \chi^\pm_{ab}, \quad \mathrm{tr}(\chi) = 0 \quad \longrightarrow \quad A_{\mu\nu\rho\sigma} = 2\delta_{\mu[\rho}\delta_{\sigma]\nu} + a_{\mu\nu\rho\sigma} + \mathcal{O}(h^2, \dots)$$

$a \leftrightarrow (h, \chi^+, \chi^-)$ relation can be inverted to $h(a), \chi^+(a), \chi^-(a)$

area metric induced by flat length metric

1st-order area metric fluctuations

$$S^{(2)}[a] \equiv S^{(2)}_{\text{mod. Pleb. eff.}}[h_{\mu\nu}, \chi^+_{ab}, \chi^-_{ab}] = S^{+(2)}_{\text{su(2)-BF}}[h_{\mu\nu}, \chi^+_{ab}] + S^{-(2)}_{\text{su(2)-BF}}[h_{\mu\nu}, \chi^-_{ab}] + \int V^{(2)}[h_{\mu\nu}, \chi^+_{ab}, \chi^-_{ab}]$$
$$\equiv \frac{\gamma_+}{2}\int \mathcal{L}^{(2)}_{\text{BF}}(h_{\mu\nu} + \chi^+_{\mu\nu}) + \frac{\gamma_-}{2}\int \mathcal{L}^{(2)}_{\text{BF}}(h_{\mu\nu} + \chi^-_{\mu\nu}) + \frac{\gamma_+}{8}\int m_+^2 \chi^+_{\mu\nu}\chi^{+\mu\nu} + \frac{\gamma_-}{8}\int m_-^2 \chi^-_{\mu\nu}\chi^{-\mu\nu}$$

BF shift symmetry … … broken by mass terms

<u>su(2) chiral theory:</u> (non-local) field redefinition $(h_{\mu\nu}, \chi^\pm_{ab}) \to (H^\pm_{\mu\nu} \equiv h_{\mu\nu} + \chi^\pm_{\mu\nu}, \chi^\pm_{ab})$ & integrating out auxiliary $\chi^\pm_{ab}$ gives linearized Einstein-Hilbert with “shifted graviton” $H^\pm_{\mu\nu}$
<u>so(4) non-chiral theory:</u> graviton + massive d.o.f.

Effective Action for the Length Metric

Linearized Effective Action for the Length Metric

Integrate out $\chi^{\pm}_{\mu\nu}$: (in Fourier space)

$$L^{(2)}_{\text{eff}}[h] = \bar{h}_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}h_{\rho\sigma} - \frac{1}{4}\left(\frac{1}{m_+^2+\Delta}+\frac{1}{m_-^2+\Delta}\right){}^{(1)}\bar{C}_{\mu\nu\rho\sigma}(\bar{h})^{(1)}C^{\mu\nu\rho\sigma}(h)$$

*no ghost!

(linearized) **Einstein-Hilbert Lagrangian**

lowest-order corrections: **Einstein-Weyl gravity**

reproduces effective dynamics for the length metric fluctuations from **continuum limit of Area-Regge action** [Dittrich & Kogios, 2022]

Summary

Effective action for the continuum limit of spin foams...

1. starting from the **non-chiral Plebanski action**
2. imposing only a **subset of the simplicity constraints** sharply as in spin foam quantization & replacing the others by a **quadratic potential**
3. identifying an **area metric** from the remaining d.o.f. of the B-fields
4. interpreting the (linearized) effective Plebanski action as an **action for an area metric**
5. resulting in **effective action for a length metric** which **reproduces GR** plus **non-local and ghost-free Weyl squared correction**

$$S_{\text{Pleb.}}[B,\omega,\phi] \quad \longmapsto \quad S_{\text{mod. Pleb. eff.}}[e^I_\mu, q^+_{ab}, q^-_{ab}] \quad \Leftrightarrow \quad S[A] \quad \cong \quad S[a] \quad \longrightarrow \quad S_{\text{eff}}[h] = S_{\text{GR}}[h] + \text{Weyl}[h]^2 + \cdots$$

Prospect

enlargement of the configuration space for gravity (– from length to area metrics)

- **IR phenomenology** (e.g. consequences of absence of no-hair theorem?)
- **connection to other QG approaches** (using translation to field-theoretic language)
- classification of area metric actions & canonical analysis
- 21st area metric component?

Area Metric

Area metric [Schuller & Wohlfarth, 2005; Punzi, Schuller & Wohlfarth, 2006]

$$A : (T_pM)^{\otimes\, 4} \to \mathbb{R} \quad (\text{or } A : \Lambda^2 T_pM \otimes \Lambda^2 T_pM \to \mathbb{R})$$

i. $A_{\mu\nu\rho\sigma} = -A_{\nu\mu\rho\sigma} = A_{\rho\sigma\mu\nu}$

(ii.) $A_{\mu[\nu\rho\sigma]} = 0$ cyclicity

(iii.) invertibility as symmetric 6x6 matrix $A_{\mathcal{A}\mathcal{B}},\ \mathcal{A},\mathcal{B} \in \{[01],[02],[03],[12],[13],[23]\}$

$A_{\mu\nu\rho\sigma}$:

21-1 = 20 d.o.f.