Double parton scattering in QCD

Jonathan Gaunt

Manchester Bohr Lunch Seminar, 23/10/20

OUTLINE

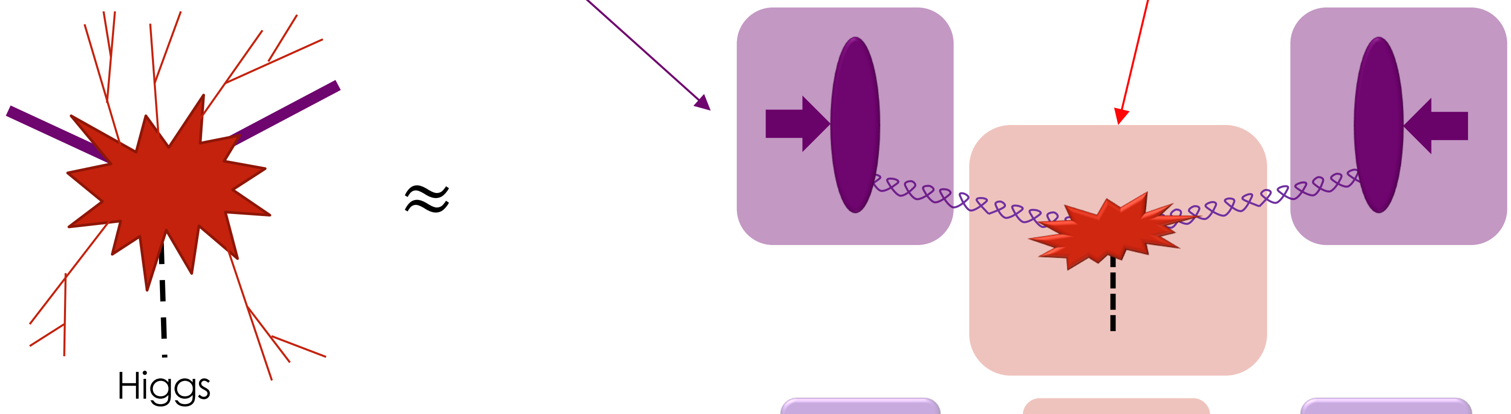
- What is double parton scattering (DPS)?
- Why double scattering is important and interesting, with reference to specific processes and experimental measurements.
- Crudest phenomenogical approach to DPS: ‘the pocket formula’. Extension of the pocket formula to arbitrarily many scatters: ‘eikonal model for multiple scattering’.
- Proper QCD framework for description of DPS. Recent developments – Monte Carlo simulation, NLO corrections. Correlations in colour and spin.

LHC FACTORISATION FORMULA

Standard framework for computing pp → some hard final state, say a Higgs boson, assumes this is produced via a single parton-parton collision (SPS):

Long distance proton structure: gluon in proton

Short distance scattering:
gluon + gluon → Higgs

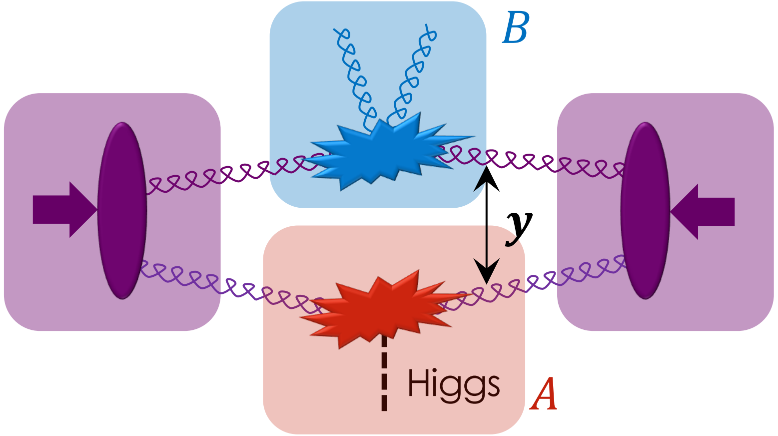


$$\sigma_{pp\to H} = \boxed{f_g(x)} \otimes \boxed{\hat{\sigma}_{gg\to H}} \otimes \boxed{f_g(x')}$$

Parton density (PDF)

DOUBLE PARTON SCATTERING

But proton is composite! If the hard process can be divided into two hard subsets A & B, this can also be produced via double parton scattering (DPS):



From parton model analysis (no QCD radiation):

$$\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \boldsymbol{y}) \otimes \hat{\sigma}_{ij}^{A} \hat{\sigma}_{kl}^{B} \otimes F_{jl}(x'_1, x'_2, \boldsymbol{y})\ d^2\boldsymbol{y}$$

Double parton density (DPD)

Paver, Treleani, Nuovo Cim. A70 (1982) 215.
Mekhfi, Phys. Rev. D32 (1985) 2371.
Blok, Dokshitzer, Frankfurt, Strikman, Phys.Rev. D83 (2011) 071501
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

POWER COUNTING

What is the rough power behaviour of these mechanisms?

$$\sigma_{SPS}^{(A,B)} = f_i(x) \otimes \hat{\sigma}_{ij\to AB} \otimes \; f_j(x')$$

$${}^{1}/_{Q^2}$$

$$\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \boldsymbol{y}) \otimes \; \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B} \; \otimes \; F_{jl}(x'_1, x'_2, \boldsymbol{y}) \;\; d^2\boldsymbol{y}$$

POWER COUNTING

What is the rough power behaviour of these mechanisms?

$$\sigma_{SPS}^{(A,B)} = f_i(x) \otimes \underset{{}^{1}/_{Q^2}}{\hat{\sigma}_{ij\to AB}} \otimes\ f_j(x')$$

$$\sigma_{DPS}^{(A,B)} = \int \underset{\Lambda^2_{\mathrm{QCD}}}{F_{ik}(x_1, x_2, \boldsymbol{y})} \otimes\ \underset{{}^{1}/_{Q^2}}{\hat{\sigma}_{ij}^{A}}\ \underset{{}^{1}/_{Q^2}}{\hat{\sigma}_{kl}^{B}} \otimes\ \underset{\Lambda^2_{\mathrm{QCD}}}{F_{jl}(x'_1, x'_2, \boldsymbol{y})}\ \underset{{}^{1}/_{\Lambda^2_{\mathrm{QCD}}}}{d^2\boldsymbol{y}}$$

POWER COUNTING

What is the rough power behaviour of these mechanisms?

$$\sigma_{SPS}^{(A,B)} = f_i(x) \otimes \underset{1/Q^2}{\hat{\sigma}_{ij\to AB}} \otimes \; f_j(x')$$

$$\sigma_{DPS}^{(A,B)} = \int \underset{\Lambda_{\mathrm{QCD}}^2}{F_{ik}(x_1, x_2, \boldsymbol{y})} \otimes \; \underset{1/Q^2}{\hat{\sigma}_{ij}^{A}} \, \underset{1/Q^2}{\hat{\sigma}_{kl}^{B}} \; \otimes \; \underset{\Lambda_{\mathrm{QCD}}^2}{F_{jl}(x'_1, x'_2, \boldsymbol{y})} \;\; \underset{1/\Lambda_{\mathrm{QCD}}^2}{d^2\boldsymbol{y}}$$

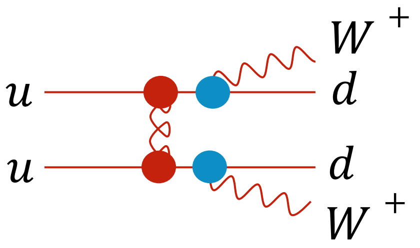
$\Rightarrow \sigma_{DPS}^{(A,B)} \big/ \sigma_{SPS}^{(A)} \approx \Lambda_{\mathrm{QCD}}^2 \big/ Q^2$, DPS is formally power suppressed at the level of the total cross section! Why then should we care about DPS?

WHY STUDY DPS?

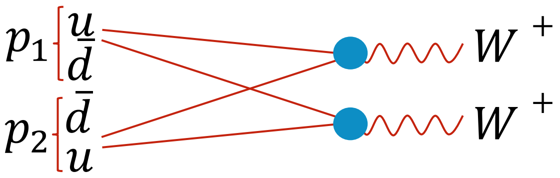
(1) DPS can be a significant background to processes suppressed by small/multiple coupling constants.

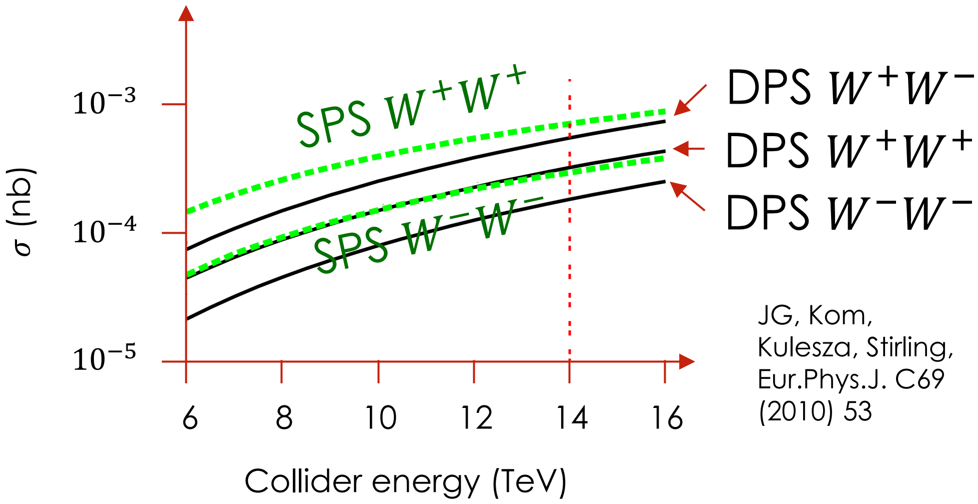
‘Classic’ SM example: same-sign WW production.

SPS:



DPS:





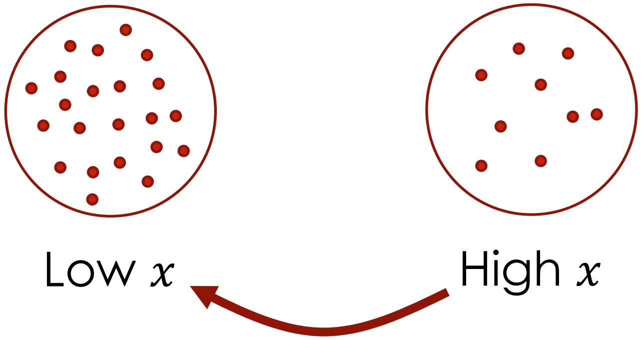
JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53

N.B. same-sign dilepton production an important channel for various new physics searches (doubly charged Higgs, SUSY,…)

WHY STUDY DPS?

(2) DPS grows faster than SPS as collider energy grows.

For a process with given scale, an increase in collider energy means a decrease in x

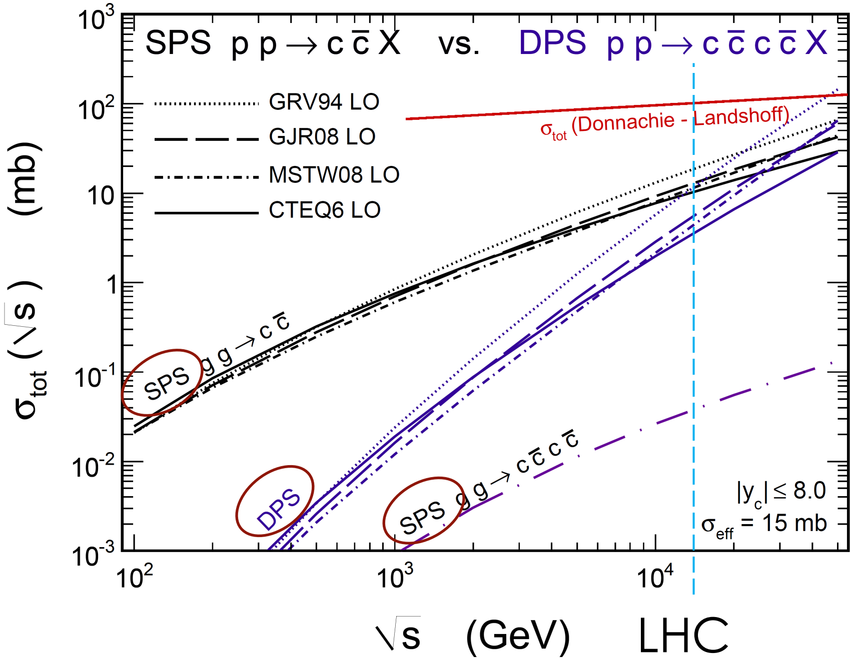


Low x High x

DPS probability increases

Growth particularly strong for low-scale processes

Łuszczak, Maciuła, Szczurek, Phys. Rev. D79, 094034 (2012)



DPS particularly important for processes involving charm and bottom quarks. '10% of all "hard" events have an additional charm pair' v. *Belyaev, MPI@LHC 2017*

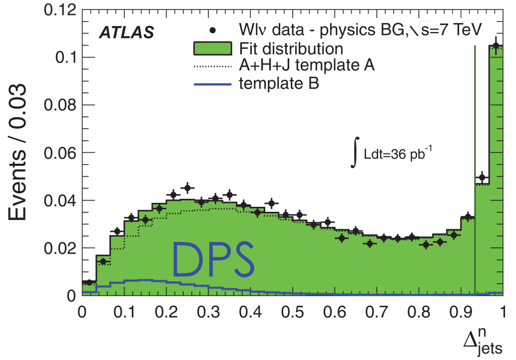
WHY STUDY DPS?

(3) DPS populates phase space in a different way to SPS. Can compete with SPS in certain regions.

Small $p_{T,A}$, $p_{T,B}$

'Double back-to-back' config preferred for DPS

ATLAS W + jj

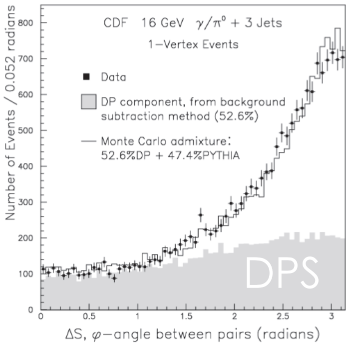


Phys.Rev. D56 (1997) 3811-3832

Angle between $p_{T,A}$, $p_{T,B}$

DPS almost flat

CDF $\gamma + 3j$



New J.Phys. 15 (2013) 033038

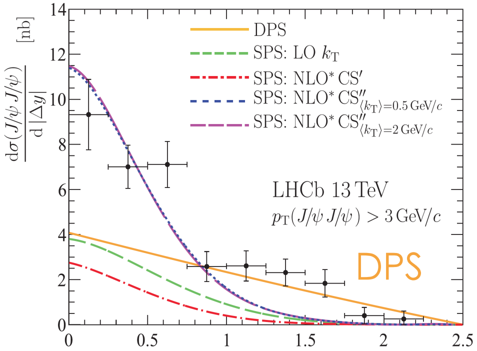
Large rapidity separation of A&B

Large Δy

→ large m_{AB}

→ SPS suppression

LHCb $J/\psi J/\psi$



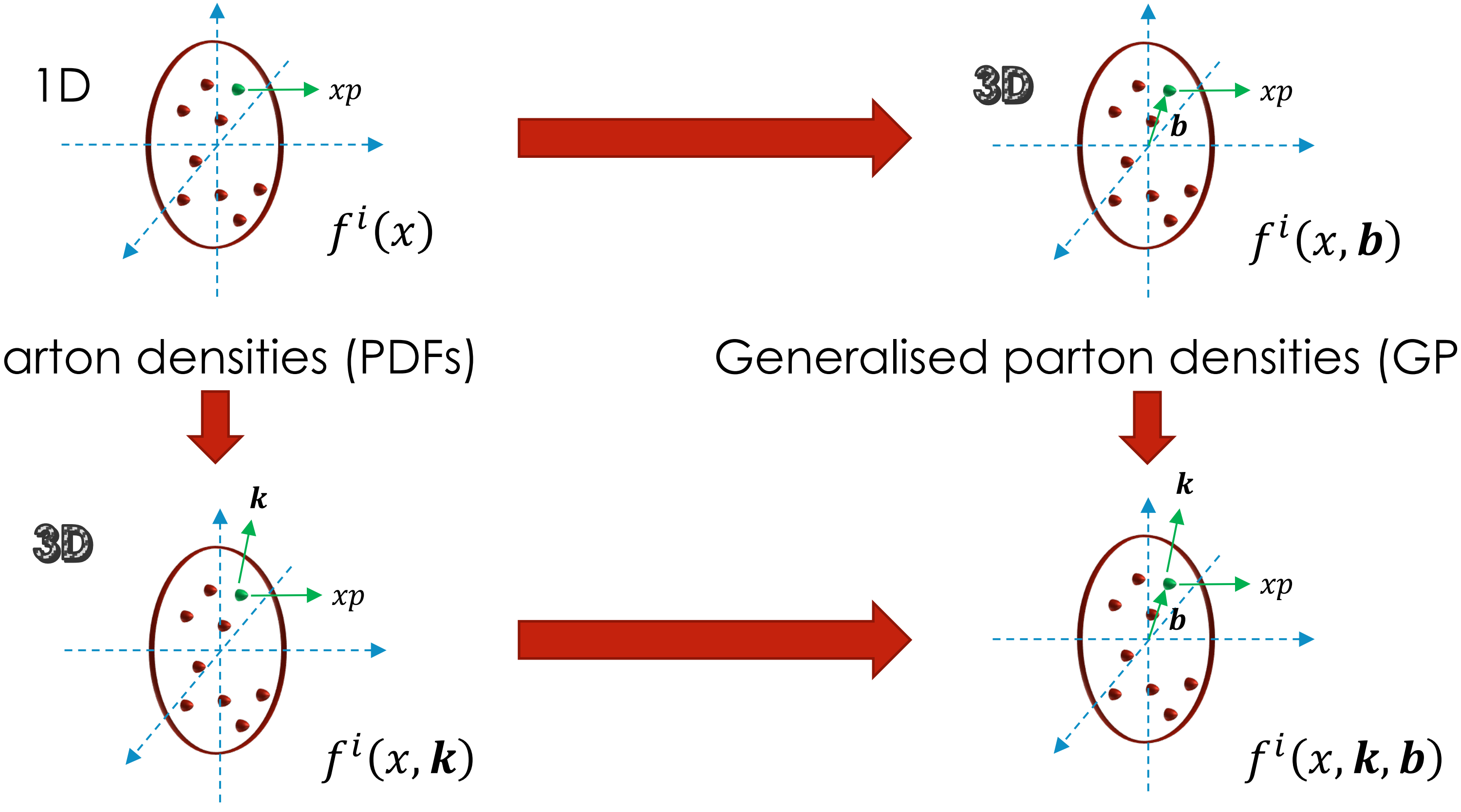
$\Delta y = |y_A - y_B|$

JHEP 06, 047, (2017)

WHY STUDY DPS?

(4) DPS gives us new information on hadron structure.

From current measurements, one-particle picture of proton:



Parton densities (PDFs)

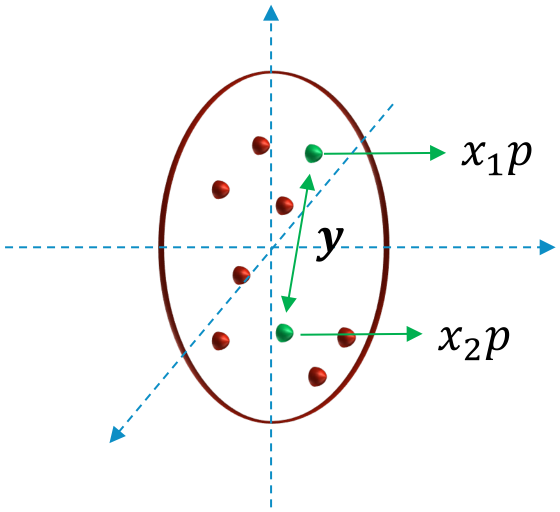
Generalised parton densities (GPDs)

Transverse momentum densities (TMDs)

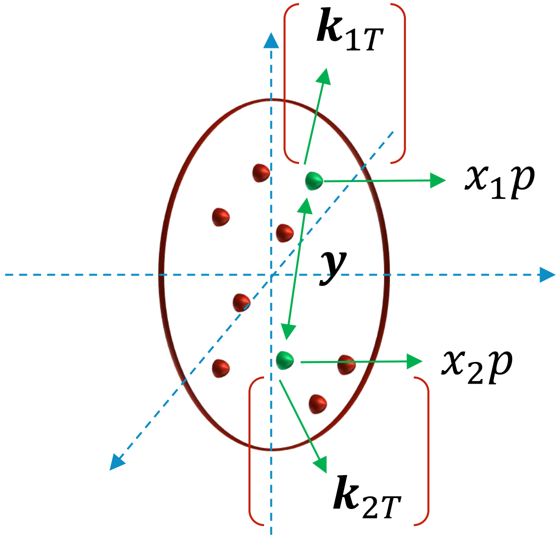
Generalised transverse momentum dependent densities (GTMDs)

WHY STUDY DPS?

Double parton scattering gives us information, for the first time, on correlation between partons!



3D⊗3D

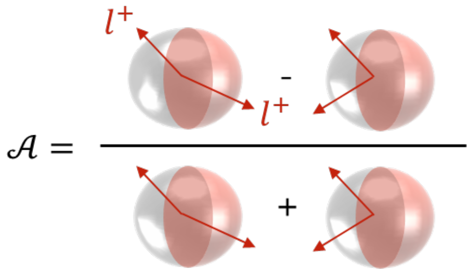


Double parton distributions (DPDs)

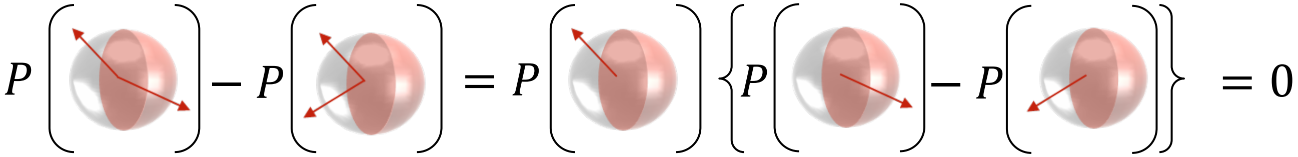
Double parton transverse momentum distributions (DTMDs)

MEASURING CORRELATIONS

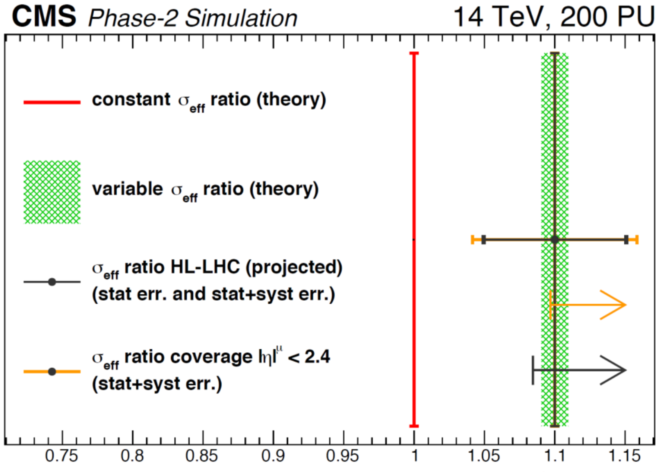
One observable to measure in detail the correlations: $\mathcal{A}$ in $W^{\pm}W^{\pm} \to l^{\pm}l^{\pm}\nu\nu$



If no correlations:



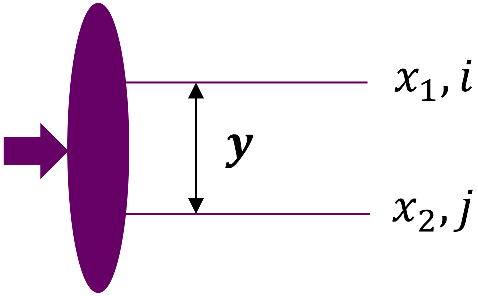
$\mathcal{A} \neq 0$ implies correlations! $\mathcal{A}$ values of $\simeq 0.1$ are measurable at hi-lumi LHC



CMS-TDR-016

DPS 'POCKET FORMULA'

DPD $F_{ik}(x_1, x_2, \boldsymbol{y})$ is a complex object!

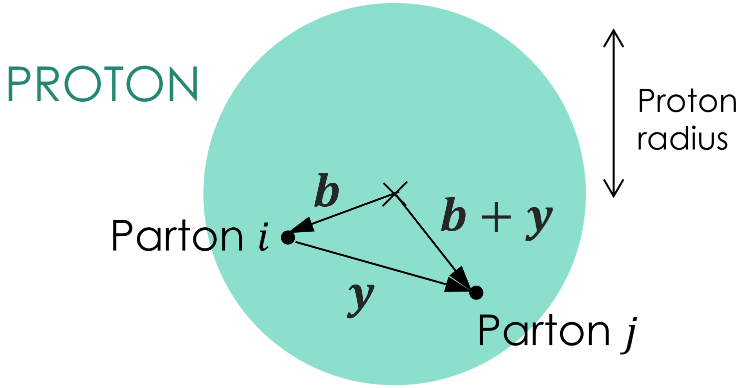


Historically several approximations, for rough estimates of DPS.

(1) Ignore correlations between partons

$$F^{ij}(x_1, x_2, \boldsymbol{y}) \to \int d^2\boldsymbol{b}\, f^i(x_1, \boldsymbol{b})\, f^j(x_2, \boldsymbol{b}+\boldsymbol{y})$$

GPD



DPS ‘POCKET FORMULA’

(2) Assume GPD can be written as $f^i(x_1, \boldsymbol{b}) = f^i(x_1) G(\boldsymbol{b})$

Then $F^{ij}(x_1, x_2, \boldsymbol{y}) = f^i(x_1)\, f^j(x_2) \int d^2\boldsymbol{b}\, G(\boldsymbol{b})\, G(\boldsymbol{b} + \boldsymbol{y})$

Inserting into $\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \boldsymbol{y}) \otimes \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B} \otimes F_{jl}(x'_1, x'_2, \boldsymbol{y})\; d^2\boldsymbol{y}$ …

$$\longrightarrow \quad \boxed{\sigma_D^{(A,B)} = \frac{\sigma_S^{(A)}\, \sigma_S^{(B)}}{\sigma_{\text{eff}}}}$$

“DPS pocket formula”

Most pheno estimates of DPS use this!

$[\sigma_{\text{eff}} \approx 10 - 20$ mb$]$

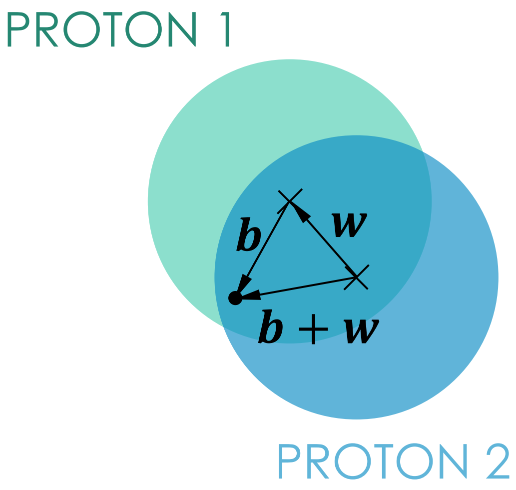
EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Can rewrite pocket formula cross section:

$$\sigma_{\mathrm{D}} = \int \frac{1}{2!}\left(\underbrace{\int f(x_1) f(\bar{x}_1)\hat{\sigma}(x_1,\bar{x}_1) G(\boldsymbol{b}) G(\boldsymbol{b}+\boldsymbol{w}) d^2\boldsymbol{b}}\right)^2 d^2\boldsymbol{w}$$

(For identical particles)

$$= \int \frac{1}{2!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^2 d^2\boldsymbol{w}$$



EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Generalise to N scatters:

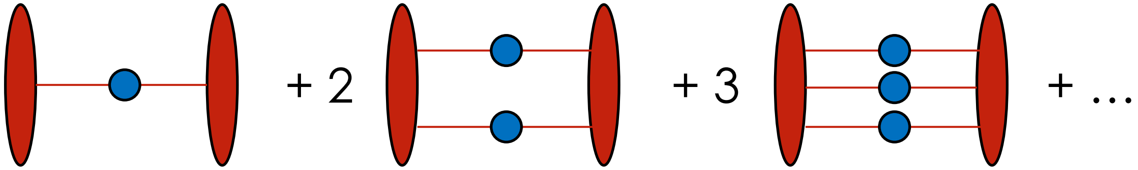
$$\sigma_{\mathrm{N}} = \int \underbrace{\frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N}_{\text{INCLUSIVE N-PARTON SCATTERING PROBABILITY}} d^2\boldsymbol{w}$$

EIKONAL MODEL FOR MULTIPLE INTERACTIONS

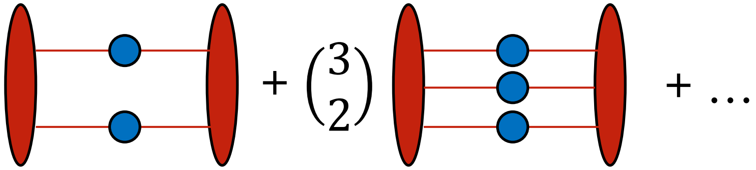
Generalise to N scatters:

$$\sigma_{\mathrm{N}} = \int \underbrace{\frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N}_{\text{INCLUSIVE N-PARTON SCATTERING PROBABILITY}} d^2\boldsymbol{w}$$

SPS =



DPS =



EIKONAL MODEL FOR MULTIPLE INTERACTIONS

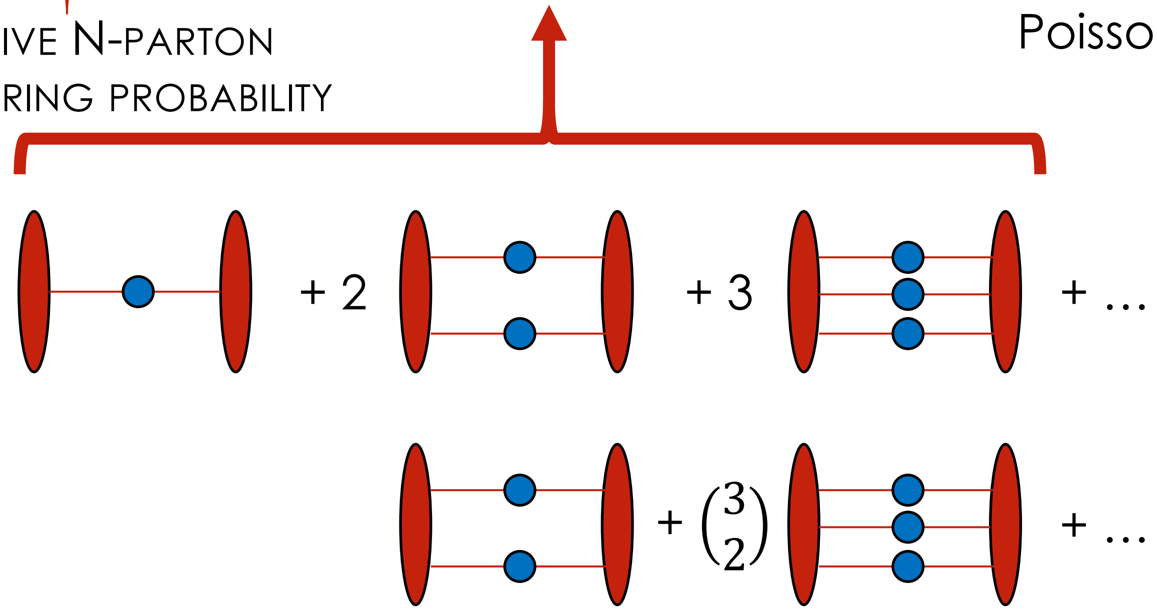
Generalise to N scatters:

EXCLUSIVE M-PARTON SCATTERING PROBABILITY

$$\sigma_{\mathrm{N}} = \int \underbrace{\frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N}\, d^2\boldsymbol{w} = \int \sum_{M \geq N} \binom{M}{N} P_M(\boldsymbol{w})\, d^2\boldsymbol{w} \qquad P_M(\boldsymbol{w}) = \frac{\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^M}{M!} e^{-\sigma_S \mathcal{G}(\boldsymbol{w})}$$

INCLUSIVE N-PARTON SCATTERING PROBABILITY

Poisson distribution



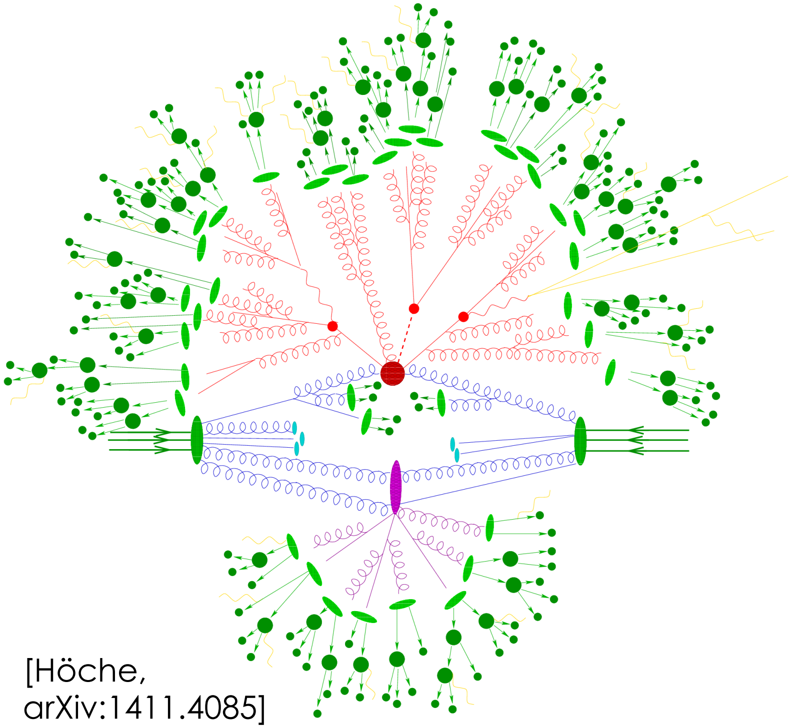
Seymour, Siodmok, arXiv:1308.6749
Calucci, Treleani, Phys.Rev. D79 (2009) 034002, Phys.Rev. D79 (2009) 074013

EIKONAL MODEL FOR MULTIPLE INTERACTIONS

Generalise to N scatters:

$$\sigma_\mathrm{N} = \int \frac{1}{N!}\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^N d^2\boldsymbol{w} = \int \sum_{M \geq N} \binom{M}{N} P_M(\boldsymbol{w})\, d^2\boldsymbol{w} \qquad P_M(\boldsymbol{w}) = \frac{\left(\sigma_S \mathcal{G}(\boldsymbol{w})\right)^M}{M!} e^{-\sigma_S \mathcal{G}(\boldsymbol{w})}$$

Poisson distribution



[Höche, arXiv:1411.4085]

This eikonal model is the basis of the multiple interactions models in Monte Carlo event generators!

Herwig model ≈ eikonal model.

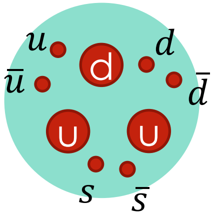
Butterworth, Forshaw, Seymour, Z.Phys. C72 (1996) 637
Borozan, Seymour, JHEP 0209 (2002) 015
Bahr, Gieseke, Seymour, JHEP 0807 (2008) 076

MULTIPLE SCATTERING IN PYTHIA

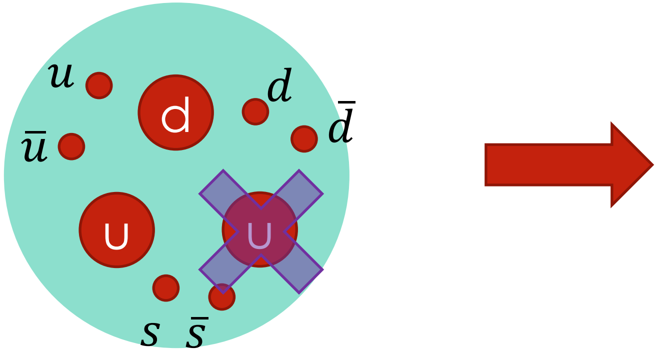
Pythia model has some improvements to this picture.

Sjöstrand, van Zijl, Phys.Rev. D36 (1987) 2019,
Sjöstrand, Skands, JHEP 0403 (2004) 053
Eur.Phys.J. C39 (2005) 129-154

Start at hardest interaction and work ‘backwards’. Start with normal PDFs: $\int f^{u_v}(x)\mathrm{d}x = 2\,,\int f^{d_v}(x)\mathrm{d}x = 1,\ \sum_i \int f^i(x)\,x\,\mathrm{d}x = 1$

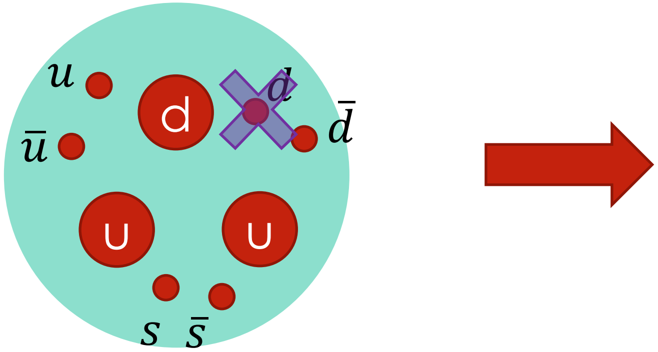


Interaction 1 involves valence u parton with momentum z



Adjust PDFs for remaining interactions: Total momentum $1 - z$, number of u valence = 1.

Interaction 1 involves sea d parton with momentum z

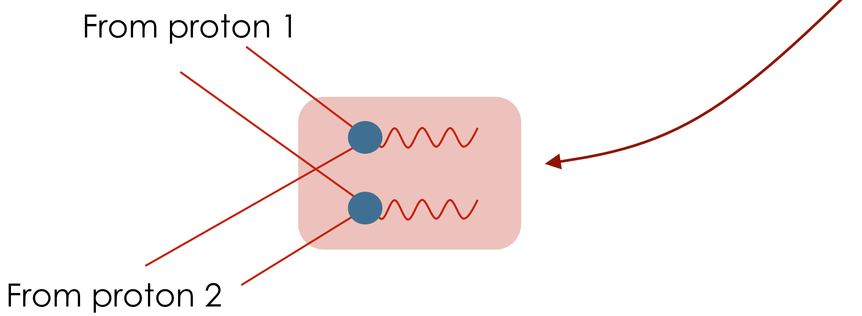


Adjust PDFs for remaining interactions: Total momentum $1 - z$, add to $\bar{d}$ distribution ‘companion quark distribution’

QCD EVOLUTION EFFECTS IN DPS

Now let’s try to develop a more sophisticated QCD treatment.

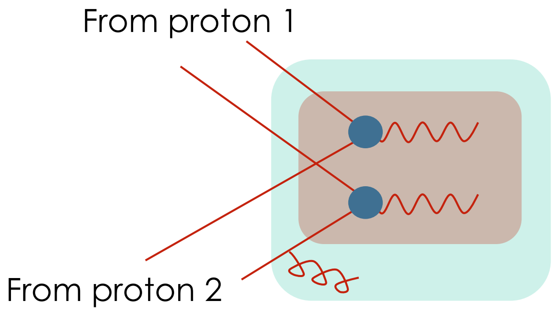
Consider “zooming out” from the hard processes. What kind of QCD effects can occur?



QCD EVOLUTION EFFECTS IN DPS

Now let's try to develop a more sophisticated QCD treatment.

Consider "zooming out" from the hard processes. What kind of QCD effects can occur?

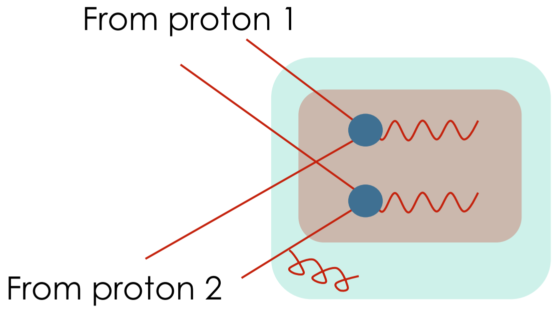


Emission from single leg. Familiar from single scattering.

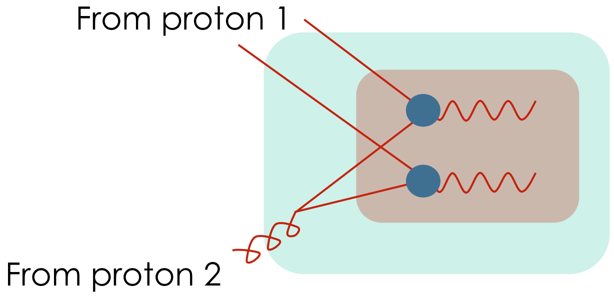
QCD EVOLUTION EFFECTS IN DPS

Now let's try to develop a more sophisticated QCD treatment.

Consider "zooming out" from the hard processes. What kind of QCD effects can occur?



Emission from single leg. Familiar from single scattering.



'1→2 splitting'. New effect!

Single PDF

Perturbative calculation at small ***y***

$$F(x_1, x_2, y) \propto \alpha_s \frac{f(x_1 + x_2)}{x_1 + x_2} P\left(\frac{x_1}{x_1+x_2}\right) \frac{1}{y^2}$$

Dimensionful part

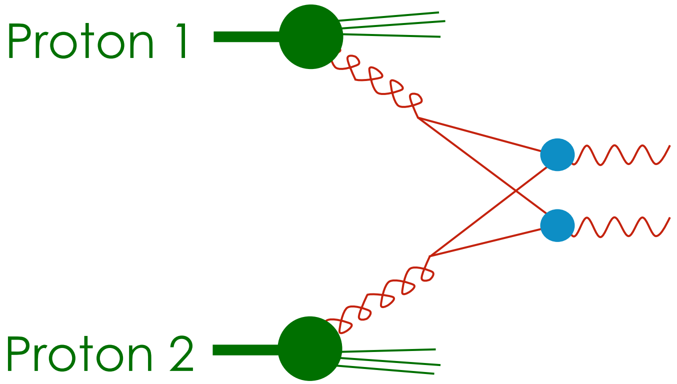
Perturbative splitting kernel

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

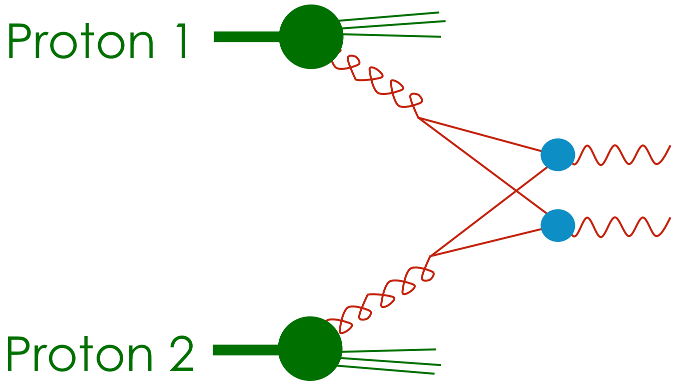
DOUBLE COUNTING PROBLEMS

Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2 y}{y^4} = ?$$



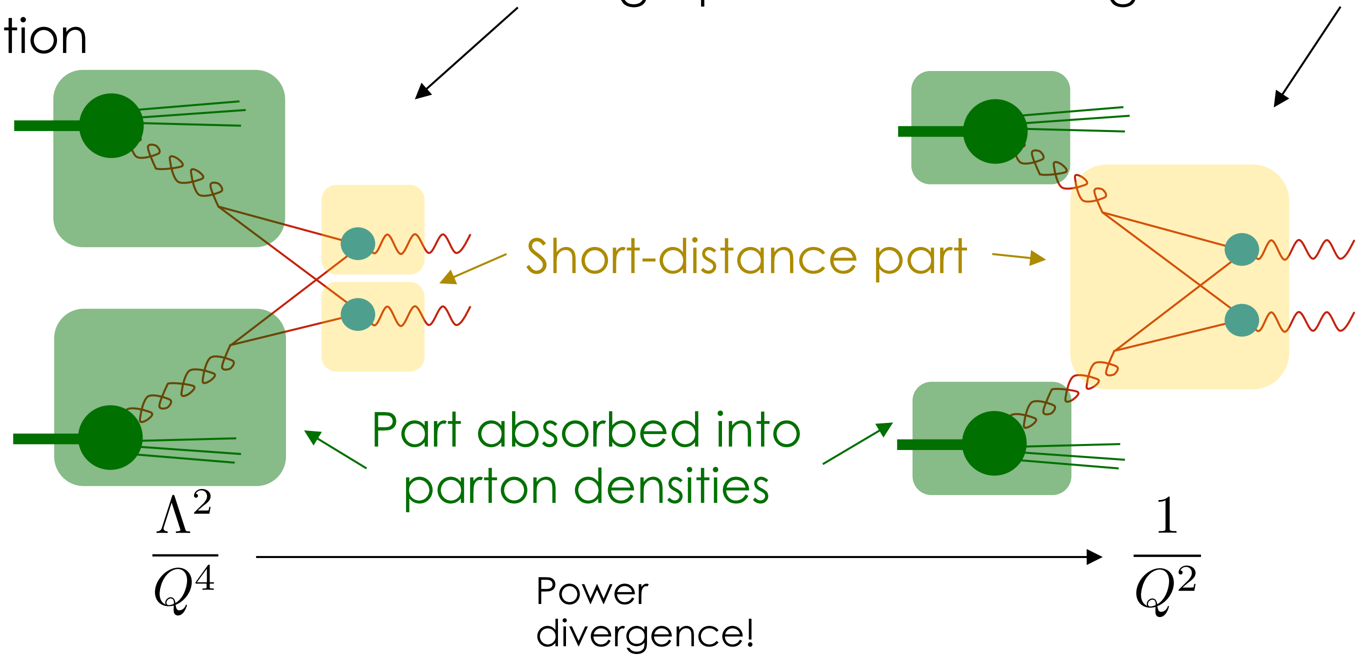
DOUBLE COUNTING PROBLEMS



Perturbative splitting can occur in both protons (1v1 graph) – gives power divergent contribution to DPS cross section!

$$\int \frac{d^2y}{y^4} = ?$$

This is related to the fact that this graph can also be regarded as an SPS loop correction



Short-distance part

Part absorbed into parton densities

$$\frac{\Lambda^2}{Q^4} \longrightarrow \frac{1}{Q^2}$$

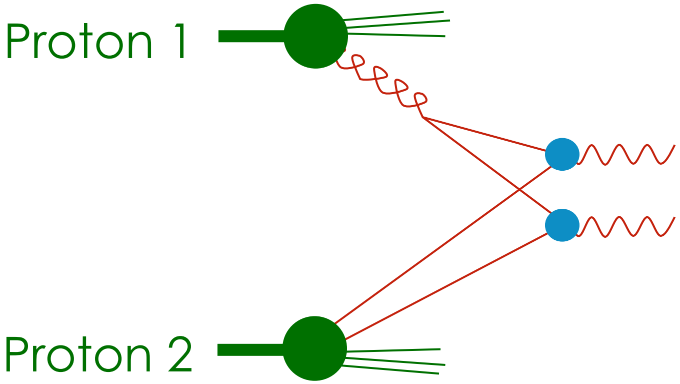
Power divergence!

Diehl, Ostermeier and Schafer (JHEP 1203 (2012)), Manohar, Waalewijn Phys.Lett. 713 (2012) 196, **JG and Stirling, JHEP 1106 048 (2011)**, Blok et al. Eur.Phys.J. C72 (2012) 1963 Ryskin, Snigirev, Phys.Rev.D83:114047 ,2011, Cacciari, Salam, Sapeta JHEP 1004 (2010) 065

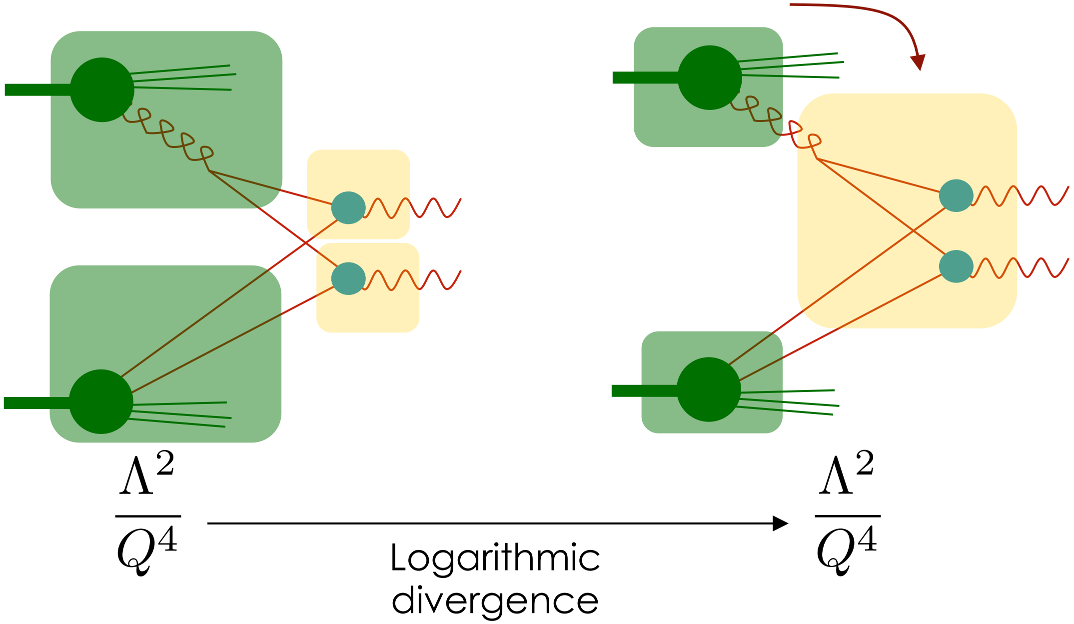
DOUBLE COUNTING PROBLEMS

Also have graphs with perturbative 1→2 splitting in one proton only (2v1 graph).

This has a log divergence: $\int d^2y/y^2 F_{\text{non-split}}(x_1, x_2; y)$



Related to the fact that this graph can also be thought of as an NLO correction to collision of one parton with two



Blok et al., Eur. Phys. J. C72 (2012) 1963
Ryskin, Snigirev, Phys. Rev. D83:114047,2011,
JG, JHEP 1301 (2013) 042

DOUBLE COUNTING PROBLEMS

Desired features of a solution to these issues:

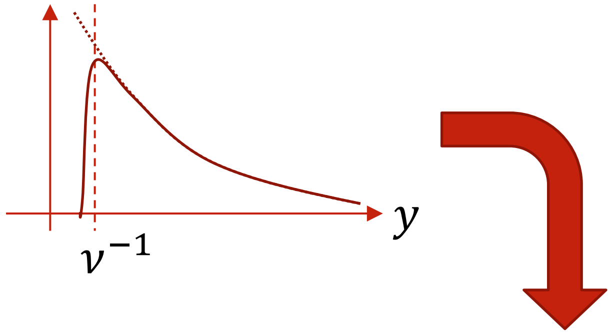
- DPS contribution finite + no double counting between DPS and SPS.
- Retain concept of the DPD for an individual hadron, with rigorous definition beyond perturbation theory.
- Should resum DGLAP logarithms in all types of diagram (1v1, 2v1, 2v2) where appropriate.
- All-order formulation, with corrections that are practicably computable.
- Re-use as many SPS results as possible.

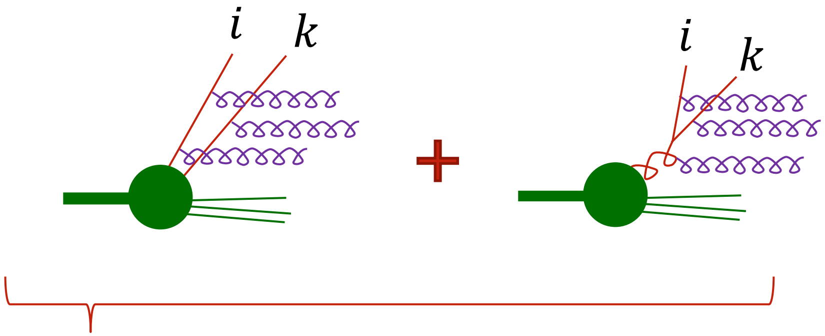
Solution with these features achieved in 'DGS framework' Diehl, JG, Schönwald JHEP 1706 (2017) 083.

DPS WITHOUT DOUBLE COUNTING

I focus on SPS & 1v1 DPS overlap. Removal of overlap between 2v1 DPS & 3 particle collision is similar.

<u>Step 1:</u> insert cut-off function into DPS cross section formula





$$\sigma_{DPS}^{(A,B)} = \int dx_i dx'_i d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{ik}(x_1, x_2, \boldsymbol{y}, \mu_A, \mu_B)\, F_{jl}(x'_1, x'_2, \boldsymbol{y}, \mu_A, \mu_B)$$

$$\times \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B}$$

Choose $\nu \sim Q$ in practice.

Removed divergence. Double counting up to scale ν.

DPS WITHOUT DOUBLE COUNTING

Step 2: For total cross section for production of *AB*, include a subtraction term to remove double counting.

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

σ_{sub}: DPS cross section with DPDs replaced by fixed order splitting expression – i.e. combining the approximations used to compute double splitting piece in two approaches.

$$F_{ij}(x_1, x_2, y, \mu^2) \rightarrow \frac{1}{\pi y^2} \frac{f_k(x_1 + x_2, \mu^2)}{x_1 + x_2} \frac{\alpha_s(\mu^2)}{2\pi} P_{k \rightarrow ij}\left(\frac{x_1}{x_1 + x_2}\right)$$

General subtraction philosophy used in many QCD calculations (proofs of factorisation, SCET, NLO + PS matching…)

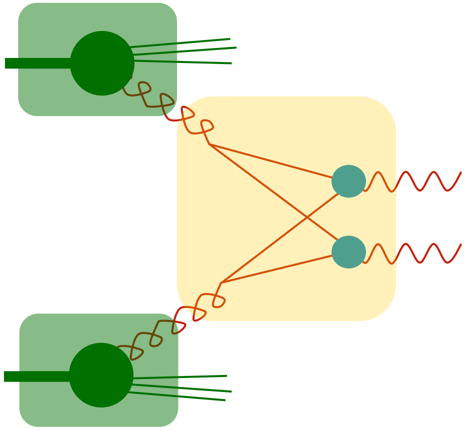
HOW THE SUBTRACTION WORKS

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

For small $\boldsymbol{y}$ (of order $1/Q$) the dominant contribution to σ_{DPS} comes from the (fixed order) perturbative expression $\Rightarrow \sigma_{DPS} \approx \sigma_{sub}$

& $\sigma_{tot} \approx \sigma_{SPS}$ ✓

Dependence on ν cancels order-by-order between σ_{DPS} & σ_{sub}:



For large $\boldsymbol{y}$ (much larger than $1/Q$) the dominant contribution to σ_{SPS} is the region of the 'double splitting' loop where DPS approximations are valid

$\Rightarrow \sigma_{SPS} \approx \sigma_{sub}$

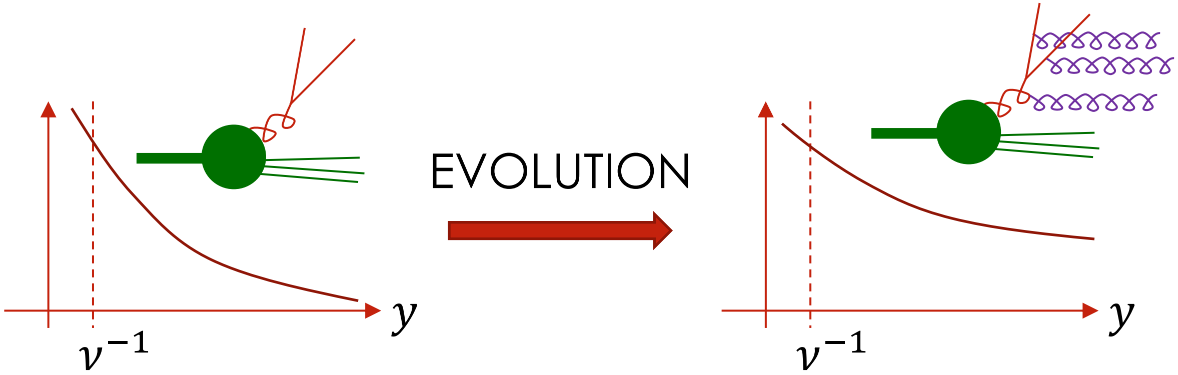
& $\sigma_{tot} \approx \sigma_{DPS}$ ✓

CUTOFF DEPENDENCE

Important: σ_{DPS} is not really ‘meaningful’ on its own. Can only measure $\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$

Generically $\propto \nu^2$

IN CERTAIN CASES:

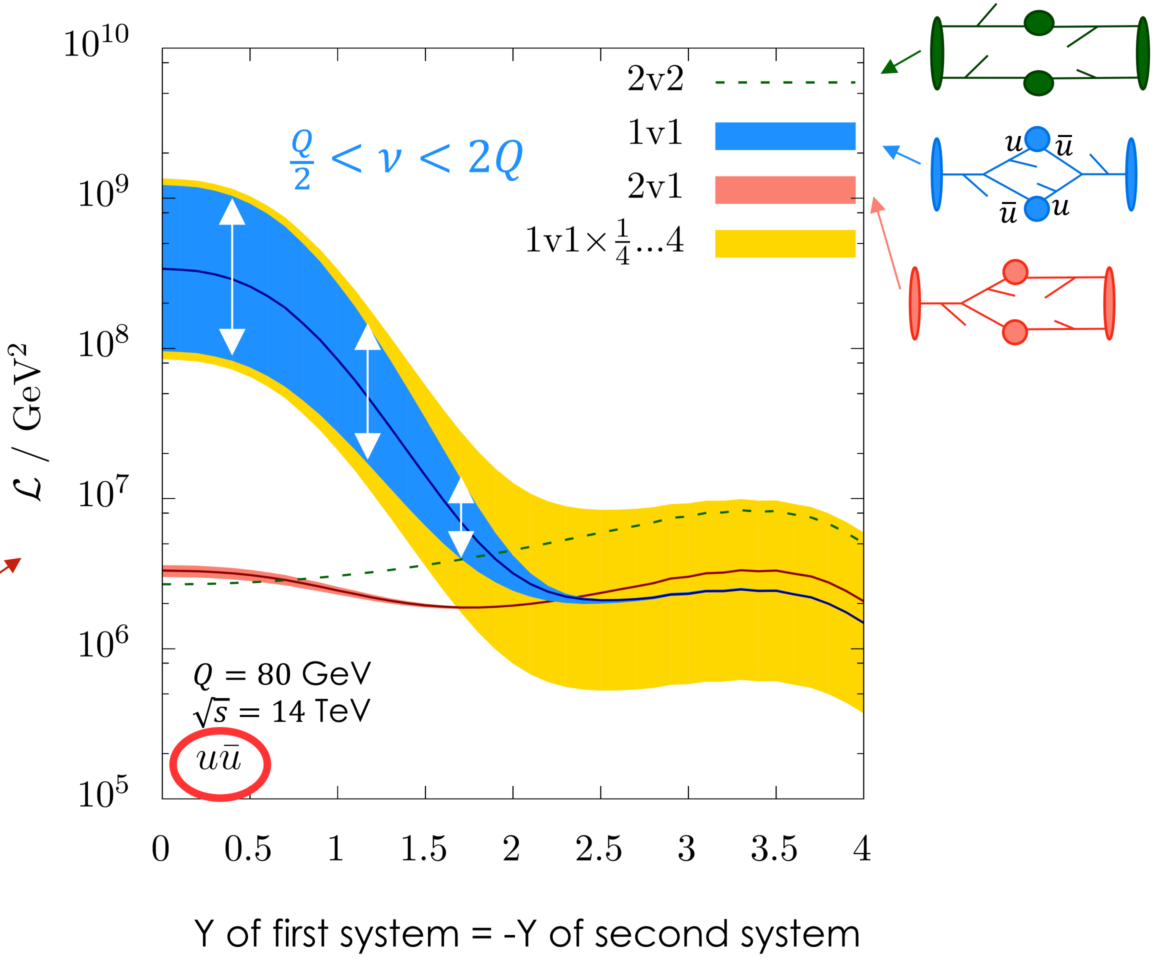


Bulk of σ_{DPS} shifts to large $\boldsymbol{y}$ where DPS approximations are valid. Small $\boldsymbol{y}$ is less important → reduced ν dependence, σ_{sub} and two-loop σ_{SPS} less important.

REDUCED CUTOFF DEPENDENCE

Example: two systems widely separated in rapidity.

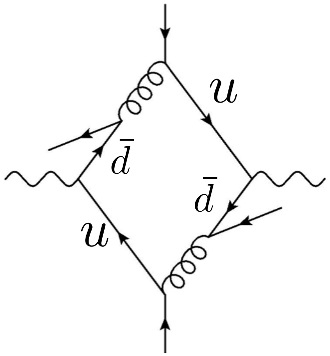
$$\mathcal{L} = \int \Phi(\nu y)^2 F_{u\bar{u}}(y) F_{\bar{u}u}(y)$$

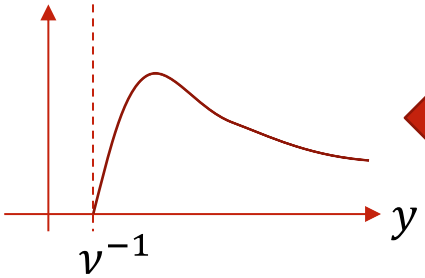


REDUCED CUTOFF DEPENDENCE

Another example where overlap considerations are less important: processes with no two-loop box contribution

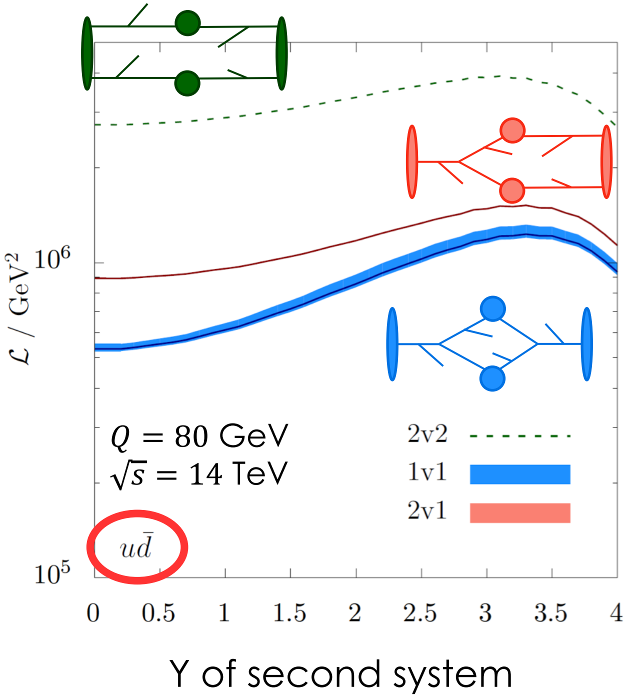
E.g. Same-sign WW production





Splitting DPD profile

Effect of 2v1 and 1v1 graphs seem to be less pronounced.



PHENO TOOLS FOR DPS

DPS theory developments have been rapid in past 10 years. Development of phenomenological tools has lagged behind.

Many experimental extractions of DPS use theoretical predictions of DPS shapes in multiple distributions (‘templates’).

Typically provided by Monte Carlo event generators.

11 variables in same-sign WW:

$$p_T^{l_1}, p_T^{l_2}, p_T^{miss}, \eta_1\eta_2, |\eta_1 + \eta_2|,$$
$$m_{T(l_1,p_T^{miss})}, m_{T(l_1,l_2)}, \left|\Delta\phi_{(l_1,l_2)}\right|,$$
$$\left|\Delta\phi_{(l_2,p_T^{miss})}\right|, \left|\Delta\phi_{(ll,l_2)}\right|, m_{T2}^{ll}$$

CMS-PAS-SMP-18-015

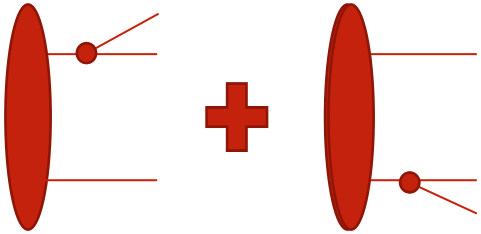
Would be very useful to have a Monte Carlo event generator for DPS that includes latest theory developments!

A DPS PARTON SHOWER

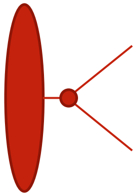
Motivated a parton shower implementation of the DGS framework: dShower. Cabouat, JG, Ostrolenk, JHEP 1911 (2019) 061

Key features:

- Account of $\boldsymbol{y}$ dependence, 1→2 splittings consistently included.
- Shower evolution 'guided' by a set of DPDs. Correlations encoded by these DPDs are fed into the shower.
- Backward evolution from hard process with emissions from two legs. Angular ordered shower, as in Herwig.



- $2 \rightarrow 1$ 'mergings' in backward evolution at scale $\mu_y \sim 1/y$, with a probability determined by [splitting part of DPD] / [total DPD].

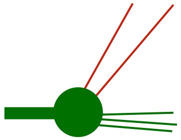


SOME FIRST NUMERICS

- same-sign WW $pp \to W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$
- 3 quark flavours
- DPDs from JHEP 1706 (2017) 083 (Diehl, JG, Schönwald):

Initialise at low scale $\mu_0 = 1\ \mathrm{GeV}$

$$F_{\mathrm{int}}^{ij}(x_1, x_2, y, \mu_0) = \frac{1}{4\pi h_{ij}}\, e^{-\frac{y^2}{4h_{ij}}}\, f_i(x_1,\mu_0)\, f_j(x_2,\mu_0)(1-x_1-x_2)^2(1-x_1)^{-2}(1-x_2)^{-2}$$



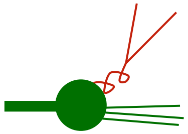
Smooth transverse y profile, radius ~ R_p

'Usual' product of PDFs

Factor to suppress DPD near phase space limit $x_1 + x_2 = 1$

Initialise at scale $\mu_y \sim \frac{1}{y}$

$$F_{\mathrm{spl}}^{ij}(x_1, x_2, y, \mu_y) = e^{-\frac{y^2}{4h_{ij}}}\, \frac{1}{\pi y^2}\, \frac{\alpha_s(\mu_y)}{2\pi} \sum_k \frac{f_k(x_1+x_2,\mu_y)}{x_1+x_2}\, P_{k\to i}\left(\frac{x_1}{x_1+x_2}\right)$$

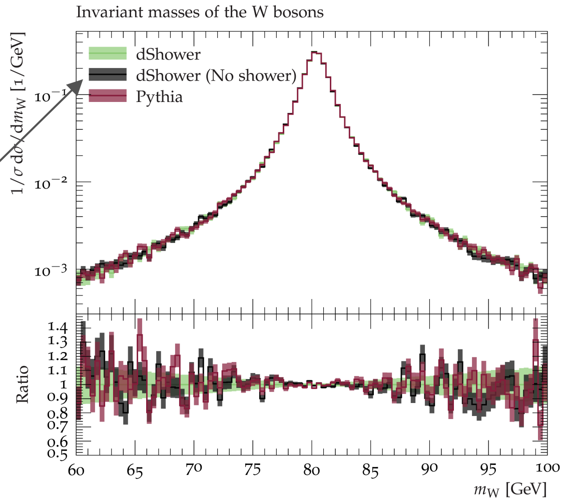


Gaussian suppression at large y

Perturbative splitting expression

with modifications to very approximately take account of finite valence number [$uu \to uu - \frac{1}{2}u_v u_v$, $dd \to dd - d_v d_v$ in intrinsic]

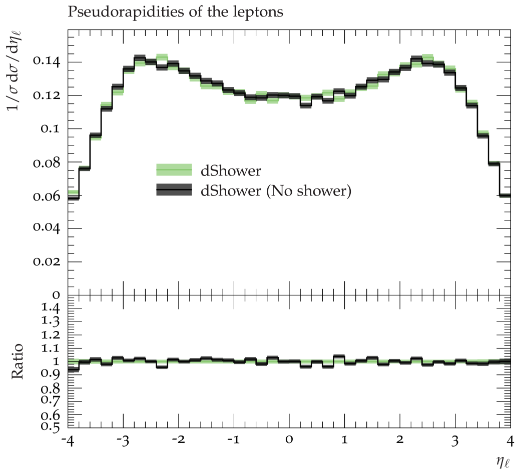
VALIDATION OF DSHOWER

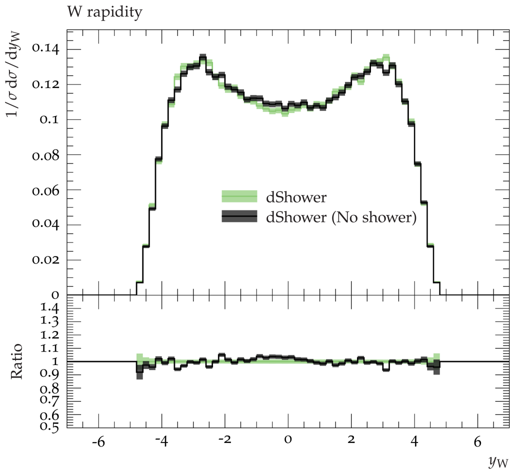


DPS cross section formula

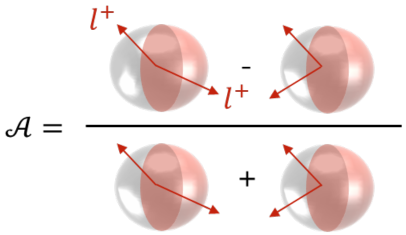
dShower preserves invariant mass spectrum of *W*s

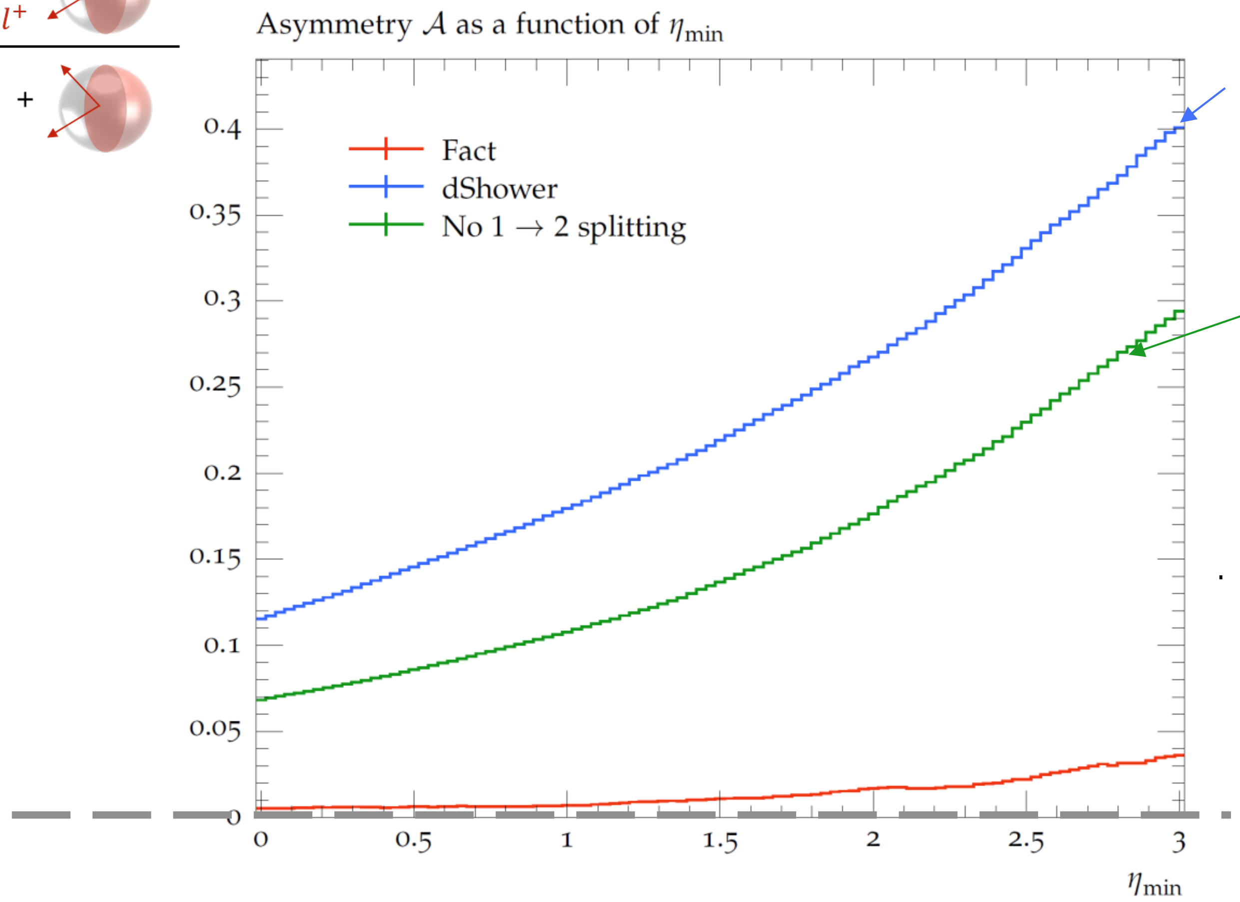
Rapidity distributions of leptons and *W*s preserved





RESULTS: ASYMMETRY





Includes 1→2 splittings
+ valence number effects

Simple valence number effects

No parton-parton correlations

DSHOWER: COMBINING SPS AND DPS

In general will need to combine DPS shower with an SPS shower in an appropriate way to obtain physical results.

Need ‘fully differential’ formulation of subtraction formalism:

Cabouat, JG, JHEP 10 (2020) 012

Usual SPS shower

$$\frac{d\sigma^{tot}_{A+B}}{dO} = \overbrace{\boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma^{SPS}_{A+B}}{dO}\right.} - \left.\frac{d\sigma^{sub}_{(A,B)}}{dO}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma^{DPS}_{(A,B)}}{dO\, d^2\boldsymbol{y}}$$

Single parton shower

Double parton shower

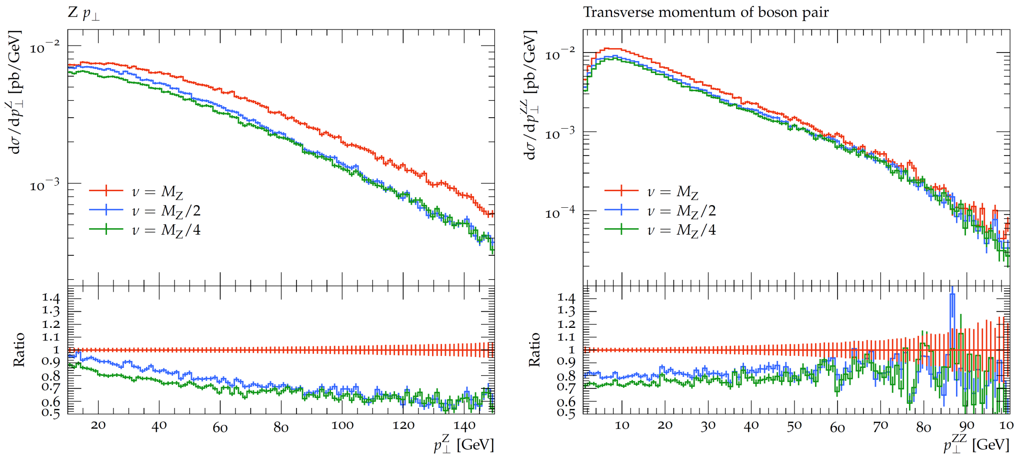
Hard cross section in this term is DPS shower expanded to $\mathcal{O}(\alpha_s^2)$, keeping only merging terms in each proton, integrated over y

[Inspired by methods to match shower with NLO calculations: Frixione, Webber, JHEP 06 (2002) 029, Frixione, Nason, Oleari, JHEP 11 (2007) 070, Nason, JHEP 11 (2004) 040,…]

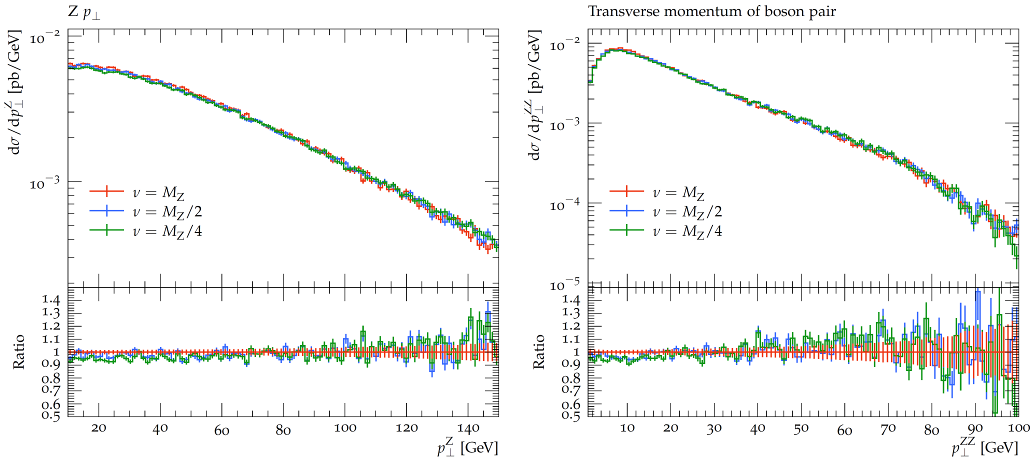
VALIDATION: DPS & SUB AT SMALL Y

Study for ZZ production. SPS is loop induced gg→ZZ only, divided by 10

No subtraction:



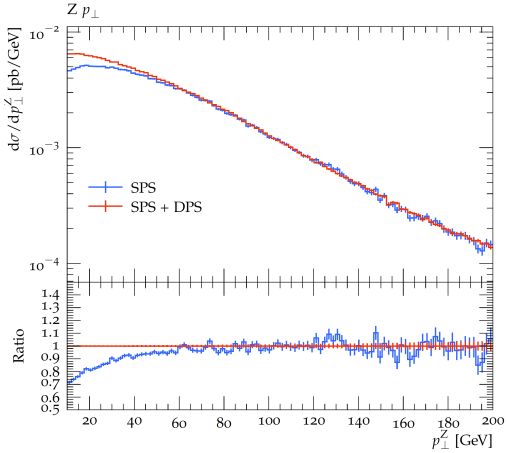
Subtraction included:

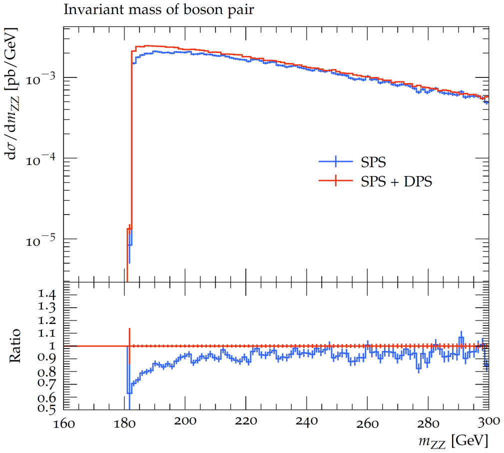


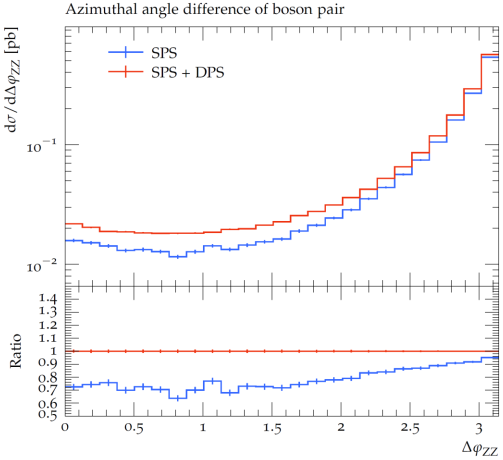
DISTINGUISHING SPS AND DPS IN ZZ

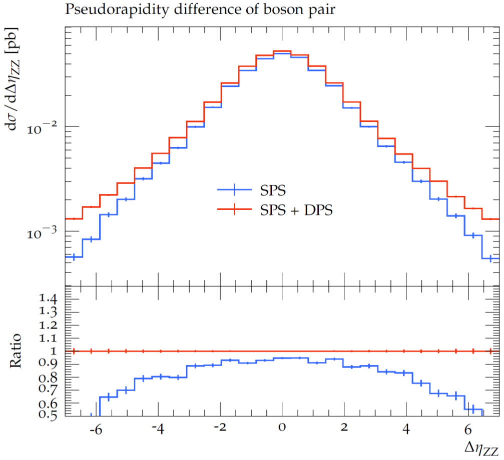
“Toy” study: SPS is loop induced only, divided by 10 (& 3 quark flavours)

Small p_T of bosons, small invariant mass of pair







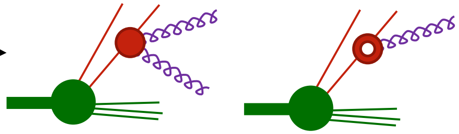


Small(ish) angle between bosons, large rapidity separation

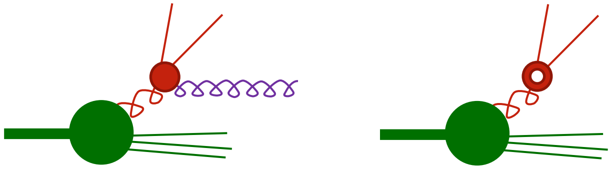
NLO CORRECTIONS TO DPS

DGS framework opens the way for the first NLO computations of DPS. What is needed for these computations?

- NLO corrections to partonic cross sections: already known for many processes from SPS calculations ✓
- NLO 'usual' splitting functions - needed for evolution of $F(\boldsymbol{y})$: already known since the 80s ✓ Curci, Furmanski, Petronzio, Nucl. Phys. B175, 27 (1980), Furmanski, Petronzio, Phys. Lett. 97B, 437 (1980),…



- NLO corrections to the splitting - recently computed! ✓



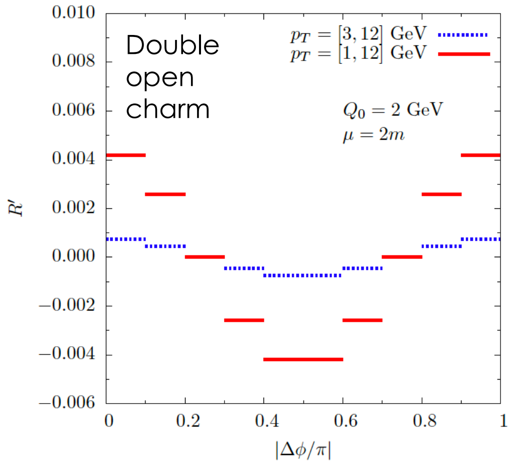
Diehl, JG, Plößl, Schäfer, SciPost Phys. 7 (2019) 2, 017

CORRELATIONS

Partons in DPS can also be correlated in spin & colour.

Can have interesting effects beyond a change in rate – e.g. transverse spin correlations can cause φ distribution to have a non-flat shape.

Echevarria, Kasemets, Mulders, Pisano, JHEP 04 (2015) 034



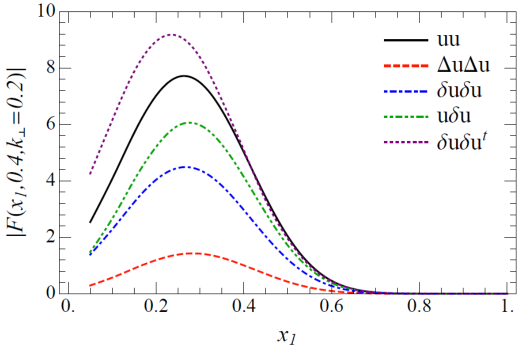
Framework for incorporating these correlations is known.

Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

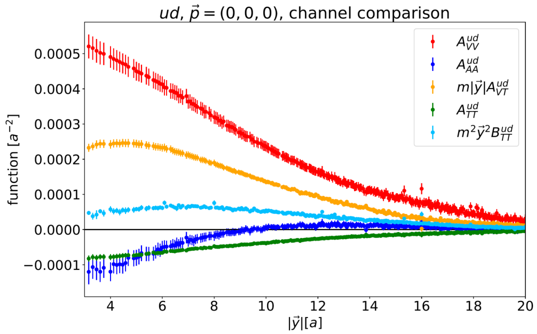
How important are these effects?

SPIN CORRELATIONS

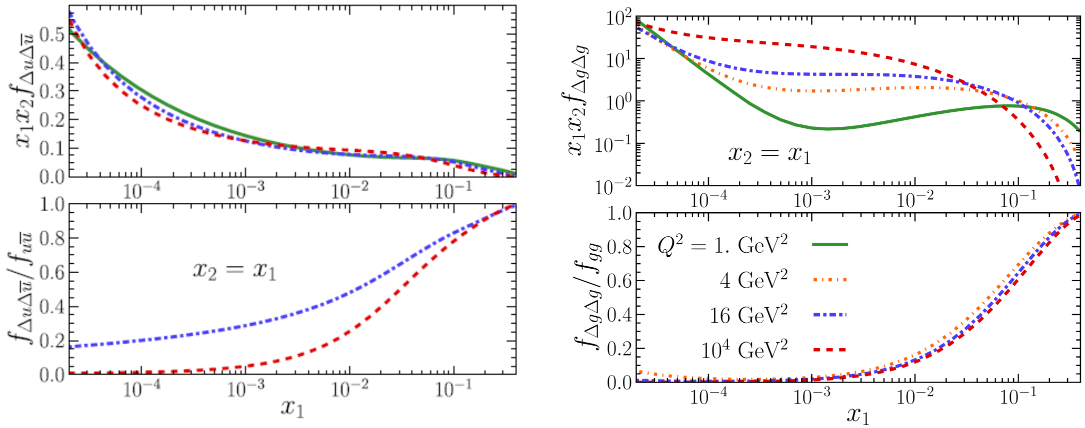
Model and lattice results indicate spin correlations large at larger x and low scale.



Chang, Manohar, Waalewijn, Phys.Rev. D87 (2013) no.3, 034009



C. Zimmermann, talks at LATTICE2019, MPI@LHC 2019



Diehl, Kasemets, Keane, JHEP 1405 (2014) 118

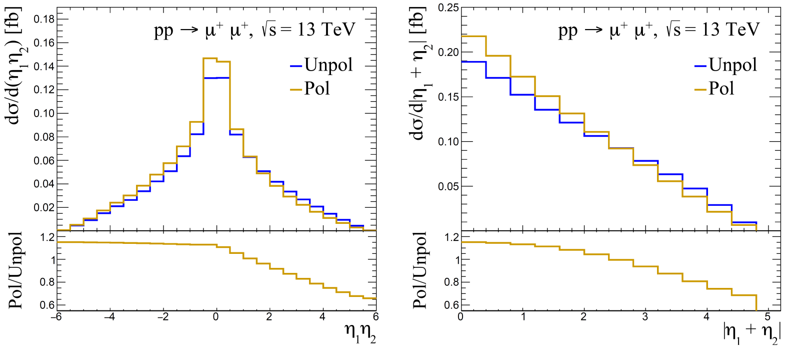
Evolution tends to wash out the correlations. Slowest at high x, and for quark channels.

SPIN CORRELATIONS IN $W^{\pm}W^{\pm}$

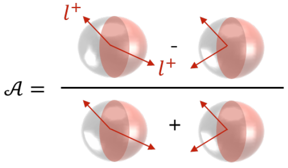
Recently identified that spin polarisation effects may have a measurable effect in same-sign WW [Cotogno, Kasemets, Myska, Phys.Rev. D100 (2019) 1, 011503, arXiv:2003.03347]

Good process in terms of spin polarisation:

- involves quarks.
- Ws couple only to left-handed quarks



Input at 1 GeV for polarised DPD chosen to yield maximum possible effect



$\lvert\eta_i\rvert$	> 0	> 0.6	> 1.2
A	0.07	0.11	0.16
σ [fb]	0.51	0.29	0.13

Few percent effect on lepton pseudorapidity asymmetry

SUMMARY

- DPS can compete with SPS for certain processes ($W^{\pm}W^{\pm}$, processes involving charm) and in certain kinematic regions. Relative importance grows with $\sqrt{s}$, and reveals new info on proton structure.
- Simplest approach: neglect correlations → 'pocket formula'. Models of general MPI in event generators based on this.
- Full QCD framework for DPS now developed, including proper effect of $1 \rightarrow 2$ splittings. Implementation as parton shower event generator ongoing. Ingredients for NLO corrections computed.
- First investigation in $W^{\pm}W^{\pm}$: effects of both 1 → 2 splittings and finite valence number on asymmetry $\mathcal{A}$. Measurable at hi-lumi LHC.
- Potential effects of spin and colour correlations on DPS. Spin effects at high scale and low x. Spin correlations could also contribute to $\mathcal{A}$.

BACKUP SLIDES

DPD OPERATOR DEFINITION

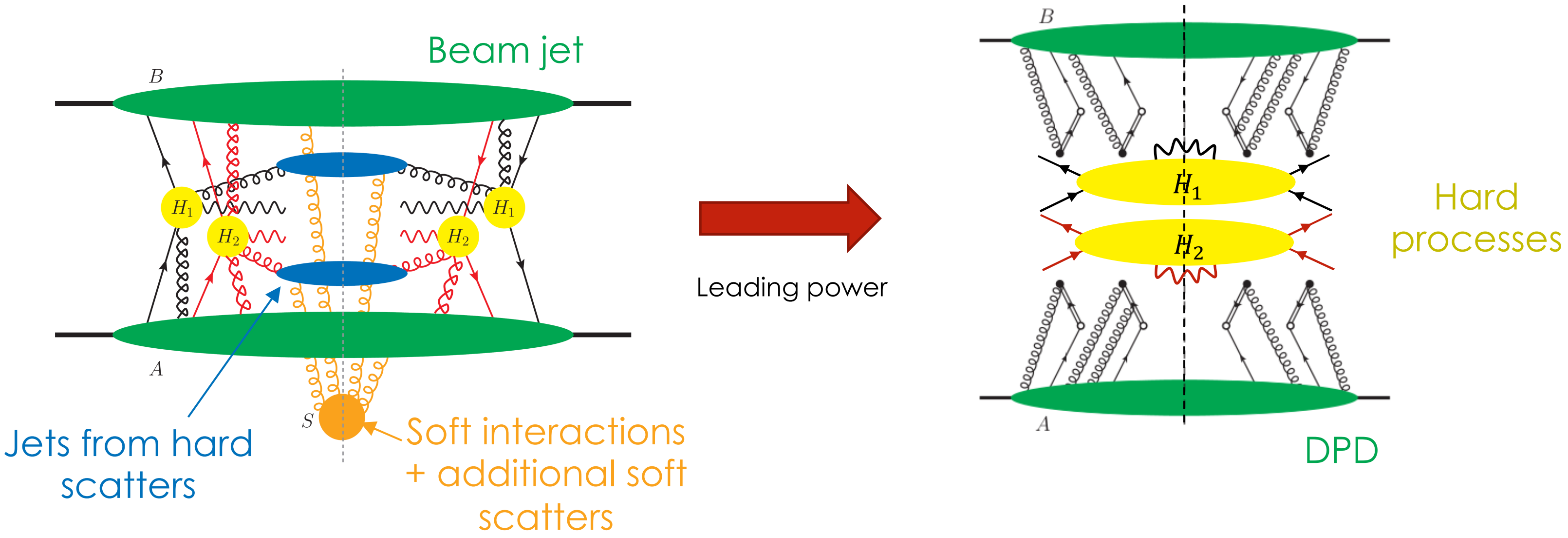
$$F_{ik}(x_1, x_2, \boldsymbol{y}, \mu_A, \mu_B) \propto \int \mathrm{d}y^- \mathrm{d}z_i^- e^{ix_i p^+ z_i^-} \langle p|\mathcal{O}_i\left(y + \tfrac{1}{2}z_1, y - \tfrac{1}{2}z_1\right) \mathcal{O}_j\left(\tfrac{1}{2}z_2, -\tfrac{1}{2}z_2\right)|p\rangle\Big|_{y^+=0,\, z_i^+=0,\, \boldsymbol{z}_i=\boldsymbol{0},}$$

PDF: $f_i(x, \mu) \propto \int \mathrm{d}z^- e^{ixp^+z^-} \langle p|\mathcal{O}_i\left(\tfrac{1}{2}z, -\tfrac{1}{2}z\right)|p\rangle\Big|_{\boldsymbol{z}=\boldsymbol{0},\, z^+=0}$

FACTORISATION IN DPS

FACTORISATION IN DPS

To prove factorisation for DPS inclusive cross section, need to show:



Beam jet

Jets from hard scatters

Soft interactions + additional soft scatters

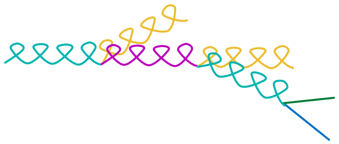
Leading power

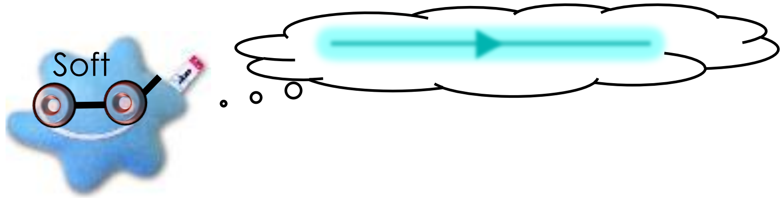
Hard processes

DPD

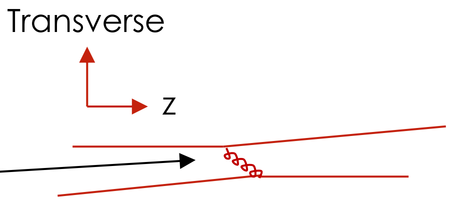
Key step: need to separate off all soft connections entangling beam and final state jets.

For ‘normal’ soft exchanges, this can be achieved via Ward identities:





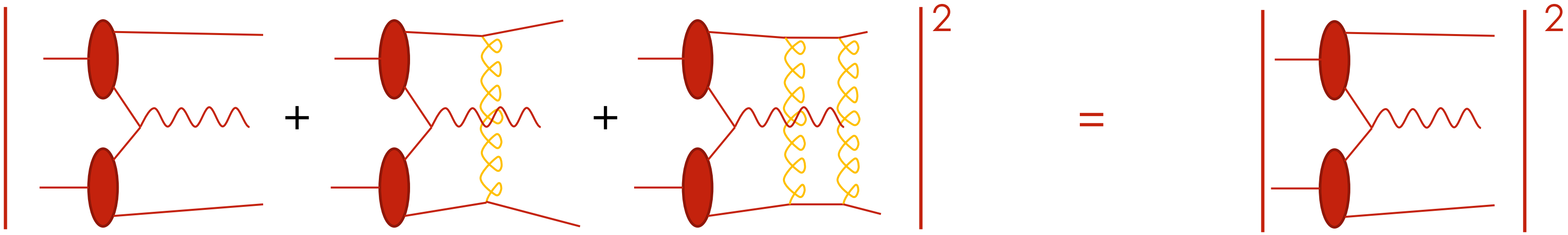
FACTORISATION: SOFT EXCHANGES



However, there is a particular type of soft exchange for which this doesn't work: Glauber exchanges. Soft particles mediating forward scattering.

Treatment of Glauber exchanges is the trickiest part of a factorisation proof!

Single scattering production of colour singlet V: Collins, Soper, Sterman showed that effect of Glauber exchanges cancels if we measure only properties of V, and sum over everything else!



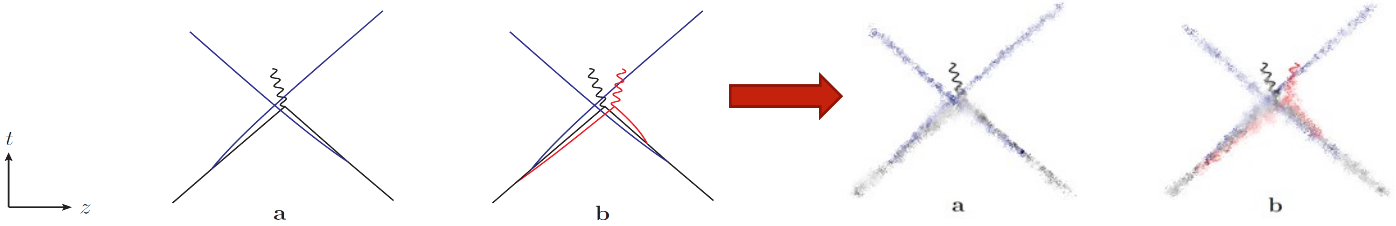
If one starts measuring properties of radiation accompanying V (e.g. global event shape variables), this argument breaks down!

JG, JHEP 1407 (2014) 110
Zeng, JHEP 1510 (2015) 189
Schwartz, Yan, Zhu, Phys.Rev. D97 (2018) no.9, 096017

GLAUBER CANCELLATION IN DPS

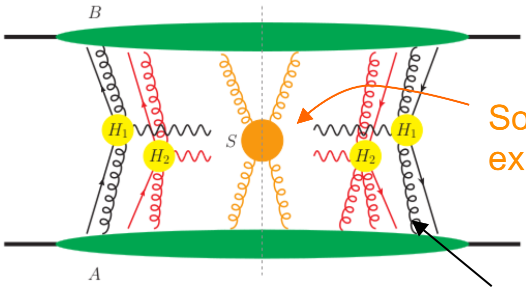
In JHEP 1601 (2016) 076 (Diehl, JG, Schäfer, Ostermeier, Plößl) we adapted the methodology of Collins, Soper, Sterman to show that Glauber exchanges also cancel for DPS production of two colourless systems.

Full proof is very technical, but can get some insight as to why it works by looking at spacetime pictures of single and double scattering:



Other important steps towards factorisation proof made in Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089 Vladimirov, JHEP 1804 (2018) 045, Diehl, Nagar, arXiv:1812.09509.

FACTORISATION IN DPS

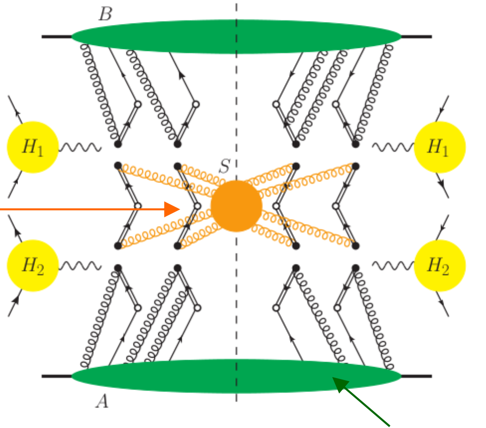


Soft and Glauber exchanges

Extra (unphysically polarised) gluon connections to hard

Initial picture

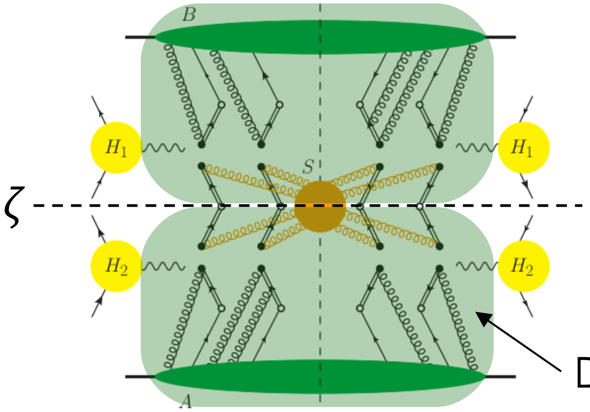
Diehl, JG, Ostermeier, Plößl, Schafer, JHEP 1601 (2016) 076, Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089, Diehl, Nagar, JHEP 1904 (2019) 124.



Soft factor

Collinear factor

Vladimirov, JHEP 1804 (2018) 045



DPDs

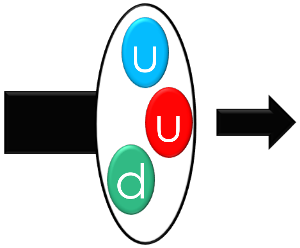
$$\sigma \sim F \otimes F \otimes \hat{\sigma} \otimes \hat{\sigma}$$

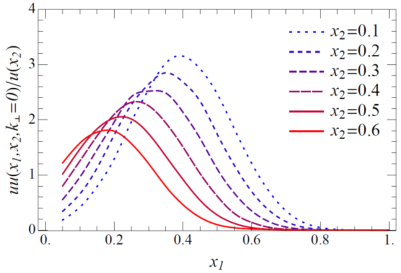
Proven, at least for double Drell-Yan production!

NONPERTURBATIVE DPD CALCULATIONS

NONPERTURBATIVE DPDS

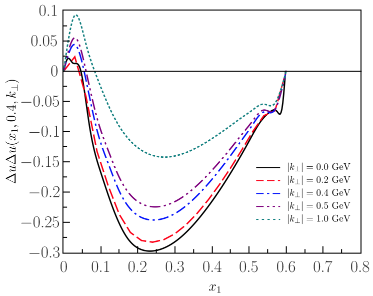
Model calculations:





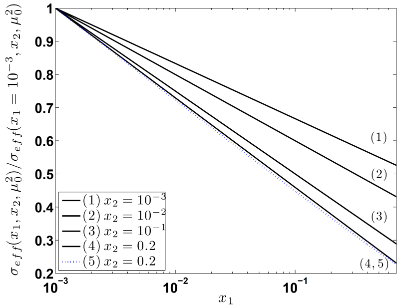
Bag model

[Phys. Rev. D 87, 034009 (2013), Manohar, Waalewijn, Chang]



Light-front CQM

[Rinaldi, Scopetta, Traini, Vento, JHEP 12 (2014) 028]



AdS/QCD

[Traini, Rinaldi, Scopetta, Vento, Phys. Lett. B 768 (2017) 270-273]

General message: factorisation of DPD into separate x_1, x_2, $\boldsymbol{y}$ pieces fails strongly at high x_i, low μ_i where these models are relevant.

Momentum and number sum rules:

[JG, Stirling, JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, Eur.Phys.J. C79 (2019) no.3, 253]
Construction of DPDs to satisfy rules in e.g. JG, Stirling, JHEP 1003 (2010) 005, Golec-Biernat et al. Phys.Lett. B750 (2015) 559-564, Diehl, JG, Lang, Plößl, Schafer Eur.Phys.J.C 80 (2020) 5, 468

$$\sum_{j_2} \int_0^{1-x_1} \mathrm{d}x_2\, x_2\, F^{j_1 j_2}(x_1, x_2; \mu) = (1-x_1) f^{j_1}(x_1; \mu)$$

$$\int_0^{1-x_1} \mathrm{d}x_2\, F^{j_1 j_{2,v}}(x_1, x_2; \mu) = \left(N_{j_{2,v}} + \delta_{j_1, \overline{j_2}} - \delta_{j_1, j_2}\right) f^{j_1}(x_1; \mu)$$

$$F(x_1, x_2; \mu) = \int d^2\boldsymbol{y}\Phi(\mu y) F(x_1, x_2, \boldsymbol{y}; \mu) + \mathcal{O}(\alpha_s)$$

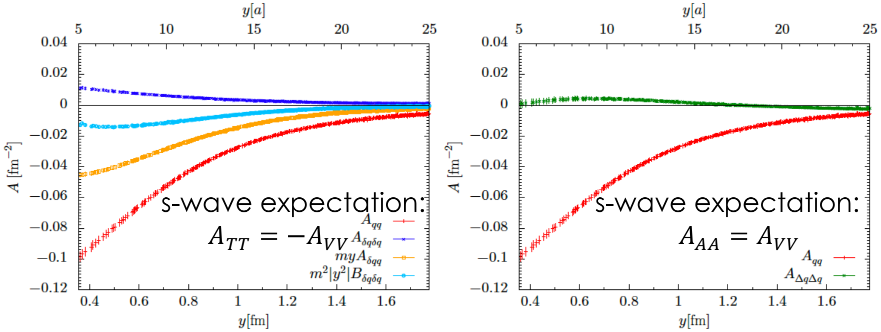
NONPERTURBATIVE DPDS

Of course, best theory input would be from lattice calculations!

Ongoing programme to compute DPD Mellin moments. Results so far only for the pion, but calculation with proton is WIP. Bali, Castagnini, Diehl, JG, Gläßle, Schäfer, Zimmermann

Test of classical s-wave picture of the pion:

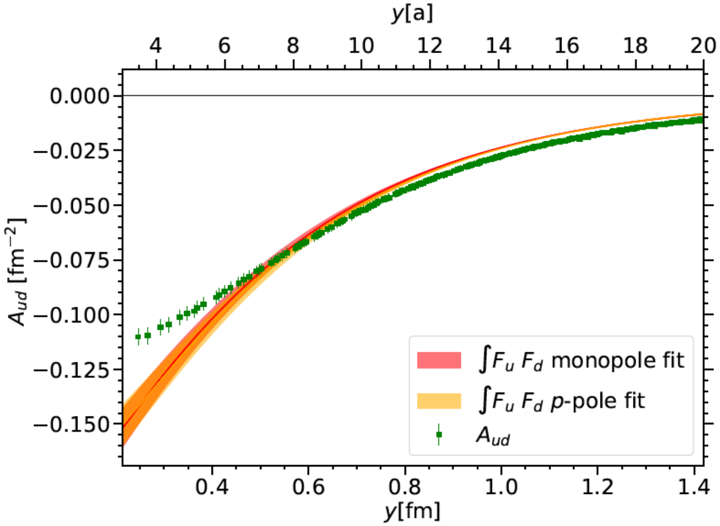
$$-A_{VV} \sim u^+d^+ + u^-d^- + u^+d^- + u^-d^+$$
$$+A_{AA} \sim u^+d^+ + u^-d^- - u^+d^- - u^-d^+$$
$$-A_{TT} \sim u^{\vec{s}}d^{\vec{s}} + u^{-\vec{s}}d^{-\vec{s}} - u^{\vec{s}}d^{-\vec{s}} - u^{-\vec{s}}d^{\vec{s}}$$



s-wave expectation: $A_{TT} = -A_{VV}$

s-wave expectation: $A_{AA} = A_{VV}$

Factorisation test:



arXiv:2006.14826

LATTICE DPDS – SOME DETAILS

$$\underbrace{F(x_1, x_2, \boldsymbol{y})} \propto \int \mathrm{d}y^- \mathrm{d}z_i^- e^{ix_i p^+ z_i^-} \langle p | \mathcal{O}\left(y + \tfrac{1}{2} z_1, y - \tfrac{1}{2} z_1\right) \mathcal{O}\left(\tfrac{1}{2} z_2, -\tfrac{1}{2} z_2\right) | p \rangle \Big|_{y^+=0,\, z_i^+=0,\, \boldsymbol{z}_i=\boldsymbol{0}}$$

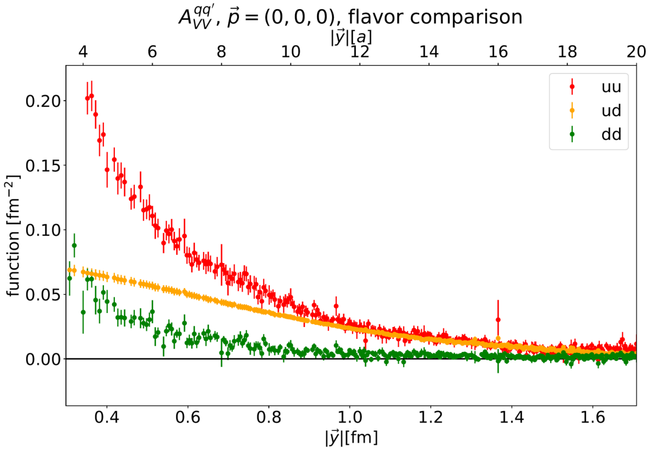
$$\int \mathrm{d}x_1 \mathrm{d}x_2 \, F(x_1, x_2, \boldsymbol{y}) \propto \int \mathrm{d}y^- \langle p | \mathcal{O}(y) \mathcal{O}(0) | p \rangle \Big|_{y^+=0}$$

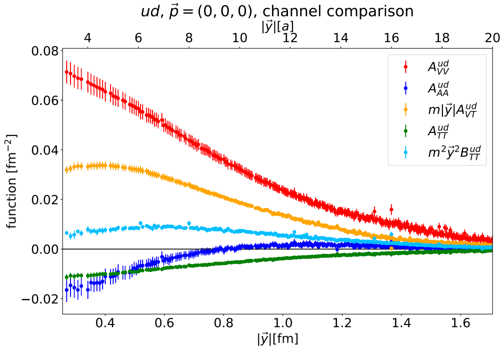
$$\propto \int d(p \cdot y) \, \underbrace{\langle \mathcal{O}\mathcal{O} \rangle (p \cdot y, y^2)} \Big|_{y^2 = -\boldsymbol{y}^2}$$

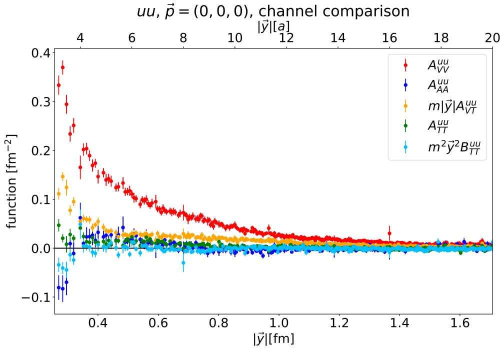
Can compute in Euclidean region on lattice. Implies:

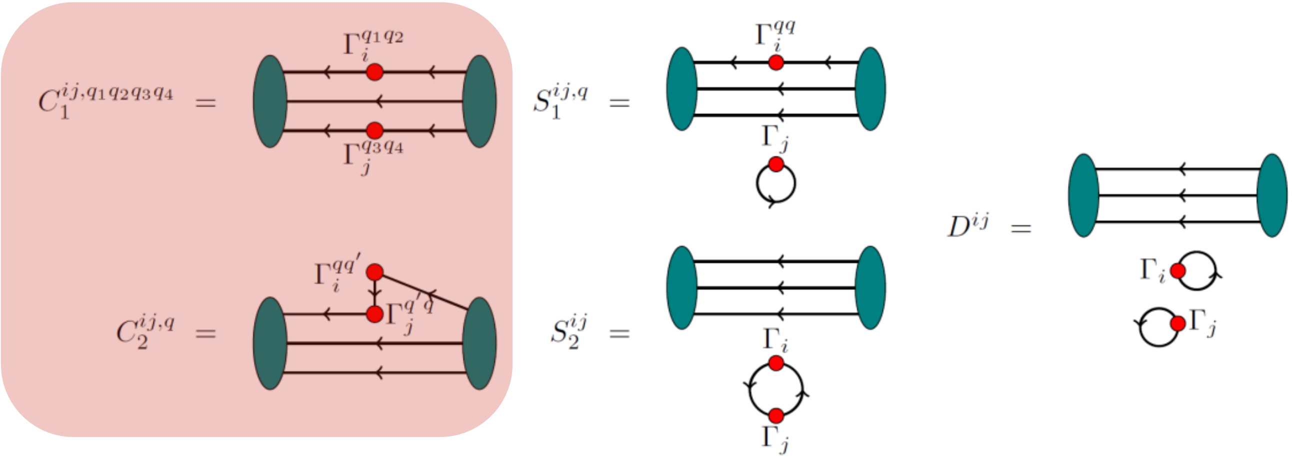
$$\frac{(p \cdot y)^2}{-y^2} = \frac{(\vec{p} \cdot \vec{y})^2}{\vec{y}^2} \leq \vec{p}^2$$

LATTICE DPDS – SOME DETAILS

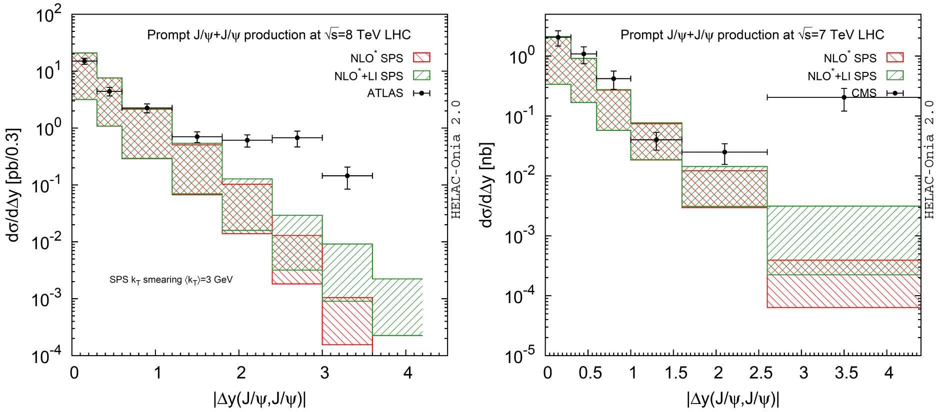




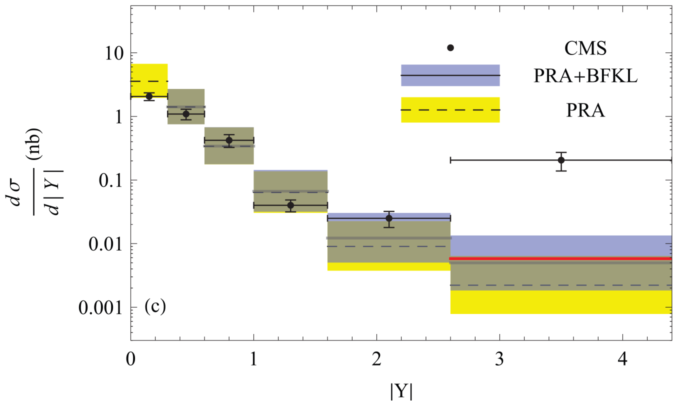




STATE-OF-THE-ART DOUBLE J/Ψ SPS



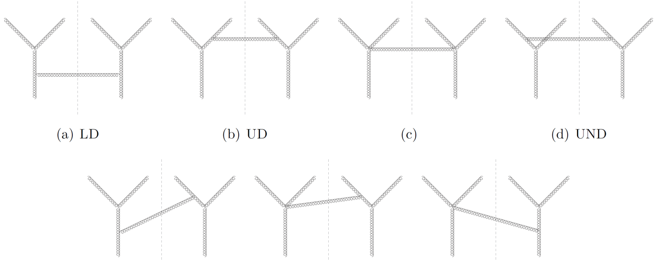
Lansberg, Shao, Yamanaka, Zhang
arXiv:1906.10049



He, Kniehl, Nefedov, Saleev
Phys.Rev.Lett. 123 (2019) no.16, 162002

NEXT-TO-LEADING ORDER

NLO: METHOD



(a) LD (b) UD (c) (d) UND

(e) T2B (f) (g)

Compute graph expressions (`FORM`, `FeynCalc`). Integrate over minus components using contours.

[Kuipers, Ueda, Vermaseren, Vollinga, Comput. Phys. Commun. 184 (2013) 1453-1467]
[Shtabovenko, Mertig, Orellana, Comput. Phys. Commun. 207 (2016) 432-444]

$$D_1 = \frac{(\boldsymbol{k}_1+\boldsymbol{\Delta})^2}{x_1} + \frac{(\boldsymbol{k}_2-\boldsymbol{\Delta})^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3} \qquad D_2 = \frac{\boldsymbol{k}_1^2}{x_1} + \frac{\boldsymbol{k}_2^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3}$$

$$D_3 = (\boldsymbol{k}_1+\boldsymbol{\Delta})^2 \qquad D_4 = \boldsymbol{k}_2^2 \qquad \tilde{D}_4 = \boldsymbol{k}_1^2 \qquad \tilde{D}_5 = (\boldsymbol{k}_1+\boldsymbol{k}_2)^2$$

$$I_1(a_1,a_2,a_3,a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}} \qquad I_2(a_1,a_2,a_3,a_4,a_5) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1...3} D_i^{a_i} \prod_{i=4...5} \tilde{D}_i^{a_i}}$$

$$I_1(1,1,0,0), I_1(0,1,1,0), I_1(1,1,1,0),$$
$$I_1(1,0,1,1), I_1(1,1,1,1), I_1(2,1,1,1)$$

$$I_2(0,1,1,0,1), I_2(1,1,1,1,0)$$

Integration-by-parts reduction to master integrals (`LiteRed`)

[Lee, J. Phys. Conf. Ser. 523 (2014)]

$$\begin{bmatrix} \frac{\partial I_1(1,1,0,0)}{\partial x_1} \\ \frac{\partial I_1(0,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,0,1,1)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,1)}{\partial x_1} \\ \frac{\partial I_1(2,1,1,1)}{\partial x_1} \end{bmatrix} = \begin{bmatrix} \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacksquare & 0 & 0 & 0 \\ 0 & \blacklozenge & 0 & \blacksquare & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \end{bmatrix} \begin{bmatrix} I_1(1,1,0,0) \\ I_1(0,1,1,0) \\ I_1(1,1,1,0) \\ I_1(1,0,1,1) \\ I_1(1,1,1,1) \\ I_1(2,1,1,1) \end{bmatrix}$$

Construct differential equations in x_1 and solve (`Fuchsia`)

[Gituliar, Magerya, Comput. Phys. Commun. 219 (2017) 329-338]

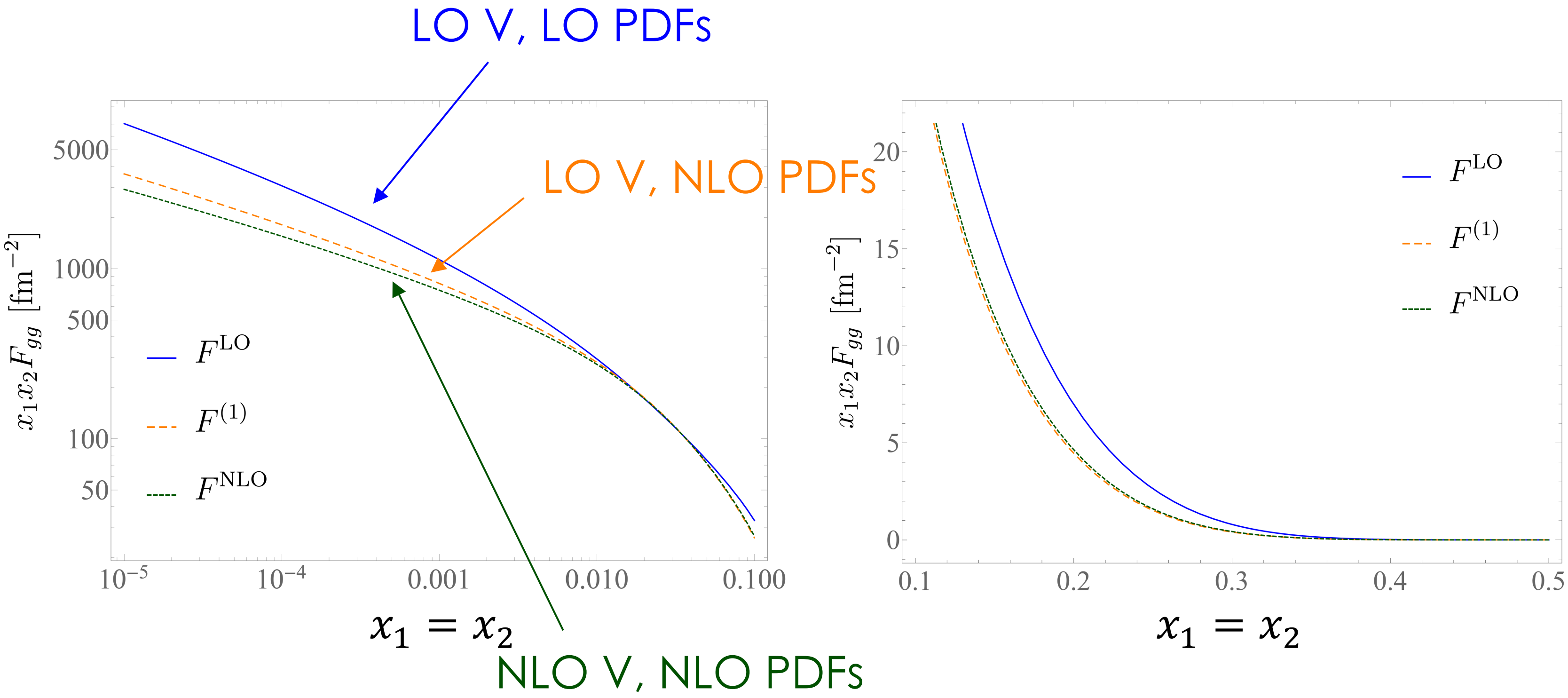
$$I_1(0,1,1,0) \to \pi^{3-2\epsilon} x_3^{1-\epsilon} (x_1 x_2)^{\epsilon} \frac{\Gamma[-\epsilon]}{\sin[2\pi\epsilon]\Gamma[1-3\epsilon]}$$

Computation of $x_3 \to 0$ limit of master integrals using method of regions (boundary conditions)

Results for bare graphs!

NLO: SOME NUMERICS

Scale $10\ \mathrm{GeV}$, splitting contribution only, no evolution after splitting

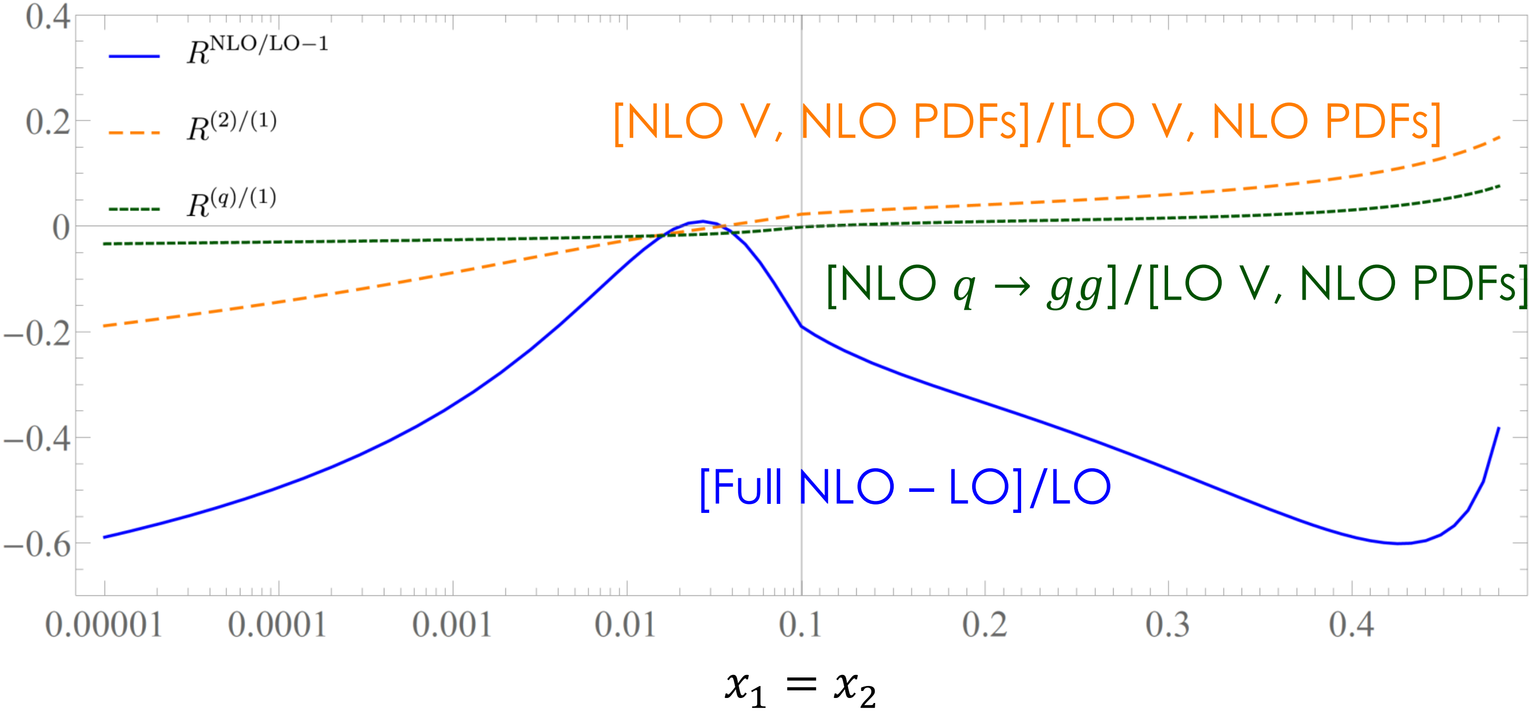


LO V, LO PDFs

LO V, NLO PDFs

NLO V, NLO PDFs

NLO: SOME NUMERICS



[NLO V, NLO PDFs]/[LO V, NLO PDFs]

[NLO $q \to gg$]/[LO V, NLO PDFs]

[Full NLO – LO]/LO

TRANSVERSE MOMENTUM IN DPS

TRANSVERSE MOMENTUM IN DPS

Small $\boldsymbol{q}_i$ region particularly important for DPS – DPS & SPS same power

Parton model analysis: $$\frac{d\sigma^{(A,B)}}{d^2\boldsymbol{q_1} d^2\boldsymbol{q_2}} \sim \int d^2\boldsymbol{y} d^2\boldsymbol{z}_i e^{-i\boldsymbol{z}_1\cdot\boldsymbol{q}_1 - i\boldsymbol{z}_2\cdot\boldsymbol{q}_2} \underbrace{F(\boldsymbol{z_1},\boldsymbol{z_2},\boldsymbol{y})}_{\text{DTMDs}}\ F(\boldsymbol{z_1},\boldsymbol{z_2},\boldsymbol{y})$$

Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

QCD treatment of transverse momentum in DPS (including DGS-style double counting subtraction) developed in Buffing, Diehl, Kasemets JHEP 1801 (2018) 044. DPS cross section in QCD:

$$\frac{d\sigma_{\mathrm{DPS}}}{dx_1 dx_2 d\bar{x}_1 d\bar{x}_2 d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} = \frac{1}{C} \cdot \sum_{a_1,a_2,b_1,b_2} \hat{\sigma}_{a_1 b_1}(Q_1,\mu_1)\, \hat{\sigma}_{a_2 b_2}(Q_2,\mu_2) \times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2} d^2\boldsymbol{y} \cdot e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2} W_{a_1a_2b_1b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu),$$

Cut-off functions

$$W_{a_1a_2b_1b_2}(\bar{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu) = \Phi(\nu \boldsymbol{y}_+)\, \Phi(\nu \boldsymbol{y}_-) \times \sum_R \eta_{a_1a_2}(R)\, {}^R F_{b_1b_2}(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \bar{\zeta}) \cdot {}^R F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta).$$

Dependence on ren. scales μ_i AND a rapidity scale ζ.

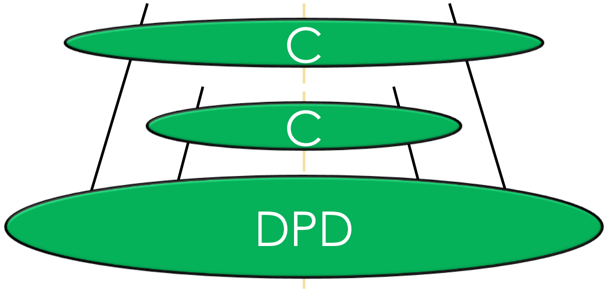
Evolution of DTMDS in all of these scales known at one loop.

TRANSVERSE MOMENTUM IN DPS

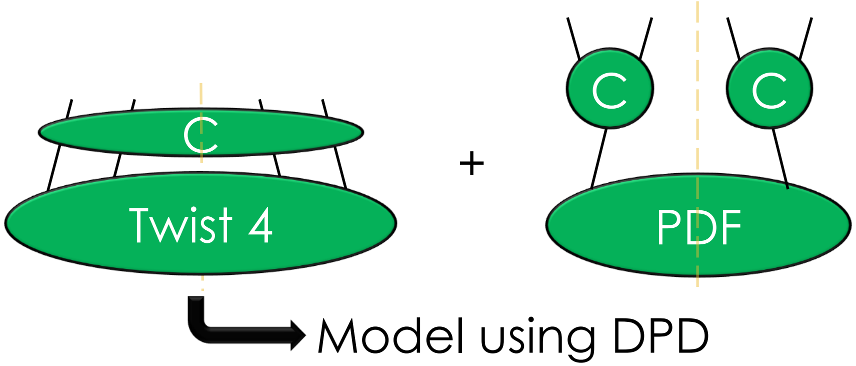
Still need some 'initial' expressions for the DTMDs. Function of many arguments $(x_i, \boldsymbol{y}, \boldsymbol{z}_i)$. Hopeless?

For perturbative $|\boldsymbol{q}_i| \gg \Lambda$ can expand DTMDs in terms of collinear quantities:

Large $\boldsymbol{y} \sim 1/\Lambda$:



Small $\boldsymbol{y} \sim 1/q_T \sim |\boldsymbol{z_i}|$:



Model using DPD

So then, need only DPDs and PDFs: very good prospects for phenomenology at perturbative $|\boldsymbol{q}_i|$!

Brief overview of transverse momentum in DPS given in JG, Kasemets, Advances in High Energy Physics, 2019, 3797394

DSHOWER ALGORITHM

DSHOWER ALGORITHM

(1) Select x_i of initiating partons and y using DPS formula:

$$\sigma^{\mathrm{DPS}}_{(A,B)}(s) = \frac{1}{1+\delta_{AB}} \sum_{i,j,k,l} \int \mathrm{d}\tau_A \, \mathrm{d}Y_A \, \mathrm{d}\hat{t}_A \, \mathrm{d}\tau_B \, \mathrm{d}Y_B \, \mathrm{d}\hat{t}_B \, \frac{\mathrm{d}\hat{\sigma}_{ij\to A}}{\mathrm{d}\hat{t}_A} \, \frac{\mathrm{d}\hat{\sigma}_{kl\to B}}{\mathrm{d}\hat{t}_B}$$

$$\times \int 2\pi \, y \, \mathrm{d}y \, \Phi^2(y\nu) \, F_{ik}(x_1, x_3, \boldsymbol{y}, \mu^2) \, F_{jl}(x_2, x_4, \boldsymbol{y}, \mu^2)$$

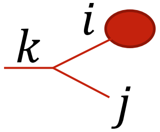
DPDs

Cut-off of DPS for y values $\lesssim 1/\nu \sim 1/Q$

DSHOWER ALGORITHM

(2) Generate QCD emissions, going backwards from hard process

In shower must select an evolution variable. We make the same choice as Herwig:



For ISR:

$$Q^2 = \tilde{q}_{ISR}^2 = -\frac{\left(p_i^2 - m_i^2\right)}{(1-z)} \approx E_k^2 \theta_j^2$$

Angular ordering

Probability that partons ij survive from Q_h to Q, and then at Q there is an emission from one leg:

$$\mathrm{d}\mathcal{P}_{ij}^{\mathrm{ISR}} = \mathrm{d}\mathcal{P}_{ij} \exp \underbrace{\left(- \int_{Q^2}^{Q_h^2} \mathrm{d}\mathcal{P}_{ij} \right)}_{\text{‘Sudakov factor’}}$$

Emission probability

Emission from leg 1

$$\mathrm{d}\mathcal{P}_{ij} = \frac{\mathrm{d}Q^2}{Q^2} \left(\sum_{i'} \int_{x_1}^{1-x_2} \frac{\mathrm{d}x_1'}{x_1'} \frac{\alpha_\mathrm{s}(p_\perp^2)}{2\pi} P_{i' \to i}\left(\frac{x_1}{x_1'}\right) \frac{F_{i'j}(x_1', x_2, \boldsymbol{y}, Q^2)}{F_{ij}(x_1, x_2, \boldsymbol{y}, Q^2)} + \sum_{j'} \int_{x_2}^{1-x_1} \frac{\mathrm{d}x_2'}{x_2'} \frac{\alpha_\mathrm{s}(p_\perp^2)}{2\pi} P_{j' \to j}\left(\frac{x_2}{x_2'}\right) \frac{F_{ij'}(x_1, x_2', \boldsymbol{y}, Q^2)}{F_{ij}(x_1, x_2, \boldsymbol{y}, Q^2)} \right)$$

Emission from leg 2

Use ‘competing veto algorithm’ to decide which leg emits

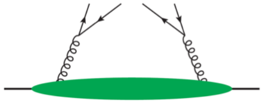
DSHOWER ALGORITHM

(3) At scale $\mu_y \sim 1/y$, decide whether to merge partons i and j. Merging is done with a probability given by:

$$p_{Mrg} = F_{ij}^{spl}(x_1, x_2, y, \mu_y^2) / F_{ij}^{tot}(x_1, x_2, y, \mu_y^2)$$

Total DPD

$$F_{ij}^{spl}(x_1, x_2, y, \mu_y^2) = \frac{1}{\pi y^2} \frac{f_k(x_1+x_2, \mu_y^2)}{x_1+x_2} \frac{\alpha_s(\mu_y^2)}{2\pi} P_{k\to ij}\left(\frac{x_1}{x_1+x_2}\right) \times\text{large } \boldsymbol{y} \text{ suppression}$$

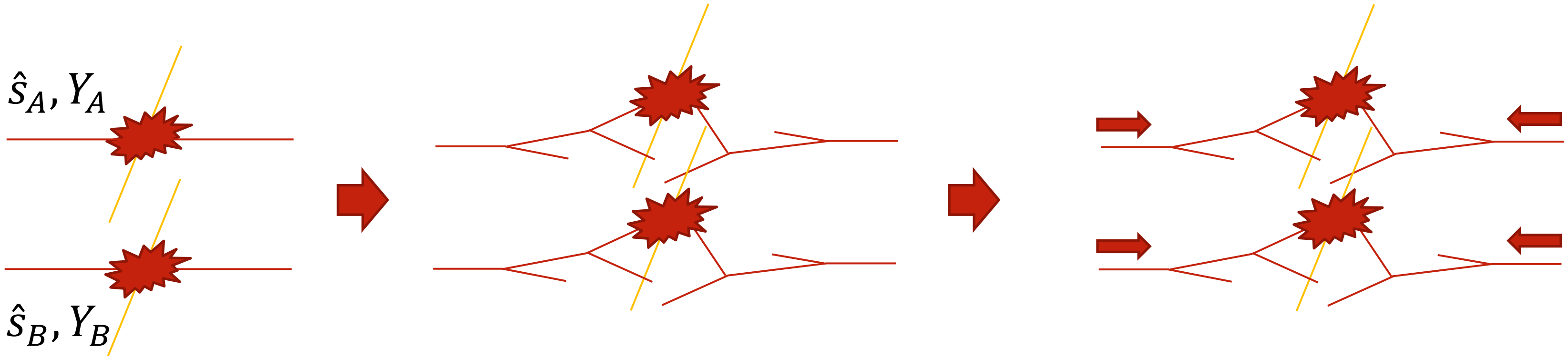


If no merging: continue with two parton branching algorithm from (2), using only ‘intrinsic’ DPDs.

If merging: shower single parton a la Herwig.

KINEMATICS: NO MERGING

No merging:



Generate hard process using DPS σ

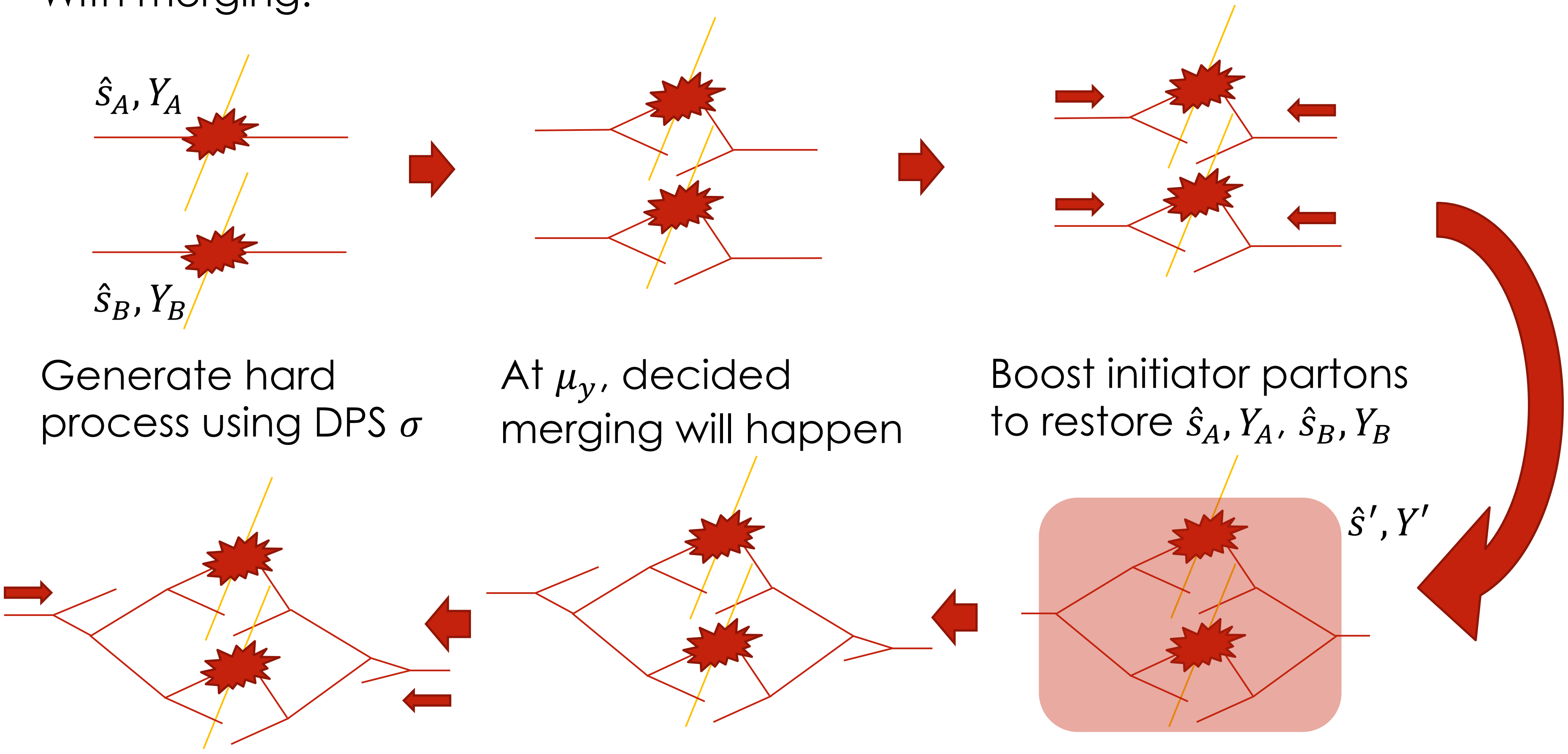
Add shower, kinematics of hard processes altered

Boost initiator partons to restore $\hat{s}_A, Y_A, \hat{s}_B, Y_B$

Works as 4 variables (boosts) and 4 constraints! What about if there is a merging? 2/3 initiator partons → overconstrained system!

KINEMATICS: MERGING

With merging:



Generate hard process using DPS σ

At μ_y, decided merging will happen

Boost initiator partons to restore $\hat{s}_A, Y_A$, $\hat{s}_B, Y_B$

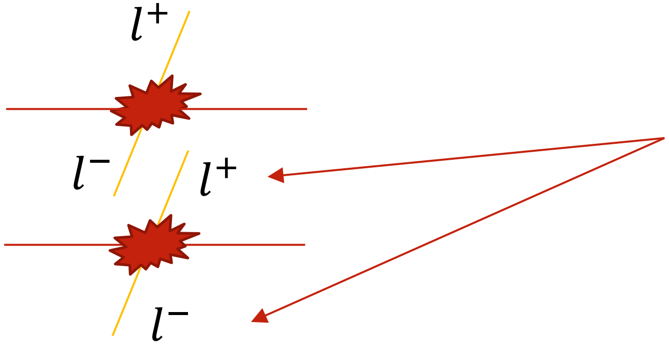
Merge (zero p_T, or $p_T \sim \mu_y$). Define new hard system.

Continue shower

Boost initiator partons to restore $\hat{s}', Y'$

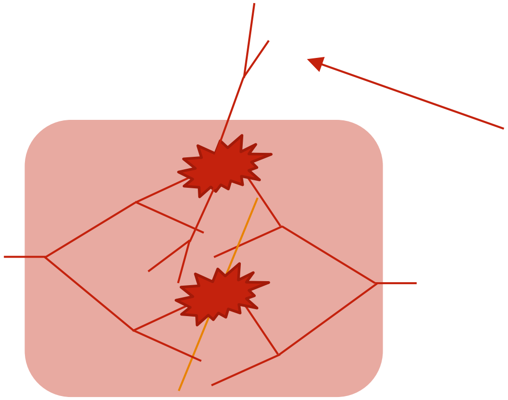
KINEMATICS: MERGING

Merging method works nicely when colourless particles produced, & these decay into further colourless particles, e.g. $pp \rightarrow ZZ \rightarrow 4l$



Keep momentum of these together, boost system in all further steps. Preserves invariant mass!

Some potential issues if final states coloured & emit FSR:

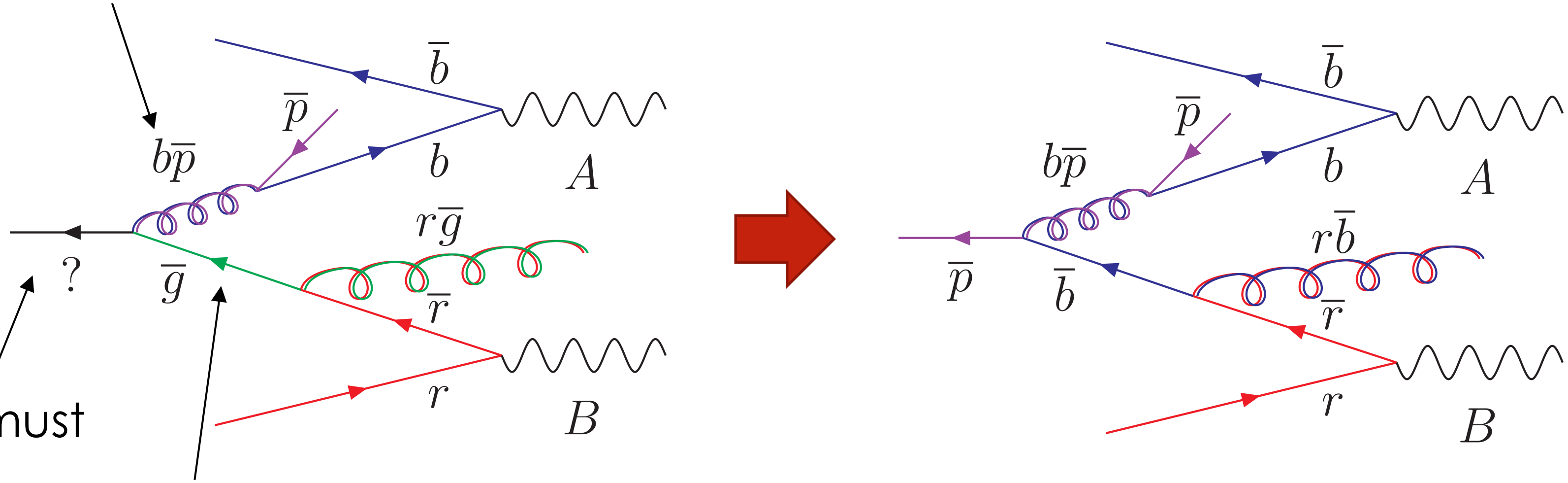


After adding FSR shower, boost jets. Here preserves $\hat{s}'$ and not $\hat{s}_A, \hat{s}_B$.

COLOUR WITH MERGING

Shower uses large N_c approximation. Each new emission → new colour. Independent showers before merging.

Mergings require some colour reshuffling. We impose minimal colour disruption.



Change $b \to g$?

This must be $\bar{p}$

Change $\bar{g} \to \bar{b}$?

Not so important for parton-level simulation, but could be important when we add hadronisation

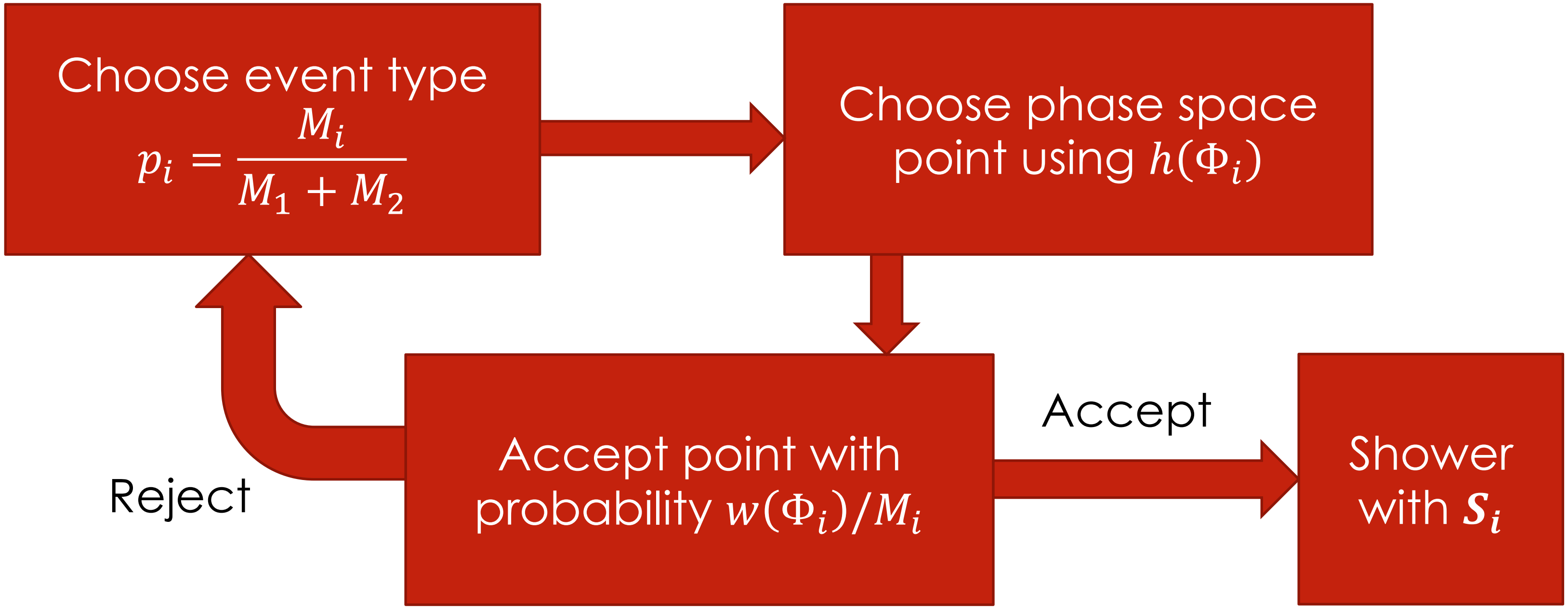
COMBINING DPS AND SPS IN THE SHOWER

IMPLEMENTATION

For each event type, define weight: $w(\Phi_i) = \frac{1}{h(\Phi_i)} \frac{d\sigma_i}{d\Phi_i}$ Dimension = $[\sigma]$

$$M_i = \max_{\Phi_i}[w(\Phi_i)]$$

$$\int h(\Phi_i) d\Phi_i = 1$$



Choose event type $p_i = \frac{M_i}{M_1 + M_2}$

Choose phase space point using $h(\Phi_i)$

Accept point with probability $w(\Phi_i)/M_i$

Accept

Shower with $\boldsymbol{S_i}$

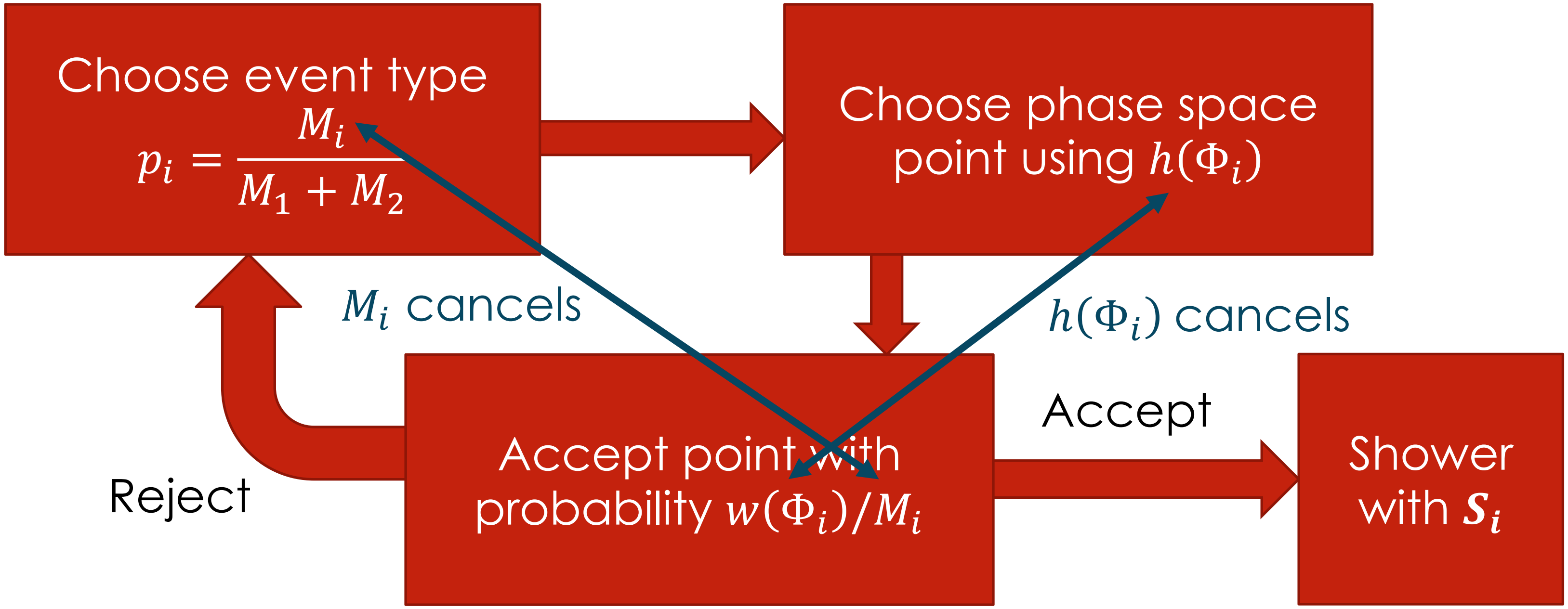
Reject

IMPLEMENTATION

For each event type, define weight: $w(\Phi_i) = \frac{1}{h(\Phi_i)} \frac{d\sigma_i}{d\Phi_i}$ Dimension = $[\sigma]$

$$\int h(\Phi_i) d\Phi_i = 1$$

$$M_i = \max_{\Phi_i}[w(\Phi_i)]$$



Choose event type $p_i = \frac{M_i}{M_1 + M_2}$

Choose phase space point using $h(\Phi_i)$

M_i cancels

$h(\Phi_i)$ cancels

Accept point with probability $w(\Phi_i)/M_i$

Accept

Reject

Shower with $\boldsymbol{S_i}$

THE SUBTRACTION: LARGE & SMALL Y

$$\frac{d\sigma_{A+B}^{tot}}{dO} = \boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma_{A+B}^{SPS}}{dO} - \frac{d\sigma_{(A,B)}^{sub}}{dO}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma_{(A,B)}^{DPS}}{dO\, d^2\boldsymbol{y}}$$

If sub kinematics correctly reproduces double splitting kinematics of DPS term → DPS & sub cancel at small y, give $d\sigma_{A+B}^{SPS}/dO$

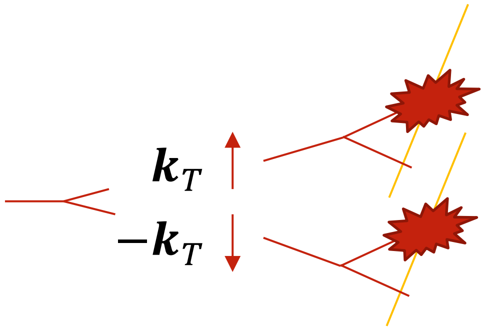
Want sub and SPS loop-induced term to cancel at large y (also differential in O). But we don't have SPS differential in y.

One thing we can look at is p_T of Z bosons – small p_T behaviour dominated by large y!

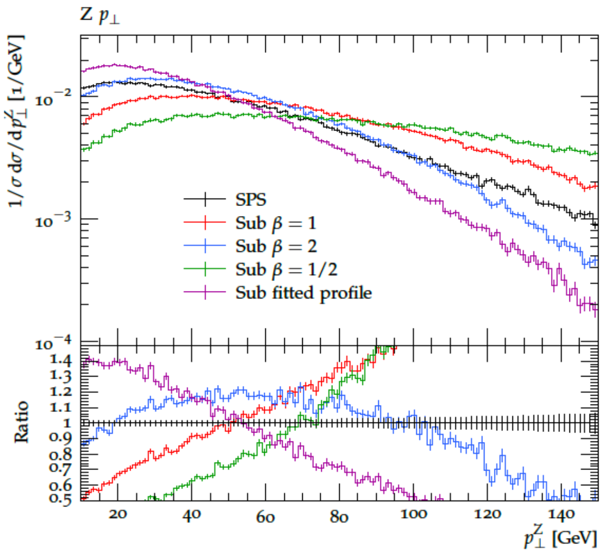
JG, Stirling,
JHEP 06 (2011) 048

THE SUBTRACTION: LARGE & SMALL Y

Want sub and SPS to coincide as closely as possible at small p_T - constrains splitting p_T kinematics in sub & DPS terms.



$\boldsymbol{k}_T$ distributed according to $g(\boldsymbol{k}_T, y)$



Options: (a) Gaussian $g(\boldsymbol{k}_T, y)$:

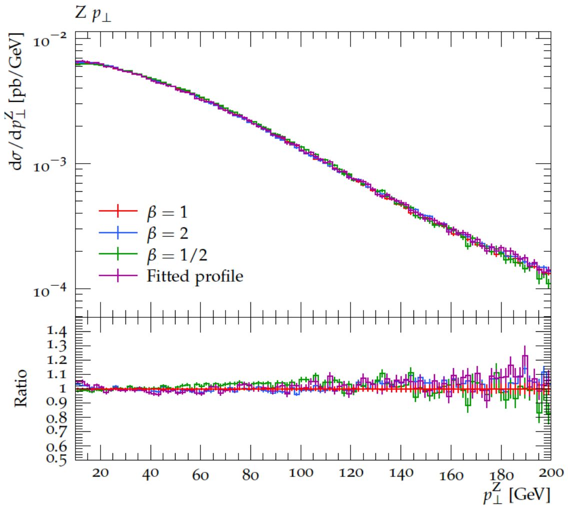
$$g(\boldsymbol{k}_T, y) = \frac{\beta}{\pi} y^2 exp(-\beta y^2 k_T^2)$$

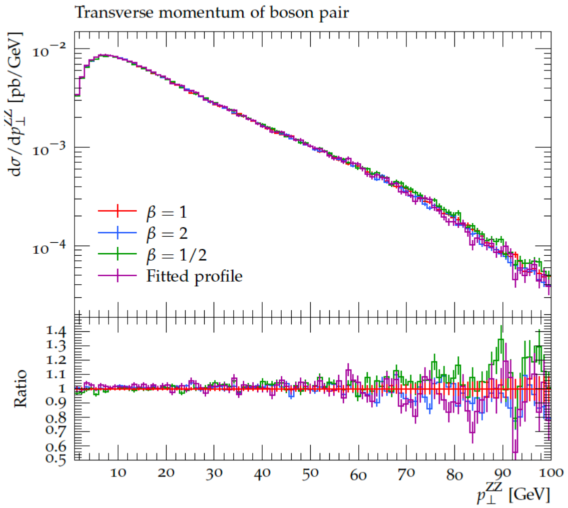
(b) ‘Decreasing Gaussian’ (more realistic)

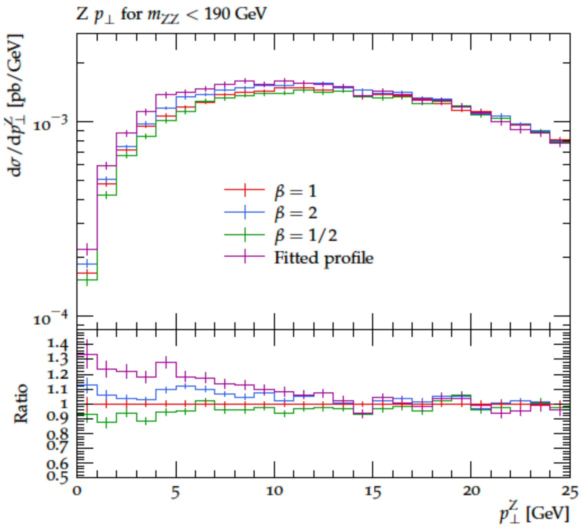
$$g(\boldsymbol{k}_T, y) = \frac{1}{\pi\sqrt{2}} \frac{y}{k_T} exp\left(-\frac{\pi}{2} y^2 k_T^2\right)$$

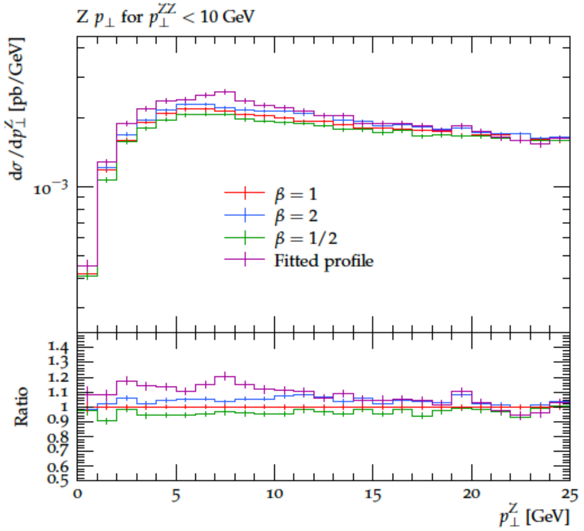
DIFFERENT PROFILES

Many distributions: ~ no difference









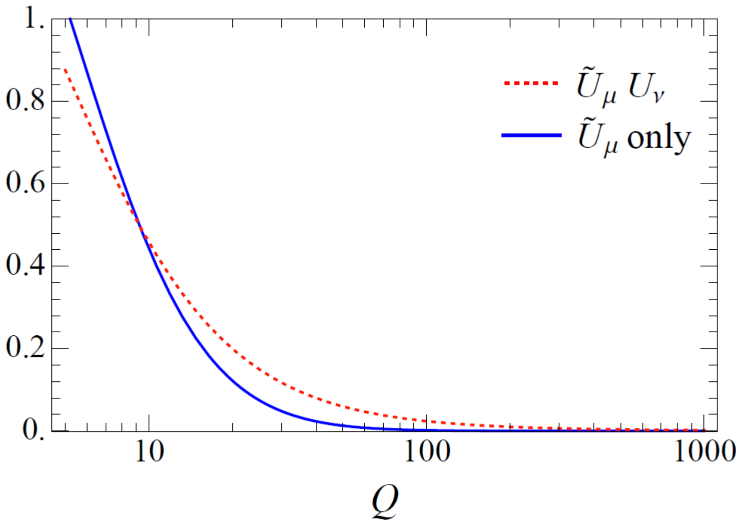
Can see some small differences focussing on region where p_Ts of both bosons are small

COLOUR CORRELATIONS

COLOUR CORRELATIONS

Colour correlations are strongly suppressed at high scales

[Technically: Sudakov suppression due to movement of colour between amplitude & conjugate by distance $\boldsymbol{y}$.]



First estimate: negligible at 100 GeV, but could be relevant at moderate scales ~10 GeV.

Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009