Precision Perturbative QCD

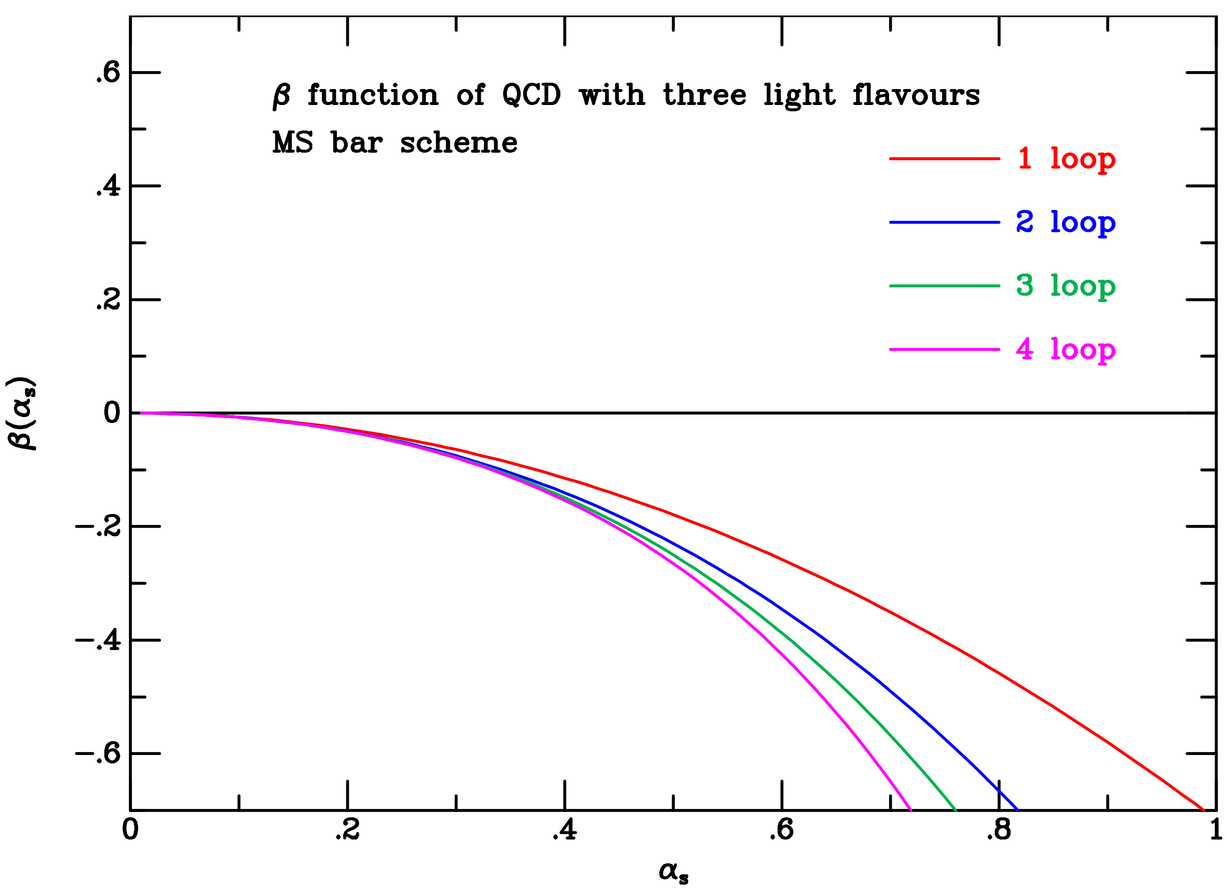
Keith Ellis
IPPP, Durham

One of the cardinal elements in the success of the LHC (and the Tevatron before it), has been the program to make precision predictions from the QCD Lagrangian. I will review the basis for these predictions and highlight the successes and the areas where further work is still needed.

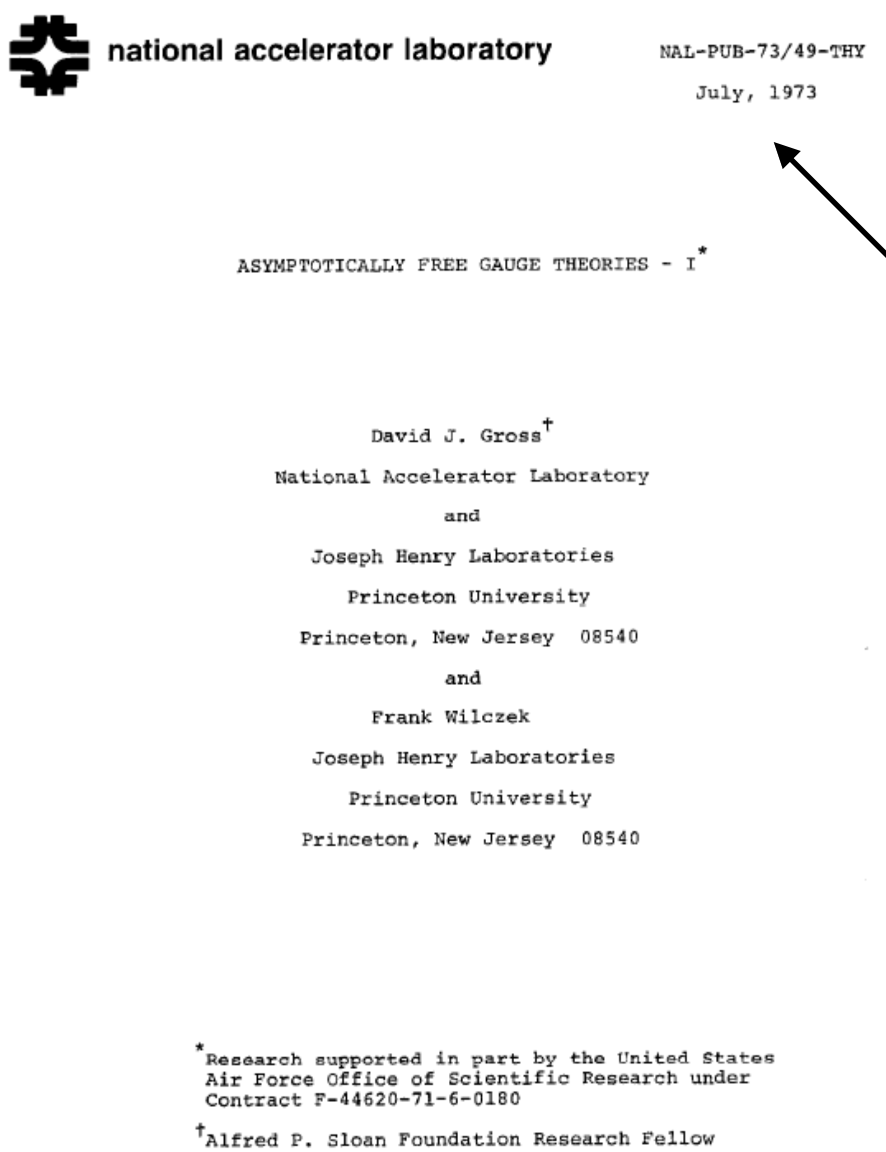
Asymptotic freedom

Back in 1973….

$$\mu^2 \frac{\partial \alpha_S}{\partial \mu^2} = \beta(\alpha_S)$$



Because of the all-important sign, the coupling decreases as we increase the energy


national accelerator laboratory

NAL-PUB-73/49-THY

July, 1973

ASYMPTOTICALLY FREE GAUGE THEORIES - I*

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and

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and

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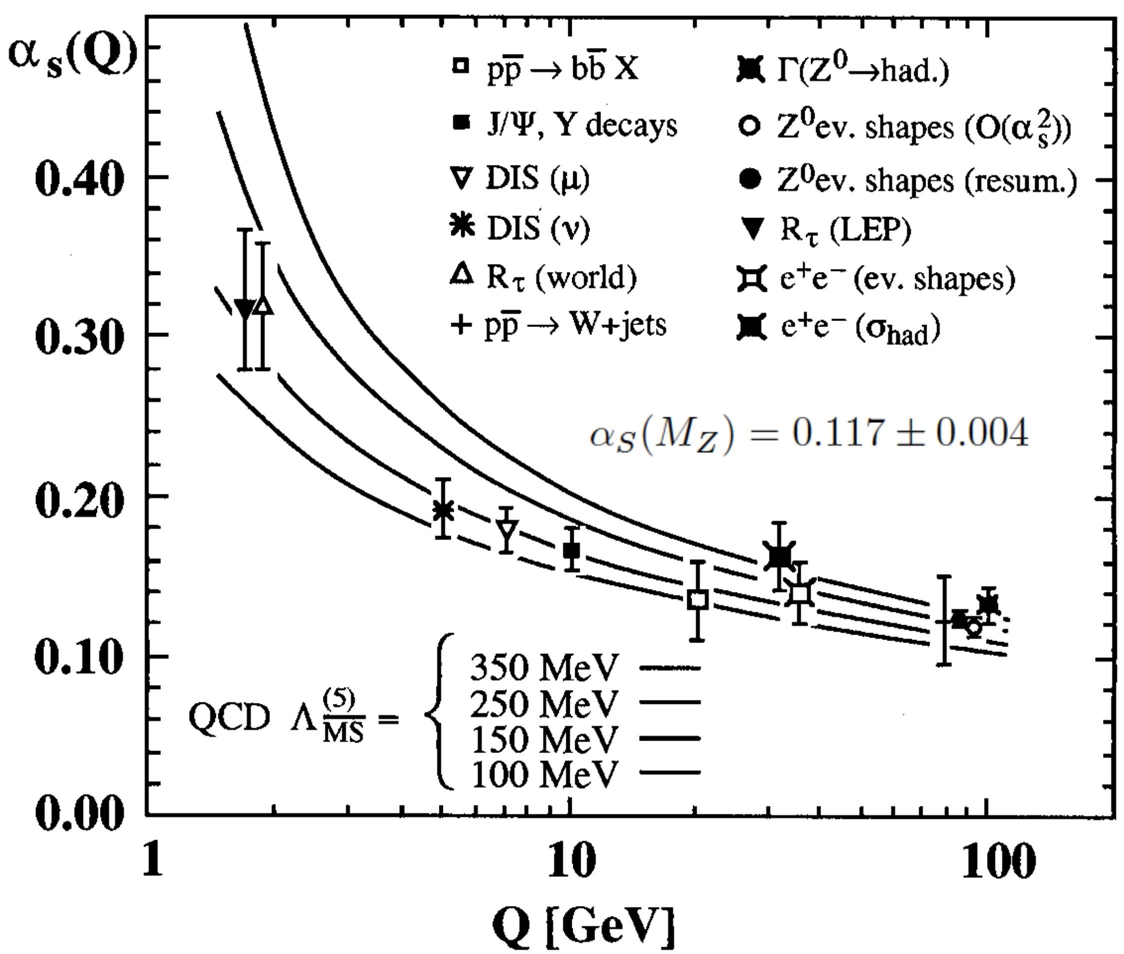
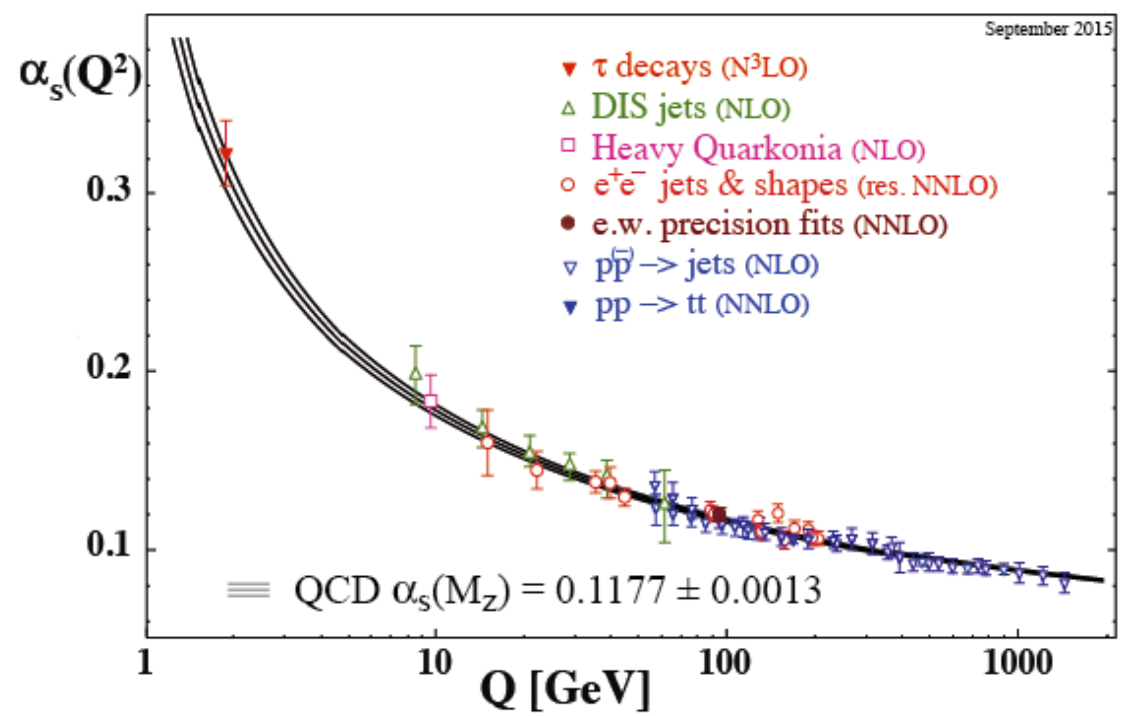
Theory summer program

Evolution of the Evolution

1992
CERN-TH.6484/92

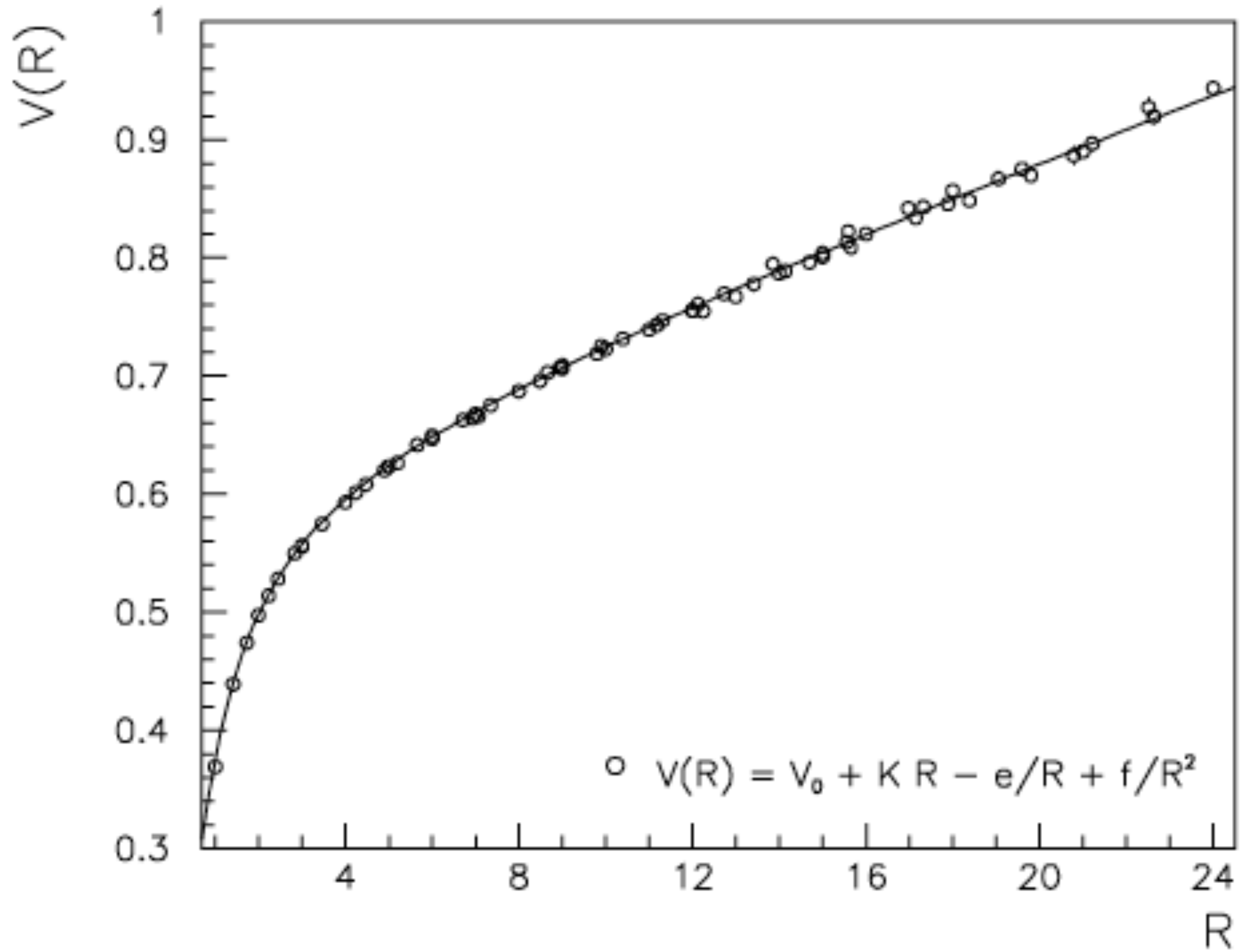


2015
1512.05194



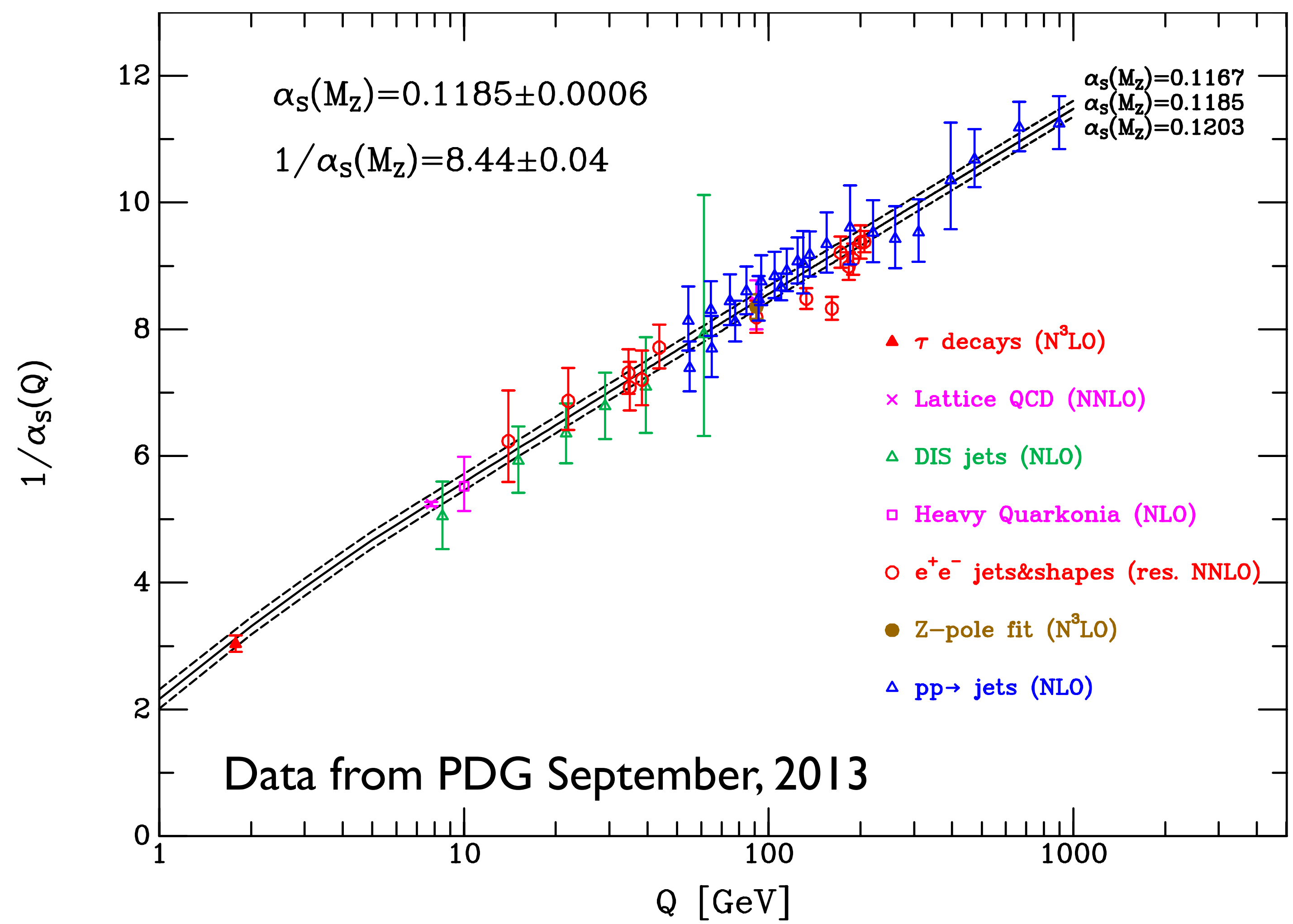
QCD potential at short distances

- Interquark forces do not become small at small distances, but only a high energy.
- Asymptotic freedom is a logarithmic modification of a coulomb potential



Charge anti-screening in QCD

- Incontrovertible fact that α_s is smallish at energies accessible with current machines.
- $1/\alpha_s$ as grows as ~log (Q).
- $1/\alpha_s(M_Z)=8.44\pm0.04$
- c.f QED: $1/\alpha=128....137$
- Radiative corrections ~15 times more important in QCD than QED.



Also some other outliers mainly from e+e- data
Abbate, 1006.3080, $\alpha_s(M_Z)=0.1135+0.0010$
Hoang, 1501.04753, $\alpha_s(M_Z)=0.1123+0.0002$

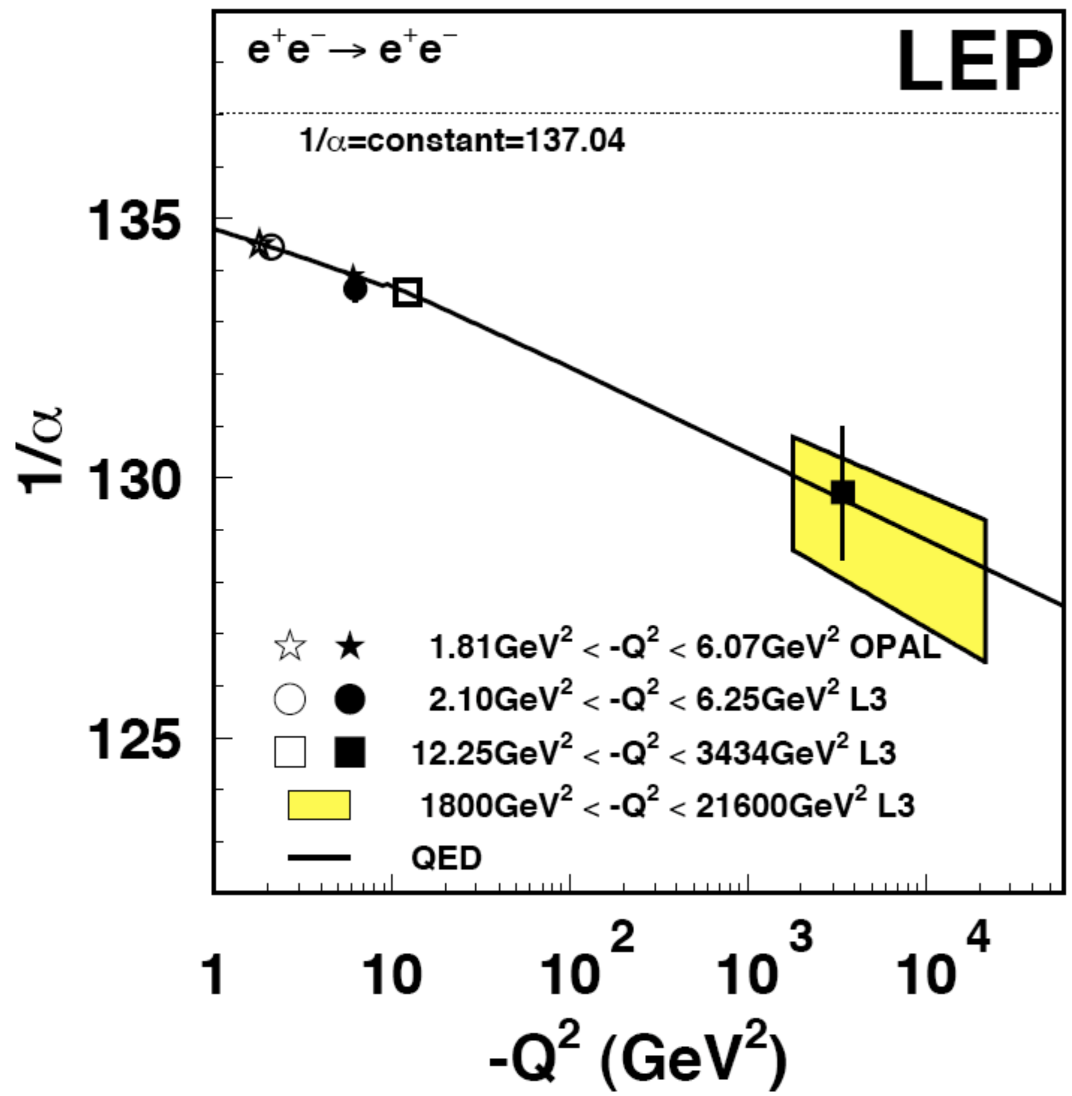
Charge screening in QED

- The expected behaviour of the electromagnetic coupling is confirmed by experiments on Bhabha scattering at LEP

$$\alpha = \frac{e^2}{4\pi}$$

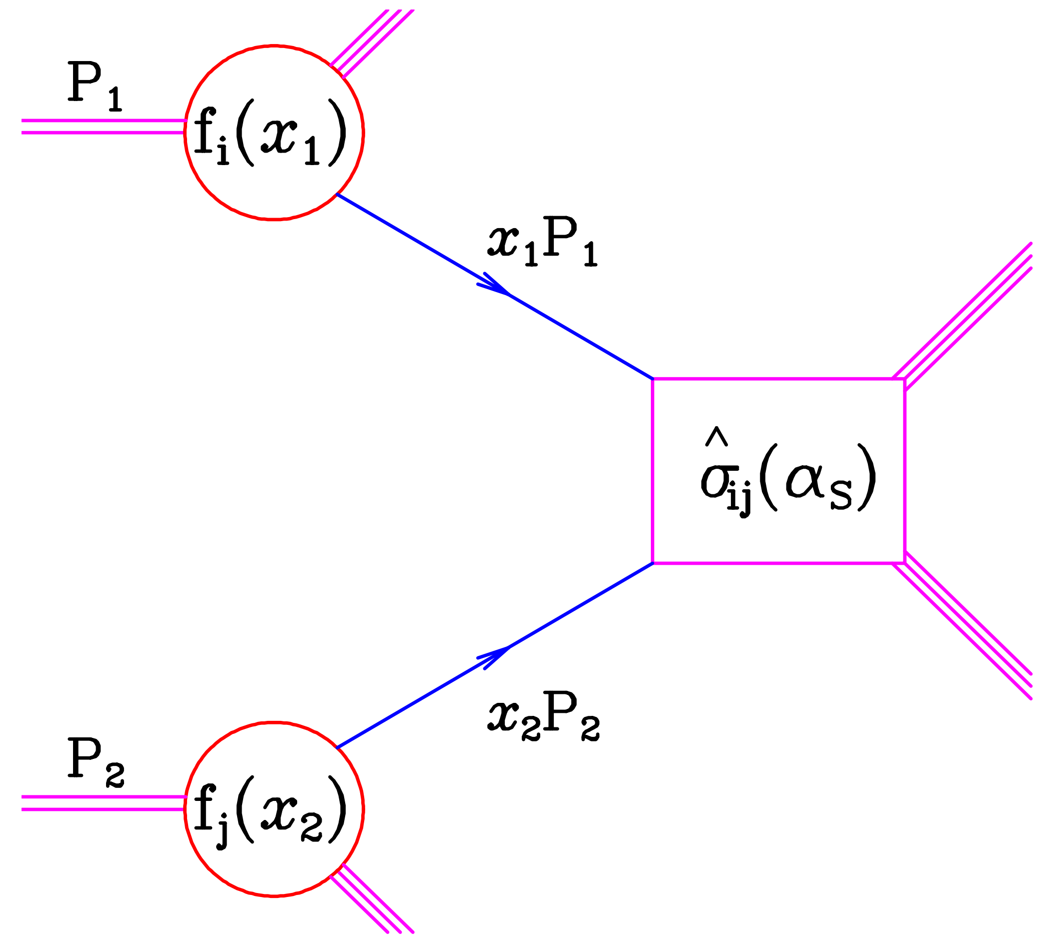
$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha_0} - \frac{2Q_f^2}{3\pi} \ln\left(\frac{Q}{m_f}\right)$$

Mele,hep-ex/0610037



QCD improved parton model

- Hard cross section is represented as a convolution of a parton scattering cross section and non-perturbative parton distribution functions.
- Physical cross section is formally independent of μ_R and μ_F through the order calculated.
- Here we shall be concerned with the parton scattering cross section.



Parton distributions

Non-perturbative effects

$$\sigma(P_1,P_2) = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1,\mu_F) f_j(x_2,\mu_F)\, \hat{\sigma}_{ij}(p_1,p_2,\alpha_s(\mu_R^2),Q^2,\mu_R,\mu_F) + O(\Lambda/Q)$$

Physical cross section

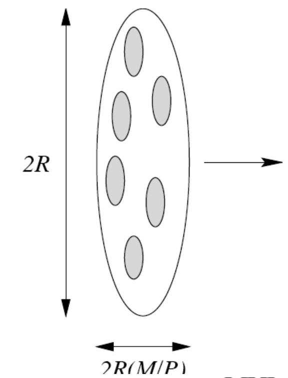
Factorization scale

Renormalization scale

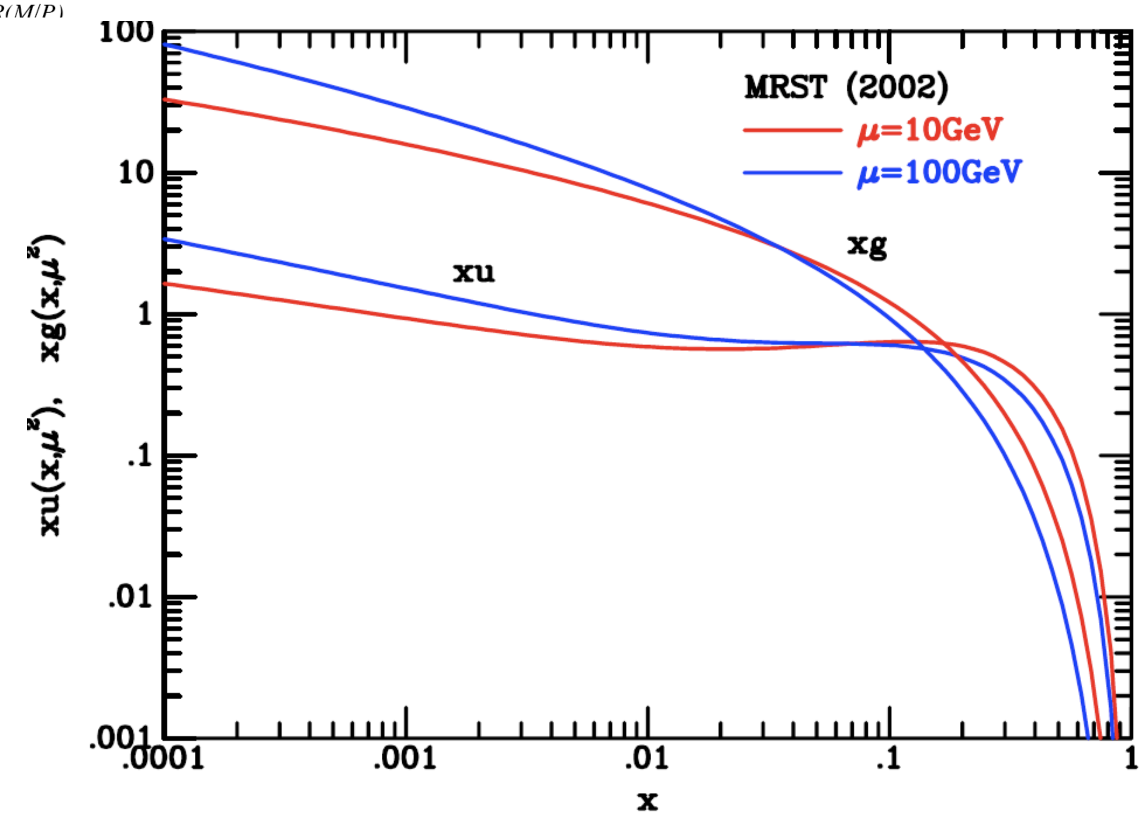
The evolution of partons within the proton

$$\sigma(P_1,P_2) = \sum_{i,j} \int dx_1 dx_2 \; f_i(x_1,\mu_F) f_j(x_2,\mu_F) \; \hat{\sigma}_{ij}(p_1,p_2,\alpha_s(\mu_R^2),Q^2,\mu_R,\mu_F) + O(\Lambda/Q)$$

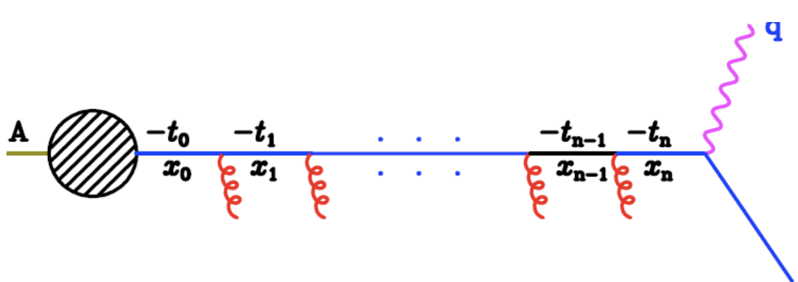
At low resolution (small) a high energy proton can be viewed as a dilute system of partons.



As we observe the proton with higher resolution (increasing μ) the number of partons grows, in a calculable way, the DGLAP equation.



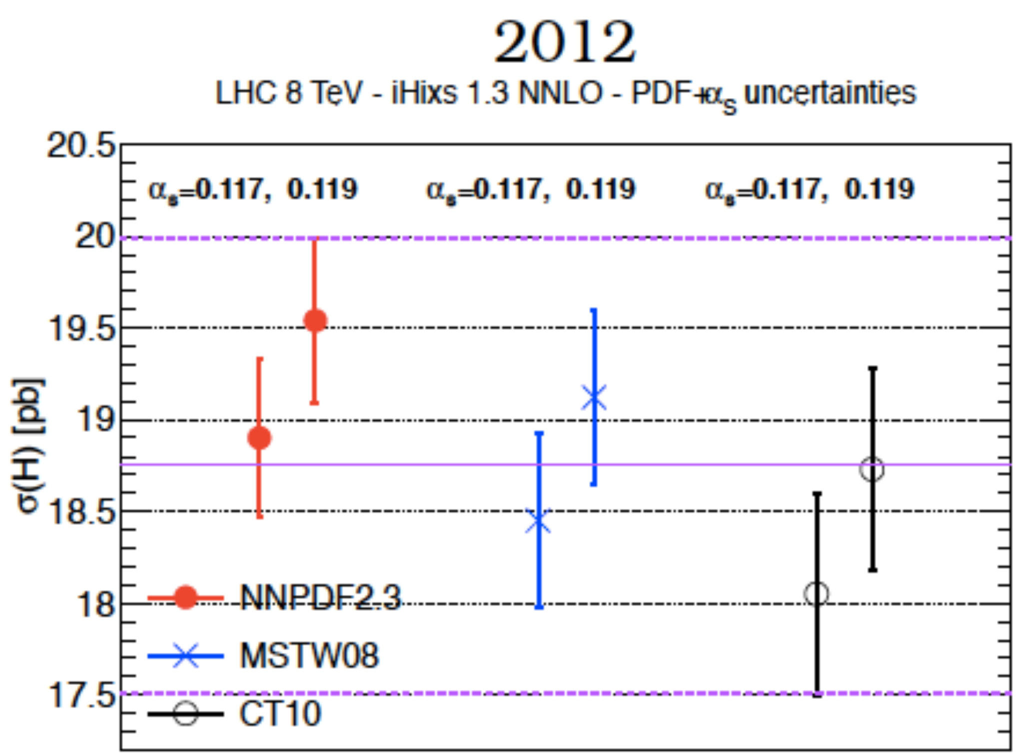
Longitudinal Momentum fraction x

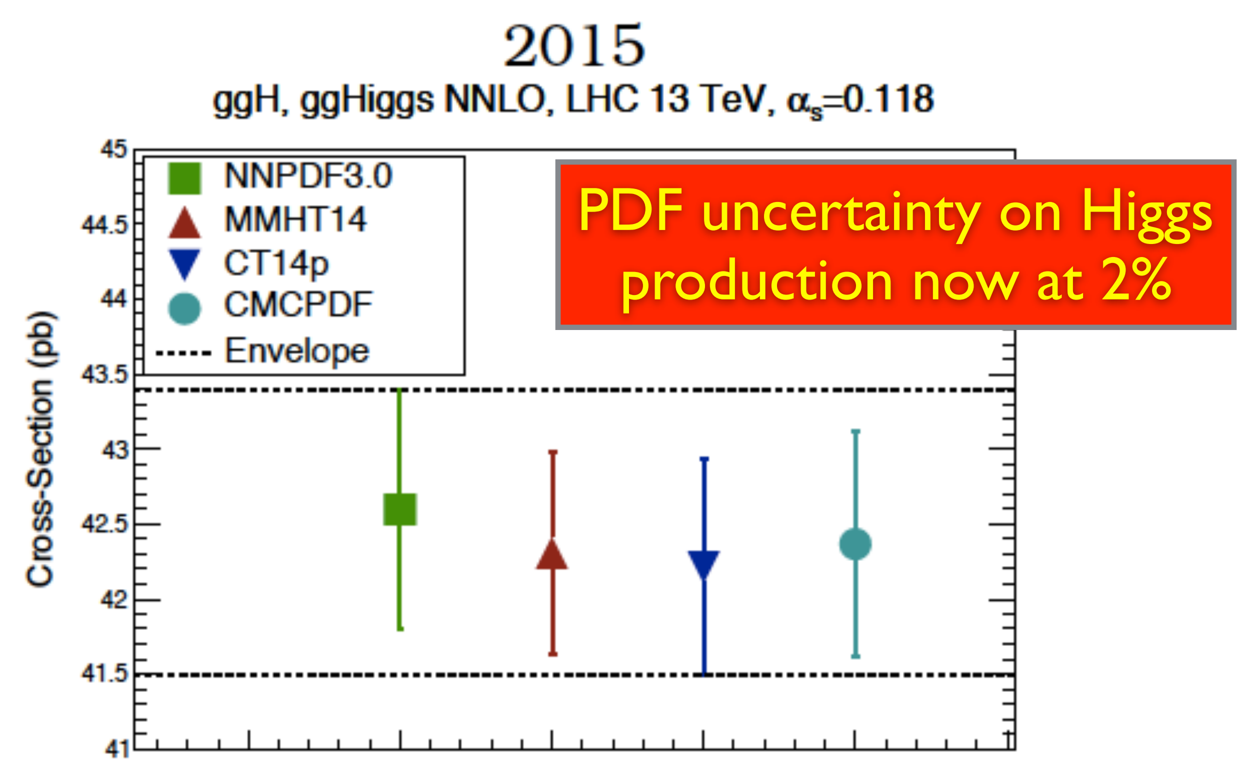


Parton shower

Concordance of parton distribution sets

Plots from S. Forte, Higgs Couplings, Durham

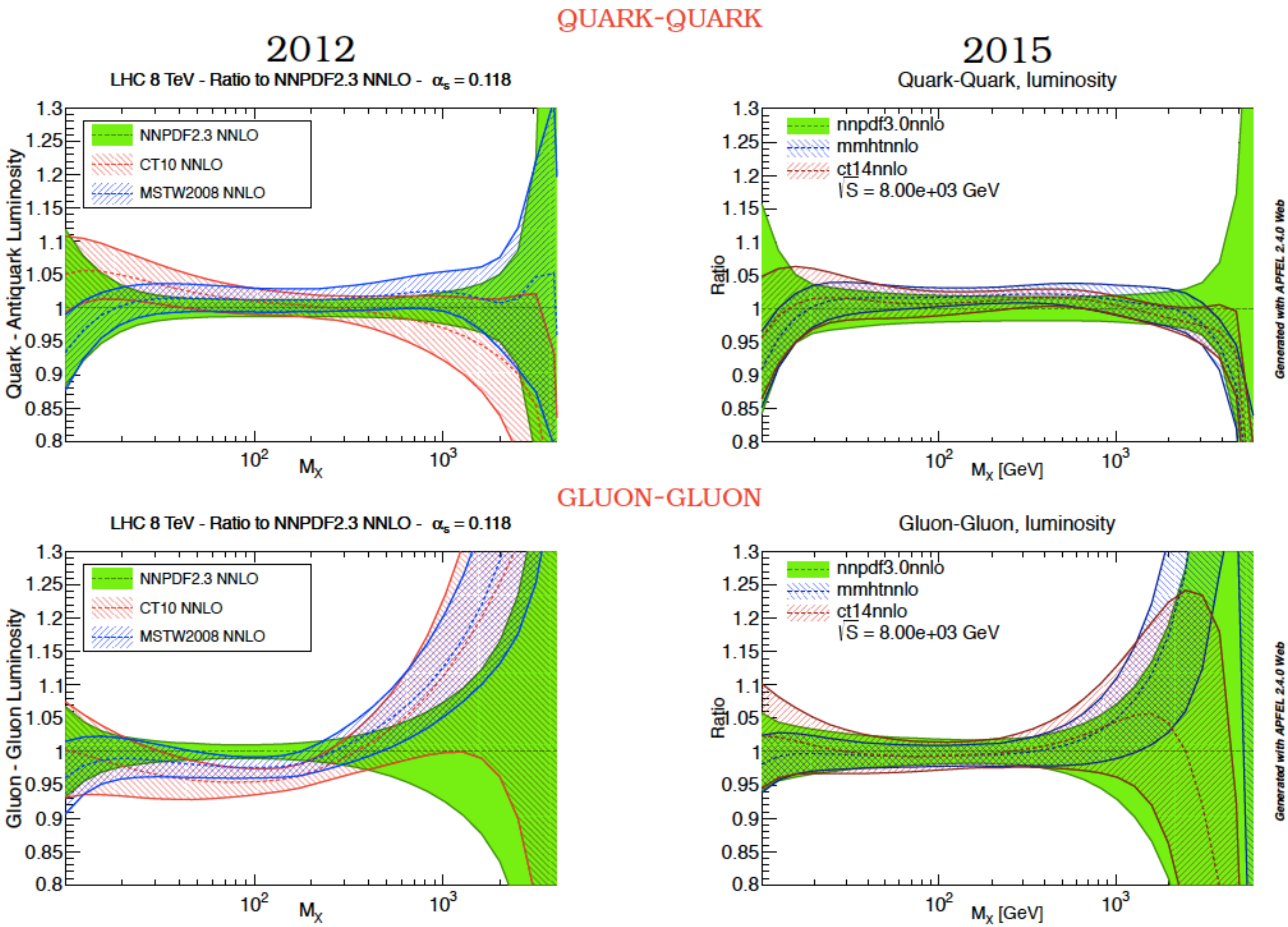




- Results shown for Higgs cross sections.
- Major upgrades by all the global fitting groups
- NNPDF2.3 →3.0, MSTW08→MMHT14, CT10→CT14
- Fits based on reduced datasets, e.g HeraPDF have larger errors

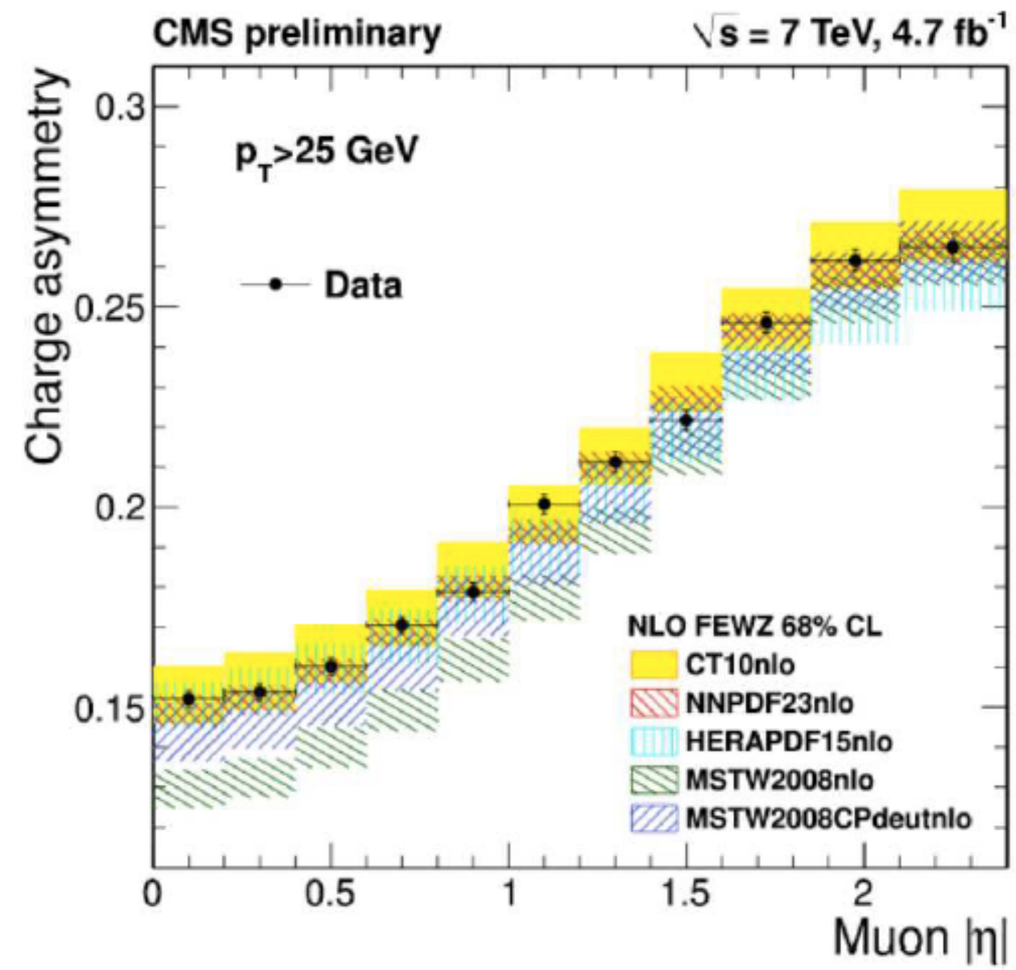
Concordance of parton distribution functions.

Plots from S. Forte, Higgs Couplings, Durham



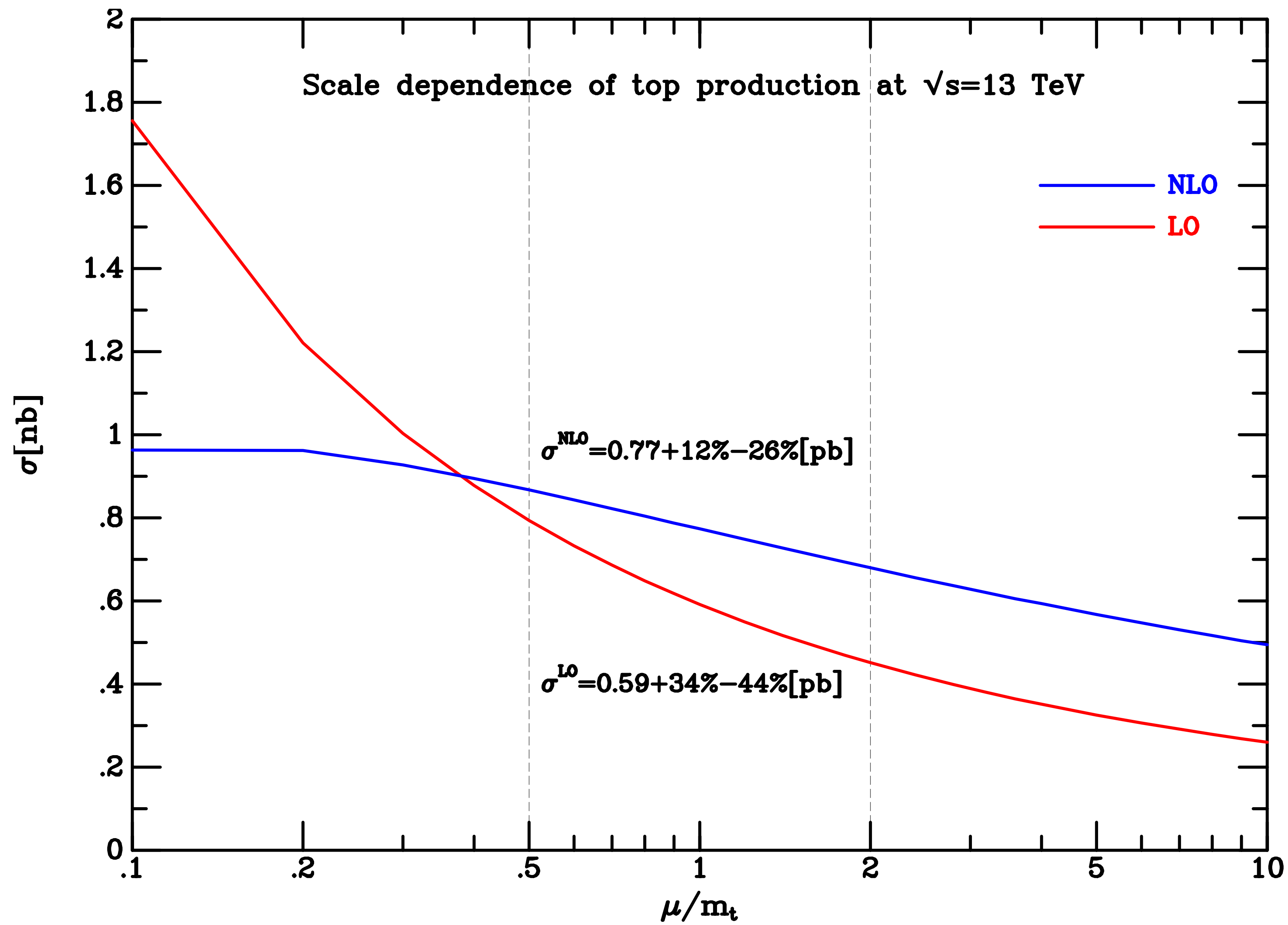
Reasons for concordance

- Influence of new data; some (arbitrary?) choices made in performing the fits are no longer tenable, (e.g. d/u ratio).
- Closure tests are able to benchmark the reliability of error estimates.
- Closure test=Generate fake experimental data, with known parton distribution functions, and check whether PDF fit to that data reproduces the "truth" parton distributions.



Higher order perturbative QCD: why bother?

- Take top pair production at 13 TeV.
- Higher order terms are not the 1/8=12% suggested by the size of α_s, because of the special nature of the renormalization group improved perturbation theory.
- NLO predictions are in the range ±20% as shown here for top production.



$\mu_R = \mu_F = \mu$

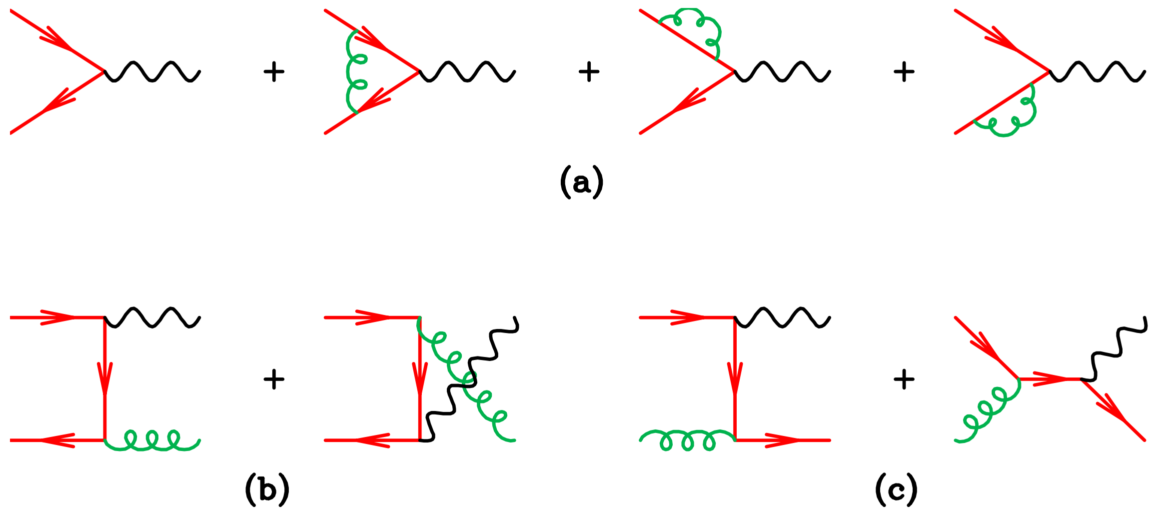
How precise should precision be?

- PDF uncertainties are at the few percent level
- Given that e.g. the luminosity measurement at the LHC is in the range 2-5%, this sets are plausible target.
- (If the predictions for hard processes are better than this, we can use them to monitor luminosity).
- Non-perturbative corrections (QCD also!) are of order Λ/Q which for current scales are of order a few %.
- Until we develop a theory for non-perturbative effects, this sets a practical limit.

One loop diagrams: NLO revolution

Ingredients in a NLO calculation

- Consider vector boson production



Virtual diagrams

Real diagrams

- Real and virtual diagrams live in different phase spaces

- For the virtual diagrams (lower multiplicity) the infrared poles are explicit, whereas as for real diagrams (higher multiplicity), they appear after integration.

- The necessity to integrate to cancel poles, is at variance with the desire for a differential distribution.

- Possible, because divergences come from LO-like regions of soft and collinear emission.

α_s corrections to the Drell-Yan process

- the birth of precision hadronic collider physics
- resolved ambiguities associated with the colour degree of freedom.
- The first 'K' factor calculation
- No agreement with data without NLO contributions.
- State of the art until NNLO was calculated NPB382 (1992) 11

W.L. van Neerven and E.B. Zijistra

LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

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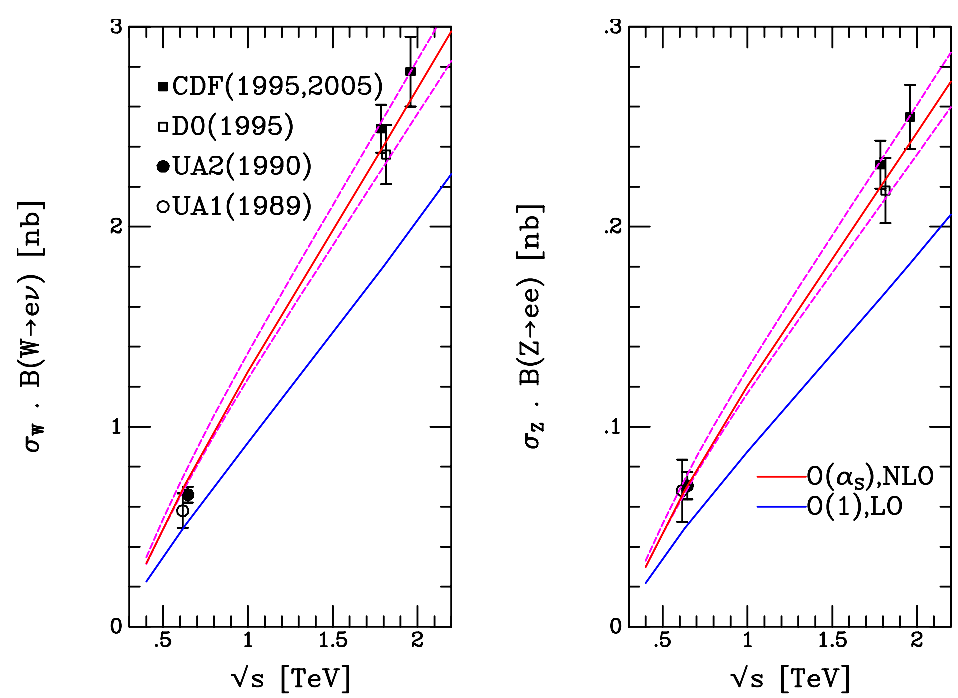
Guido Altarelli, Guido Martinelli



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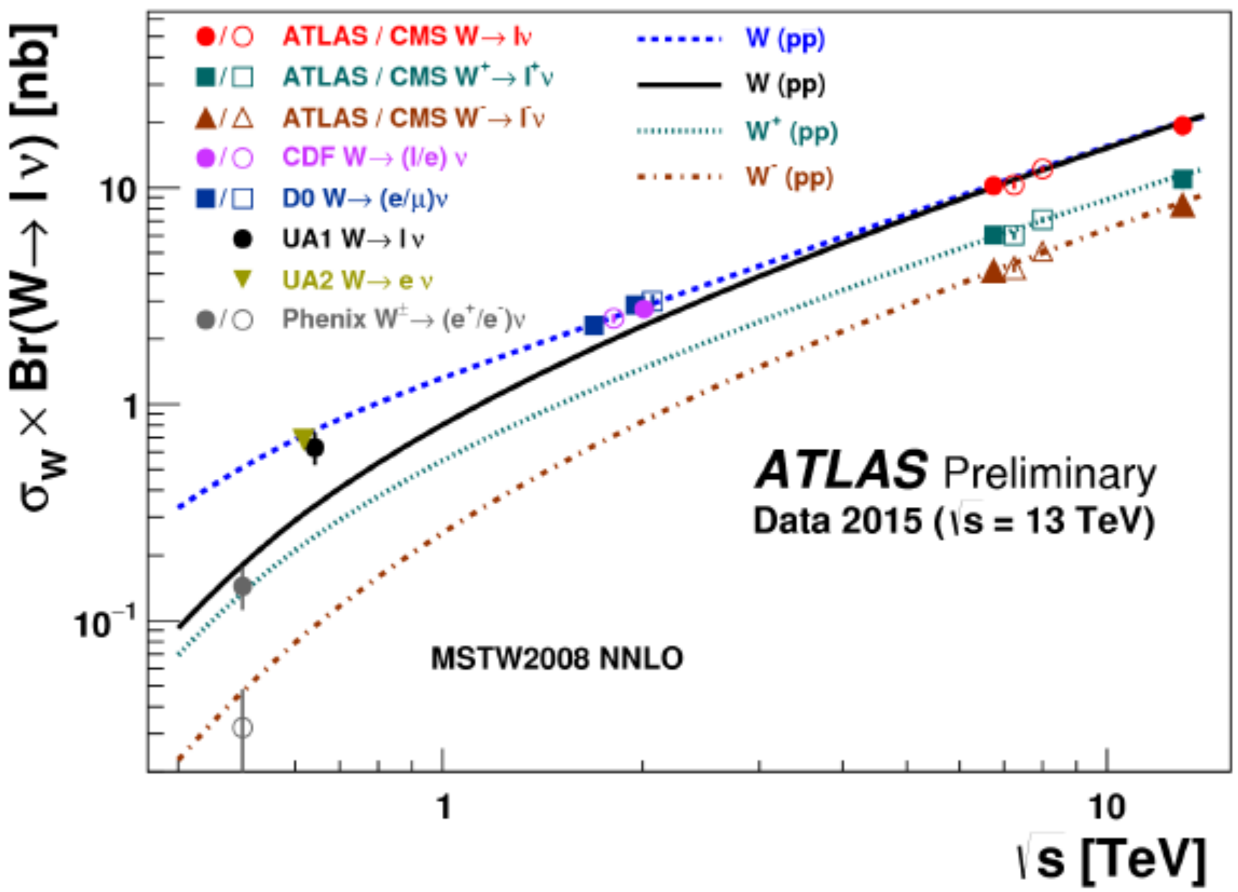
Comparison with data from low to high energy

- Basic DY mechanism is the same for W,Z production.
- Beautifully confirmed by W± production from √s=0.54-13TeV.



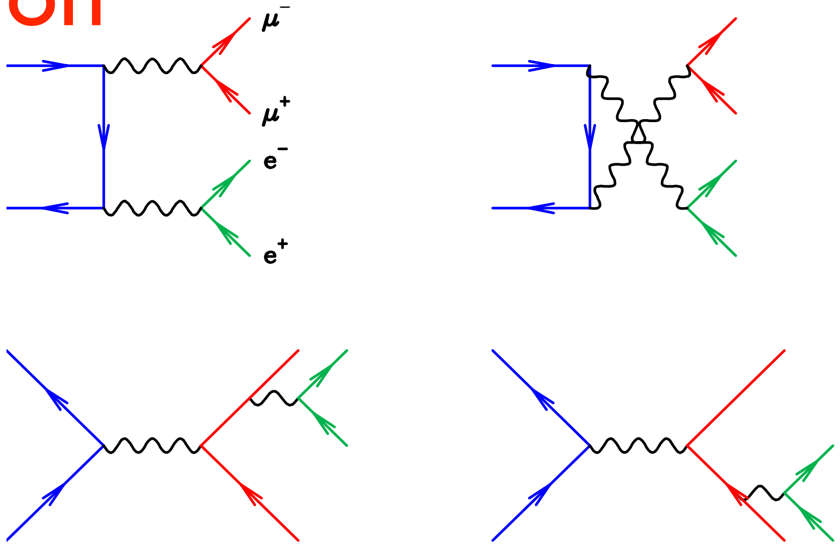
But we need to go beyond total cross sections…..

John Campbell
Ciaran Williams

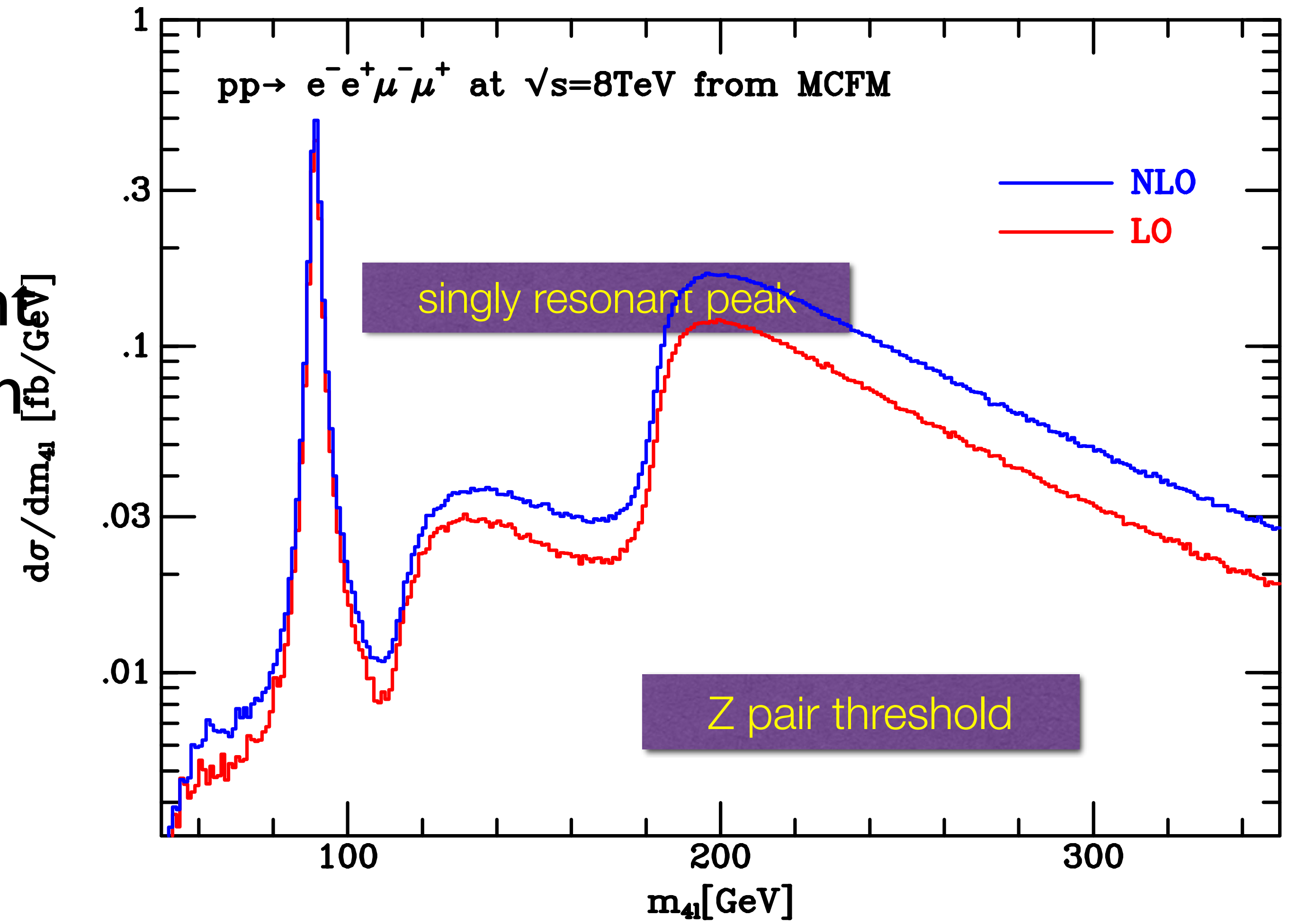
MCFM (Monte Carlo for FeMtobarn processes) 1998-present

- MCFM is a parton-level Monte Carlo program that computes hadron-collider cross sections at NLO [Campbell, RKE, Williams]
- Gives access to explicit final states, distributions.
- Implements analytic results for matrix elements, so fast and numerically stable.
- Flexible, freely distributed code, widely used in the community
- Theoretical predictions for more than 300 processes, (extensive use at Tevatron and LHC, (cited by > 650 experimental papers).
- Significant role as a catalyst for other theoretical efforts.
- Eight updates to the code in the last eight years.

Vector boson pair production



- For the final states that we are interested in, we go beyond the doubly resonant approximation.
- Z-peak coming from singly resonant diagrams, important check of resolution in search for Higgs boson.
- NLO includes gg->ZZ, (which is formally NNLO, but no Higgs yet, see later).



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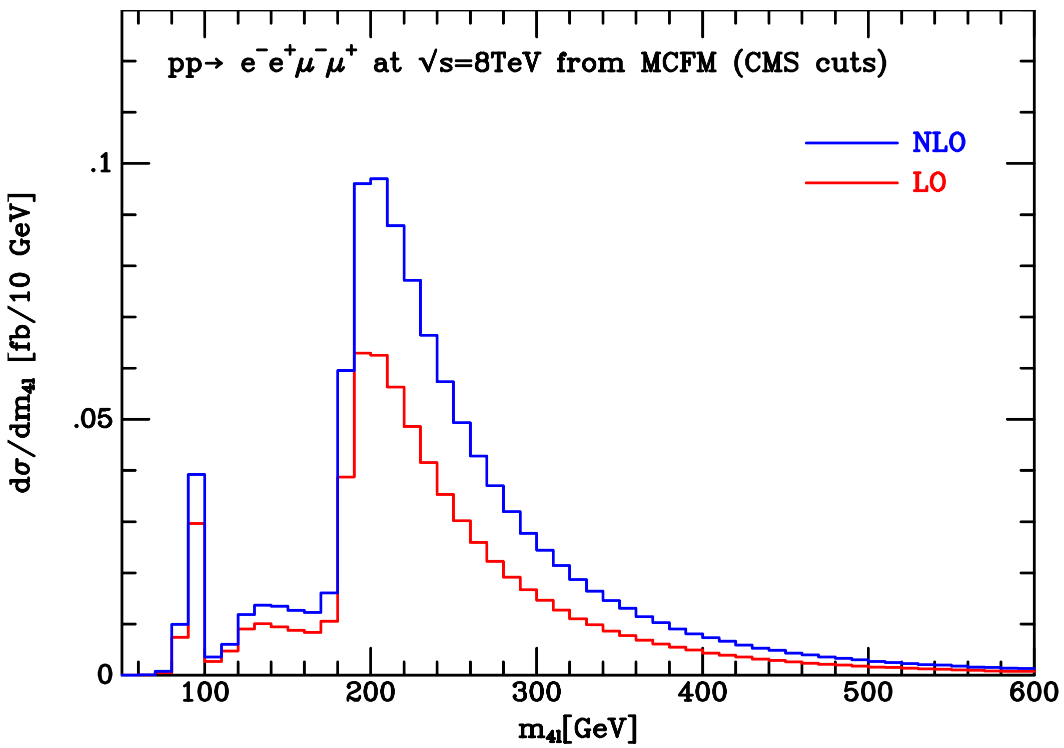
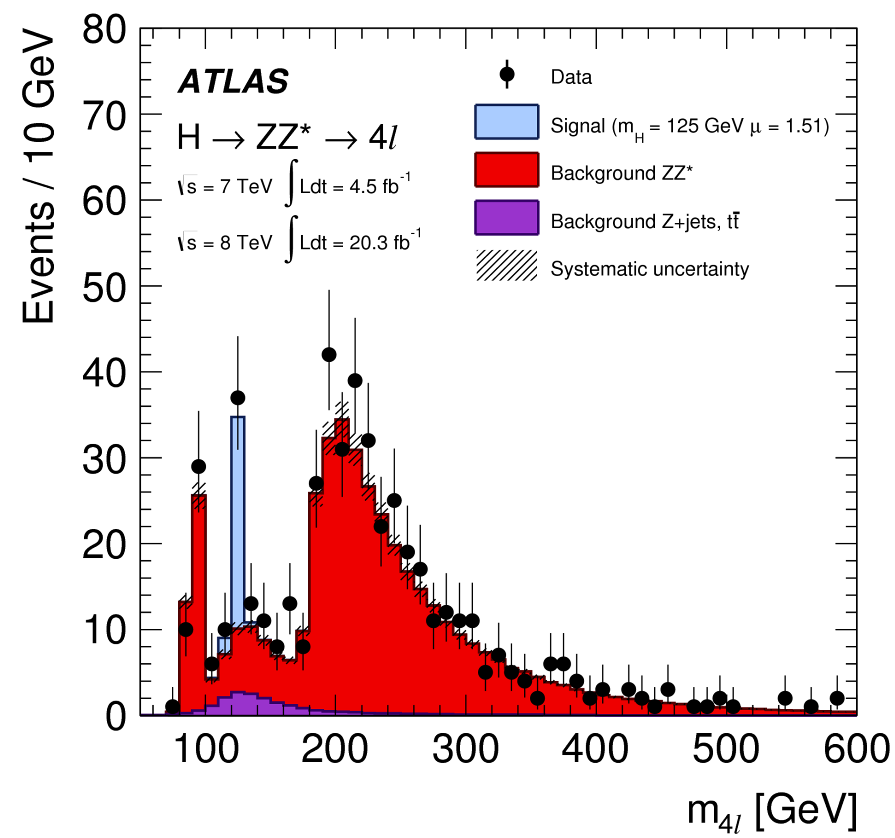
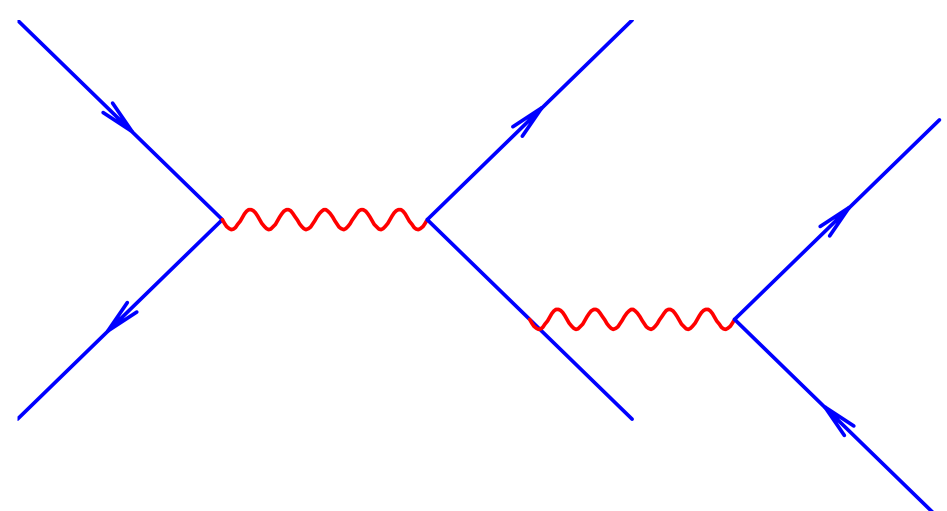
Singly resonant contribution and Higgs discovery



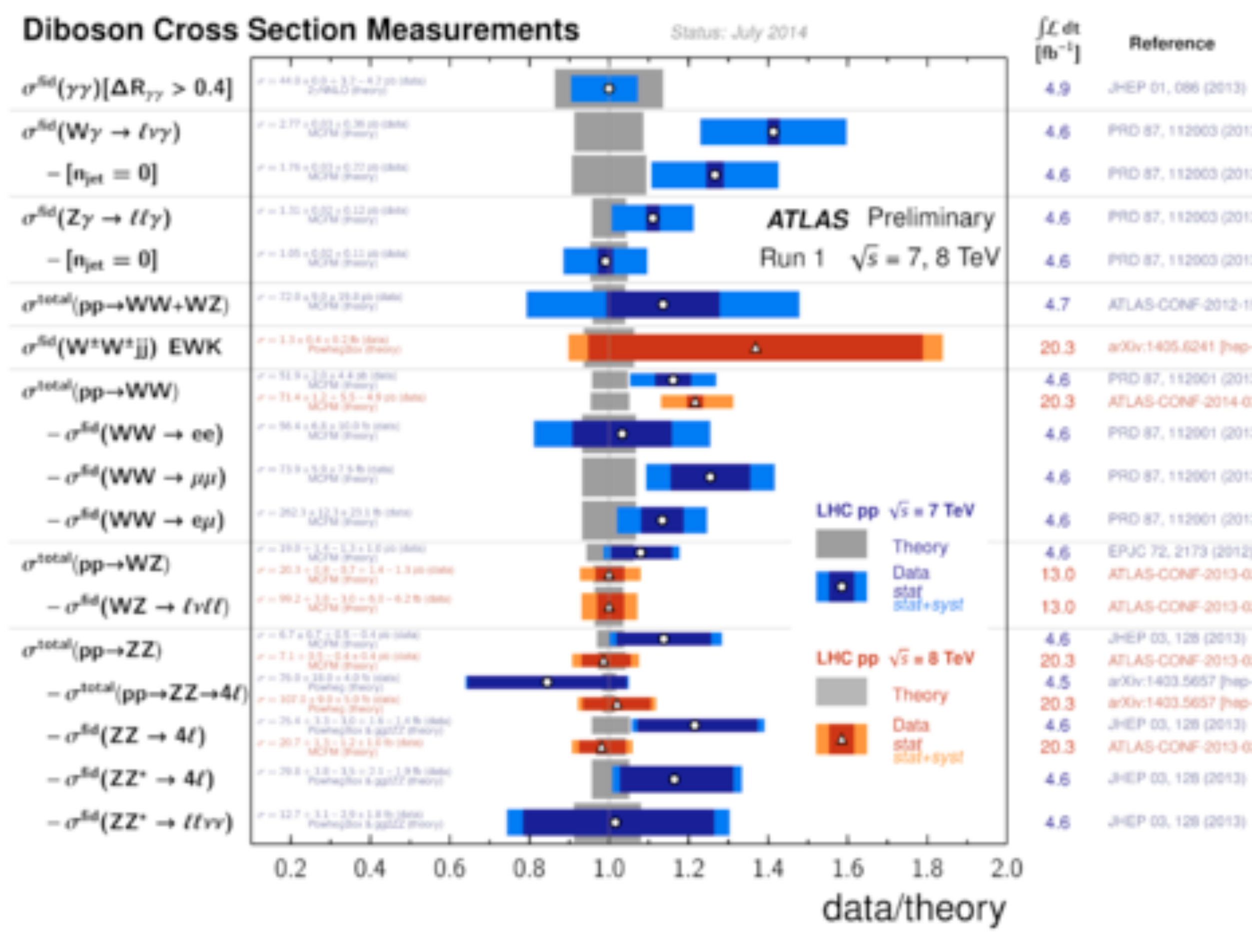


- Relative size of peaks depends sensitively on the cuts



Vector boson pair production

ATLAS results



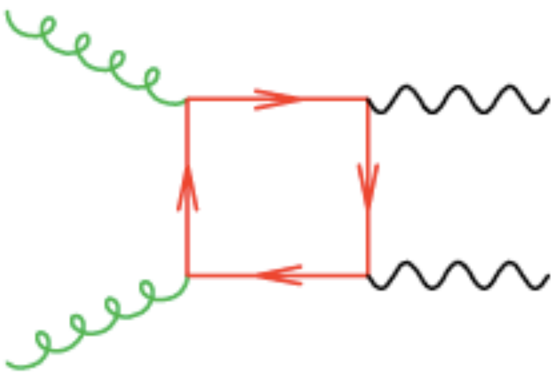
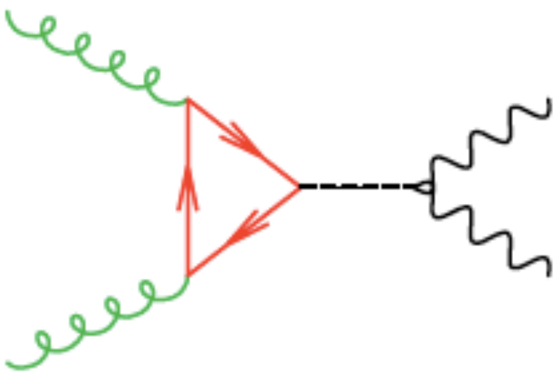
- Growth of Boson pair cross section with energy is an important check of gauge structure.
- For W^+W^-, no discrepancy in fiducial cross section.
- Emphasizes the importance of going beyond total rates.

	ATLAS @ 8 TeV	$pp \to l^+l^-\nu\bar{\nu}$	$pp \to H \to l^+l^-\nu\bar{\nu}$	total
$e^+\mu^- + e^-\mu^+$	$377.8^{+6.9}_{-6.8}(\text{stat.})^{+25.1}_{-22.2}(\text{syst.})^{+11.4}_{-10.7}(\text{lumi.})$	$332.4^{+4.7}_{-2.3}$	$9.8^{+0.0}_{-1.2}$	$342.2^{+4.7}_{-2.6}$
e^+e^-	$68.5^{+4.2}_{-4.1}(\text{stat.})^{+7.7}_{-6.6}(\text{syst.})^{+2.1}_{-2.0}(\text{lumi.})$	$63.7^{+0.8}_{-0.4}$	$2.2^{+0.0}_{-0.2}$	$65.9^{+0.8}_{-0.4}$
$\mu^+\mu^-$	$74.4^{+3.3}_{-3.2}(\text{stat.})^{+7.0}_{-6.0}(\text{syst.})^{+2.3}_{-2.1}(\text{lumi.})$	$69.3^{+0.9}_{-0.4}$	$2.4^{+0.0}_{-0.2}$	$71.7^{+0.9}_{-0.5}$

Monni, Zanderighi 1410.4745

MCFM results

The necessity of the Higgs boson

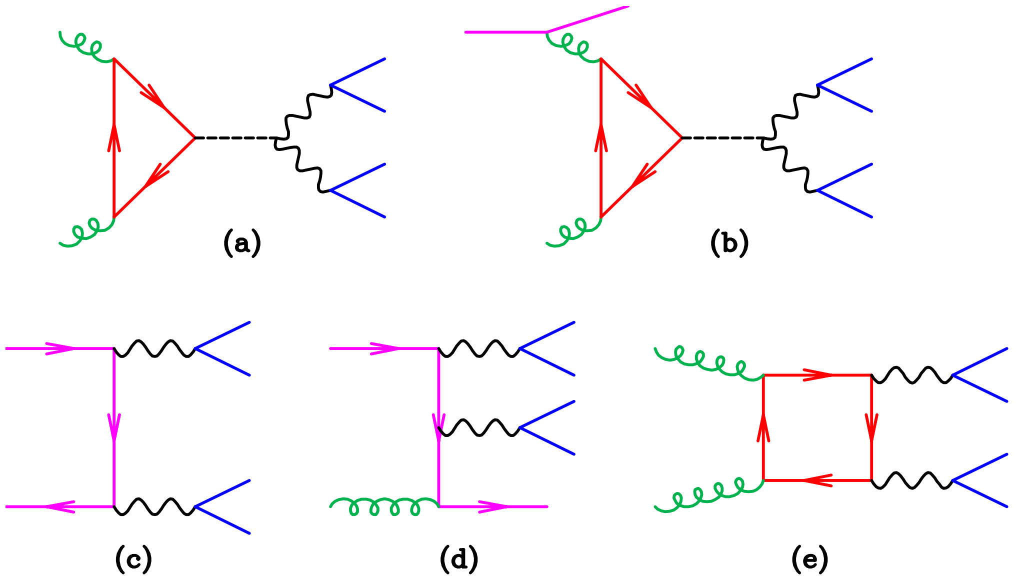
 

- The Higgs boson serves to cancel the bad high energy behaviour (E^2)
- The first cancellation comes from the gauge structure
- The second calculation requires the Higgs boson

pp→e⁻e⁺μ⁻μ⁺ in the standard model

- Mishmash of orders in perturbation theory

$(a): g(-p_1)+g(-p_2)\to H\to e^-(p_3)+e^+(p_4)+\mu^-(p_5)+\mu^+(p_6)$	$O(g_s^2e^4)$
$(b): q(-p_1)+g(-p_2)\to H\to e^-(p_3)+e^+(p_4)+\mu^-(p_5)+\mu^+(p_6)+q(p_7)$	$O(g_s^3e^4)$
$(c): q(-p_1)+\bar{q}(-p_2)\to e^-(p_3)+e^+(p_4)+\mu^-(p_5)+\mu^+(p_6)$	$O(e^4)$
$(d): q(-p_1)+g(-p_2)\to e^-(p_3)+e^+(p_4)+\mu^-(p_5)+\mu^+(p_6)+q(p_7)$	$O(g_se^4)$
$(e): g(-p_1)+g(-p_2)\to e^-(p_3)+e^+(p_4)+\mu^-(p_5)+\mu^+(p_6)$	$O(g_s^2e^4)$



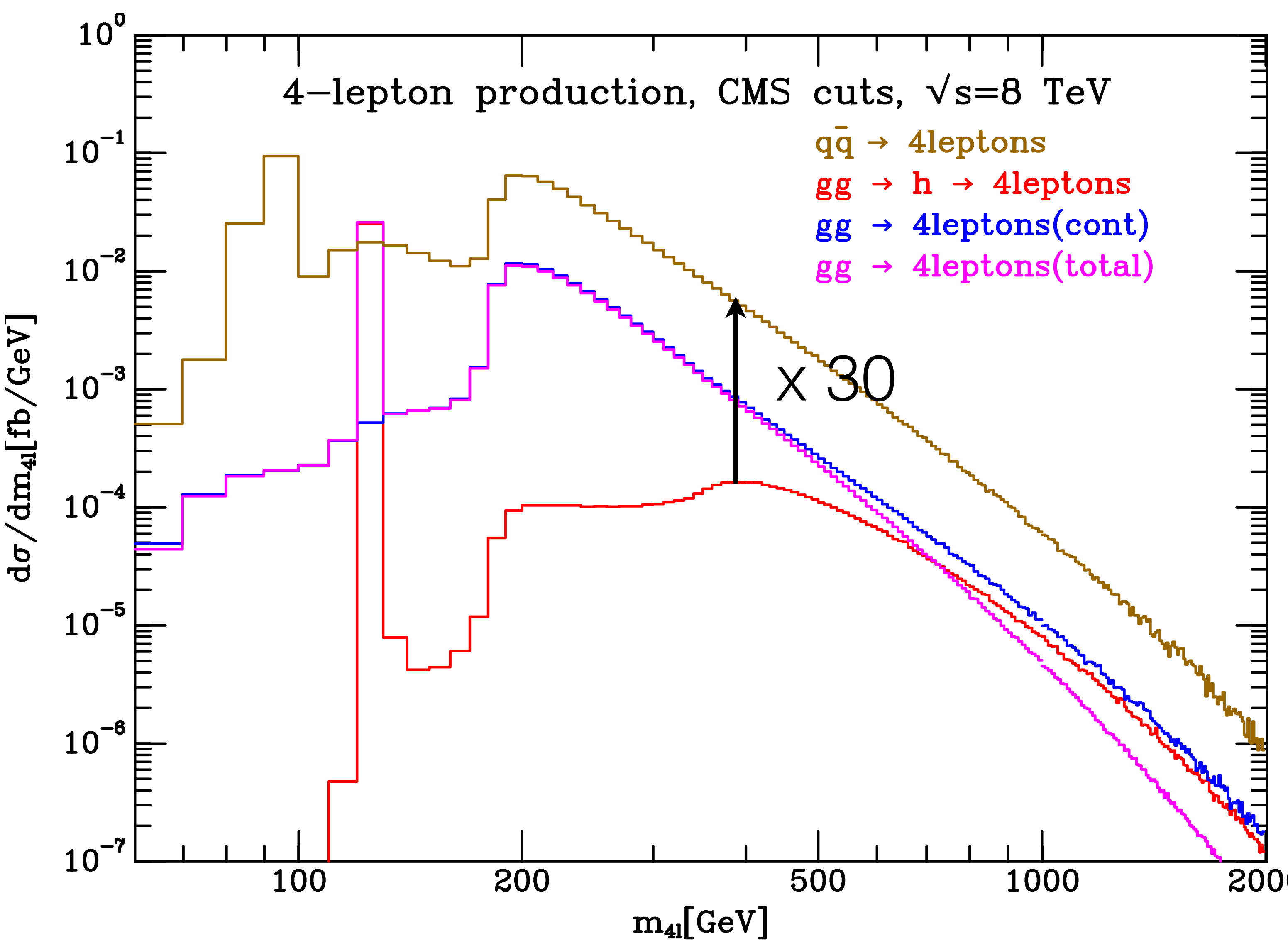
- Representative diagrams are:-
- (a) and (e), (b) and (d) can interfere.
- (b-d) interference does not overwhelm (a-e).

The big picture @ 8TeV

- Peak at Z mass due to singly resonant diagrams.
- Interference is an important effect off-resonance.
- Destructive at large mass, as expected.
- With the standard model width, Γ_H, challenging to see enhancement/deficit due to Higgs channel.
- 3 phenomena happening in the tail.

$$p_{T,\mu} > 5\ \mathrm{GeV}\,,\quad |\eta_\mu| < 2.4\,,$$
$$p_{T,e} > 7\ \mathrm{GeV}\,,\quad |\eta_e| < 2.5\,,$$
$$m_{ll} > 4\ \mathrm{GeV}\,,\quad m_{4\ell} > 100\ \mathrm{GeV}\,.$$

CMS cuts
CMS PAS HIG-13-002



Higgs couplings and width

- Off-shell tail is a valuable source of information about the Higgs production and decay couplings

$$\sigma_{\text{off}} \propto g_i^2 g_f^2$$

- Higgs cross section under the peak depends on ratio of couplings and width.

$$\sigma_{\text{peak}} \propto \frac{g_i^2 g_f^2}{\Gamma}$$

- So measurements at the peak cannot untangle couplings and width.
- Off-peak cross section is independent of the width, but still depends on $g_i^2 g_f^2$ (modulo interference, see later).
- Taking ratio

$$\frac{\left(\frac{\sigma_{\text{off}}}{\sigma_{\text{peak}}}\right)_{\text{experimental gg}}}{\left(\frac{\sigma_{\text{off}}}{\sigma_{\text{peak}}}\right)_{\text{theoretical SM}}} = \frac{\Gamma}{\Gamma^{\text{SM}}}$$

- Ratio depends linearly on the Higgs boson width.

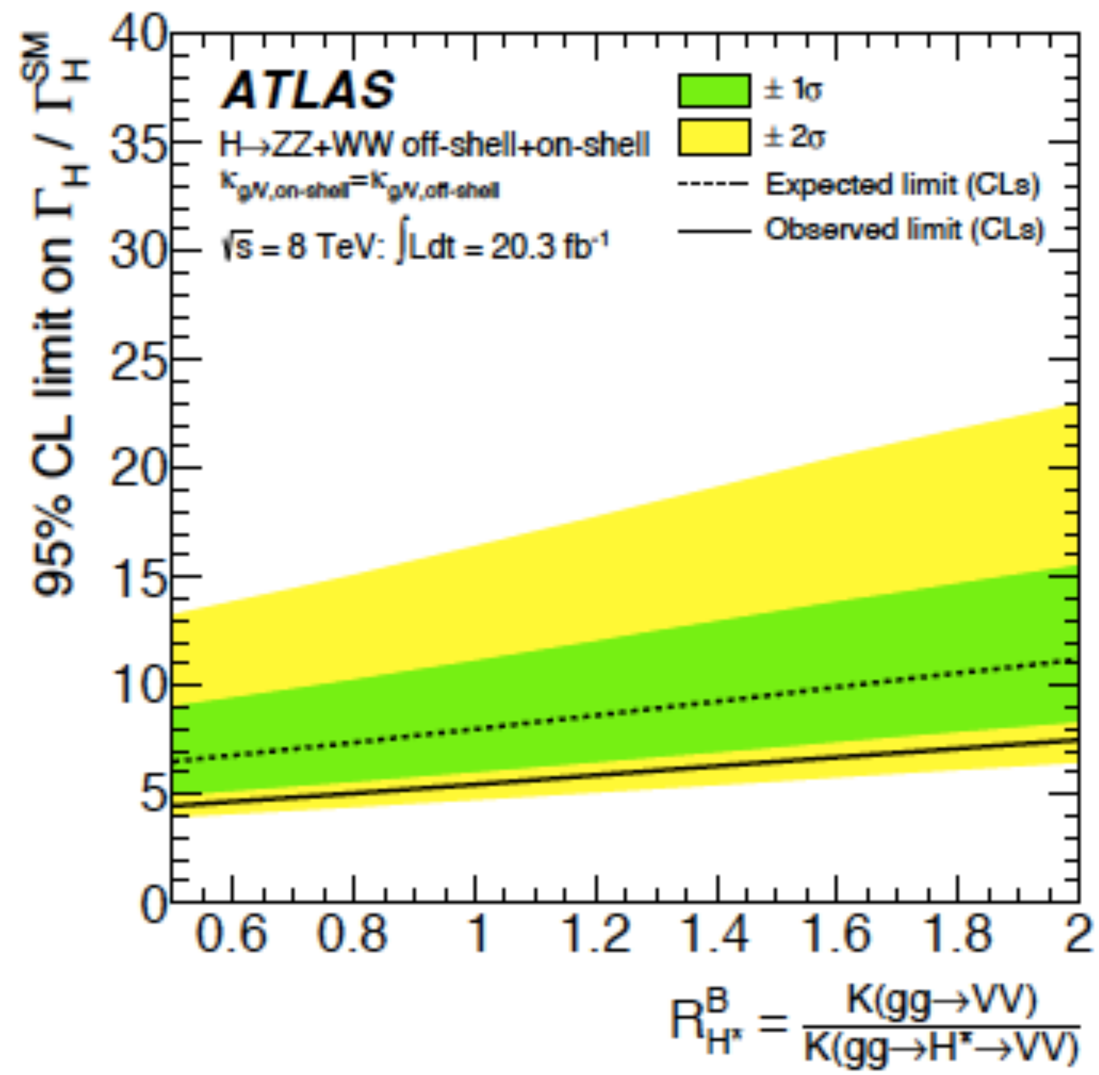
Caola-Melnikov 1307.4935

ATLAS results

- ATLAS presents their results as a function of the unknown relative K-factor for the Higgs mediated pieces and for the interference piece.

- Some partial information on this from the large mass expansion. Melnikov and Dowling 1503.01274

- Complete calculation needed



Subsequent automatic NLO programs

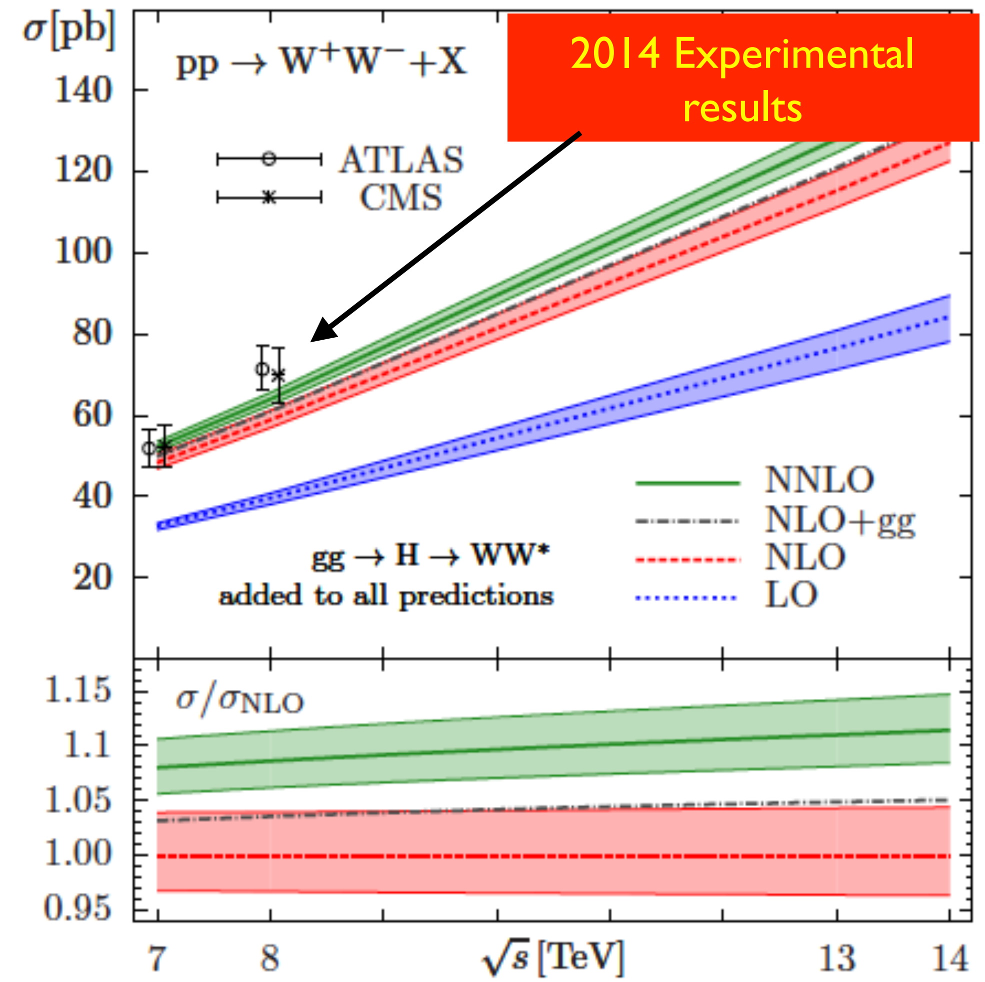
- Fully automatic procedures.
  - Madgraph5_aMC@NLO 1405.0301
  - Helac-1Loop 1502.01521
  - Go-Sam 1404.7096
- Approaches for greater number of legs of a less automatic nature.
  - Blackhat-Sherpa 1310.2808
  - Njet 1312.7140

The current workhorse: NNLO

- NNLO calculations roughly at the level of NLO in 1990.
    - NLO 2 to 2 virtual matrix elements known
    - NLO top cross section (total and differential) known
    - NLO 2 to 3 calculations just beginning to be tackled?
- NLO calculations complete ~2010
- Will we make faster progress on NNLO?

Necessity of NNLO

Gehrmann et al, 1408.5243

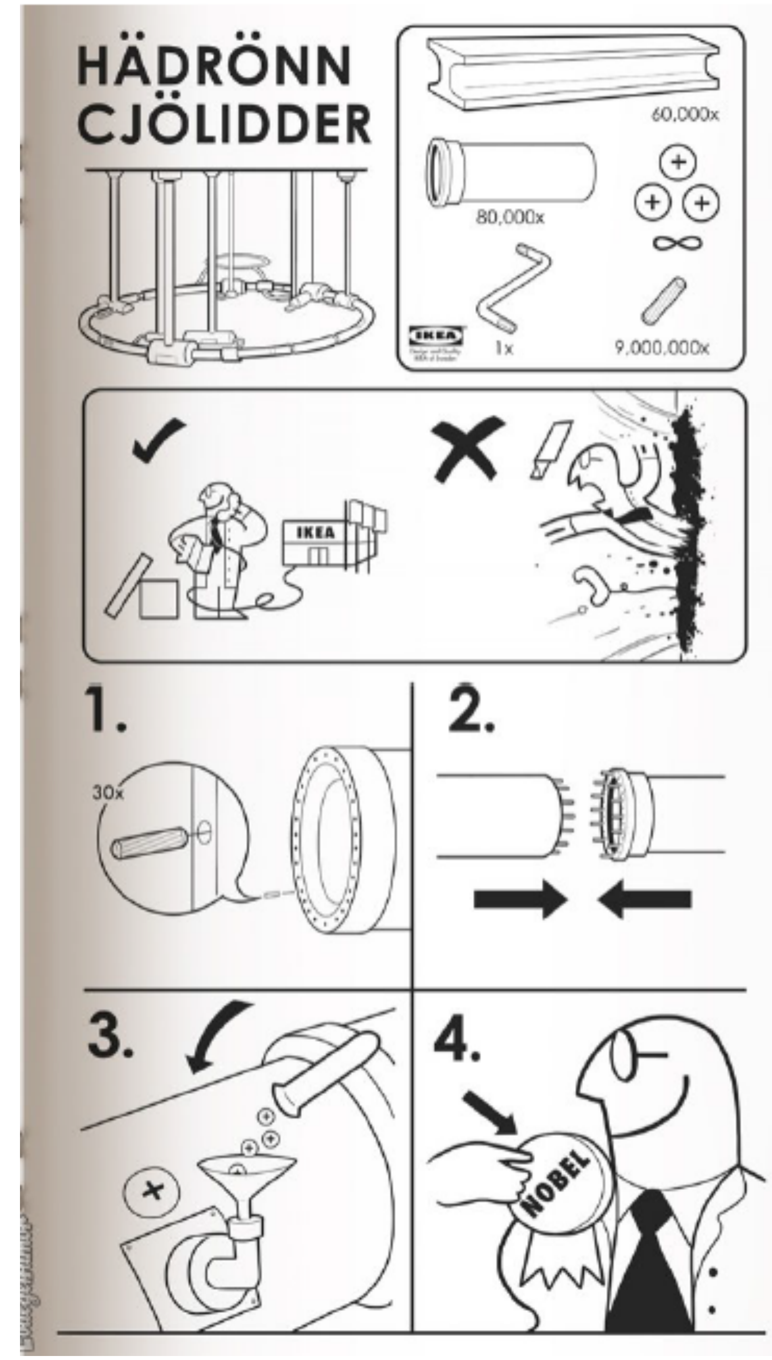


Einsweiler, Lepton Photon 2015

Di-boson production: W^+W^- Production (CMS)

- Measure σ(fid) = 60.1 ± 0.9 (stat) ± 3.2 (exp) ± 3.1 (theo) ± 1.6 (lumi) pb
- Theory σ(NNLO) = 59.8 ± 1.2 pb
- Good agreement (expt ± 8%), but NNLO calculations are really necessary.

NNLO-some assembly required

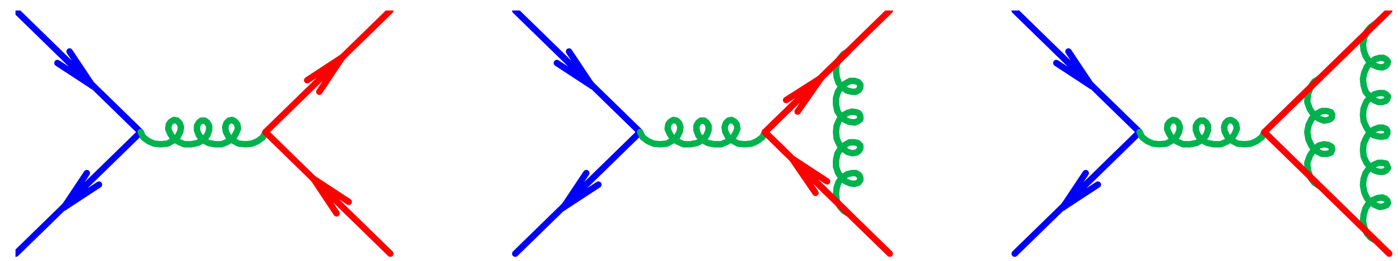


- Contributions from Real-Real, Real-Virtual and Virtual-Virtual.
- For the lower multiplicities the poles are explicit, whereas as for higher multiplicities, they appear after integration.
- Thus the requirement to cancel the poles appears to be in contradiction with the desire for a differential cross section.

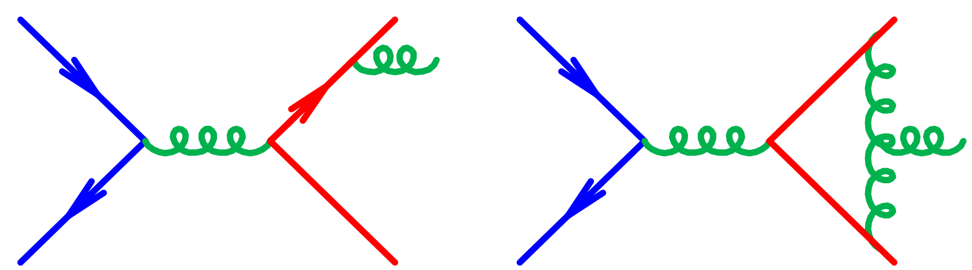
NNLO diagrams

- Challenge is not the calculation of the individual diagrams, but rather the assembly of pieces that individually contain infrared divergences

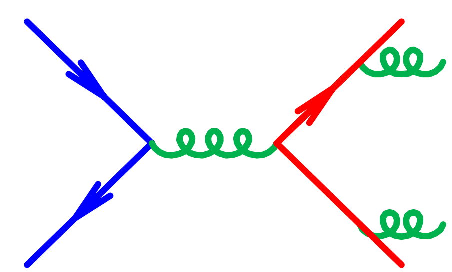
examples of 2→2 diagrams:VV



examples of 2→3 diagrams:RV



examples of 2→4 diagram:RR



- In different regions of phase space, different subsets of partons lead to singularities of the matrix elements.

NNLO - Four main combination methods

- Antenna
    - Pros:Analytic cancellation of poles, demonstrated for 2->2 colored scattering
    - Con: More challenging interface to existing NLO codes
- Sector improved residue subtraction scheme.
    - Pros:Brute force method, offers possibility of generalization to arbitrary processes, demonstrated for 2->2 colored scattering
    - Con: Numerical cancellation of poles
- qt/N-jettiness subtraction
    - Pro:Meshes well with existing NLO codes
    - Con:Slicing method, have to demonstrate independence from cutoff parameter.
- Colour subtraction
    - Pro:Local subtraction terms
    - Con: No NNLO application to processes with initial state hadrons yet.

Processes currently known through NNLO

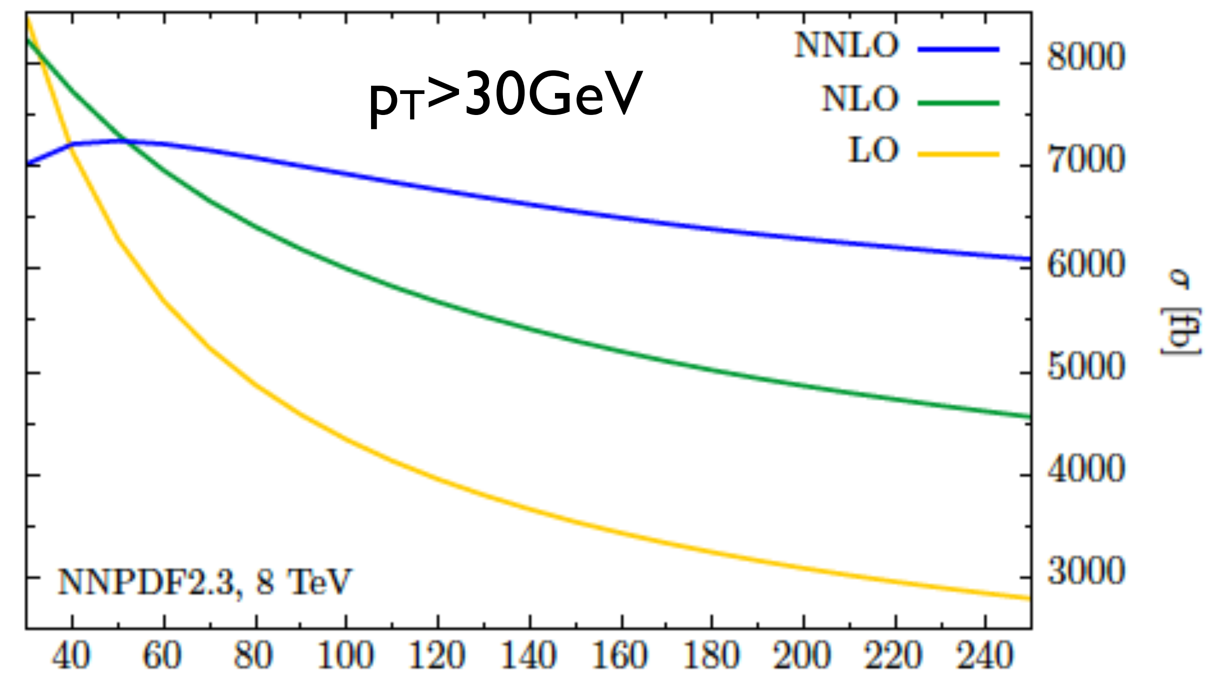
dijets	gluon-gluon	PDFs,strong couplings,BSM	1407.5558
H+0jet	fully inclusive N^{3}LO	Higgs couplings	1503.06056,
H+1jet	fully exclusive	Higgs couplings,probing GGH vertex	1408.5325,1504.07922, 1505.03893
tt pair	fully exclusive, stable tops	top cross section ,mass pt, FB asymmetry,PDFs BSM	1601.05375
single top	fully exclusive, stable tops, t-channel	Vtb,width, PDfs	1404.7116
WBF	exclusive VBF cuts	Higgs couplings	1506.02660
W+j	fully exclusive, decays	PDFs	1504.02131
Z+j	decay, off-shell effects	PDFs	1601.04569,1507.20850, 1507.02850
ZH	decays to bb at NLO	Higgs couplings	1407.4747,1601.00658
WH	fully exclusive	Higgs couplings	1312.1669, 1601.00658
ZZ	fully exclusive, off-shell	trilinear gauge couplings,BSM	1405.2219, 1507.06257
WW	fully inclusive	trilinear gauge couplings,BSM	1408.5243
$W\gamma$,$Z\gamma$	fully exclusive	trilinear gauge couplings,BSM	1601.06751
$\gamma\gamma$	fully differential	Background studies	1110.2375,1603.02663
top decay	exclusive	Top couplings	1301.7133
H-bb	exclusive, massless	Higgs couplings boosted	1110.2368

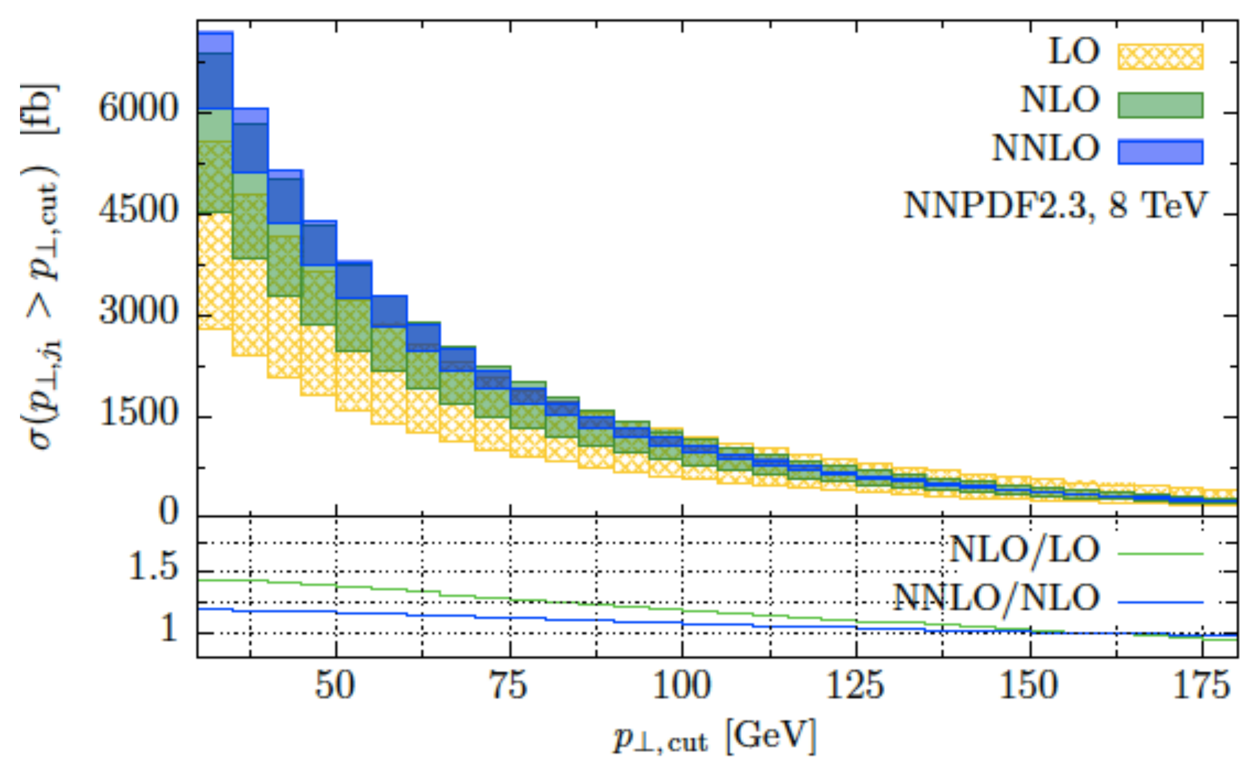
Adapted from K. Melnikov, Aspen Winter Conference 2016

Higgs+1 jet

Boughezal et al,1504.07922, 1505.03893
Caola et al, 1508.02684

- σ=6.7+0.5-0.6pb at 7 TeV
- Calculation performed in effective theory.
- QCD corrections depend on the kinematics, (K-factor dependent on p_T cut)





At 13 TeV in effective theory

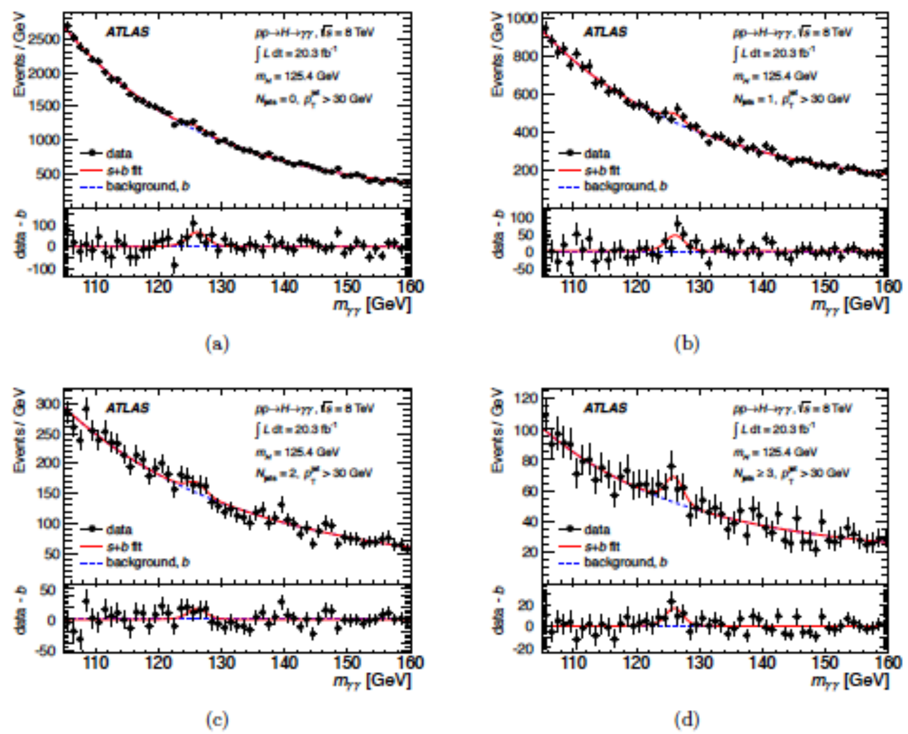
Process	α_s^2	α_s^3	α_s^4	α_s^5
$\sigma(pp \to H)$ pb [1]	13	30	40	43
$\sigma(pp \to H+$ jet) pb[2]		10	15	18
$\sigma(pp \to H+$ 2 jet) pb[3]			3.5	5.1
$\sigma(pp \to H+$ 3 jet) pb[3]				1.6

Higgs + 1 jet, (fiducial cross section)

1407.4222

- ATLAS has published Higgs cross section separated by the number of jets, in their fiducial region.

- Allow comparison of their results with new NNLO results (Caola et al, 1508.02684) in their fiducial region.



ATLAS: $\sigma^{\rm fid}_{H+j}(8\ \mathrm{TeV}) = 21.5 \pm 5.3(\mathrm{stat.}) \pm^{2.4}_{2.2}(\mathrm{syst.}) \pm 0.6(\mathrm{lumi})\ \mathrm{fb}.$

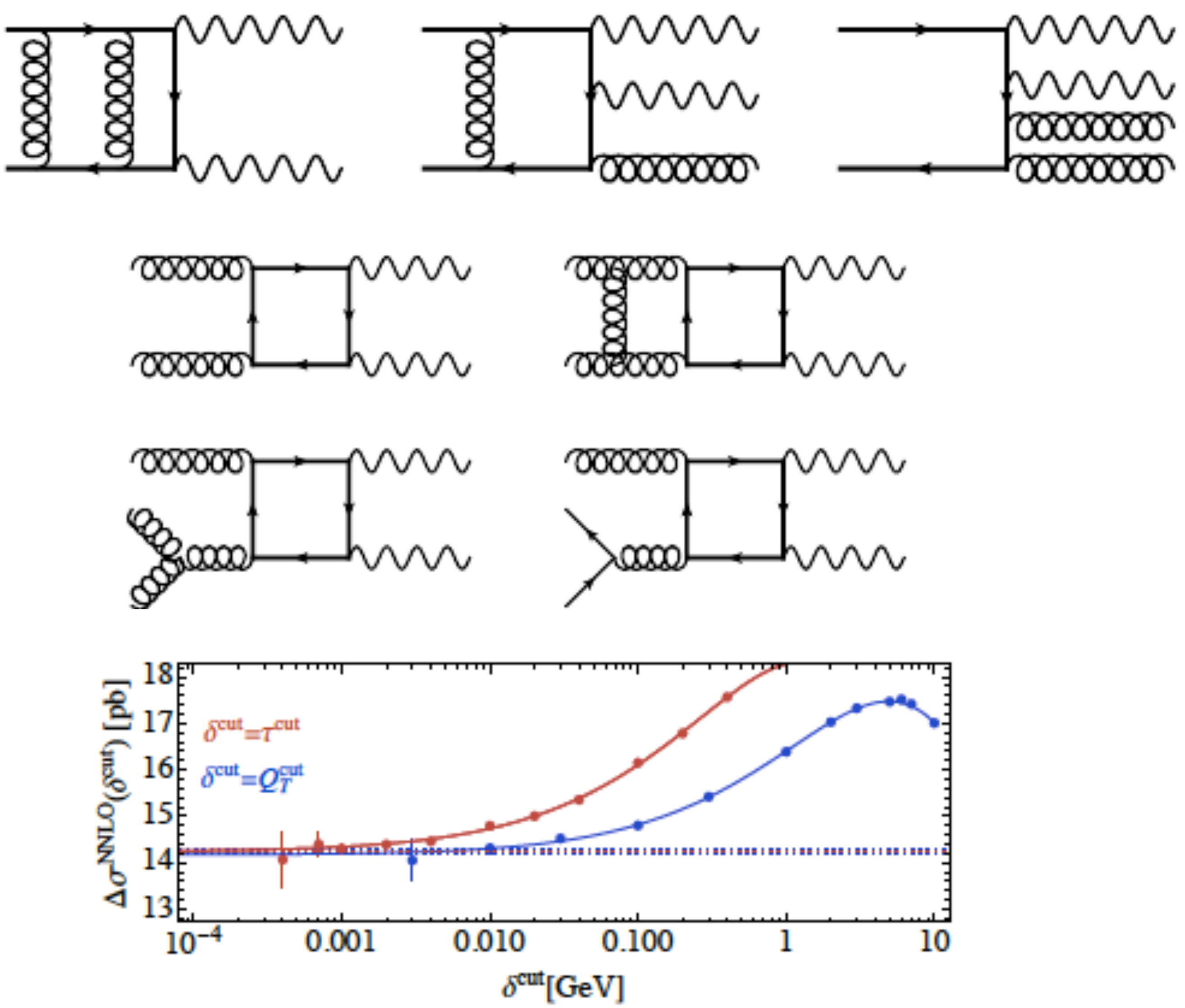
Fixed order: $\sigma^{\rm fid}_{\rm LO} = 5.43^{+2.32}_{-1.49}\ \mathrm{fb}, \quad \sigma^{\rm fid}_{\rm NLO} = 7.98^{+1.76}_{-1.46}\ \mathrm{fb}, \quad \sigma^{\rm fid}_{\rm NNLO} = 9.45^{+0.58}_{-0.82}\ \mathrm{fb},$

- ATLAS result larger by a factor of 2.1-2.5, (2.4 σ effect)

Compare and contrast

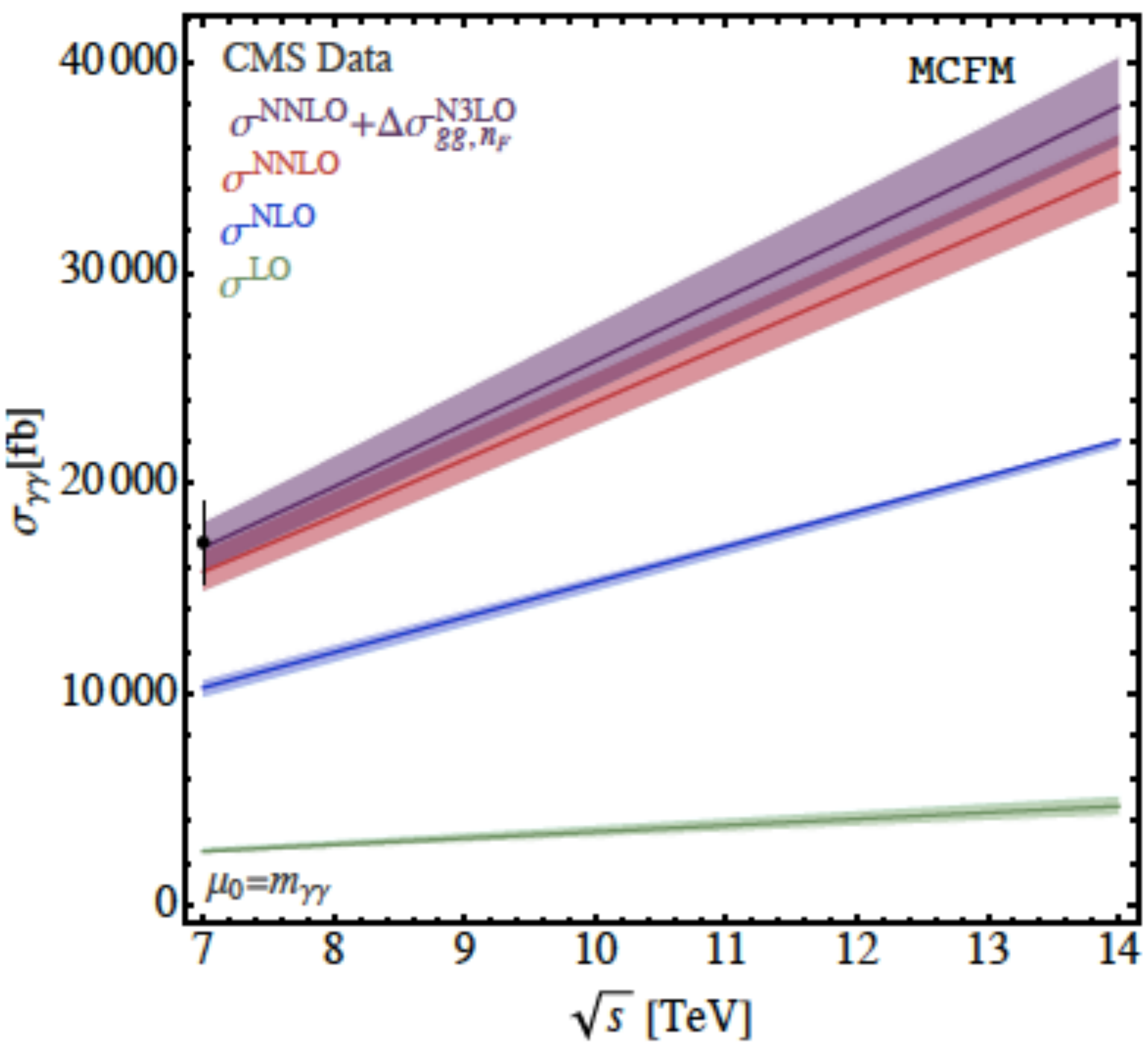
Another example: $\gamma\gamma$ production at NNLO

- Representative Feynman diagrams for NNLO
- Also include gluon initiated diagrams at NLO
- This a slicing method, must show independence from slicing parameter, (two methods).



$\gamma\gamma$ production at NNLO

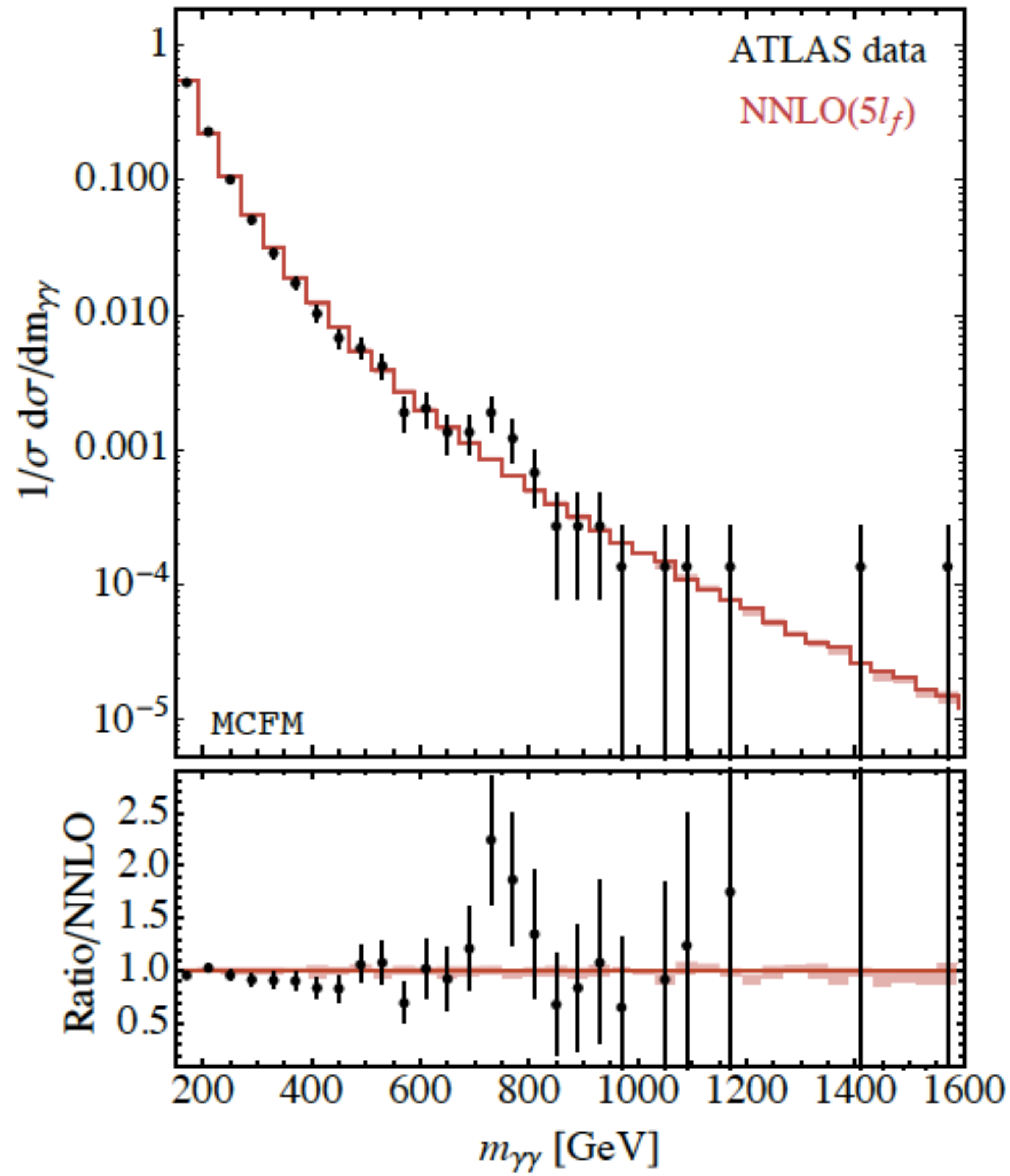
- Previous published NNLO result [Catani et al, 1110.2375] for this cross section is in error, (about 7% to big).
- Emphasises importance of benchmarking the NNLO codes.
- However NNLO corrections are still large.



- Predicted growth with energy compared to 7 TeV CMS data

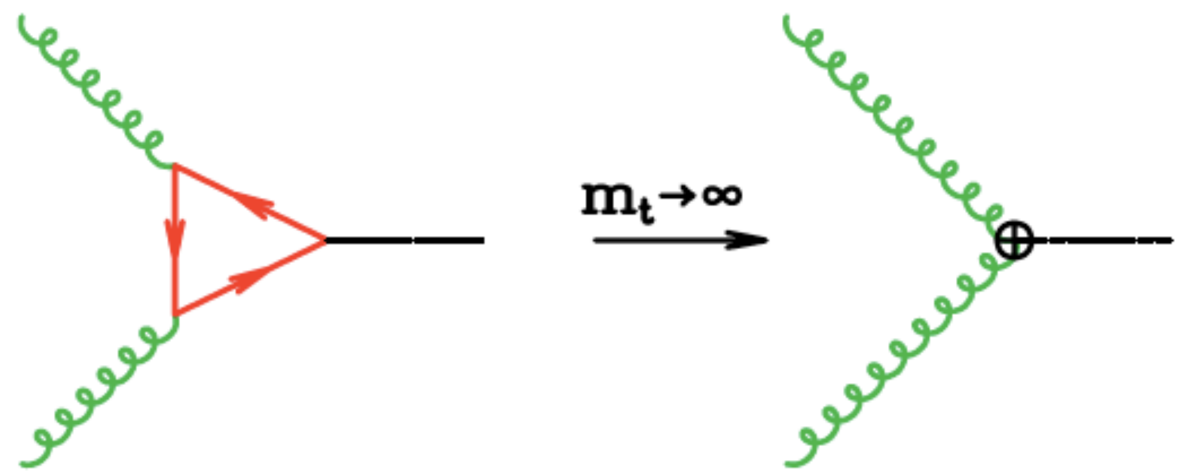
Invariant mass spectrum compared to Atlas data

- To correctly compare the prediction with the ATLAS data requires a knowledge of fake rates, photon efficiencies and acceptances.
- However, comparing shape alone, agreement is excellent.
- Theoretical prediction has more information than fitting function, and could be used as an alternative.

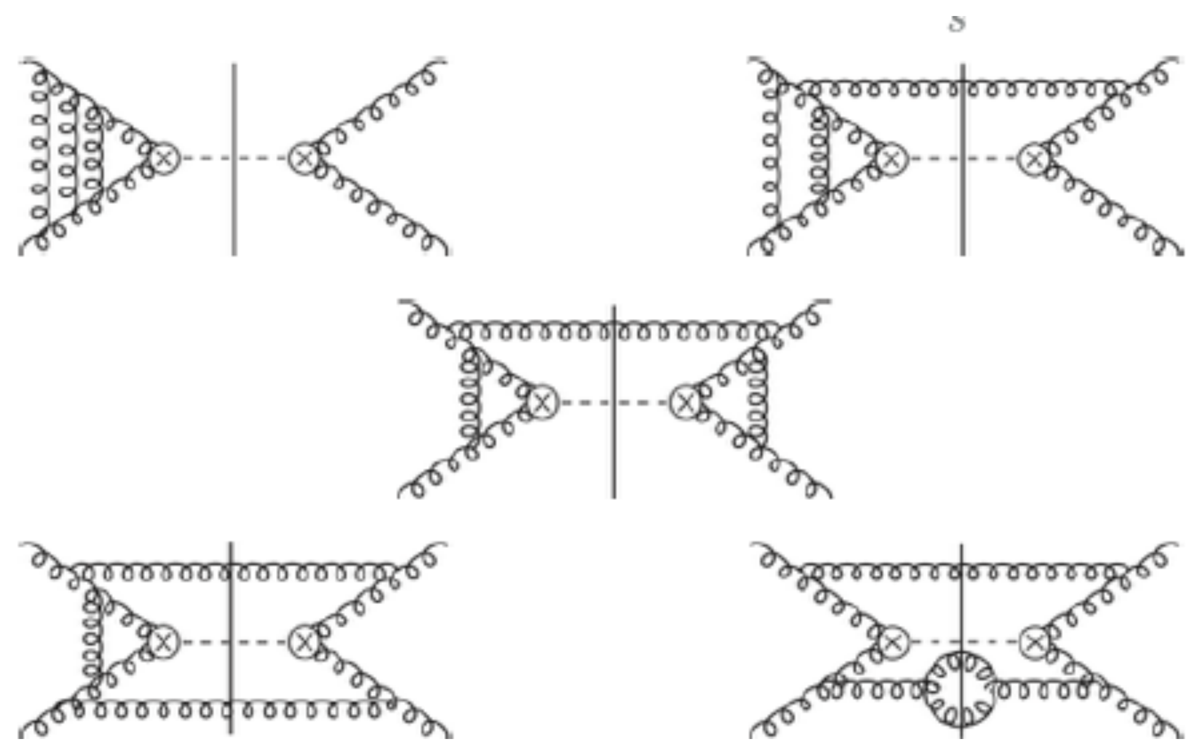


The new frontier: N^{3}LO

- A simple "Drell-Yan" process
- Great practical importance for the determination of Higgs couplings.
- Performed in effective theory



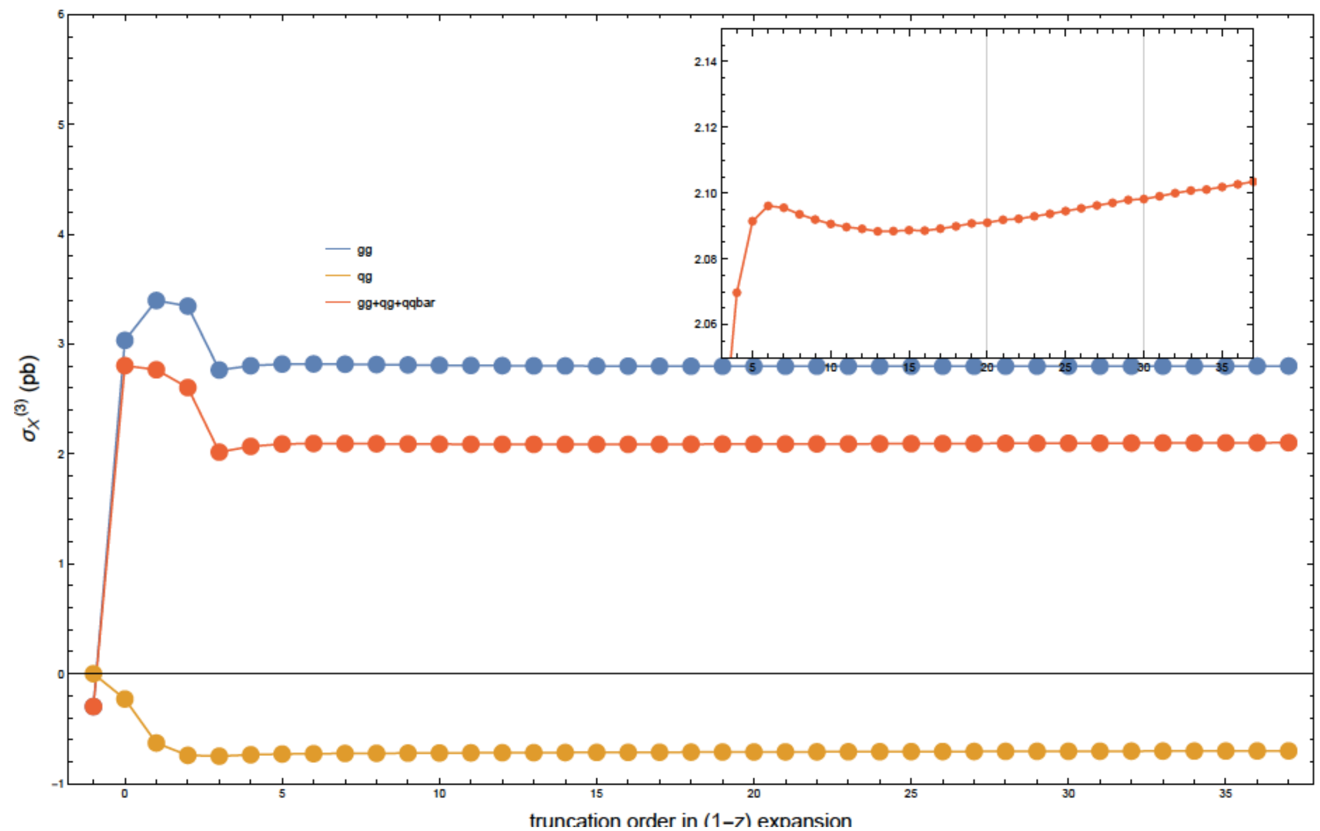
- Requires H at 3-loop
- H+parton at 2-loop
- (H+1-partons at 1-loop)^2
- H+2-partons at 1-loop
- H+3 partons at tree graph-level



N^3LO : the total Higgs production cross section

- Compute N^3LO cross section as an expansion around the soft limit (1-z), $z=m_H^2/s$
- Achieve excellent convergence with small residual growth due to high energy log z
- Plausibly claim to have calculated the total cross section.

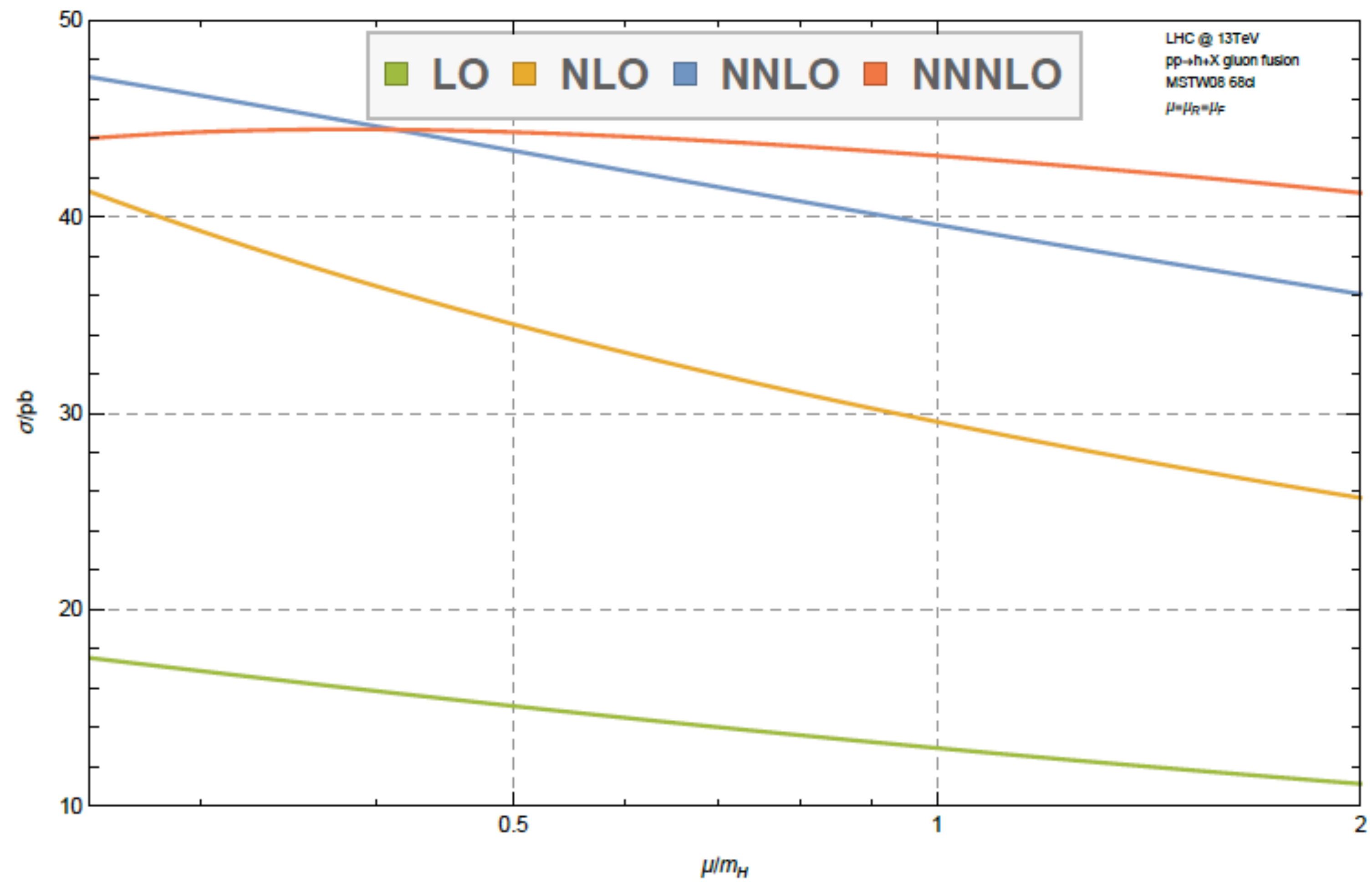
Anastasiou et al, 1503.06056



Truncation order of (1-z) expansion

N^3LO: Higgs total cross section at N^3LO

1503.06056,1505.04110

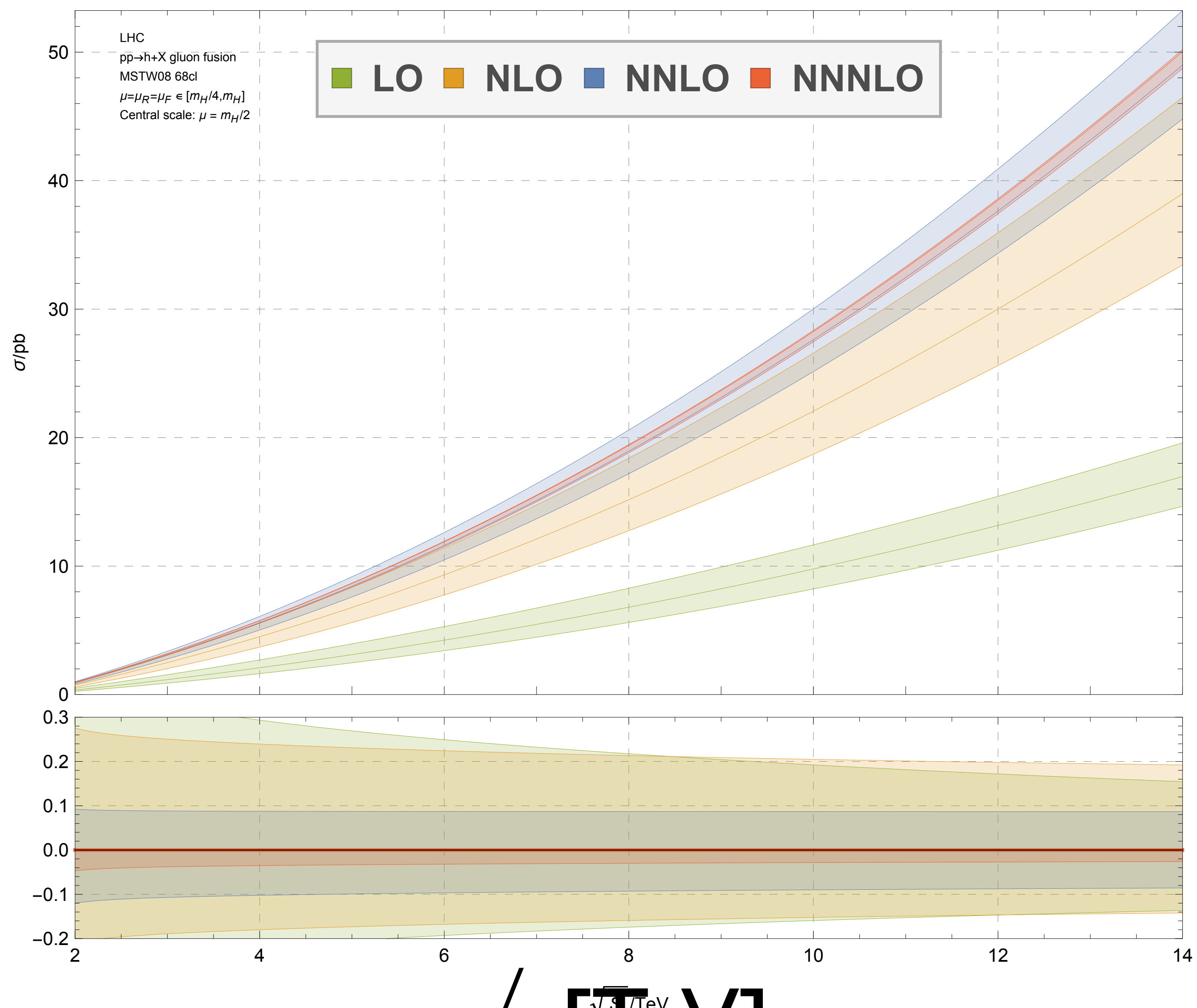


- Result in 1503.06056 is $\sigma = 44.31 + 0.31\% - 2.64\%$pb for $\mu \in [m_H/4, m_H]$ at N^3LO
- At N^2LO this uncertainty is ±9%

Convergence of perturbation series

- Perturbation series for Higgs is well-tempered at all energies

σ [pb]

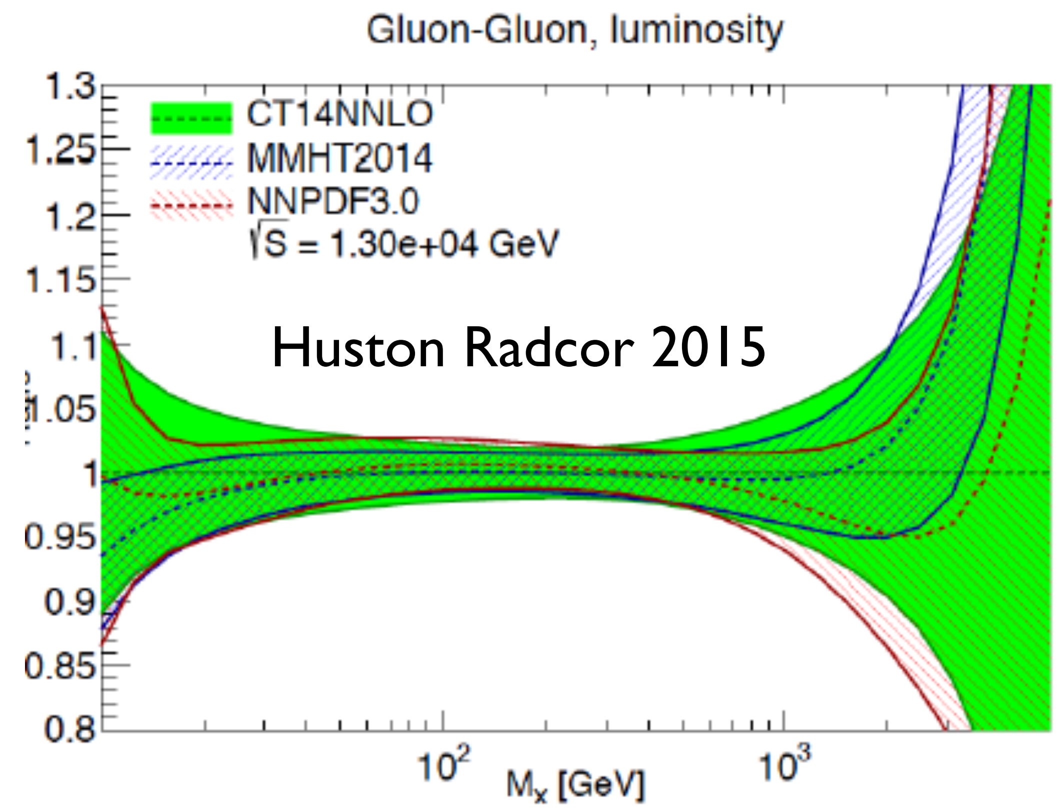


$\sqrt{s}$[TeV]

Uncertainty budget for gg->H

- According to Anastasiou et al, after the N^{3}LO calculation the dominant uncertainty is the PDF and α_s.
- However recent progress in PDF fits has reduced the uncertainty so that it is also at the 2% level.
- Most studies of the evolution of the uncertainty in the gluon distribution are targeted at larger x.

Status as of Radcor 2015



NNPDF down by 2-2.5%, CT14 up by ~1%, MMHT14 down by ~0.5%

partially data, partially corrections in fitting code, partially changes in fitting procedures

scale = m_H	CT14	MMHT2014	NNPDF3.0
8 TeV	18.66 pb -2.2% +2.0%	18.65 pb -1.9% +1.4%	18.77 pb -1.8% +1.8%
13 TeV	42.68 pb -2.4% +2.0%	42.70 pb -1.8% +1.3%	42.97 pb -1.9% +1.9%

The PDF uncertainty using this new generation of PDFs will be similar in size to the NNNLO scale uncertainty and to the $\alpha_s(m_Z)$ uncertainty.

Latest word on Higgs cross section error budget

F. Dulat, CERN, December 2015, https://indico.cern.ch/event/462111/

σ/pb	δ_{PDF}/pb	δ_{α_s}/pb	δ_{scale}/pb	δ_{trunc}/pb	δ_{pdfTH}/pb	δ_{EW}/pb	δ_{tb}/pb	δ_{1/m_t}/pb
48.48	± 0.90	± 1.26	$^{+0.09}_{-1.11}$	± 0.12	± 0.56	± 0.48	± 0.34	± 0.48
48.48	$\pm 1.86\%$	$\pm 2.60\%$	$^{+0.2}_{-2.3}\%$	$\pm 0.25\%$	$\pm 1.15\%$	$\pm 1.00\%$	$\pm 0.70\%$	$\pm 1.00\%$

- N^{3}LO pdfs are not available and not accounted for by pdf uncertainties
- Finite mass effects only known approximately beyond NLO, do not include all important interference effects
- electroweak corrections at LO known, dominant mixed effects calculated in EFT.

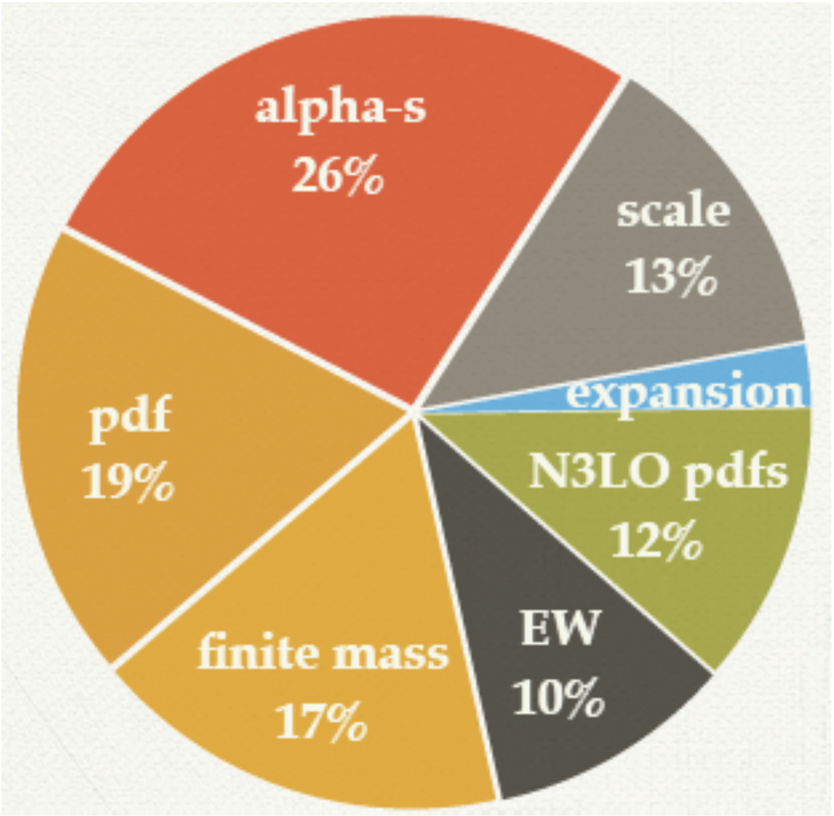
Best prediction at 13 TeV

F. Dulat, CERN, December 2015, https://indico.cern.ch/event/462111/

- The best prediction at 13 TeV, combining all sources of uncertainty

$$\sigma = 48.48^{+2.60}_{-3.47}\ \text{pb} = 48.48\ \text{pb}^{+5.36\%}_{-7.15\%}$$

- For the Higgs cross section we have finally achieved the precision initially offered by as=0.118
- Uncertainty budget indicates the areas for future improvement.
- Extension to more differential distributions?



The next frontier: fully differential cross sections mostly at NNLO

- It is important at NNLO, as it was at NLO that we can go beyond total cross sections.
- This is necessary so that we can calculate fiducial cross section for limited detector coverage.

Outlook

- It might seem that precision QCD is a game of diminishing returns; higher orders terms are harder to calculate and, if the perturbative series is well-tempered, less important.
- On the contrary it is a great time to work on radiative corrections. The Higgs boson is a central theme of run II at the LHC; it radiates copiously.
- Furthermore this effort is absolutely necessary, to achieve the results promised by the small value of $\alpha_s(M_Z) \approx 1/8.44$
- There has been an astonishing development of theoretical tools, both software and new ideas which help us in the task.