Convexity, gauge-dependence, and tunneling rates

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The effective action

The effective action Γ encodes the quantum dynamics of expectation values of scalar fields in the presence of sources.

$$\bar{\phi} \equiv \langle \phi \rangle_J, \quad \Gamma[\bar{\phi}] = \int d^D x \left[\mathcal{O}(\partial \bar{\phi}) - V_{\text{eff}}(\bar{\phi}) \right]$$

$$= \sum (\text{1PI diagrams}),$$

$$J = \frac{\delta \Gamma[\bar{\phi}]}{\delta \bar{\phi}}.$$

Its zero-momentum part, the effective potential, determines the vacuum structure of the theory.

$$J = 0, \partial \bar{\phi} = 0 \Rightarrow \frac{\partial V_{\text{eff}}(\bar{\phi})}{\partial \bar{\phi}} = 0.$$

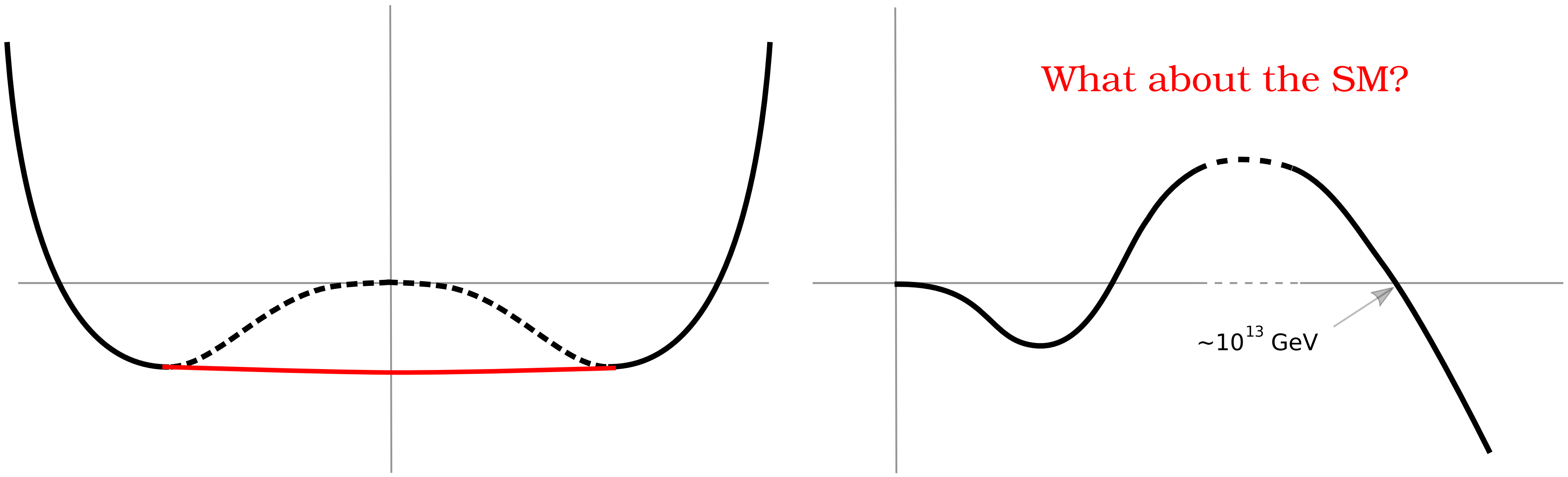
Troubling issues with the effective action

It is gauge-dependent! [Jackiw]

How can one extract gauge-independent information?

It is a convex functional, meaning the effective potential is concave (positive second derivative everywhere) [Iliopoulos, Itzykson, Martin].

How can one talk about false vacua and tunnelling rates?



Gauge-dependence: Nielsen identities

The gauge dependence of the effective action is encoded by Nielsen identities [Nielsen, Kugo, Fukuda, ...]

$$\xi \frac{\partial \Gamma}{\partial \xi}[\bar{\phi};\xi] + \Gamma_{,j}[\bar{\phi};\xi] K^j[\bar{\phi},\xi] = 0,$$

Gauge dependence ⟷ Nonlocal field redefinition

At vacua,

$$\Gamma_{,j} = 0 \to \xi \frac{\partial \Gamma}{\partial \xi}[\bar{\phi};\xi] = 0.$$

Energies and masses defined from V_{eff} at the vacuum configurations remain independent of the choice of gauge.

What about tunneling rates?

Tunneling rates

Tunneling à la Callan-Coleman

$$\gamma = \det(\text{Fluctuations})e^{-S_E[\varphi_b]}.$$

φ_b : Classical solution to Euclidean equations of motion, using $V_{clas} \neq V_{eff}$, with boundary conditions set by false vacuum

What if false vacuum or instability is generated radiatively (e.g. SM)?

V_{eff} suspected/assumed to play a role (e.g. SM)

V_{eff} should be convex (no false vacua!!)

How to avoid double-counting of fluctuations?

No formal proof of gauge-independence at all orders. [Previous work by Metaxas and Weinberg, Garny and Konstandin,...]

φ_b depends on the potential between minima, known to be gauge-dependent.

True-vacuum effective action

True-vacuum partition function Z[J]:

Identity insertions

$$Z[J] = \exp iW[J] = \lim_{T\to\infty} \langle 0|e^{-iHT}|0\rangle^J$$

$$= \lim_{T\to\infty} \int [dq][dq'] \langle 0|q\rangle^J \langle q|e^{-iHT}|q'\rangle^J \langle q'|0\rangle^J$$

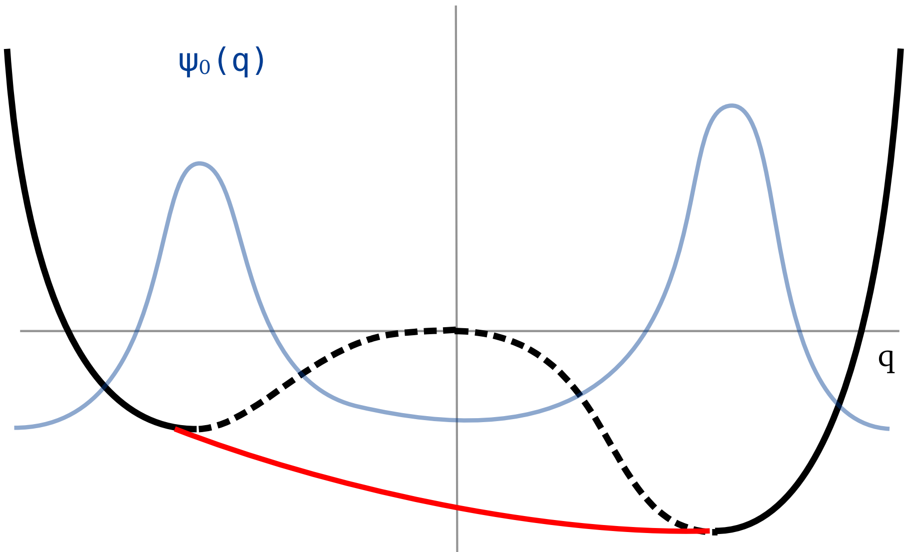
$$= \int [dq][dq'] \psi_0^J(q')\psi_0^{J\star}(q) \int_{q'}^{q} [d\phi] \exp i\left[\tilde{S}_g[\phi;\xi] + J_j\phi^j\right]$$

Usual single path integral

$$\Psi_0^J(q) \equiv \langle q|0\rangle^J$$

Vacuum wave function, usually ignored!
Expected to peak around local vacua in field-space

True-vacuum effective action



Multi-peak structure of the wave function means that Z[J] can be approximated by a sum of path integrals

$$Z[J] \approx \sum_{m,n=1}^{N} Z^{m,n}[J], \quad Z^{m,n}[J] = \mathcal{N}_{mn} \int_{q_0^{J,m}}^{q_0^{J,n}} [d\phi] \exp i \left[\tilde{S}_g[\phi;\xi] + J_j \phi^j \right]$$

Same for the true-vacuum effective action, explaining multi-path integral constructions of concave effective potentials! [Fujimoto et al, Bender et al,...]

$$\bar{\phi} = -\, i \frac{\delta \log Z[J]}{\delta J}, \;\; \Gamma[\bar{\phi}] = -i \log Z[J] - J_j\, \bar{\phi}^j\,,$$

$$\exp i\Gamma[\bar{\phi};\xi] = \sum_{m,n=1}^{N} \mathcal{N}_{mn} \int_{q_0^{J,m} - \bar{\phi}_\infty}^{q_0^{J,n} - \bar{\phi}_\infty} [d\phi]\, \exp i \left[S_g[\bar{\phi},\phi;\xi] - \Gamma_{,j}[\bar{\phi};\xi]\phi^j \right]$$

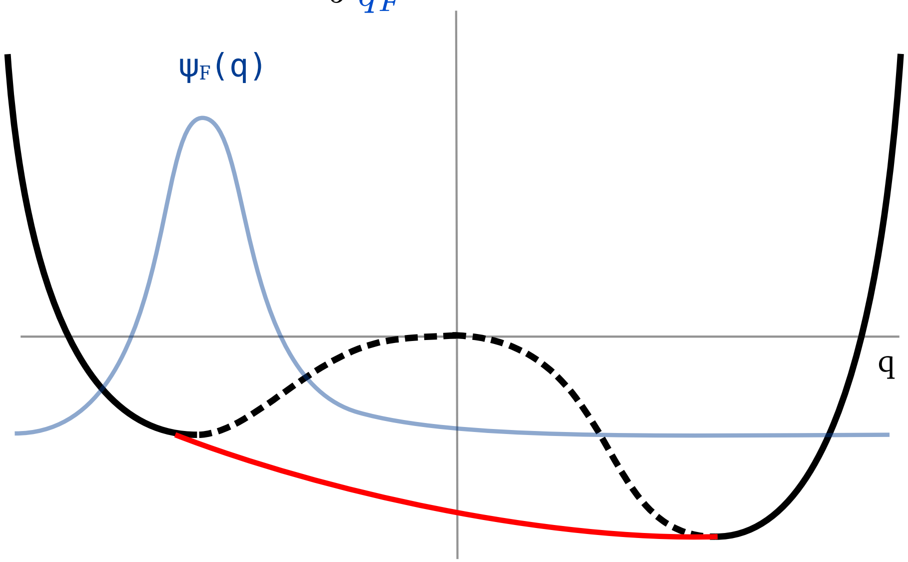
False-vacuum effective action

By definition, a false vacuum state $|F\rangle$ has a localized field-space wave-function. Thus one can define a partition function which can be approximated by a single path integral.

$$Z_F[J] = \lim_{T\to\infty} \langle F|e^{-iHT}|F\rangle^J = e^{-i\epsilon VT}$$

$$= \int [dq][dq']\psi_F^J(q')\psi_F^{J\star}(q) \int_{q'}^{q} [d\phi] \exp i \left[\tilde{S}_g[\phi;\xi] + J_j\phi^j\right]$$

$$\approx \int_{q_F}^{q_F} [d\phi] \exp i \left[\tilde{S}_g[\phi;\xi] + J_j\phi^j\right]$$



The vacuum being unstable, Z_F has an imaginary part related with the decay rate.

$$\gamma = -2\,\mathrm{Im}\epsilon = -\frac{2}{VT}\,\mathrm{Re}\,(\log Z_F[0]).$$

False-vacuum effective action

One can construct a false vacuum effective action, which will be:

Approximated by a single path integral

Complex, not convex!

$$\bar\phi = -\,i\frac{\delta \log Z_F[J]}{\delta J}, \quad \Gamma_F[\bar\phi] = \overbrace{-i\log Z_F[J]}^{-\epsilon VT,\ \text{complex!}} - J_j\,\bar\phi^j,$$

$$\exp i\Gamma_F[\bar\phi;\xi] = \int_0^0 [d\phi]\ \exp i\left[S_g[\bar\phi,\phi;\xi] - \Gamma_{F,j}[\bar\phi;\xi]\phi^j\right]$$

This is essentially the usual effective action used e.g. in the SM!

The resulting effective potential is complex, not necessarily convex , explaining why one can see radiatively generated false-vacua or instabilities (e.g. SM).

Tunneling and gauge-independence

The gauge dependence of Γ_F is encoded by its Nielsen identities. In particular, $\Gamma_F[\varphi]$ is gauge-independent if φ solves the quantum equations of motion

$$\Gamma_{F,i}[\varphi(\xi);\xi] = 0 = -J_i \Rightarrow \xi\frac{\partial\Gamma_F}{\partial\xi}[\varphi(\xi);\xi] = 0.$$

The tunneling rate is related to the effective action evaluated at a solution to the quantum equations of motion! Gauge-independence follows immediately from the Nielsen identities.

$$\left.\begin{array}{l} \gamma = -\frac{2}{VT}\,\mathrm{Re}\left(\log Z_F[0]\right) \\ \Gamma_F[\bar\phi] = -i\log Z_F[J] - J_i\bar\phi^i \end{array}\right\} \quad J_i = 0 \Rightarrow \ \Gamma_F[\varphi] = -i\log Z_F[0]$$

$$\gamma = \frac{2}{VT}\,\mathrm{Im}\,\Gamma_F[\varphi_F(\xi);\xi].$$

Which solution exactly?

From the definition of γ in terms of $Z_F[0]$, being careful with the boundary conditions, we get a result that generalizes Callan and Coleman's sum over multiple bounce solutions. After rotating to Euclidean space:

$$\gamma = \frac{2}{VT^E}\,\mathrm{Im}\; e^{-\Gamma_F^E[\varphi^{1,E}(\xi);\xi]}.$$

[see also Garbrecht, Millington in theories without gauge fields]

Single quantum bounce solution, with boundary conditions fixed by the false vacuum.

The exponential of the classical Euclidean action recovered in the semiclassical limit, $\Gamma_F = S_{cl} + O(\hbar)$.

The formula includes all quantum corrections. Clarifies how to compute tunneling rates with radiatively generated vacua or instabilities.

Summary

We clarified issues of convexity of the effective action and gauge-independence of tunneling rates.

We introduced the notion of false-vacuum effective action. In contrast to the true-vacuum effective potential, the false-vacuum one is neither real nor convex.

False vacua and tunneling rates can only be defined and found with the false-vacuum effective action.

Tunneling rates are related to the exponential of the false-vacuum effective action evaluated at a generalized bounce solution. This encodes all quantum corrections and clarifies how to compute tunneling rates with radiatively generated vacua/instabilities (e.g. SM).

Our result stresses the role of the imaginary part of Γ_F (and V_{feff}).

First formal proof of gauge-independence of tunneling rates.