# Perturbative Unitarity Bound In Composite Two Higgs Doublet Models

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Based on: [arXiv:1602.06437v1]

Joint Annual HEPP and APP Conference, Brighton

22nd March 2016

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# The idea of composite two Higgs doublet model

- Higgs boson emerges as a pseudo-Nambu-Goldstone Boson (pNGB) from a new strong interaction at the compositeness scale $f$.
- The Composite 2 Higgs Doublet Model (C2HDM) based on $SO(6)/SO(4) \times SO(2)$ coset developing 8 pNGBs, which are identified with the (composite) *two Higgs doublet fields*.
- Symmetry breaking occurs in two steps
  1. Spontaneously global symmetry breaking $SO(6) \xrightarrow{f} SO(4) \times SO(2)$ at scale $f$.
  2. Electroweak symmetry breaking is triggered by coupling of the SM particles to the composite sector via the Coleman-Weinberg (CW) potential at loop levels.
- Minimal composite Higgs model (with a single Higgs doublet) can explain hierarchy problem by its pNGB nature. It's remarkable motivation to study C2HDM for describing presence of extra Higgs particles as pNGBs and explain their mass differences.

# Effective Lagrangian approach for C2HDM

$\Rightarrow$ The $SO(6)$ invariant effective kinetic Lagrangian, can be constructed by the analogue of the construction in non-linear sigma models developed by Callan-Coleman-Wess-Zumino (CCWZ) as

$$\mathcal{L}_{kin} = \frac{f^2}{4}(d_\alpha^{\hat{a}})_\mu (d_\alpha^{\hat{a}})^\mu \quad (d_\alpha^{\hat{a}})_\mu = i\ tr(U^\dagger D_\mu U T_\alpha^{\hat{a}}), \text{ where } \alpha = 1, 2.\ \hat{a} = 1, 4$$

$$U{=}\exp(i\tfrac{\Pi}{f}),\ \Pi{\equiv}\sqrt{2}h_\alpha^{\hat{a}}T_\alpha^{\hat{a}}{=}{-i}\begin{pmatrix} O_{4\times 4} & h_1^{\hat{a}} & h_2^{\hat{a}} \\ -h_1^{\hat{a}} & 0 & 0 \\ -h_2^{\hat{a}} & 0 & 0 \end{pmatrix},\ \Phi_\alpha{\equiv}\tfrac{1}{\sqrt{2}}\begin{pmatrix} h_\alpha^2 + ih_\alpha^1 \\ h_\alpha^4 - ih_\alpha^3 \end{pmatrix}{\equiv}\begin{pmatrix} \phi_\alpha^+ \\ \phi_\alpha^0 \end{pmatrix}$$

$$i(d_\alpha^{\hat{1}})_\mu + (d_\alpha^{\hat{2}})_\mu = -\frac{2}{f}\left[\partial_\mu\phi_\alpha^+ - i\frac{g}{\sqrt{2}}\phi_\alpha^0 W_\mu^+ - ig_Z\left(\frac{1}{2} - s_W^2\right)\phi_\alpha^+\right] + \mathcal{O}(1/f^3),$$

$$-i(d_\alpha^{\hat{3}})_\mu + (d_\alpha^{\hat{4}})_\mu = \frac{2}{f}\left[\partial_\mu\phi_\alpha^0 - i\frac{g}{\sqrt{2}}\phi_\alpha^+ W_\mu^- + i\frac{g_Z}{2}\phi_\alpha^0 Z_\mu\right] + \mathcal{O}(1/f^3).$$

⇒ Modified Higgs to gauge boson couplings from SM and E2HDM

$$\frac{\lambda^{C2HDM}_{hW^+W^-}}{\lambda^{SM}_{hW^+W^-}} = \sqrt{1-\xi}\cos\theta$$

$$\frac{\lambda^{C2HDM}_{HW^+W^-}}{\lambda^{E2HDM}_{HW^+W^-}} = \sqrt{1-\xi}$$

$$\frac{\lambda^{C2HDM}_{hH^+W^-}}{\lambda^{E2HDM}_{hH^+W^-}} = \sqrt{1-\frac{1}{6}\xi}\tan\theta$$

where $\xi = \frac{v^2}{f^2}$ with $v \simeq 246 GeV$, $\theta$ is mixing angle between CP-even states.

# Perturbative Unitarity

- Perturbative unitarity gives a bound on the parameters of the model.
- Equivalence theorem the replacement $W_L, Z_L -> G^{\pm} G^0$ gives the same amplitude up to $\mathcal{O}(s^0)$.
  (Cornwall, Lewin,Tiktopoulos (1974)
- We calculate the S-wave amplitude matrix for the all possible 2-to-2 body elastic scalar boson scatterings.
- $$\mathcal{M} = 16\pi \Sigma_{J=0}^{\infty} (2J+1) P_J(\cos\theta) a_J(s)$$

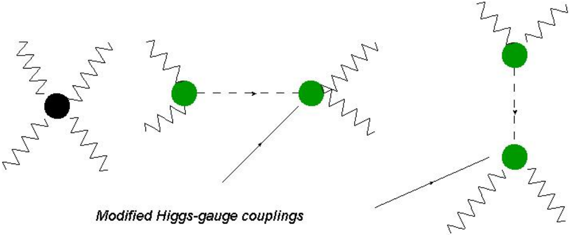
  Perturbative unitarity bound $\boxed{|a_J|^2 \leq \frac{1}{2}}$

# Perturbative Unitarity violated

$\Rightarrow$ $A(V_L V_L \to V_L V_L)$ grows with energy due to modified $hV_L V_L$, unitarity is lost in the C2HDM.

$$\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-)_{\text{Higgs}}$$
$$= -\frac{s}{2v_{\text{SM}}^2}(1 - c_\phi)(1 - \xi) - \frac{2}{v_{\text{SM}}^2}(1 - \xi)(m_h^2 c_\theta^2 + m_H^2 s_\theta^2) + \mathcal{O}(s^{-1}),$$

where $\phi$ is the scattering angle.



Modified Higgs-gauge couplings

$\Rightarrow$ S-wave amplitude $a_0$ for $W_L W_L$ scattering:

$$a_0(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s}{32\pi v^2}\xi - \frac{1}{8\pi v^2}(m_h^2 \cos^2\theta + m_H^2 \sin^2\theta)(1-\xi) \leq \frac{1}{2}$$

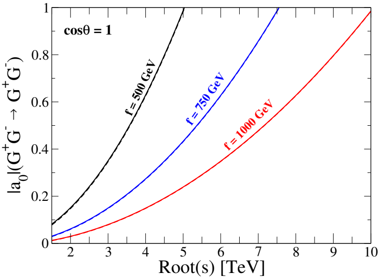



Fig: $S$-wave amplitude for the $G^+G^- \to G^+G^-$ process as a function of $\sqrt{s}$ in the case of $\cos\theta = 1$ and $f = 500$ GeV (black), 750 GeV (blue), 1000 GeV (red). The solid (dashed) curves are the result with (without) $\mathcal{O}(\xi s^0)$ term.

$$\mathcal{M}(H^+H^- \to H^+H^-) = \left[\boxed{\frac{s}{2\upsilon_{SM}^2}\xi(1+c_\phi)} - \boxed{\frac{m_{H^\pm}^2}{\upsilon_{SM}^2}\xi(\frac{2}{3}+4c_\phi)} + \boxed{\lambda_{H^+H^-H^+H^-}}\right] + \mathcal{O}(s^{-1}).$$

Kinetic Term — Kinetic and Potential Term — Emerges From Potential Term

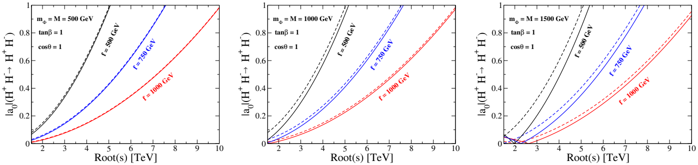



Figure : $S$-wave amplitude for the $H^+H^- \to H^+H^-$ process as a function of $\sqrt{s}$ in the case of $\cos\theta = 1$, $\tan\beta = 1$ and $f = 500$ (black), 750 (blue) and 1000 GeV (red). The solid (dashed) curves are the results with (without) $\mathcal{O}(\xi s^0)$ term. The left, center and right panels show the results for $m_\Phi (m_A = m_H = m_{H^\pm}) = M = 500$, 1000 and 1500 GeV, respectively.

$$\lambda_{H^+H^-H^+H^-} = [\frac{2}{\upsilon^2}4M^2\cot^2 2\beta - m_h^2(c_\theta + 2\cot 2\beta\sin\theta)^2) - m_H^2(s_\theta - 2\cot 2\beta c_\theta)^2](1-\frac{\xi}{3})$$
$$+\frac{4c_{2\beta}}{3\upsilon^2 s_{2\beta}^2}[m_h^2(c_\theta s_{2\beta} + 2s_\theta c_{2\beta})s_\theta + m_H^2(2c_\theta c_{2\beta} - s_\theta s_{2\beta})c_\theta]\xi.$$

# Perturbative Unitarity in $(H^+H^- \to H^+H^-)$ scattering II

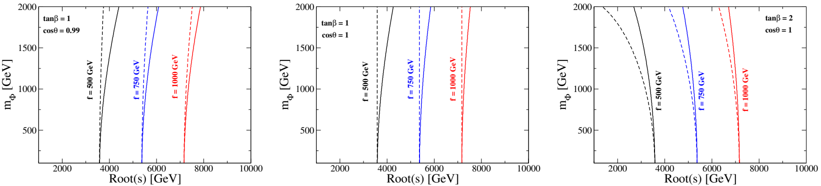



Figure : Unitarity bound on the $(\sqrt{s}\text{-}m_\Phi)$ plane from the requirement of $|a_0(H^+H^- \to H^+H^-)| < 1/2$ in the case of $M = m_\Phi$. In the left, center and right panels, we take $(\cos\theta, \tan\beta) = (1,1)$, $(0.99,1)$ and $(1,2)$, respectively. The solid (dashed) curves are the result with (without) $\mathcal{O}(\xi s^0)$ terms.

- According to the results we have shown so far, we can conclude that the $\mathcal{O}(\xi s^0)$ contributions are not so important as long as we consider the case $m_\phi \leq 1$ TeV and $\sqrt{s} \geq m_\phi$.

# Perturbative Unitarity in $(G^+G^- \to G^+G^-)$ process with and without $\mathcal{O}(1/s)$ term

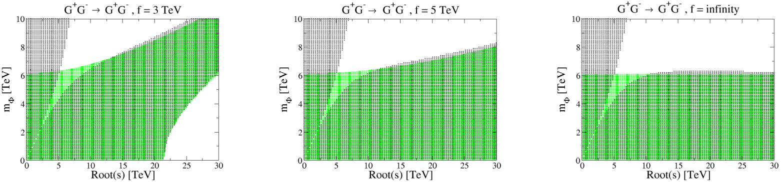



Figure : Allowed regions from perturbative unitarity in the plane $(\sqrt{s}, m_H)$ from $G^+G^- \to G^+G^-$ scattering amplitudes within the C2HDM. We take $\cos\theta = 0.99$, $\tan\beta = 1$ and $m_H = m_A = m_{H^\pm} = M$. The grey regions are obtained by using the exact formulae (with $\mathcal{O}(1/s)$ terms), the green ones by neglecting $\mathcal{O}(1/s)$ terms. The left, center and right panels show the cases with $f = 3000$ GeV, 5000 GeV and infinity (corresponding to the E2HDM).

- If we focus on the region of $\sqrt{s} \geq 1\,TeV$ and $m_\phi \leq 1\,TeV$, $\mathcal{O}(s^0\xi)$ and $\mathcal{O}(s^{-1})$ terms can be neglected safely.

# S-wave amplitude matrix for all the 2-to-2 body (pseudo)scalar boson scattering channels

⇒ We calculate all the two body scalar boson scattering amplitudes by keeping the $\mathcal{O}(\xi s)$ and $\mathcal{O}(\xi^0 s^0)$ contributions.

⇒ 14 neutral channels are expressed by

$$\boxed{G^+G^-, \frac{GG}{\sqrt{2}}, \frac{hh}{\sqrt{2}}, hG, H^+H^-, \frac{AA}{\sqrt{2}}, \frac{HH}{\sqrt{2}}, HA, hH, GA, hA, HG, G^+H^-, H^+G^-}$$

⇒ 8 singly charged channels are expressed by

$$\boxed{G^+Z, H^+A, G^+h, H^+h, G^+A, H^+Z, G^+H, H^+h}$$

⇒ 3-doubly charged channels are expressed by

$$\boxed{\frac{G^+G^+}{\sqrt{2}}, \frac{H^+H^+}{\sqrt{2}}, G^+H^+}$$

- Each of neutral, singly-charged and doubly-charged states respectively give the $14 \times 14, 8 \times 8$ and $3 \times 3$ S-wave amplitude matrix, they can be simplified to block diagonalized $2 \times 2$ sub-matrices according to their quantum numbers hyper-charge $Y$, isospin number $I$ and its third component $I_3$ and $Z_2$ charge of 2-to-2-body scattering states.

# Analytic formulae of all the independent eigenvalues values

$$16\pi a_1^{\pm} = \frac{3}{2}\frac{\xi s}{v_{SM}} - \frac{1}{2}[3(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{9(\lambda_1-\lambda_2)^2 + (\frac{\xi s}{v_{SM}} + 4\lambda_3 - 2\lambda_4)^2}],$$

$$16\pi a_2^{\pm} = -\frac{1}{2}\frac{\xi s}{v_{SM}} - \frac{1}{2}[(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1-\lambda_2)^2 + (\frac{\xi s}{v_{SM}} - 2\lambda_4)^2}],$$

$$16\pi a_3^{\pm} = \pm\frac{1}{2}\frac{\xi s}{v_{SM}} - \frac{1}{2}[(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1-\lambda_2)^2 + (\frac{\xi s}{v_{SM}} - 2\lambda_5)^2}],$$

$$16\pi a_4^{\pm} = \frac{\xi s}{v_{SM}} - (\lambda_3 + 2\lambda_4 \pm 3\lambda_5), \quad 16\pi a_5^{\pm} = \pm\frac{\xi s}{v_{SM}} - (\lambda_3 \mp \lambda_5),$$

$$16\pi a_6^{\pm} = \pm\frac{\xi s}{v_{SM}} - (\lambda_3 \mp \lambda_5).$$

$\Rightarrow$The eigenvalues listed above give the constraints $|a_i^{\pm}| \leq 8\pi$

# Constraint on the parameter space of the C2HDM

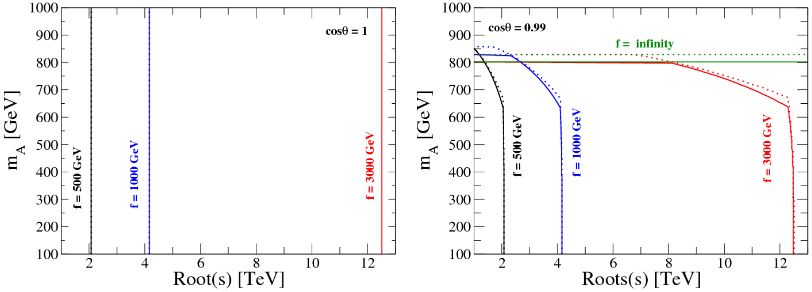



Figure : Constraint on the parameter space of the C2HDM from the unitarity and the vacuum stability in the case of $\tan\beta = 1$ and $m_{H^\pm} = m_A$ for several fixed values of $f$. The left and right panels show the case with $\cos\theta = 1$ and 0.99, respectively. The lower left region from each curve is allowed. We take the value of $m_H$ to be equal to $m_A$ for the solid curves, while we scan it within the region of $m_A \pm 500$ GeV for the dashed curves. For all the plots, $M$ is scanned.

# Constraint on $m_A$—$\tan\beta$ plane

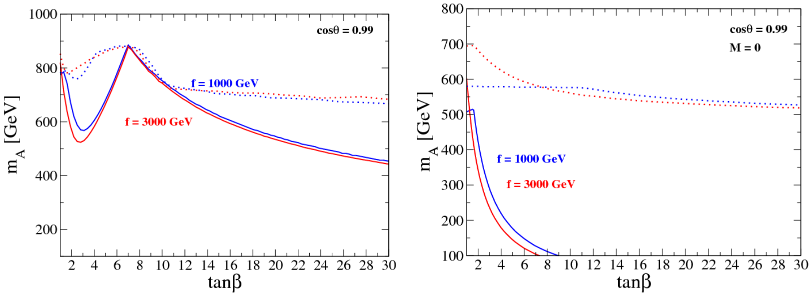



Figure : Constraint on the parameter space on the ($\tan\beta$-$m_A$) plane from the unitarity and the vacuum stability in the case of $\cos\theta = 0.99$, $\sqrt{s} = 3000$ GeV and $m_{H^\pm} = m_A$ for $f = 1000$ GeV (blue) and $f = 3000$ GeV (red). The lower left region from each curve is allowed. The left panel shows the case with $M$ to be scanned, while the right one does the case with $M = 0$. We take the value of $m_H$ to be equal to $m_A$ for the solid curves, while we scan it within the region of $m_A \pm 500$ GeV for the dashed curves.

# Unitarity and vacuum stability in the inert case

⇒ The case of second doublet without vacuum expectation value (VEV)

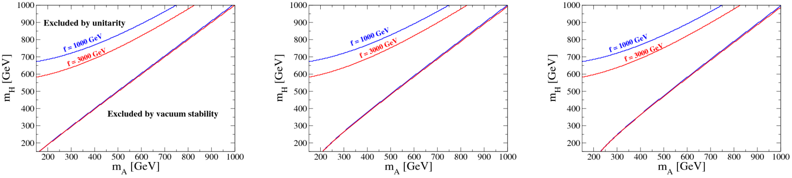



Figure : Constraint on the parameter space on the $m_A$-$m_H$ plane by unitarity and vacuum stability in the inert case of $m_{H\pm} = m_A = m_2$ and $\sqrt{s} = 3000$ GeV. We take $\lambda_2 = 0.1$, 2 and 4 in the left, center and right panels, respectively.

- We have considered $h$ as the lightest Higgs, but a choice of parameters leading to a different mass spectrum is possible. For example, we have checked $m_H = m_2 = 100 GeV$ the upper limit from unitarity on $m_A (= m_{H\pm})$ is about 700 GeV. So, a dark matter motivated scenario is available in this work.

# Conclusion

- We have explicitly shown that the amplitude grows with $\sqrt{s}$ in scattering processes, so that unitarity is broken at a certain energy scale depending on the scale $f$.
- We have discovered significant differences of the allowed parameter space in E2HDM and C2HDM that can be exploited in order to separate phenomenologically the two Higgs scenarios.
  - For $\cos\theta = 1, h = 125$ GeV and $m_H = m_{H^\pm} = m_A$ case, we got upper limit on $\sqrt{s}$ under the scan of $M^2$ e.g, $\sqrt{s} \leq 2, 4$ and 13 TeV for the case of $f = 500, 1000$ and 3000 GeV respectively.
  - For $\cos\theta = 0.99$, we got the upper limit not only on $\sqrt{s}$ but also on $m_\phi$ which can be $\mathcal{O}(1)\, TeV$.
- A through investigation of the Higgs mass patterns that may arise at the LHC could allow us to find hints of a C2HDM hypothesis and distinguish it from E2HDM one.

***Thank You!***

Potential structure

$$
\begin{aligned}
V(\Phi_1, \Phi_2) = {} & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \quad + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \\
& \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
& + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2}\lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}].
\end{aligned}
$$

# Generic Formulae for the 2-to-2-body (Pseudo) Scalar Boson Scatterings

$$\begin{aligned}
\mathcal{M}_c(AB \to CD) &= -(g_{AB,CD}\, p_{AB} + g_{CD,AB}\, p_{CD}) \\
&\quad + g_{AC,BD}\, p_{AC} + g_{BD,AC}\, p_{BD} + g_{AD,BC}\, p_{AD} + g_{BC,AD}\, p_{BC} + \lambda_{ABCD}, \\
\mathcal{M}_s(AB \to X \to CD) &= -\frac{1}{s - m_X^2}\left(g_{XA,B}\, p_{XA} + g_{BX,A}\, p_{BX} - g_{AB,X}\, p_{AB} + \lambda_{ABX}\right) \\
&\quad \times \left(g_{XC,D}\, p_{XC} + g_{DX,C}\, p_{DX} - g_{CD,X}\, p_{CD} + \lambda_{CDX}\right), \\
\mathcal{M}_t(AB \to X \to CD) &= -\frac{1}{t - m_X^2}\left(g_{AC,X}\, p_{AC} + g_{XA,C}\, p_{XA} - g_{CX,A}\, p_{CX} + \lambda_{ACX}\right) \\
&\quad \times \left(g_{BD,X}\, p_{BD} - g_{XB,D}\, p_{XB} + g_{DX,B}\, p_{DX} + \lambda_{BDX}\right), \\
\mathcal{M}_u(AB \to X \to CD) &= -\frac{1}{u - m_X^2}\left(g_{AD,X}\, p_{AD} + g_{XA,D}\, p_{XA} - g_{DX,A}\, p_{DX} + \lambda_{ADX}\right) \\
&\quad \times \left(g_{BC,X}\, p_{BC} - g_{XB,C}\, p_{XB} + g_{CX,B}\, p_{CX} + \lambda_{BCX}\right), \\
g_{ab,cd} \equiv \frac{\partial^4 \mathcal{L}_{kin}}{\partial(\partial_\mu a)\partial(\partial_\mu b)\partial(c)\partial(d)}&, \quad g_{ab,c} \equiv \frac{\partial^3 \mathcal{L}_{kin}}{\partial(\partial_\mu a)\partial(\partial_\mu b)\partial(c)}, \\
\lambda_{abcd} \equiv -\frac{\partial^4 V}{\partial a \partial b \partial c \partial d}&\quad \lambda_{abc} \equiv -\frac{\partial^3 V}{\partial a \partial b \partial c}
\end{aligned}$$

⇒**Approximate formulae** for S-wave amplitude in $G^+G^- \to G^+G^-$

$$a_0(G^+G^- \to G^+G^-) = \frac{s}{32\pi v_{\rm SM}^2}\xi - \frac{1}{8\pi v_{\rm SM}^2}(m_h^2 c_\theta^2 + m_H^2 s_\theta^2)(1-\xi) + \mathcal{O}(g^2, s^{-1}).$$

⇒ **Exact formulae** for S-wave amplitude in $G^+G^- \to G^+G^-$

$$\mathcal{M}_c(G^+G^- \to G^+G^-) = \frac{s}{2}(1-c_\phi)(g_{G^\pm G^\pm, G^\mp G^\mp} - g_{G^+G^-,G^+G^-}) + \lambda_{G^+G^-G^+G^-}$$
$$= \frac{s}{2\upsilon_{\rm SM}^2}(1-c_\phi)\xi - \frac{2}{\upsilon_{\rm SM}^2}(m_h^2 c_\theta^2 + m_H^2 s_\theta^2)\left(1+\frac{\xi}{3}\right),$$

$$\mathcal{M}_s(G^+G^- \to G^+G^-) = -\sum_{\phi=h,H}\frac{1}{s-m_\phi^2}\left[\frac{s}{2}(2g_{G^\pm\phi,G^\mp} - g_{G^+G^-,\phi}) + \lambda_{G^+G^-\phi}\right]^2$$
$$= \frac{4}{3\upsilon_{\rm SM}^2}\left(m_h^2 c_\theta^2 + m_H^2 s_\theta^2\right)\xi + \mathcal{O}(s^{-1}),$$

$$\mathcal{M}_t(G^+G^- \to G^+G^-) = -\sum_{\phi=h,H}\frac{1}{t-m_\phi^2}\left[\frac{t}{2}(2g_{G^\pm\phi,G^\mp} - g_{G^+G^-,\phi}) + \lambda_{G^+G^-\phi}\right]^2$$
$$= \frac{4}{3\upsilon_{\rm SM}^2}\left(m_h^2 c_\theta^2 + m_H^2 s_\theta^2\right)\xi + \mathcal{O}(s^{-1}).$$

$$\lambda_{G^+G^-G^+G^-} = -\frac{2}{\upsilon_{\text{SM}}^2}\left(1+\frac{\xi}{3}\right)(m_h^2 c_\theta^2 + m_H^2 s_\theta^2),$$

$$\lambda_{G^+G^-h} = -\frac{m_h^2}{\upsilon_{\text{SM}}}\left(1+\frac{\xi}{6}\right)c_\theta, \quad \lambda_{G^+G^-H} = \frac{m_H^2}{\upsilon_{\text{SM}}}\left(1+\frac{\xi}{6}\right)s_\theta,$$

$$g_{G^+G^-,G^+G^-} = -\frac{\xi}{3v_{\text{SM}}^2}, \quad g_{G^\pm G^\pm,G^\mp G^\mp} = \frac{2\xi}{3v_{\text{SM}}^2},$$

$$g_{G^+G^-,h} = -\frac{2\xi}{3\upsilon_{\text{SM}}}c_\theta, \quad g_{G^\pm h,G^\mp} = \frac{\xi}{3\upsilon_{\text{SM}}}c_\theta,$$

$$g_{G^+G^-,H} = \frac{2\xi}{3\upsilon_{\text{SM}}}s_\theta, \quad g_{G^\pm H,G^\mp} = -\frac{\xi}{3\upsilon_{\text{SM}}}s_\theta.$$