

# Comparisons of Binned vs. Unbinned Likelihood Analyses for Neutrino Oscillation Measurements in NOvA

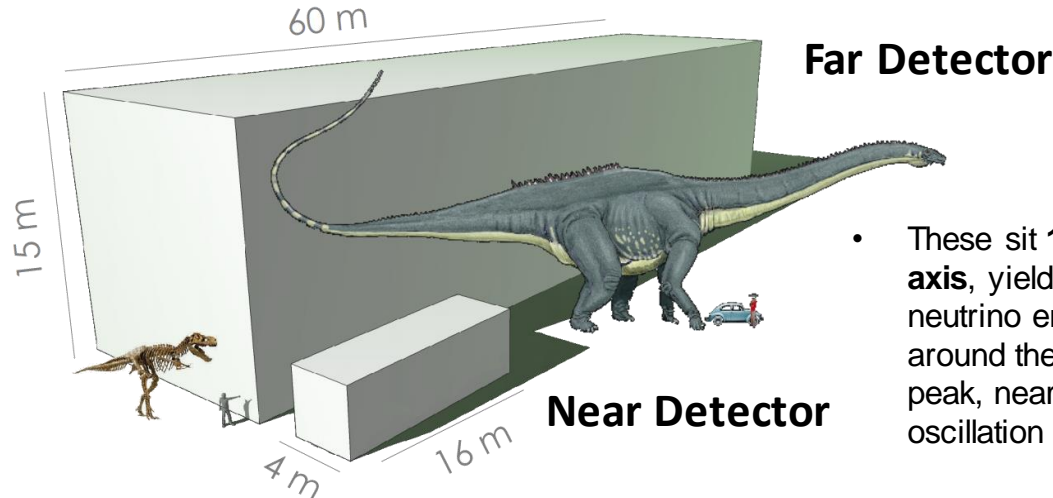
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Sebastian Bending

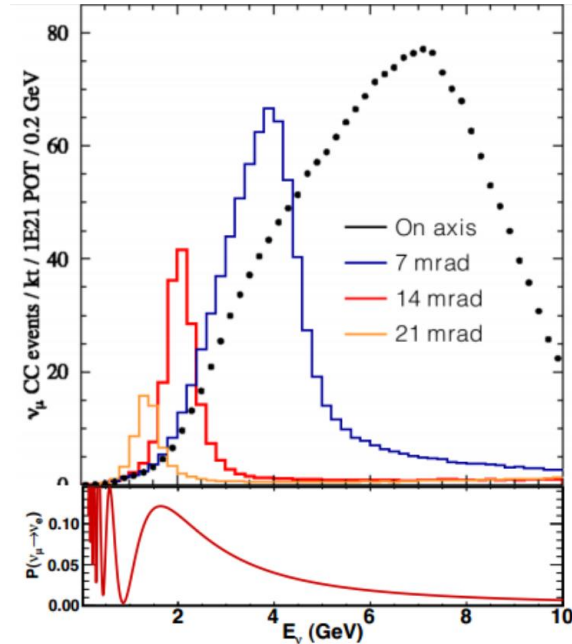
IOP Joint APP and HEPP Annual Conference  
26th March, 2018

# What is NOvA?

- Long-baseline neutrino experiment investigating the **appearance** and **disappearance** of **electron** and **muon neutrinos**, respectively, from the NuMI beamline at Fermilab.
- NuMI beam sends neutrinos (or antineutrinos) through two **functionality identical detectors** located 810km apart.



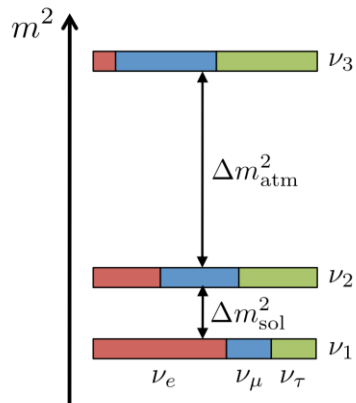
- These sit **14.6 mrad off-axis**, yielding a narrow neutrino energy band around the 2 GeV beam peak, near the 1.6 GeV oscillation maximum.



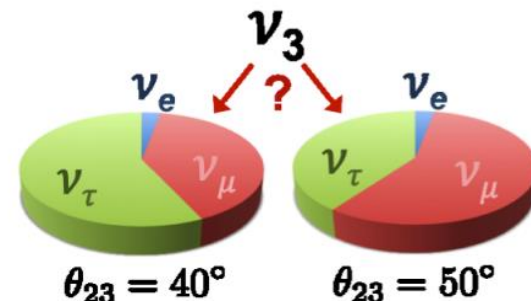
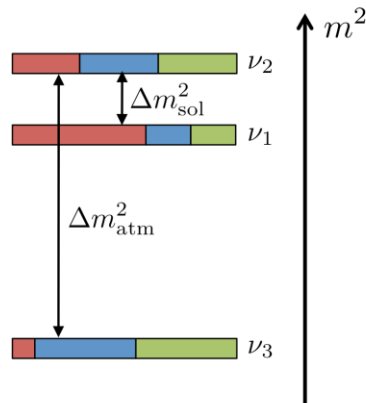
# Physics Goals

- Make **precision measurements** of neutrino oscillation parameters  $\theta_{23}$  and  $\Delta m_{32}^2$  for neutrinos and antineutrinos.

normal hierarchy (NH)



inverted hierarchy (IH)



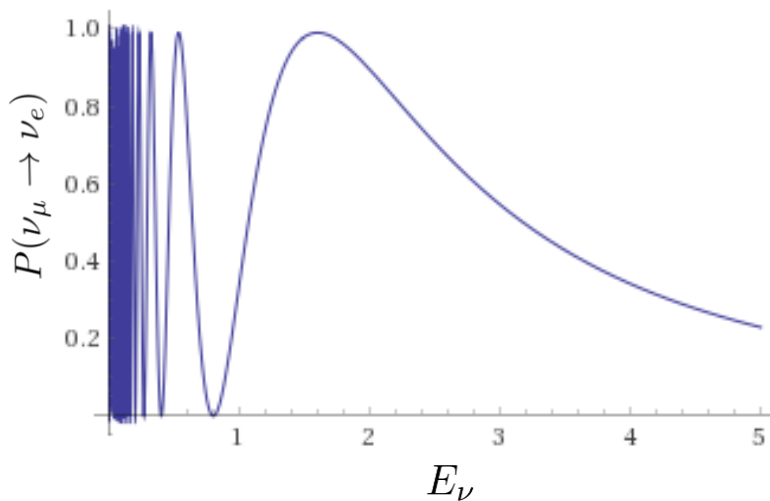
- Can constrain several other physics parameters:

$\theta_{23}$  octant

$\delta_{CP}$  CP-violating phase

neutrino mass hierarchy

# Why fit with unbinned data?



- **Oscillation probability varies greatly** as a function of energy so with binned data information may be lost as oscillation probabilities change across a bin width.
- With unbinned data we can squeeze more information from it and potentially increase our **experimental sensitivities**.
- Trade-off here is between **statistics** and **precision**.

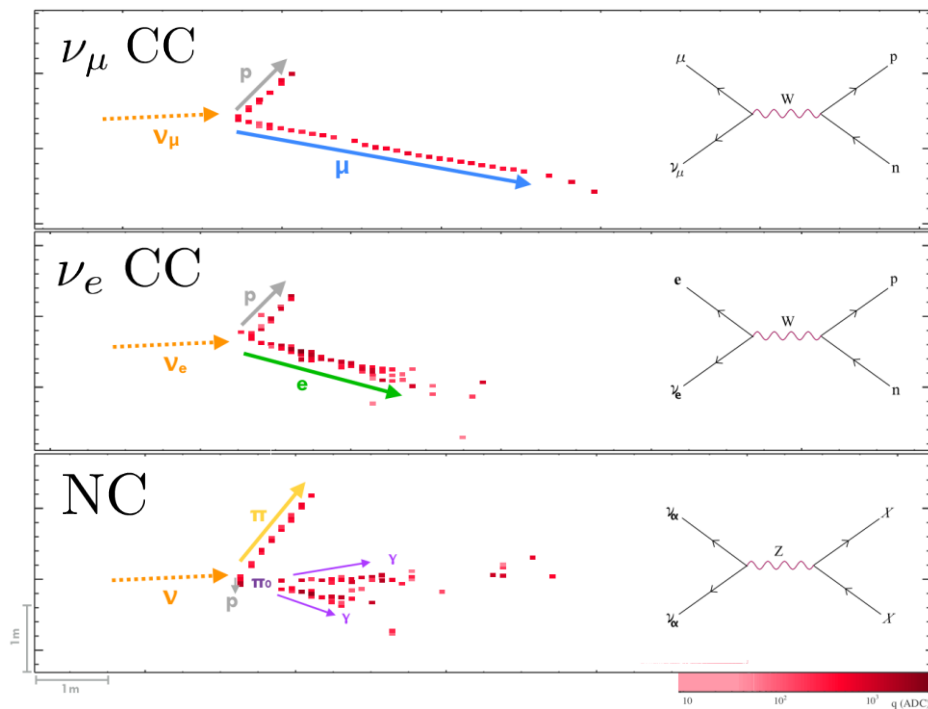
# FNEX (Far/Near EXtrapolation)

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- FNEX is the analysis framework used in this study, which differs to the one used in the official NOvA analyses.
- Performs analyses on an **event-by-event basis** rather than propagating histograms through the framework.
- This makes it ideal for performing unbinned fits since all event information is already present.



# NOvA Signal Selection



There are a number of criteria by which signal events for each analysis are selected:

- Events must pass **basic quality cuts** and must be **contained** within the detector.
- **Deep learning** and **computer vision** techniques are applied to select desired signal events.
- **Cosmic rejection** is applied to remove cosmic ray muons.
- Remaining events divided up into bins of **hadronic energy fraction** ( $\nu_\mu$  disappearance) or **convolutional visual network PID** ( $\nu_e$  appearance).

# NOvA Extrapolation

- In FNEX the **predicted reconstructed neutrino energy spectrum** at the Far Detector is acquired through **extrapolation** of the Near Detector spectrum on an **event-by-event basis**.
- Monte Carlo events are scaled in the Far Detector - first by the **Near Detector data / MC ratio** and then, for signal events, by the **oscillation probability associated with the true energy** of that event.

$$F_j^{Predicted} = \frac{N_i^{Data}}{N_i^{MC}} \times \sum_j F_j^{MC} (P_{\nu_\mu \rightarrow \nu_\mu}(E_\nu^{True}))$$

- Here  $N_i^{Data}$  is the number of ND data events in reconstructed energy bin  $i$ ,  $N_i^{MC}$  is the number of ND Monte Carlo simulated events in that bin and  $F_j^{MC}$  is the  $j$ th Monte Carlo simulated event in reconstructed energy bin  $i$  with  $E_j^{True}$  in the FD.  $F_j^{MC}$  is a function of the neutrino oscillation probability,  $P_{\nu_\mu \rightarrow \nu_\mu}$ .

# Unbinned Likelihood Fitting Implementation

- Use **unbinned extended maximum likelihood fit**\*.
- This takes into account both the **shape** and **normalisation** of the neutrino energy distribution:

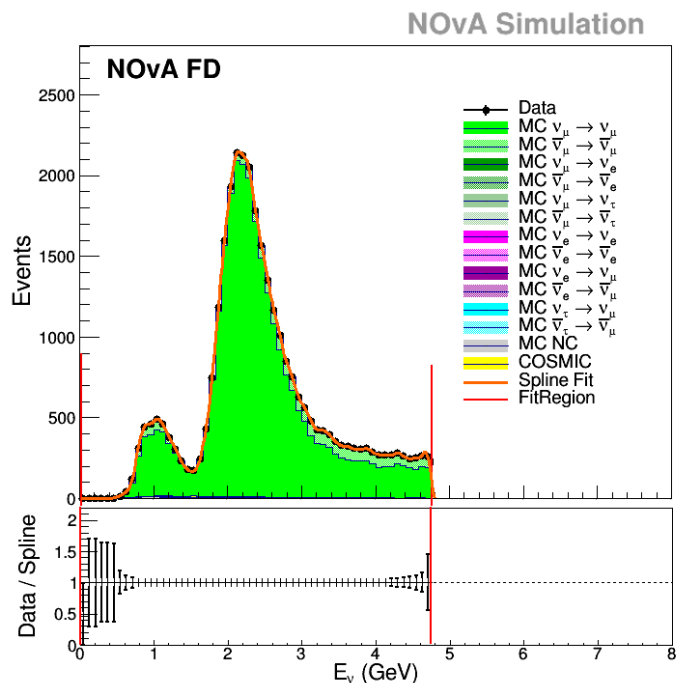
$$\ln \mathcal{L} = \sum_i \ln P(\underbrace{x_i}_{\text{red}}, \underbrace{a_j, \dots, a_n}_{\text{green}}) - \underbrace{\mathcal{N}}_{\text{blue}}(\underbrace{a_j, \dots, a_n}_{\text{green}})$$

- Here  $x_i$  are the **reconstructed neutrino energy** of each event in the data,  $a_j, \dots, a_n$  are the **neutrino oscillation parameters** floated in the fit, and  $\mathcal{N}$ , the normalisation term, is the **expected number of events** in the fit range.

\*R. Barlow, *Extended Maximum Likelihood*

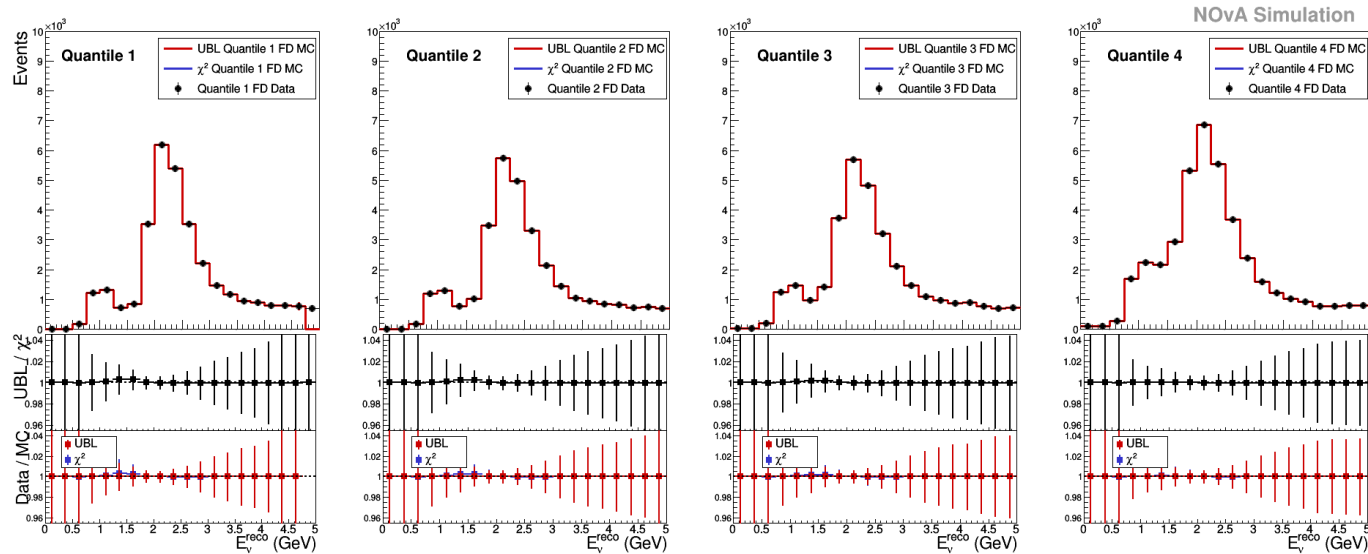


# Unbinned Likelihood Fitting Implementation



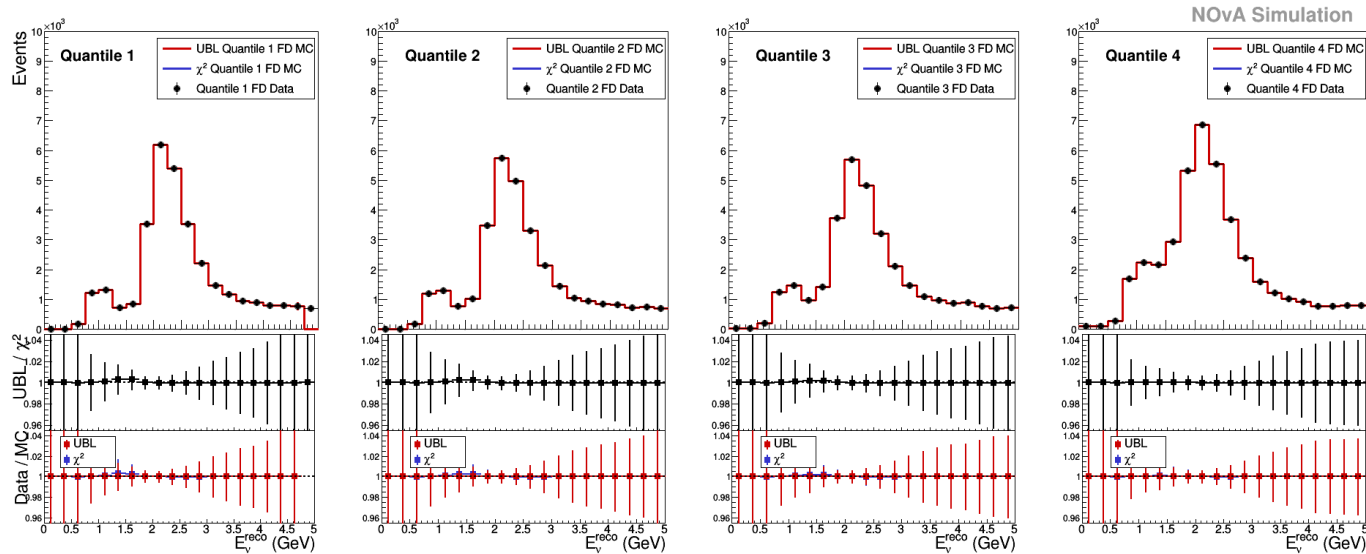
- Functions  $P(x_i; a_j, \dots, a_n)$  and  $\mathcal{N}(a_j, \dots, a_n)$  acquired by fitting a **smooth spline** to the **predicted neutrino energy distribution** for a given set of oscillation parameters.
- From this the **likelihood** of each event occurring according to these parameters is evaluated.
- **MINUIT minimisation** performed to obtain oscillation parameters with best fit.

# $\nu_{\mu}$ disappearance sensitivity best fit results (FD)



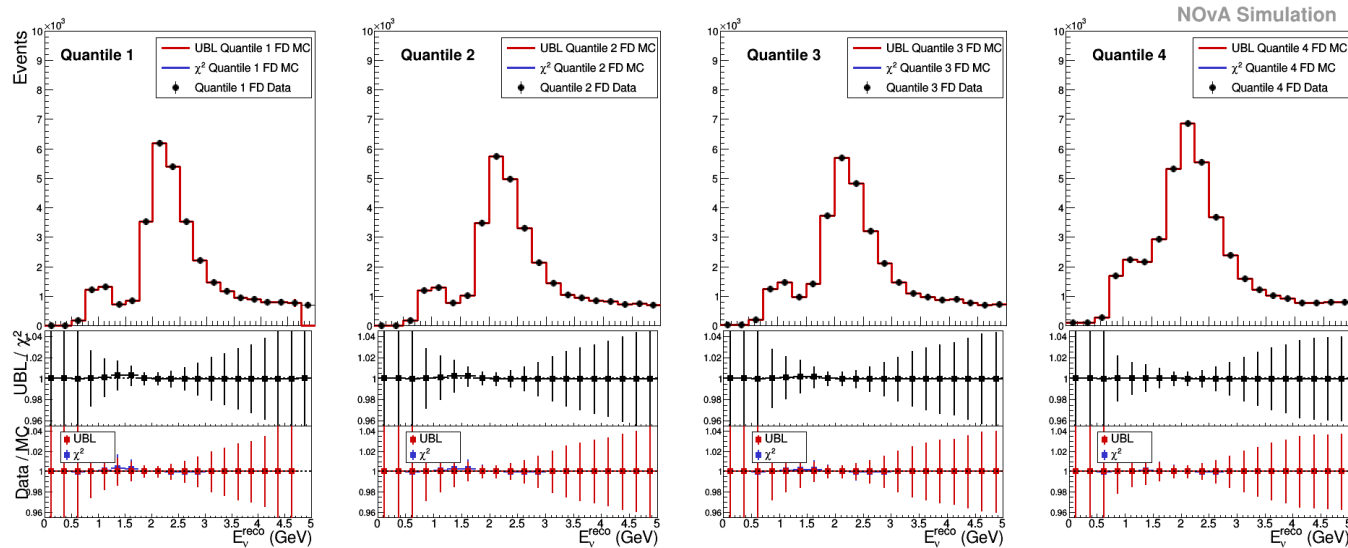
- **FD reconstructed neutrino energy spectra at best fit points found by UBL (red) and  $\chi^2$  (blue) fitting methods when fitting to high statistics fake data.**
- **Divided up into quantiles of hadronic energy fraction (in decreasing energy resolution from quantiles 1 through 4).**

# $\nu_\mu$ disappearance sensitivity best fit results (FD)



- Middle ratio plot shows the ratio of the **UBL MC best fit** to the  $\chi^2$  **MC best fit**.
- Bottom ratio is the ratio of the fake data to the UBL (**red**) and  $\chi^2$  (**blue**) fitted MC distributions.

# $\nu_\mu$ disappearance sensitivity best fit results (FD)



NOvA Simulation

## Fit values:

Fake data input:

$$\sin^2(\theta_{23}) = 0.558$$

$$\Delta m_{32}^2 = 2.444 \times 10^{-3}$$

UBL best fit point:

$$\sin^2(\theta_{23}) = 0.558 \pm 0.001$$

$$\Delta m_{32}^2 = (2.444 \pm 0.002) \times 10^{-3}$$

$\chi^2$  best fit point:

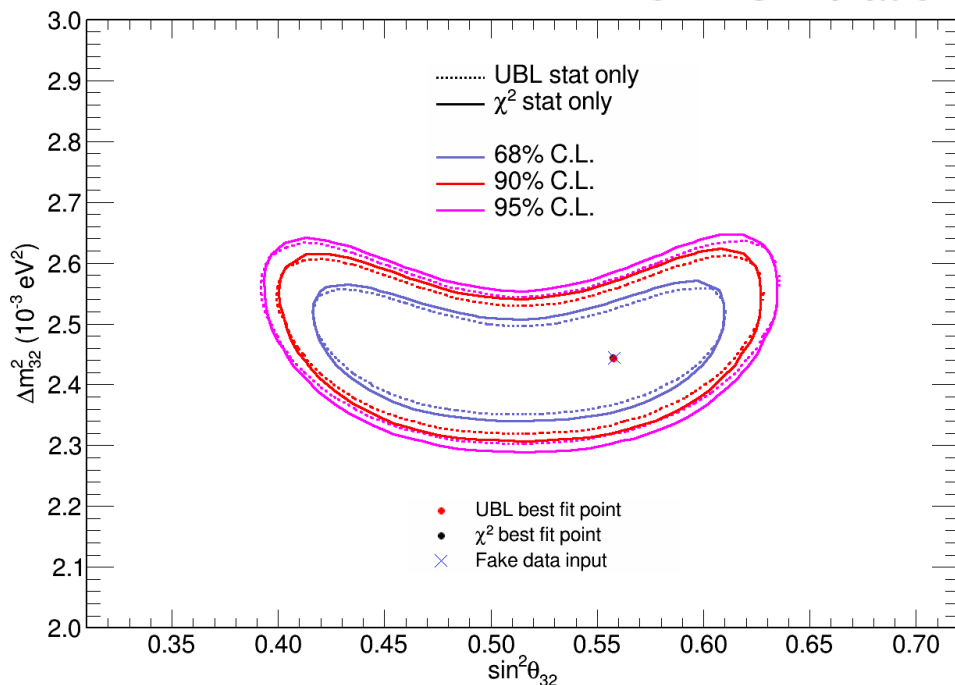
$$\sin^2(\theta_{23}) = 0.559 \pm 0.002$$

$$\Delta m_{32}^2 = (2.444 \pm 0.002) \times 10^{-3}$$

- Both methods are able to fit out the oscillation parameters input in the fake data.
- Comparing the fits to the fake data spectrum shows **< 1% differences** for both fitting techniques.

# $\nu_\mu$ disappearance sensitivities

NOvA Simulation

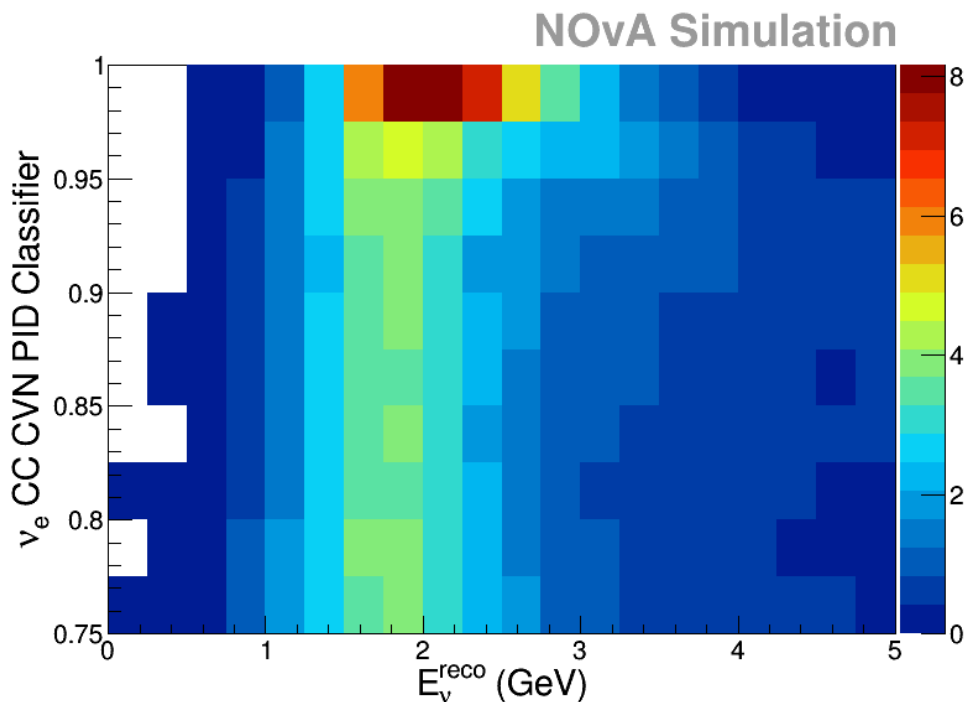


- UBL statistics only sensitivity is shown compared to the  $\chi^2$  equivalent.
- Official 3rd analysis  $\nu_\mu$  best fit point used as fake data input.
- UBL displays **increased sensitivity** in  $\Delta m_{32}^2$ , 1D limits at 68% confidence level:

$$\chi^2: (2.44 + 0.09 - 0.07) \times 10^{-3} \text{eV}^2$$

$$\text{UBL}: (2.44 + 0.08 - 0.06) \times 10^{-3} \text{eV}^2$$

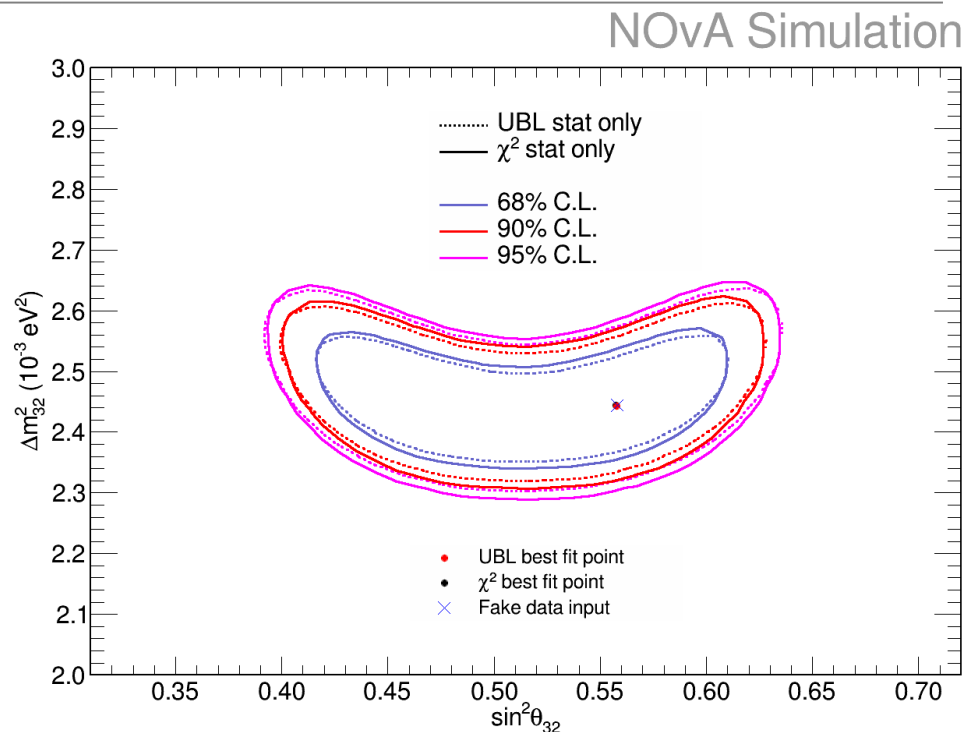
# Electron neutrino appearance



- Can also apply unbinned likelihood fitting to electron neutrino appearance analyses.
- In the official NOvA electron neutrino appearance analysis the spectra are divided into bins of CVN PID values.
- With the UBL fit can extend this to a spectrum and **fit to 2D spectra** with the potential for further gains in sensitivity.
- Work is currently underway to implement this.

# Summary and Conclusions

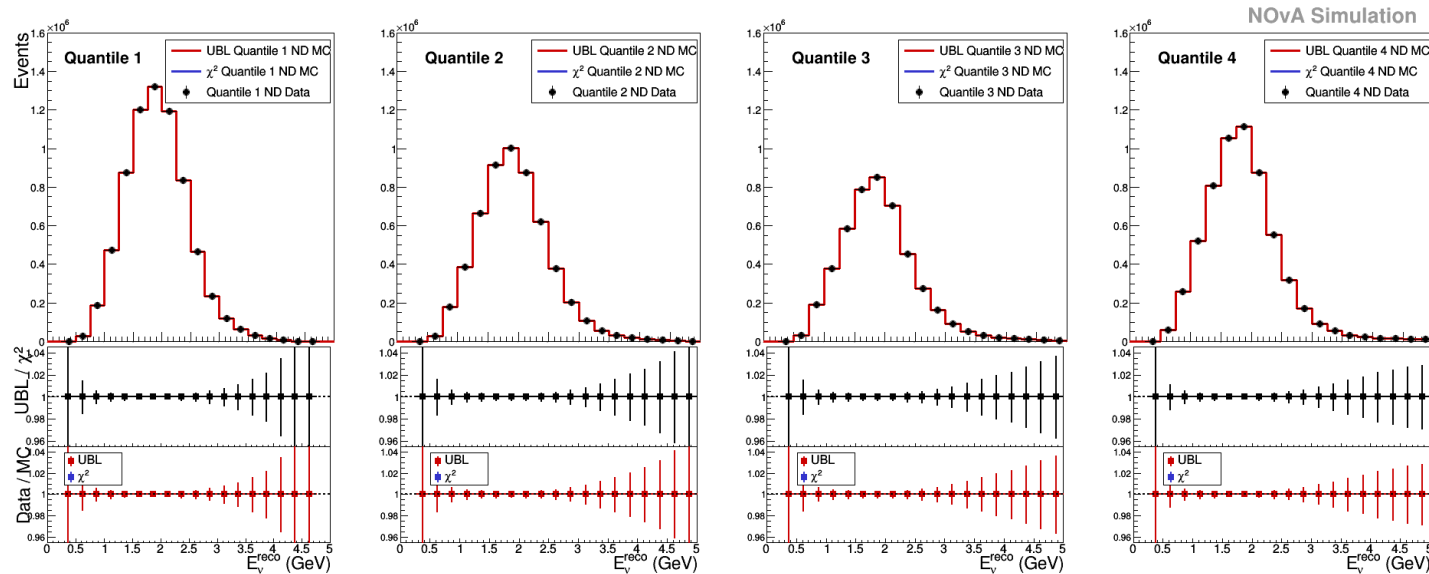
- An **unbinned likelihood fitting method** has been developed for use in NOvA analyses.
- The unbinned fitting method shows **increased sensitivity** in  $\Delta m_{32}^2$  compared to standard  $\chi^2$  fitting methods, yielding a 68% confidence interval of  $(2.44 + 0.08 - 0.06) \times 10^{-3} \text{eV}^2$ .
- Further work is underway to implement unbinned likelihood fitting to 2D spectra, for example to distributions of CVN PID against reconstructed neutrino energy.



# Backup

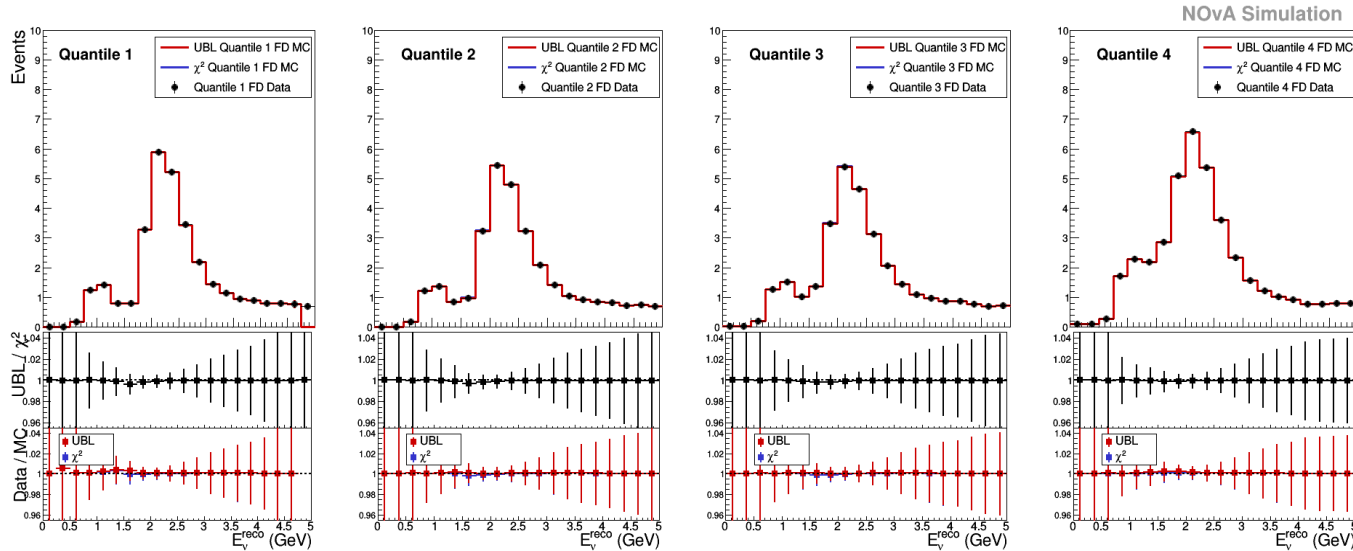


# $\nu_\mu$ disappearance sensitivity best fit results (ND)



Comparisons of the Near Detector reconstructed neutrino energy spectra at the best fit points found by the ChiSq and UBL fitting methods using fake data generated at the official NOvA numu 3rd analysis best fit point.

# $\nu_\mu$ disappearance sensitivity best fit results (FD low statistics)



## Fit values:

**Fake data input:**

$$\sin^2(\theta_{23}) = 0.558$$

$$\Delta m_{32}^2 = 2.444 \times 10^{-3}$$

**UBL best fit point:**

$$\sin^2(\theta_{23}) = 0.559 \pm 0.078$$

$$\Delta m_{32}^2 = (2.444 \pm 0.007) \times 10^{-3}$$

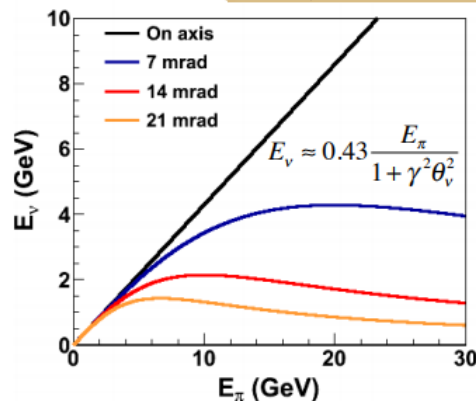
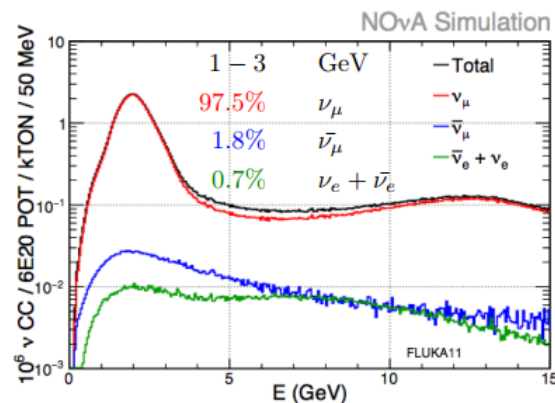
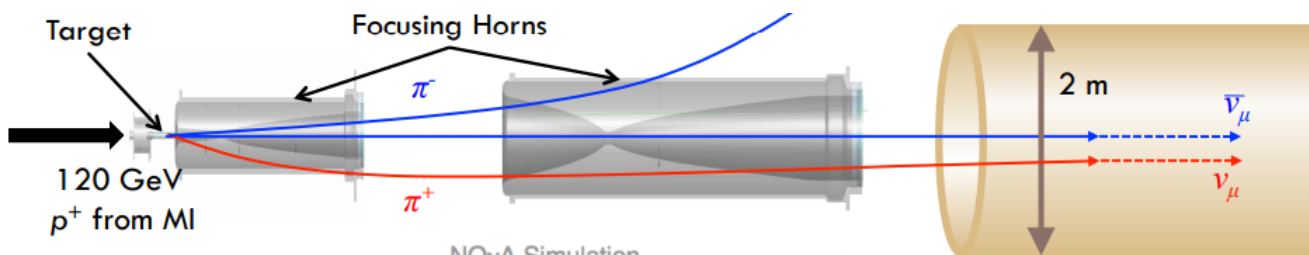
**$\chi^2$  best fit point:**

$$\sin^2(\theta_{23}) = 0.557 \pm 0.076$$

$$\Delta m_{32}^2 = (2.444 \pm 0.008) \times 10^{-3}$$

- Comparisons of the Far Detector reconstructed neutrino energy spectra at the best fit points found by the  $\chi^2$  and UBL fitting methods. Fake data generated with standard exposure at the official NOvA numu 3rd analysis best fit point is used and divided into quantiles of hadronic energy resolution. Again both methods yield very similar results with < 1% difference between them.

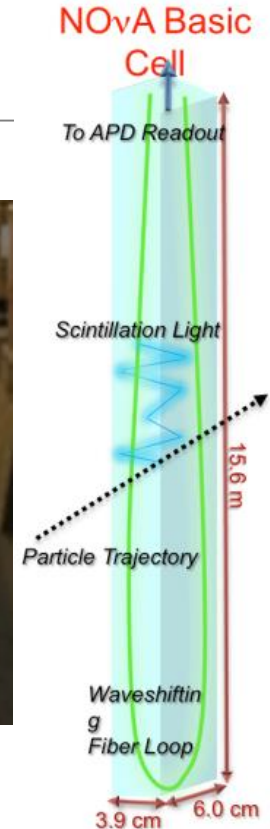
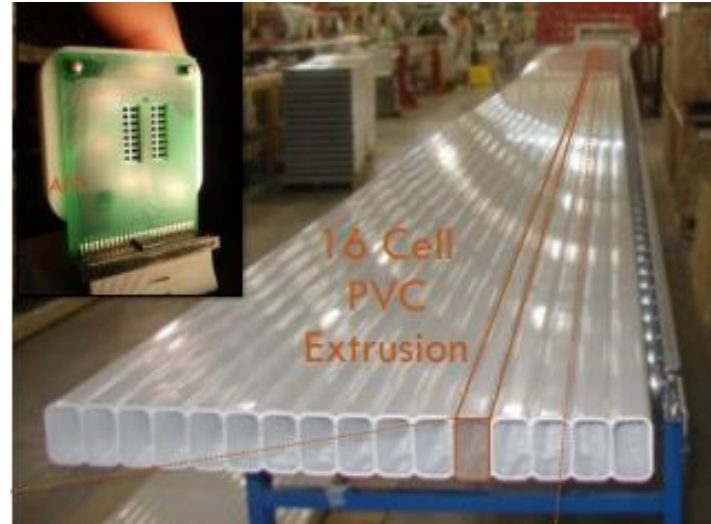
# The NuMI Beam



- 14.6 mrad off-axis, narrow band peaked at 2 GeV.
- Near oscillation maximum of 1.6 GeV.
- Few high energy NC background events.

# NOvA Detectors

- Designed for electron ID (low  $z$  materials, 65% active).
- PVC extrusion + liquid scintillator (mineral oil + 5% pseudocumene).
- Readout via wavelength-shifting fibre to avalanche photodiodes.
- 344,064 channels in FD.
- Layed planes with orthogonal views.
- 150kHz cosmic events at FD.

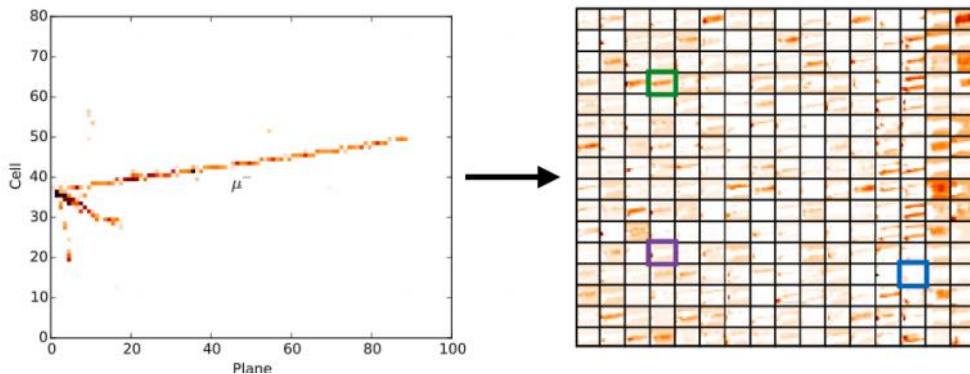
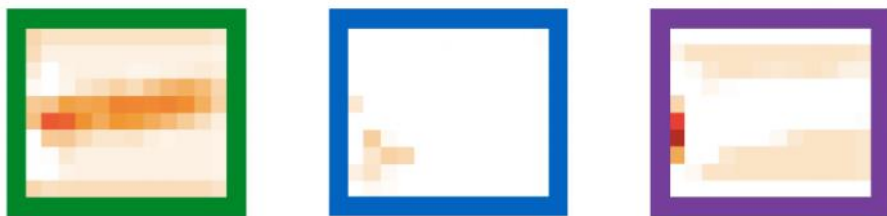


# 2017 Analysis – what's new?

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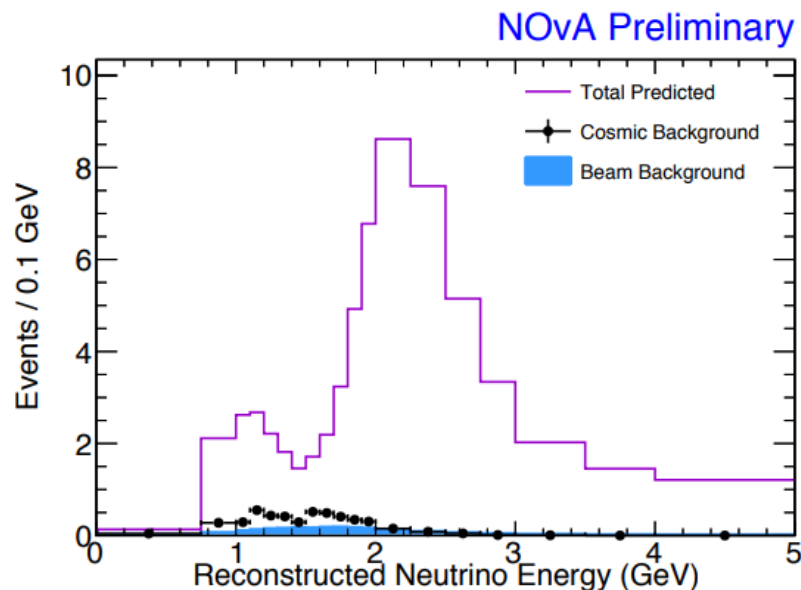
- **50% more data** than previous analysis.
- Further development of **deep learning tools** and binning in energy resolution to better exploit information in existing data.
- **Retuned cross-section modelling** (in particular multi-nucleon effects).
- **Detector simulation improvements**, reducing some of the larger systematics from previous analyses.
- **Data driven flux estimates** from MINERvA.

# Convolutional visual network PIDs



- Calibrated hit maps input to convolutional visual network (CVN).
- Series of image processing transformations applied to extract abstract features.
- Extracted features used as inputs to a convolutional neural network to classify the event.

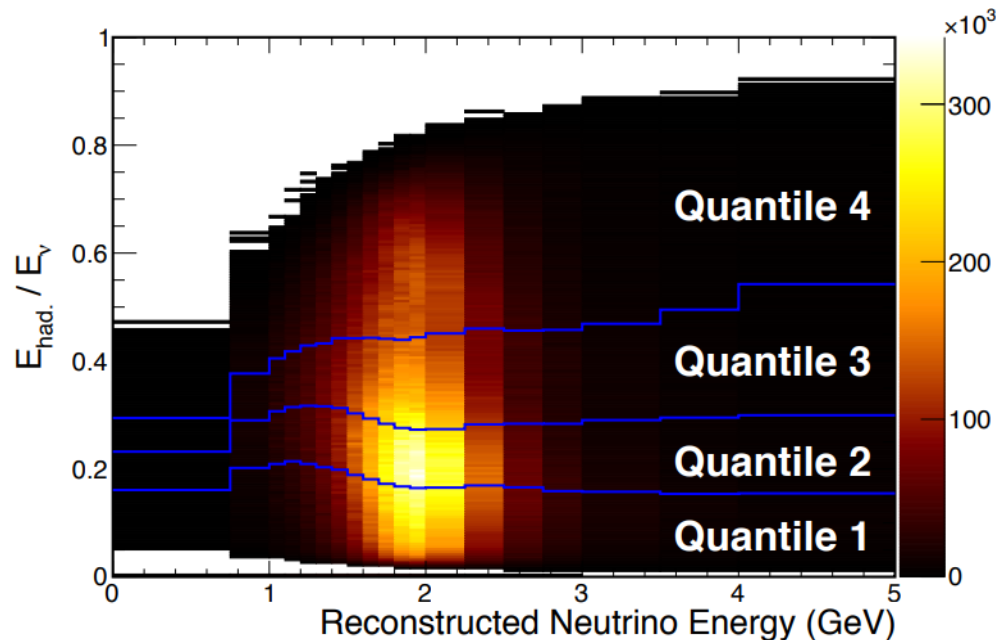
# Cosmic background rejection



- Cosmic rejection boosted decision tree (BDT) used to reject cosmic events.
- Characterised using cosmic activity recorded adjacent to beam spill to ensure equivalent detector performance.
- BDT trained on: reconstructed event direction, reconstructed muon direction, reconstructed length of muon track, y-position of the muon track, **CVN's cosmic output**, projected distance to edge of the muon track, and fraction of event hits in the reconstructed muon track.

# Energy resolution bins

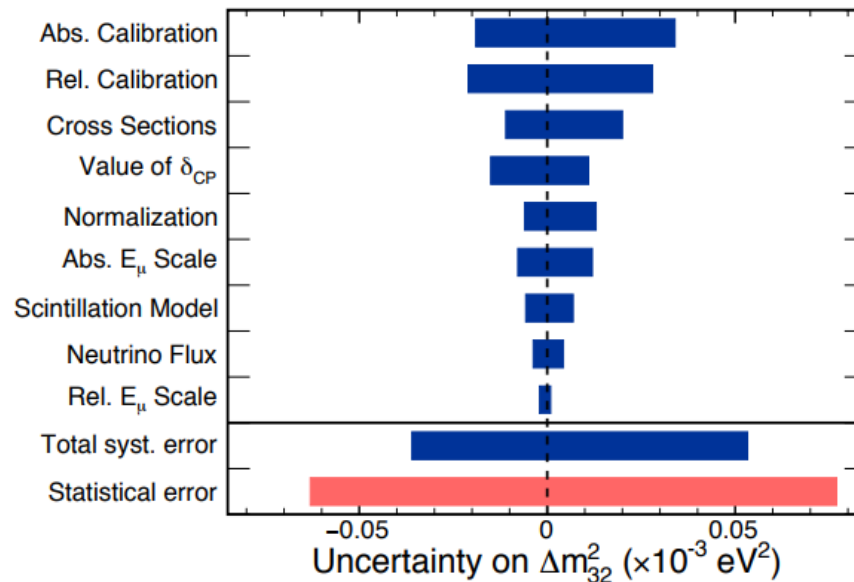
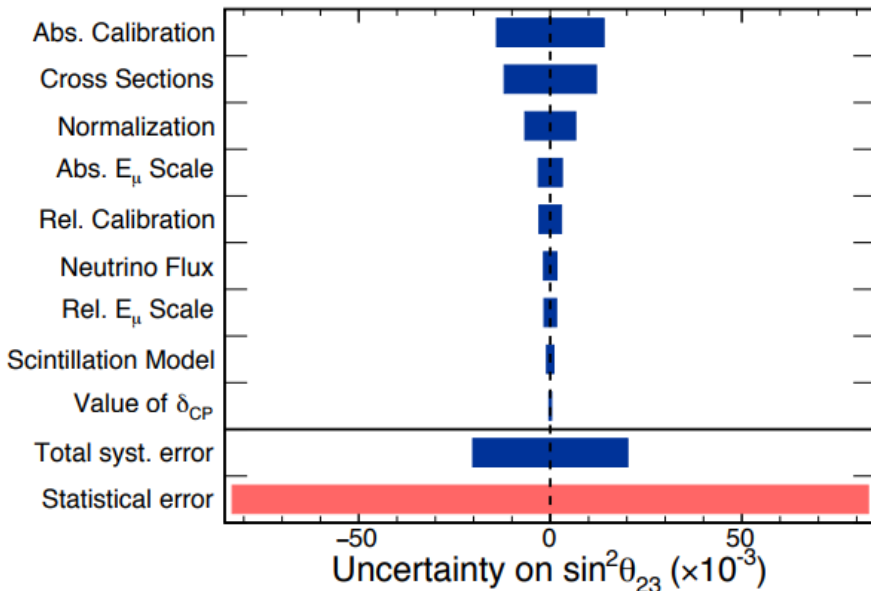
- Four bins of equal populations in FD, split in hadronic energy fraction as a function of reconstructed neutrino energy.
- Resolution varies from ~6% to ~12% from the best to worst resolution bins.





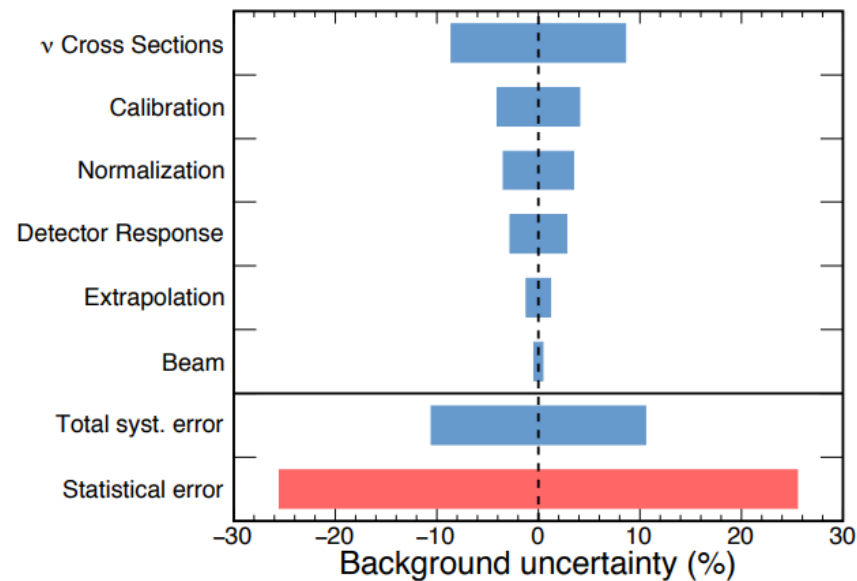
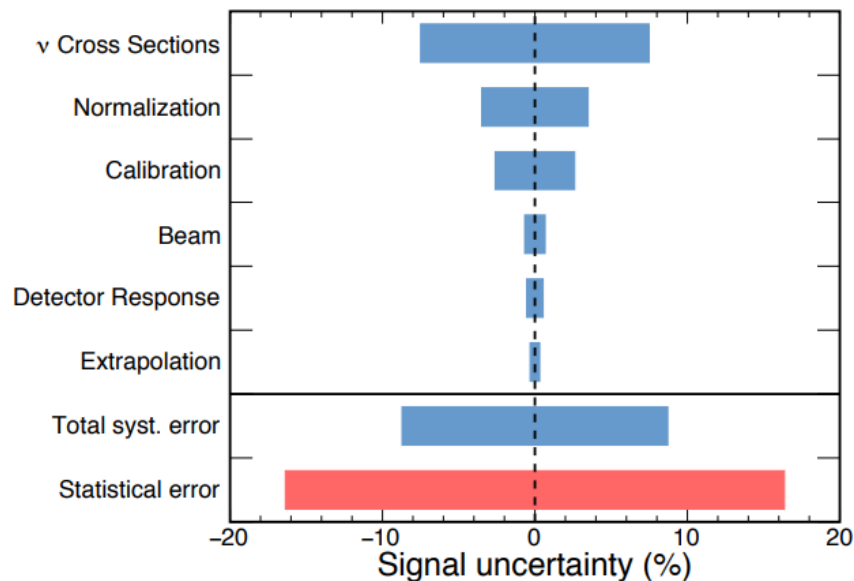
# Muon neutrino disappearance systematics

- Assessed by generating sets of shifted MC to assess impact on final result.

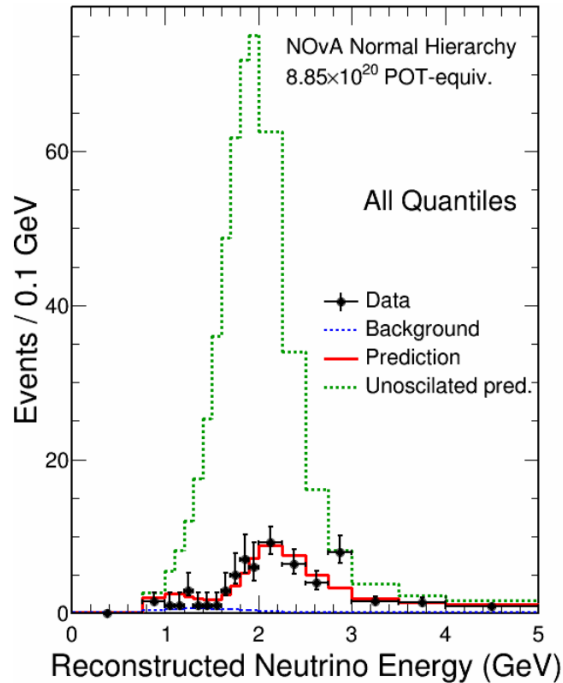


# Electron neutrino appearance systematics

- Assessed by generating sets of shifted MC to assess impact on final result.



# Muon neutrino disappearance results

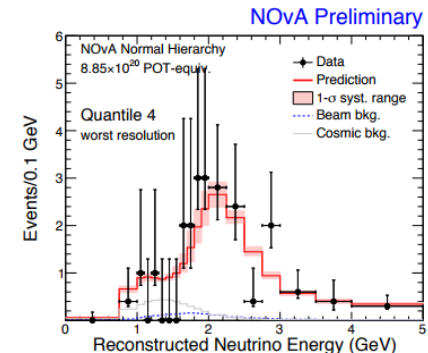
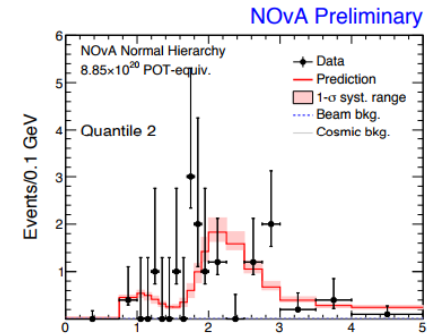
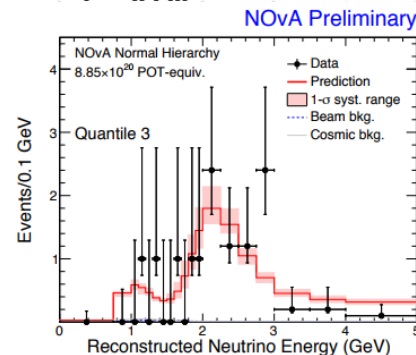
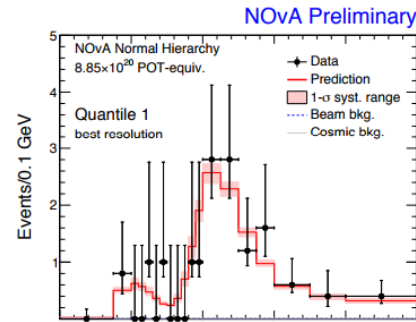
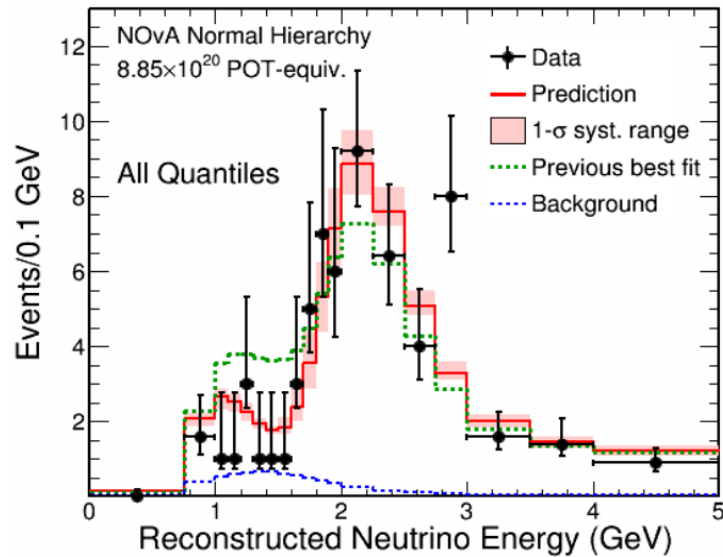


$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - \sin^2(2\theta_{23}) \sin^2\left(\frac{1.27\Delta m_{atm}^2 L}{E}\right)$$

(2 flavour approximation)

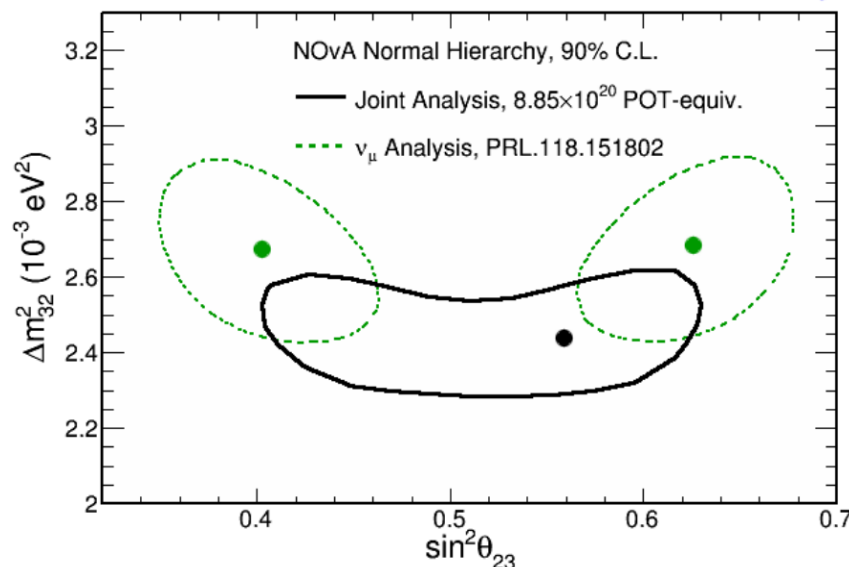
In the absence of oscillations we expect **763** events.  
**126** were observed.

# Muon neutrino disappearance results



# Muon neutrino disappearance results

NOvA Preliminary



- Full joint fit with appearance analysis. Feldman Cousins corrections in 2D & 1D limits.
- All systematics, oscillation pull terms shared.

$$\Delta m^2_{32} = 2.444^{+0.079}_{-0.077} \times 10^{-3} \text{ eV}^2$$

Best fit:

UO preferred at  $0.2\sigma$

$$\sin^2 \theta_{23} =$$

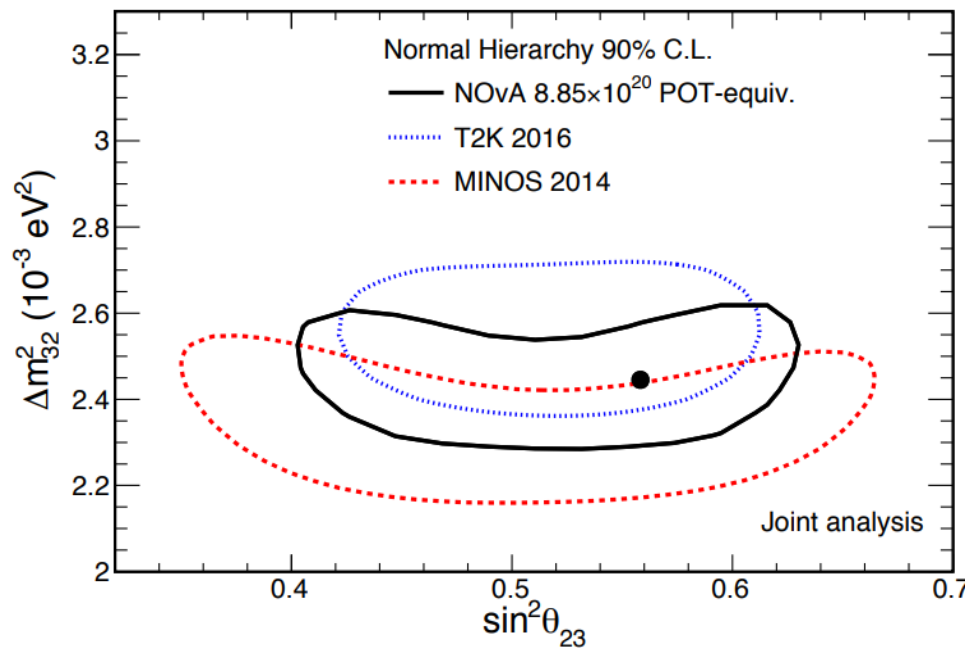
$$\text{UO: } 0.558^{+0.041}_{-0.033}$$

$$\text{LO: } 0.475^{+0.036}_{-0.044}$$

Rejection of maximal mixing from  $2.6\sigma$  to  $0.8\sigma$  compared to previous result.

# Comparison to other experiments

NOvA Preliminary



# Electron neutrino disappearance results

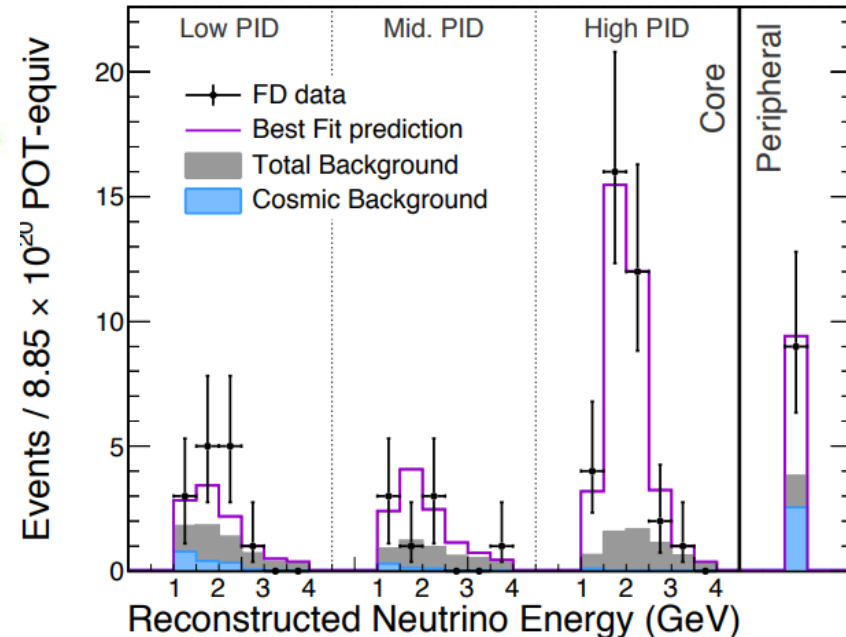
$$P(\nu_\mu \rightarrow \nu_e) \approx \left| \sqrt{P_{atm}} e^{-i\left(\frac{\Delta m_{32}^2 L}{4E} + \delta_{cp}\right)} + \sqrt{P_{sol}} \right|^2$$

$$P_{atm} = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

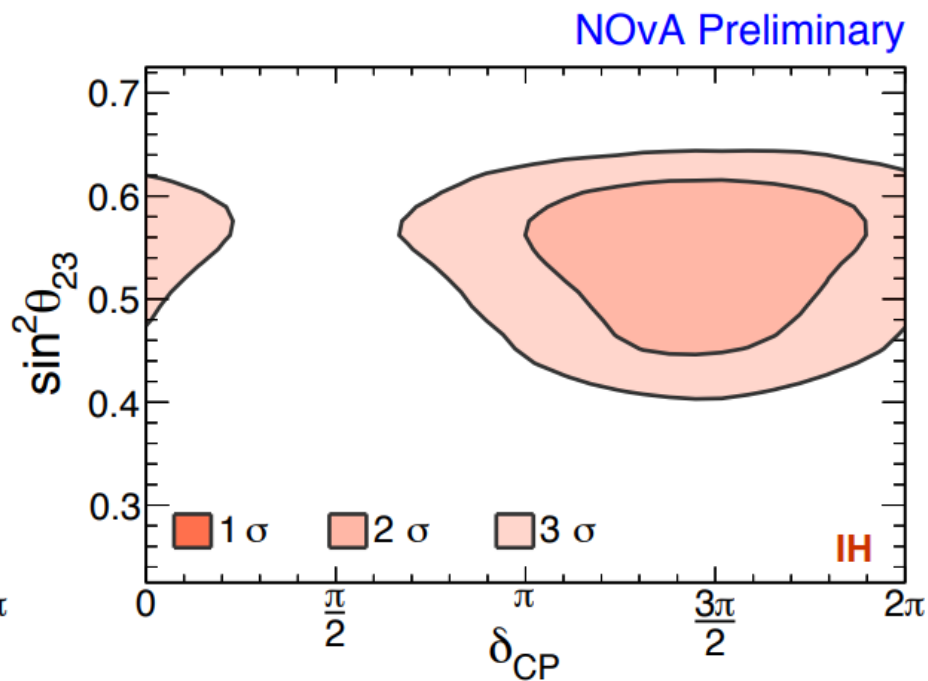
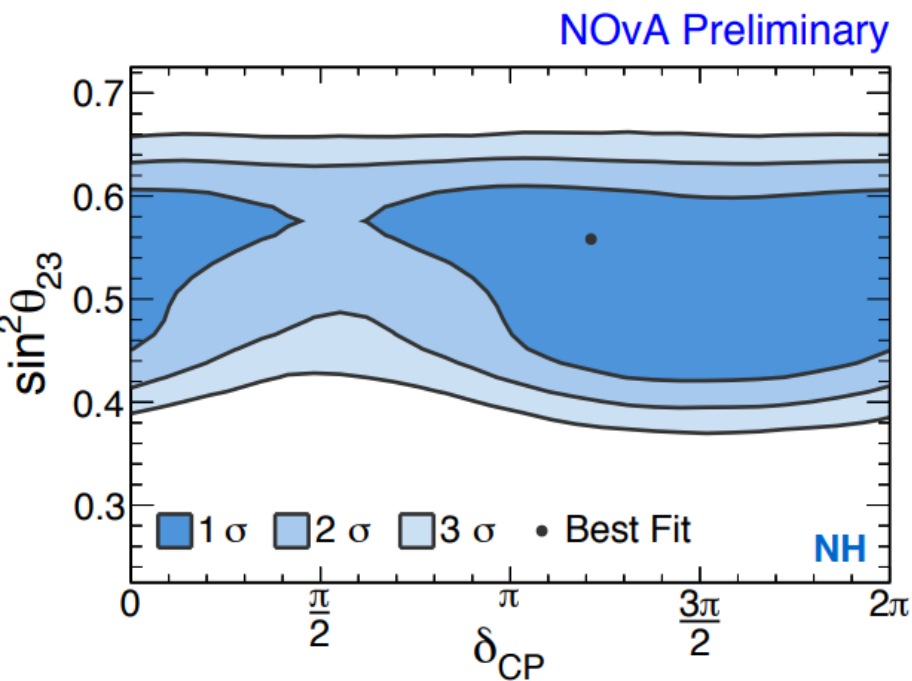
Solar term contributes <1% at ~400 L/E

Observe **66** events in FD. Background expectation **20.5 ± 2.5**.

NOvA Preliminary



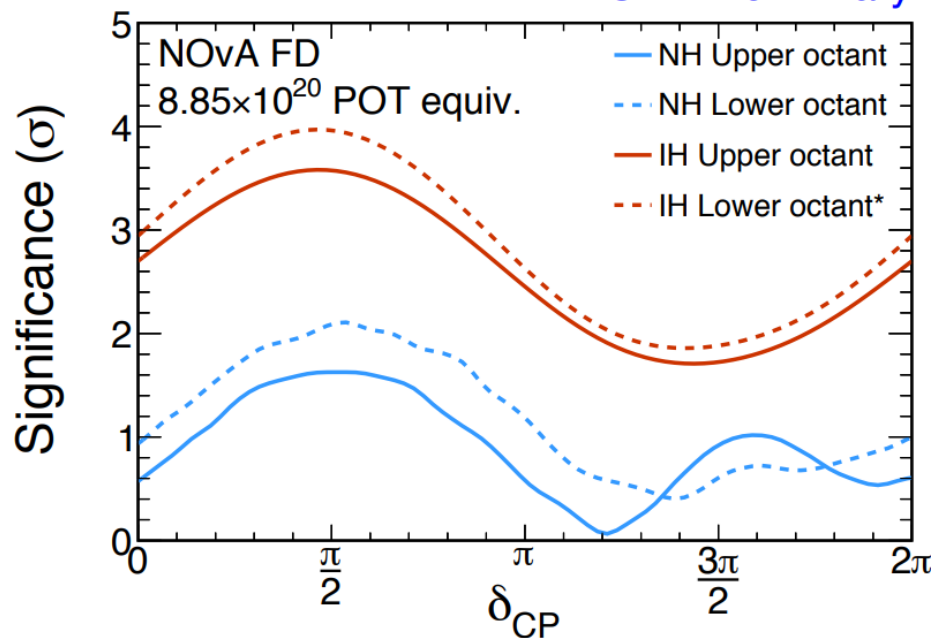
# Electron neutrino disappearance results





# Electron neutrino disappearance results

NOvA Preliminary



IH at  $\delta_{CP} = \pi/2$   
disfavored at greater  
than  $3\sigma$ .

Approaching IH  
rejection at  $2\sigma$ .

