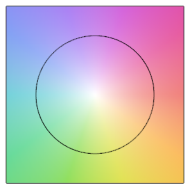
SAGEX Secondments at Wolfram

Devendra Kapadia
Wolfram Research, Inc.

Overview

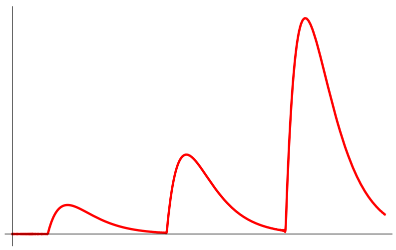
- Eight SAGEX ESRs received training at Wolfram during 2019–2022.
- The projects were all related to symbolic integration in Mathematica.
- The underlying theme for most projects was the use of complex analysis.

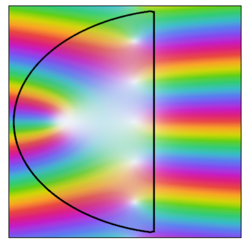


Inverse Laplace Transforms

Laplace transforms play an important role in control theory and other fields.

Gabriele Dian implemented the Bromwich method for the inverse transform.

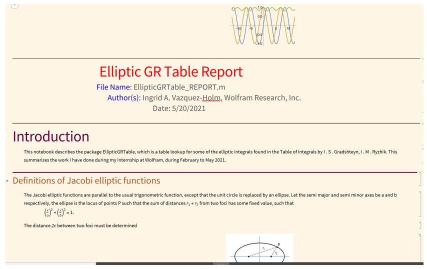




Elliptic Integrals

Elliptic functions and integrals occur in QFT and other applications.

Ingrid Holm worked on table look up for around 650 elliptic integrals.



Elliptic GR Table Report

File Name: EllipticGRTable_REPORT.m

Author(s): Ingrid A. Vazquez-Holm, Wolfram Research, Inc.

Date: 5/20/2021

Introduction

This notebook describes the package EllipticGRTable, which is a table lookup for some of the elliptic integrals found in the Table of Integrals by I . S . Gradshteyn, I . M . Ryzhik. This summarizes the work I have done during my internship at Wolfram, during February to May 2021.

Definitions of Jacobi elliptic functions

The Jacobi elliptic functions are parallel to the usual trigonometric function, except that the unit circle is replaced by an ellipse. Let the semi major and semi minor axes be a and b respectively, the ellipse is the locus of points P such that the sum of distances $r_1 + r_2$ from two foci has some fixed value, such that

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1.$$

The distance $2c$ between two foci must be determined

Hypergeometric Integration

Riccardo Gonzo worked on improvements for hypergeometric integrals.

He prepared a benchmark comparison for this class with Integrate.

```
In[•]:= Simplify[Integrate[-(16^ϵ mom^ϵ (mom - x4)^(-1-2ϵ) x4^ϵ (mm2 - x2 + x4)^(-1-2ϵ))/(π Gamma[1/2 - ϵ]^2 x2), {x4, x2 - mm2, mom},
          Assumptions → {mom > 0 && ϵ < 0}]]

Out[•]= ConditionalExpression[
          -(16^ϵ Gamma[-2 ϵ]^2 Hypergeometric2F1Regularized[-2 ϵ, -ϵ, -4 ϵ, 1 + mom/(mm2-x2)] (mm2 + mom - x2)^(-1-4ϵ) (mom (-mm2 + x2))^ϵ)/(π Gamma[1/2 - ϵ]^2 x2),
          Re[mm2] < x2 < mm2 + mom]
```

Loop by loop <u>Baikov</u> parametrization for the sunrise (following https://arxiv.org/pdf/1705.03478.pdf)

```
In[•]:= SetAttributes[MP, Orderless]

In[•]:= extmom1 = {k2};
        extmom2 = {p};
```

```
{ArcTanh[x], Erfc[x]}, {ArcTanh[x], Hypergeometric0F1[1, a x/4]}, {ArcTanh[x], HypergeometricU[a, b, x]}, {ArcTanh[x], EllipticK[b x]},
{ArcTanh[x], EllipticE[b x]}, {ArcTanh[x], HermiteH[n, x]}, {ArcTanh[x], BesselI[n, x]}, {ArcTanh[x], BesselK[n, x]},
{ArcTanh[x], KelvinBei[0, x]}, {ArcTanh[x], KelvinKer[0, x]}, {ArcTanh[x], (1 + Sqrt[1 + x])^beta}, {ArcTanh[x], AiryBi[x]},
{ArcTanh[x], AiryAi[x]}, {ArcTanh[x], SphericalBesselJ[n, x]}, {Erfc[x], Hypergeometric0F1[1, a x^2/4]}, {Erfc[x], HypergeometricU[a, b, x]},
{Erfc[x], EllipticK[b x]}, {Erfc[x], EllipticE[b x]}, {Erfc[x], HermiteH[n, x]}, {Erfc[x], BesselI[n, x]}, {Erfc[x], BesselK[n, x]},
{Erfc[x], KelvinBei[0, x]}, {Erfc[x], KelvinKer[0, x]}, {Erfc[x], (1 + Sqrt[1 + x])^beta}, {Erfc[x], AiryBi[x]}, {Erfc[x], AiryAi[x]},
{Erfc[x], SphericalBesselJ[n, x]}, {Hypergeometric0F1[1, a x^2/4], HypergeometricU[a, b, x]}, {Hypergeometric0F1[1, a x^2/4], EllipticK[b x]},
{Hypergeometric0F1[1, a x^2/4], EllipticE[b x]}, {Hypergeometric0F1[1, a x^2/4], HermiteH[n, x]}, {Hypergeometric0F1[1, a x^2/4], BesselI[n, x]},
{Hypergeometric0F1[1, a x^2/4], BesselK[n, x]}, {Hypergeometric0F1[1, a x^2/4], KelvinBei[0, x]}, {Hypergeometric0F1[1, a x^2/4], KelvinKer[0, x]},
{Hypergeometric0F1[1, a x^2/4], (1 + Sqrt[1 + x])^beta}, {Hypergeometric0F1[1, a x^2/4], AiryBi[x]}, {Hypergeometric0F1[1, a x^2/4], AiryAi[x]},
{Hypergeometric0F1[1, a x^2/4], SphericalBesselJ[n, x]}, {HypergeometricU[a, b, x], EllipticK[b x]}, {HypergeometricU[a, b, x], EllipticE[b x]},
{HypergeometricU[a, b, x], HermiteH[n, x]}, {HypergeometricU[a, b, x], BesselI[n, x]}, {HypergeometricU[a, b, x], BesselK[n, x]},
{HypergeometricU[a, b, x], KelvinBei[0, x]}, {HypergeometricU[a, b, x], KelvinKer[0, x]}, {HypergeometricU[a, b, x], (1 + Sqrt[1 + x])^beta},
{HypergeometricU[a, b, x], AiryBi[x]}, {HypergeometricU[a, b, x], AiryAi[x]}, {HypergeometricU[a, b, x], SphericalBesselJ[n, x]},
{EllipticK[b x], EllipticE[b x]}, {EllipticK[b x], HermiteH[n, x]}, {EllipticK[b x], BesselI[n, x]}, {EllipticK[b x], BesselK[n, x]},
{EllipticK[b x], KelvinBei[0, x]}, {EllipticK[b x], KelvinKer[0, x]}, {EllipticK[b x], (1 + Sqrt[1 + x])^beta}, {EllipticK[b x], AiryBi[x]},
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{EllipticE[b x], BesselK[n, x]}, {EllipticE[b x], KelvinBei[0, x]}, {EllipticE[b x], KelvinKer[0, x]}, {EllipticE[b x], (1 + Sqrt[1 + x])^beta},
{EllipticE[b x], AiryBi[x]}, {EllipticE[b x], AiryAi[x]}, {EllipticE[b x], SphericalBesselJ[n, x]}, {HermiteH[n, x], BesselI[n, x]},
{HermiteH[n, x], BesselK[n, x]}, {HermiteH[n, x], KelvinBei[0, x]}, {HermiteH[n, x], KelvinKer[0, x]}, {HermiteH[n, x], (1 + Sqrt[1 + x])^beta},
{HermiteH[n, x], AiryBi[x]}, {HermiteH[n, x], AiryAi[x]}, {HermiteH[n, x], SphericalBesselJ[n, x]}, {BesselI[n, x], BesselK[n, x]},
```

Integrals of Special Functions

Lorenzo Quintavalle created an initial table for special function integrals.

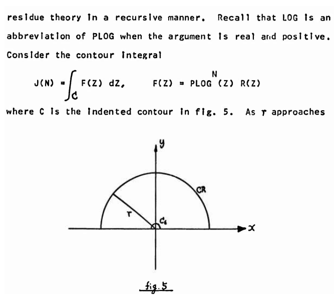
Nikolai Fadeev and Lorenzo automated the table lookup procedure.

Finally, Stefano De Angelis worked on convergence for these integrals.

Complex Contour Integration

Manuel Accettulli Huber implemented contour-based integration methods.

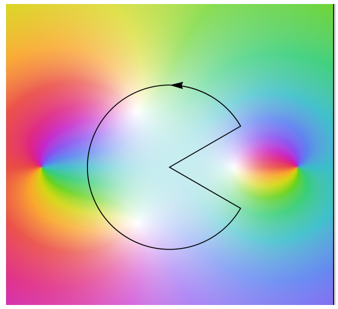
Luke Corcoran worked on a prototype for complex contour integration.


residue theory in a recursive manner. Recall that LOG is an abbreviation of PLOG when the argument is real and positive. Consider the contour integral

$$J(N) = \int_C F(Z)\, dZ, \qquad F(Z) = PLOG^N(Z)\, R(Z)$$

where C is the indented contour in fig. 5. As r approaches

fig. 5



Conclusion

- The SAGEX–Wolfram partnership has been a great success!
- We look forward to similar collaborations in the near future.
- Many thanks to all the participating institutions for their support.
- Well done, SAGEX ESRs!