Novel Bootstraps via Duality btw Color & Kinematics

22 June 2022

Based on ongoing work with Many Early Career Researchers.

SAGEX

Scattering Amplitudes: from Geometry to Experiment

I. Introduction

double copy invites us to construct & combine our favorite theories using building blocks

$$\text{SYM} = \text{color} \otimes \text{vector} + \text{susy}$$
$$\text{NLSM} = \text{color} \otimes \pi$$
$$\text{SDBIVA} = \pi \otimes \text{vector} + \text{susy}$$
$$\text{SG} = \text{vector} + \text{susy} \otimes \text{vector} + \text{susy}$$

$$\otimes \equiv (\text{KLT} = \text{BCJ} = \text{CHY})$$

(tree level) (tree level)

Loop Levels

Indeed much of this talk: $L < 2$ loops

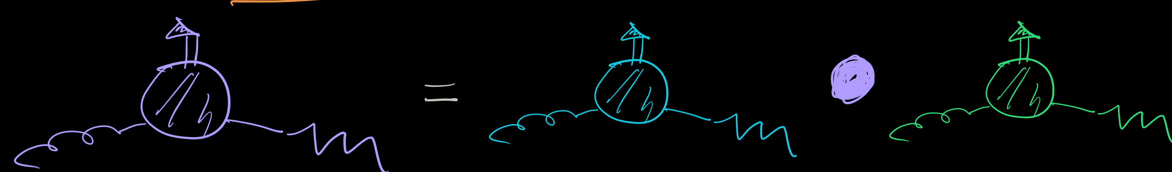
but borrowing insight & approaches from

Multi Loop Level.

Bootstrap: Legs & Loops

Mass Dim?

Will refine language around double copy w/ Something New: alg. compositon



EFT

- Higher derivative operators (or counterterms) let you parameterize your ignorance of new physics with available fields

$$\mathcal{L} = \partial^2\varphi + g\varphi^3 + c_2\frac{(\partial^2\varphi^2)}{\Lambda} + c_4\frac{\partial^4\varphi^4}{\Lambda^2} + \cdots$$

Goal:

$$\partial^n F^m, \partial^n R^m$$

Summary:

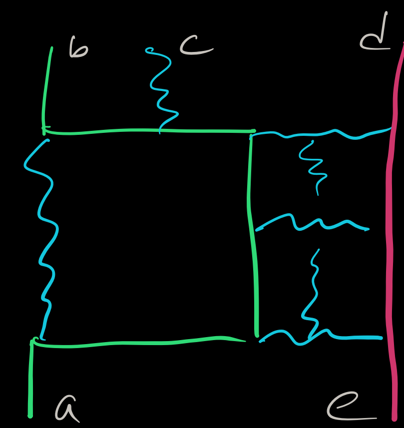
- color-kinematics lets us bootstrap [≡ GI.] 2010.1345, 2108.06798, 2112.05178
- composition lets us build & classify EFT operators for gauge & gravity theories with SMALL set of Building Blocks 1910.12850, 2104.08370
- $\mathcal{N}=4$ SG may have a non-local color-dual fate: 2203.03592
  - Berkovits Witten CSG
  - Heterotic String

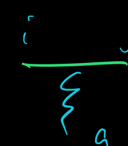
How we calculate at loops

- Not Feynman diagrams
  - off shell unweildy
  - factorial # of diagrams
- But Graph Organized
  - Unitarity methods efficient for VERIFICATION & Cut construction
  - Virtual integration wants local reps.

(Gauge) Loop Integrand Map

Mapping from labeled topology

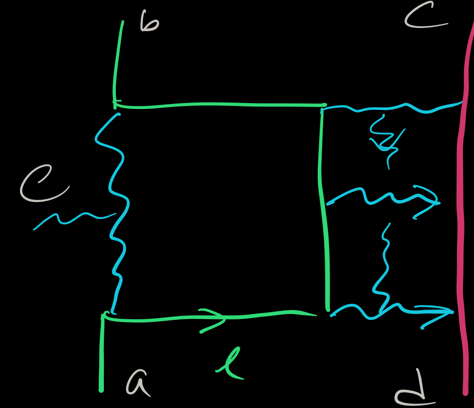


to color weights ( f^{abc}  T^{a}_{ij}) $C_{\text{(diagram)}}$

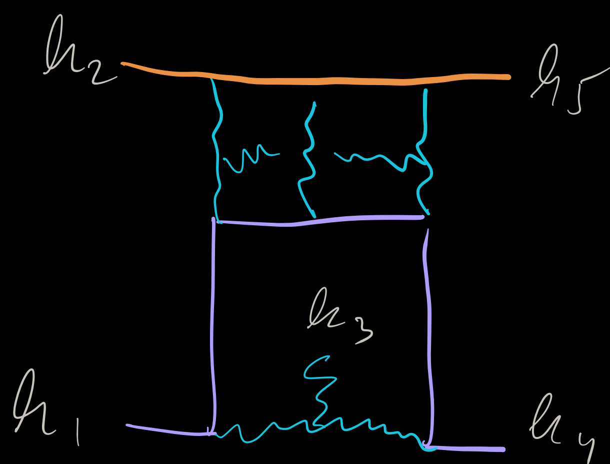
propagator weights ($\xrightarrow{\ell} = \frac{1}{\ell^2 - m^2}$)

numerator weights ($k_a \cdots k_e, \varepsilon_c, \ell_1 \cdots \ell_4$)

Weights should obey symmetries of topology



no



?

$a \to k_1$, $b \to k_4$, $e \to k_3$ or $a \to k_4$, $b \to k_1$, $e \to k_3$

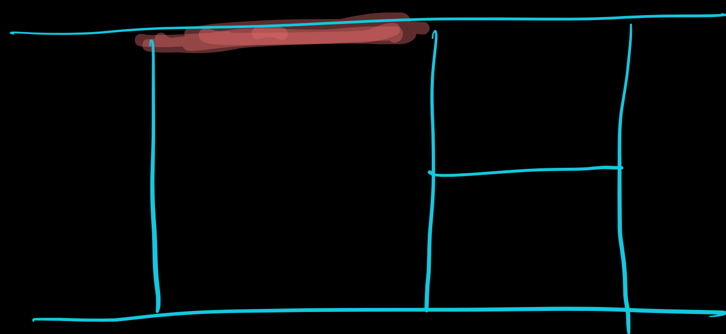
& unitarity cuts.

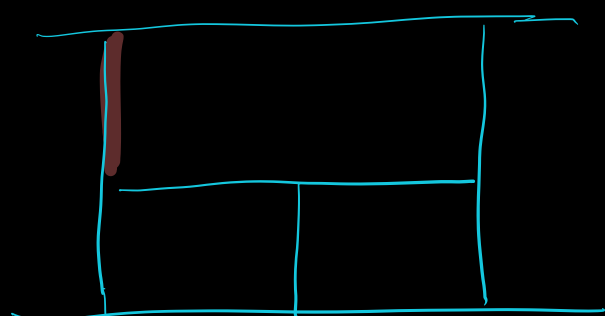
Recipe for Calculation

- Give Ansätze to topologies
- Fix on Symmetries & Cuts
- Duality between color & kinematics
  can help!

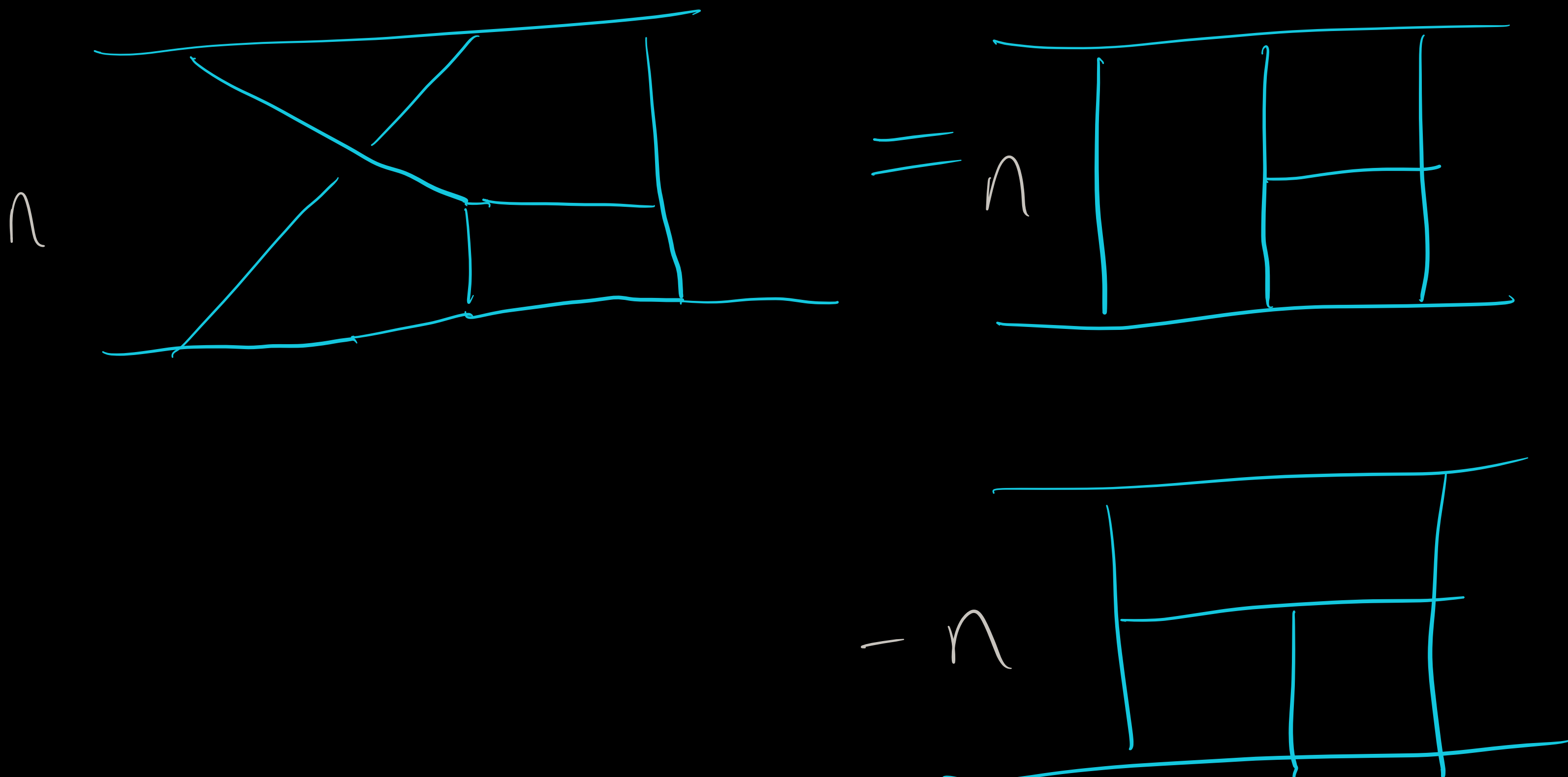
Adjoint-type color weights

f^{abc}

C  $=$

C  $+\ C$ 

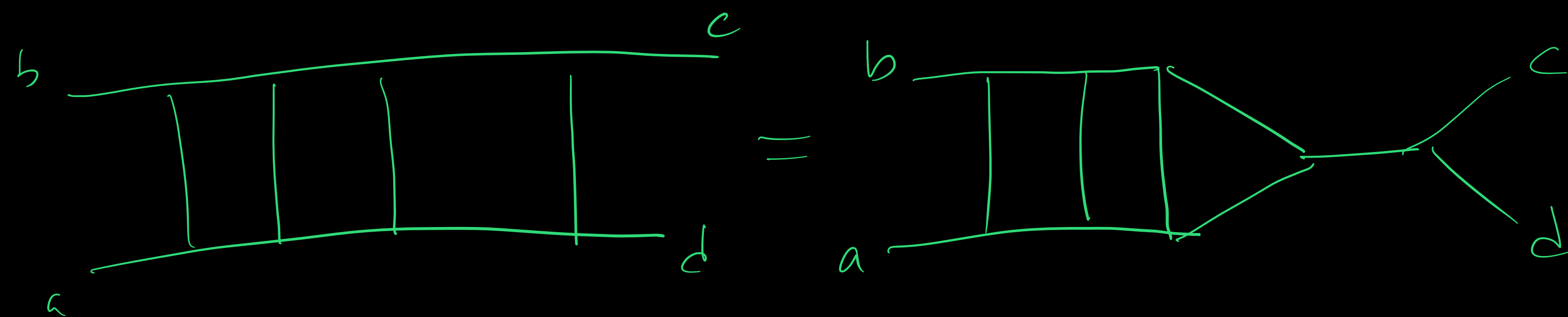
Demand this holds for n as well!



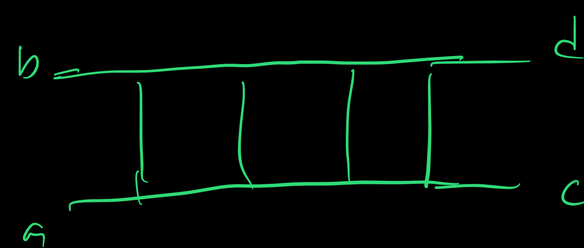
All of 3-loop 4pt $\mathcal{N}=4$ SYM in one topology!

Bern, JJMC, Johansson 2010

Note feedback constraints:



$n_L(a\,b\,c\,d) = n_L(a\,b\,d\,c) +$



Jacobi relations are <u>functional</u> at Loop Level

so Ansätze are <u>natural</u>. (but can get too expensive)

This can take you

far – multiplicity

– loop level

– even away from adjoint

Ingrid Vazquez-Holm

2010.1345
2108.06798

- Bootstrap massive scalars in arb rep to 1-loop 5pt
- C/K = GI
- trivialized extraction of Einstein
- Judicious choices ⇒ universal kinematics

Aslan Seifi

(to appear)

- Massive fermions, arb reps? No problem!
- Color-dual D-dim bootstrap gives all D-dim cut constructable info.!

Bootstrap:

trees $\to m+1$

loops $\to L+1$

EFT??

Higher Derivative Operators

- ∂_i in $\mathcal{L}$ $\longrightarrow$ k_i in $\mathcal{A}$
- Game is to figure out all distinct ways of sprinkling in some order in MD k into your scattering amplitude.

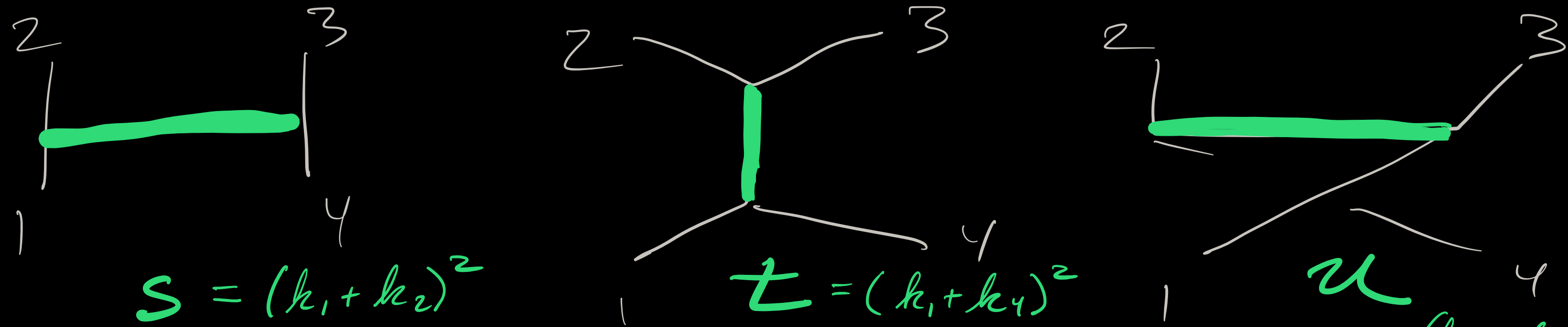
  w/out messing w/ symmetries.

- Easy: scalar permutation invariants

Suna Zehioğlu

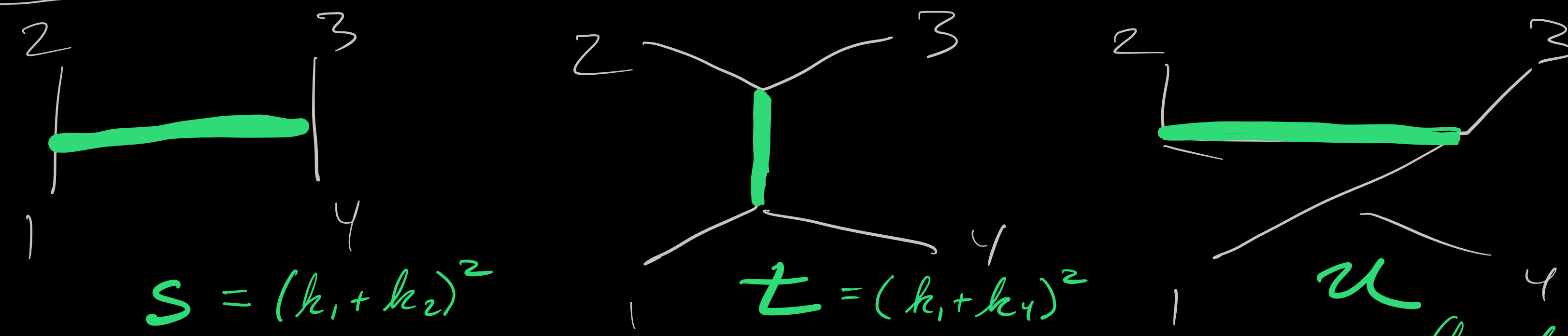
Laurentiu Rodina

4 pt tree level adjoint type $n(1-1)$



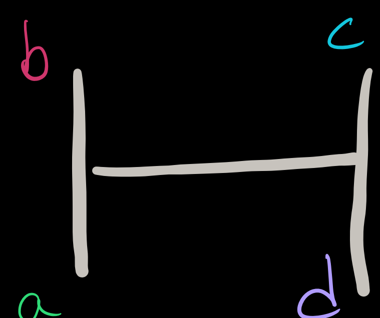
$s = (k_1 + k_2)^2$ $\quad$ $t = (k_1 + k_4)^2$ $\quad$ $u = (k_1 + k_3)^2$

4 pt tree level adjoint type $n(\vdash\!\dashv)$



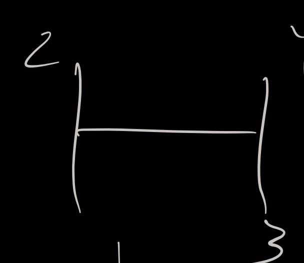
$s = (k_1 + k_2)^2$ $\quad t = (k_1 + k_4)^2$ $\quad u = (k_1 + k_3)^2$

$g(s,t,u)$



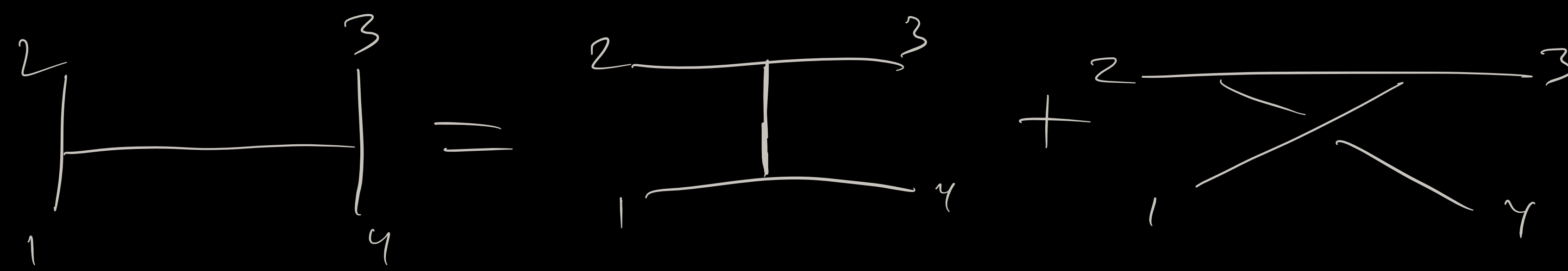
$$= g(s_{ab}, s_{ad}, s_{ac})$$

$$= g(s,t,u)$$



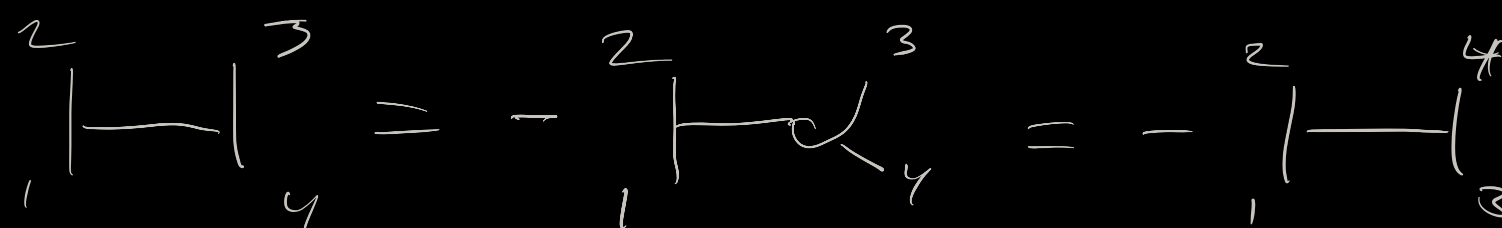
$$g(s,u,t)$$

Adjoint conditions



$$n_a(s,t,u) = n_a(t,s,u) + n_a(u,t,s)$$

&

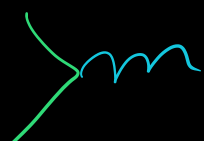


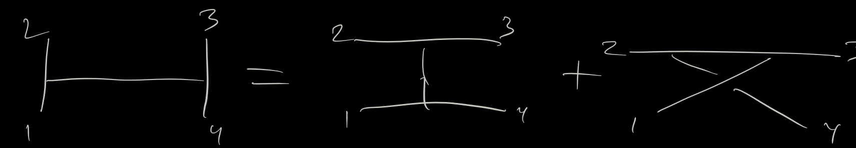
$$n_a(s,t,u) = - n_a(s,u,t)$$

Color free scalar

$$(\partial\phi)^2 \sim$$

$$(\partial_\mu \phi_a) A^\mu_b \phi_c f^{abc}$$





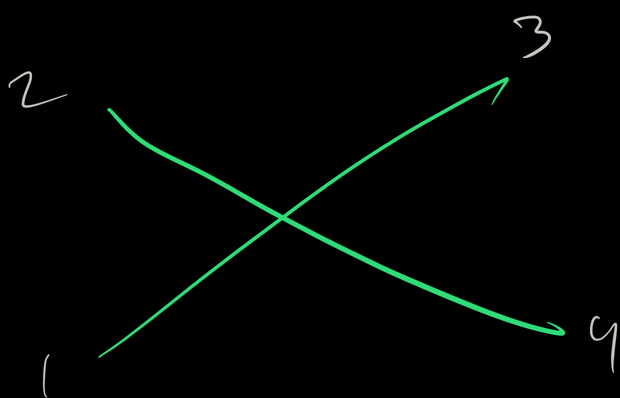
$$n_a(s,t,u) = n_a(t,s,u) + n_a(u,t,s)$$

$$n_a(s,t,u) = -\,n_a(s,u,t)$$

$$n^{ss}(a,b,c) = t-u$$

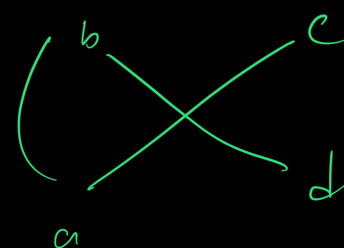
Adjoint type? $\left[\, t-u \overset{?}{=} (\not{s}-u) + (t-\not{s}) = t-u \,\right]$

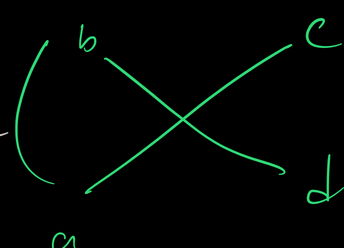
Permutation Invt Conditions



P_4 doesn't care about leg order

invt under S_4.

$c\Big($  $\Big) = d^{abcd}$

$n\Big($  $\Big) = s\,t\,A^{YM}(s,t)$

Note, no linear perm invt for massless

$$s + t + u = 0$$

But can compose adjoint $\tilde{n}$ to get perm invt.

$$n^{adj} \circledast \tilde{n}^{adj} = n_s \tilde{n}_s + n_t \tilde{n}_t + n_u \tilde{n}_u$$

$$P_2 = n^{ss} \circledast n^{ss} = s^2 + t^2 + u^2 = \sigma_2$$

Note also, trivially

$$n_s = \tilde{n}^{adj} \circled{a}_s P = P \circled{a}_s \tilde{n}^{adj} = P \, \tilde{n}_s^{adj}$$

Adjoint Comp at 4pt!

$$n_s = \tilde{n}^{adj} \circledast_s \check{n}^{adj} = \tilde{n}_t \check{n}_t - \tilde{n}_u \check{n}_u$$

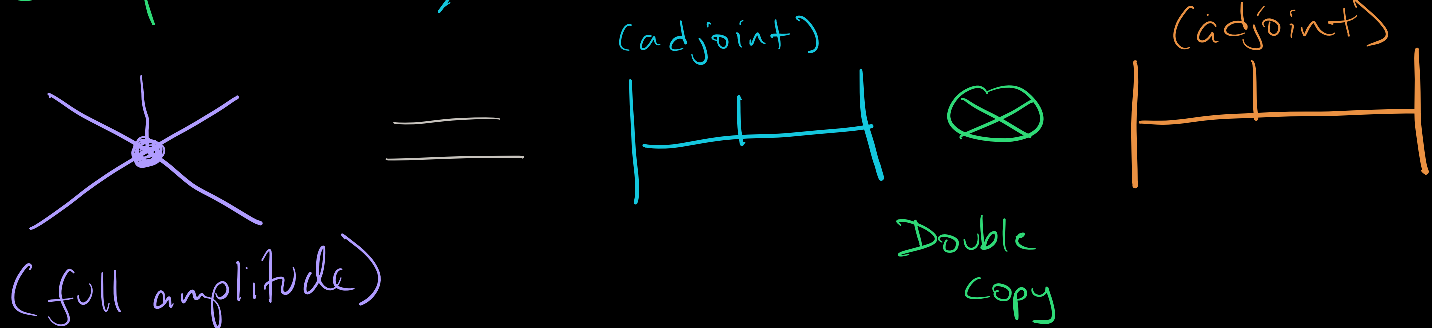
n_t, & n_u just come functionally by relabeling.

$$\Pi_s = n^{ss} \circledast_s n^{ss} \propto s(t-u)$$

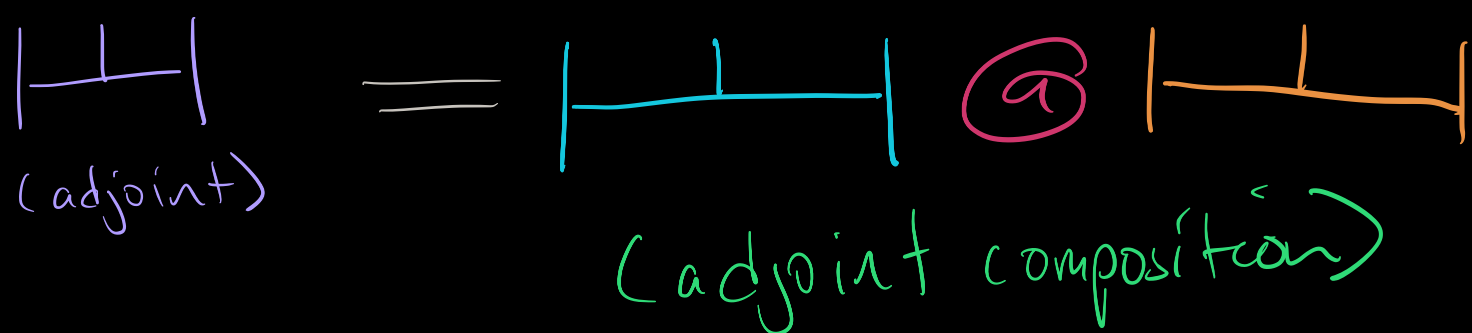
$$P_3 = \underbrace{n^{ss}}_{1} \circledast \underbrace{\Pi}_{2} \propto stu = \sigma_3$$

$$P_4 = \Pi \circledast \Pi \propto \sigma_2 \circledast \sigma_2 = (s^2+t^2+u^2)^2$$

Composition ≠ Double Copy



v s.



E.g.

LL (PIONS) $= (D\varphi)^2 \circledast (D\varphi)^2$

covariantized free scalar

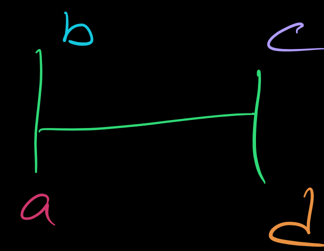
Simple Covariantized Scalar

spans all scalar adjoint type BB under composition.

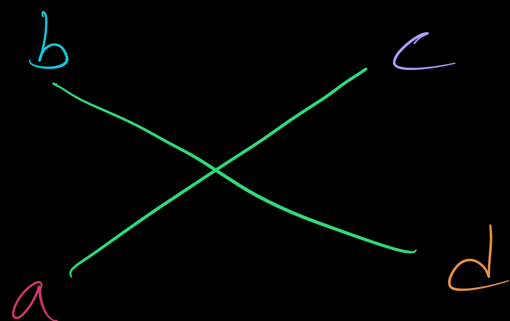
- Climbs a ladder MD by MD
- Closes after 2nd rung under perm invariants.

$$n_S^{MD} = \sum_{X,Y}^{3Y+2X=MD-1} C_{XY}^{SS}\, \sigma_2^{X} \sigma_3^{Y}\, n_S^{SS} + \sum_{X,Y}^{3Y+2X=MD-2} C_{XY}^{\Pi}\, \sigma_2^{X} \sigma_3^{Y}\, \Pi_S$$

Can add color: $c_s^{adj} = f^{abe} f^{ecd}$



$\varepsilon_i \quad d^{abcd}$



to n's using composition in relevant ways.

eg:

$$\Pi_s^{abcd} = d^{abcd}\,\Pi_s$$

Note: $A^{\mathrm{tr}(F^4)} = d^{abcd}\, s\, t\, A^{\mathrm{YM}}(1234)$

$= T^{abcd} \otimes \text{vector}$

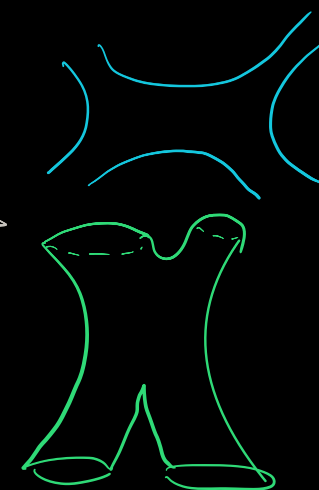
Its ordered amplitudes don't satisfy $(n-3)!$ BCJ amplitude relations (they're perm inv!)

Rather it is a double copy

This allows us to recover

Open & Closed Superstrings

at Tree Level as



Field Theory Double Copies

from scalar perm invariants (σ_2, σ_3)

scalar adjoint numerators (n^{ss}, Π)

single trace color weights (f^{abc}, d^{abcd})

& sym numerator ($n^{(s)ym}$)

$$OSS = \underbrace{\sum} \otimes sYM$$

$$CSS = sYM \otimes sYM^{(SU)}$$

$$\underbrace{\sum}_s = (\underbrace{C^{adj}} @ n^{ss})_s \, 6_2^x 6_3^y C^{ss}_{x,y}$$

$$+ (\underbrace{C^{adj}} @ \Pi)_s \, 6_2^x 6_3^y C^{\Pi}_{x,y}$$

$$+ (\underbrace{d^{abcd}} \Pi)_s \, 6_2^x 6_3^y C^{d}_{x,y}$$

$$sYM^{(SU)}_s = n^{YM}_s \, 6_2^x 6_3^y C^{SU}_{x,y}$$

Composition Very Powerful

- Could verify up to insanely high MD $\partial^{50}F^4$ $(\alpha')^{25} \sim (s_{ij})^{25}$ before finding the right way to express this in terms of OSS directly in closed $\Gamma(\ )$ form at 4pt.

- Check out paper for Examples & Proofs

Why no $(C^{ss}\,@\,n^{YM})$ or

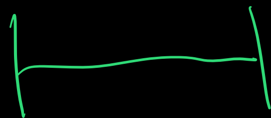
$(C^{\Pi}\,@\,n^{YM})$?

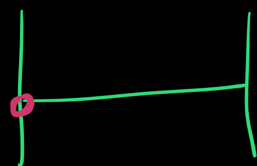
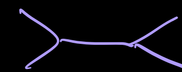
Resulting numerators satisfy adjoint constraints but NOT Gauge Inv.

What about eg other tensor structures?

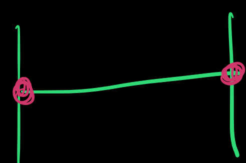
turns out $\exists$ 8 spanning building blocks at 4pts.

Vector Building Blocks

$(\alpha')^0$  YM

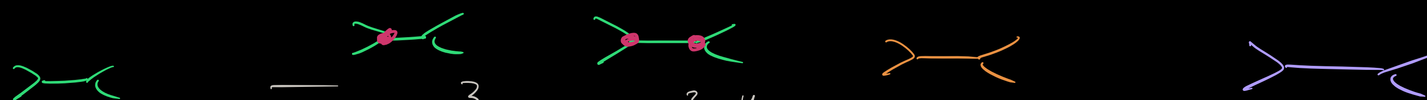
$(\alpha')^1$  $1\ \mathrm{tr}\, F^3$ + 2 more ( $/ G_3$)

$((DF)^2)^2$

$(\alpha')^2$  $2\ \mathrm{tr}\, F^3 + \mathrm{tr}(F^4)$ + 2 more

$(\alpha')^3$  $\partial^2 F^4$

Eg Bosonic String: $\overset{n^{gs},\ f^3 f^3,\ d^{abcd}}{\downarrow} Z \otimes (DF)^2 + YM$



$$n^{(DF)^2+YM} = n_{YM} + \frac{\left[\alpha'\left(n^{F^3} + \alpha'\left(n^{2F^3+F^4} + \alpha'\left(n^{\partial^2 F^4} + \alpha' n^{(DF)^4}\right)\right)\right)\right]}{1 - \alpha'^2 G_2 - \alpha'^3 G_3}$$

FINE

But can these be algebraically constructed?

= STAY TUNED =

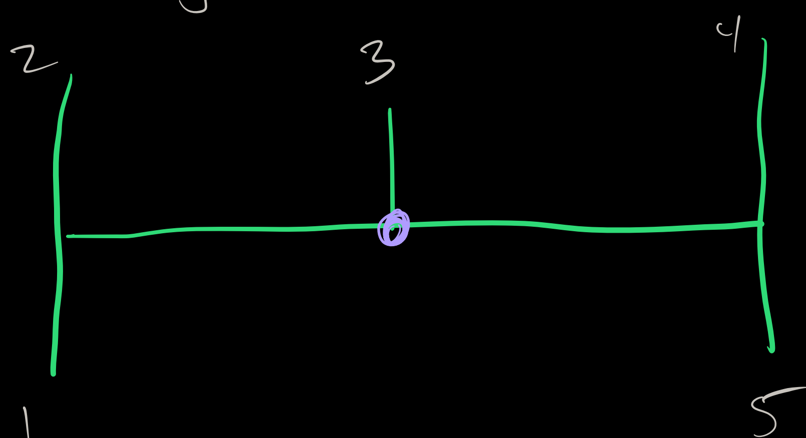
JJMC, LR, SŽ

Is this just a feature of 4pt?

No! 5pt even richer &

a fascinating surprise

No adjoint linear building block



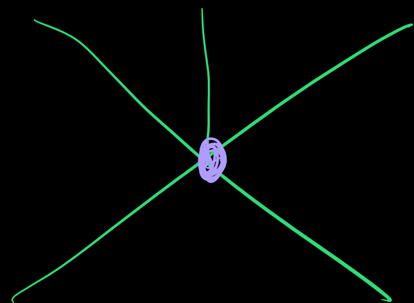
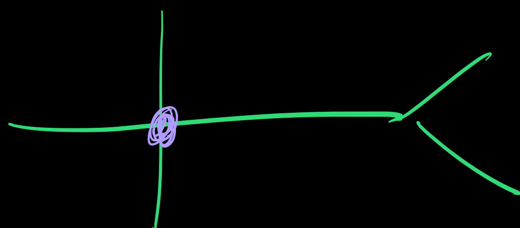
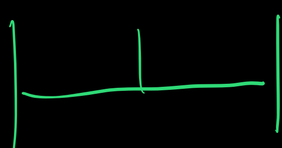
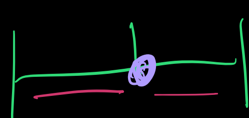
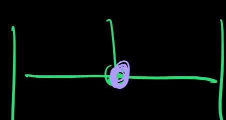
Relaxed Instead

$$(k_1-k_2)\cdot(k_5-k_4)$$

JJMC, LR, SŽ

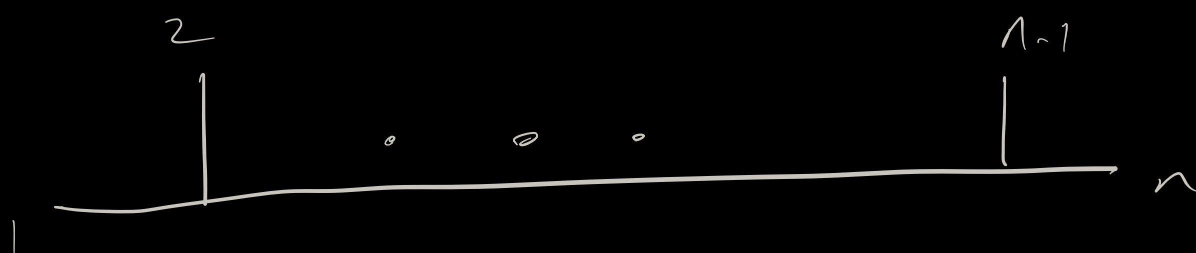
2104.08370

5 pt Algebraic Structures

	P_5	d^5	(P)
	Hybrid	$d^4 f^3$	(h)
	Adj	$f^3 f^3 f^3$	(a)
	Relaxed Adj		(r)
	Sandwhich	$f^3 d^3 f^3$	(s)

Lesson from five points.

- Odd multiplicity EFT
  linear building block: Relaxed
- Even multiplicity: Adjoint



$(k_1 - k_2) \cdot (k_n - k_{n-1})$

# of vector adjoint color-dual BB?

Open Question

- even 5pt exhausting via ansatz, motivating construction approach!

STAY TUNED!

Color-dual Fate of $N=4$ SG

JJMC, ML, NP 2203.03592

- Diverges in 4D at 4-loops
  Bern, Davies, Denen Smirnov, Smirnov '13
- Seems tied to anomalous behavior at 1-loop
  JJMC, Kallosh Roiban, Tseytlin '13
- Counterterm related to $\underline{\mathrm{tr}\, F^3}$
  - does resolve 1-loop anomalous bhvr.
  - doesn't mess up UV at 1 & 2 loops.

'17 Bern, Edison, Kosower, Parra-Martinez
'17 Bern, Parra-Martinez, Roiban
'19 Bern, Kosower, Parra-Martinez

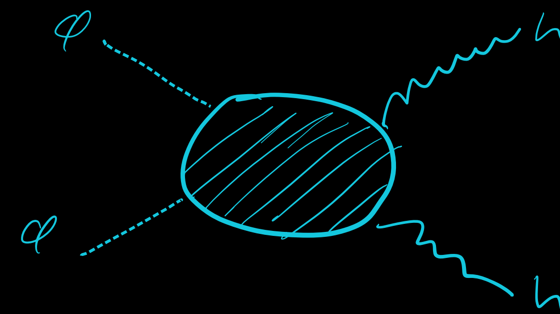
Nic Pavao

Matt Lewandowski

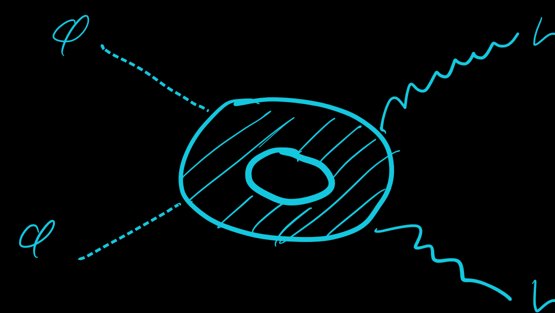
Anomalous Behavior

Marcus '85

$$A^{SG}_{tree}(\varphi \varphi h^+ h^+) = 0$$



$$\overset{\mathcal{N}=4}{A^{SG}_{loop}}(\varphi \varphi h^+ h^+) \neq 0$$



Can understand behavior via double-copy

half max SG = maximal SYM $\otimes$ pure YM

Anomalous Behavior

$g^+ \otimes g^+ = h^+$
$g^+ \otimes g^- = \varphi$
$g^- \otimes g^- = h^-$

$$A^{SG}(\varphi\varphi h^+ h^+) = A^{SYM}(g^- g^- g^+ g^+) \otimes A^{YM}(g^+ g^+ g^+ g^+)$$

A^{SYM}: B/C SUSY: ONLY VALID HEL STRUCTURE TREE OR LOOP

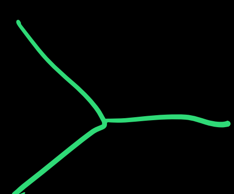
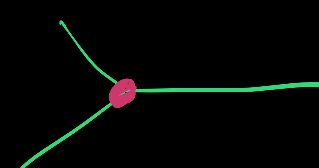
A^{YM}: Zero at tree level (secretly SUSY at tree) NON-ZERO AT 1 LOOP!

To cancel anomaly need COUNTERTERM that admits $4g^+$ even at tree level $\underbrace{tr(F^3)}$

So is $\mathrm{tr}F^2 + \alpha' \,\mathrm{tr}F^3$ Double Copy Consistent?

(ie color dual & factorizing for all multiplicity at tree level)

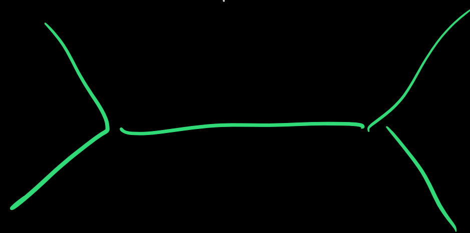
YM $\qquad\qquad\qquad\qquad$ F^3

 $+ \ \alpha'$ 

3pt sure!

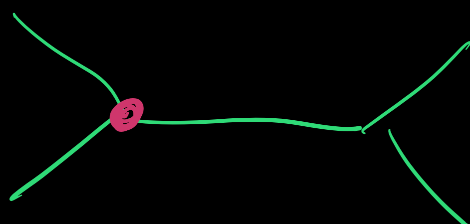
Color Dual?

4pt



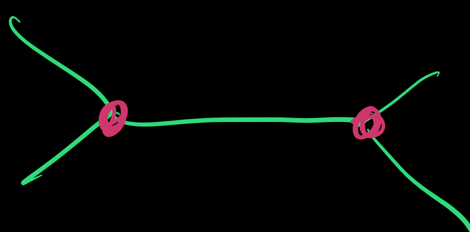
(YM) ✓

α'

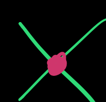


(single insertion) ✓

α'^2



(double insertion) ✗



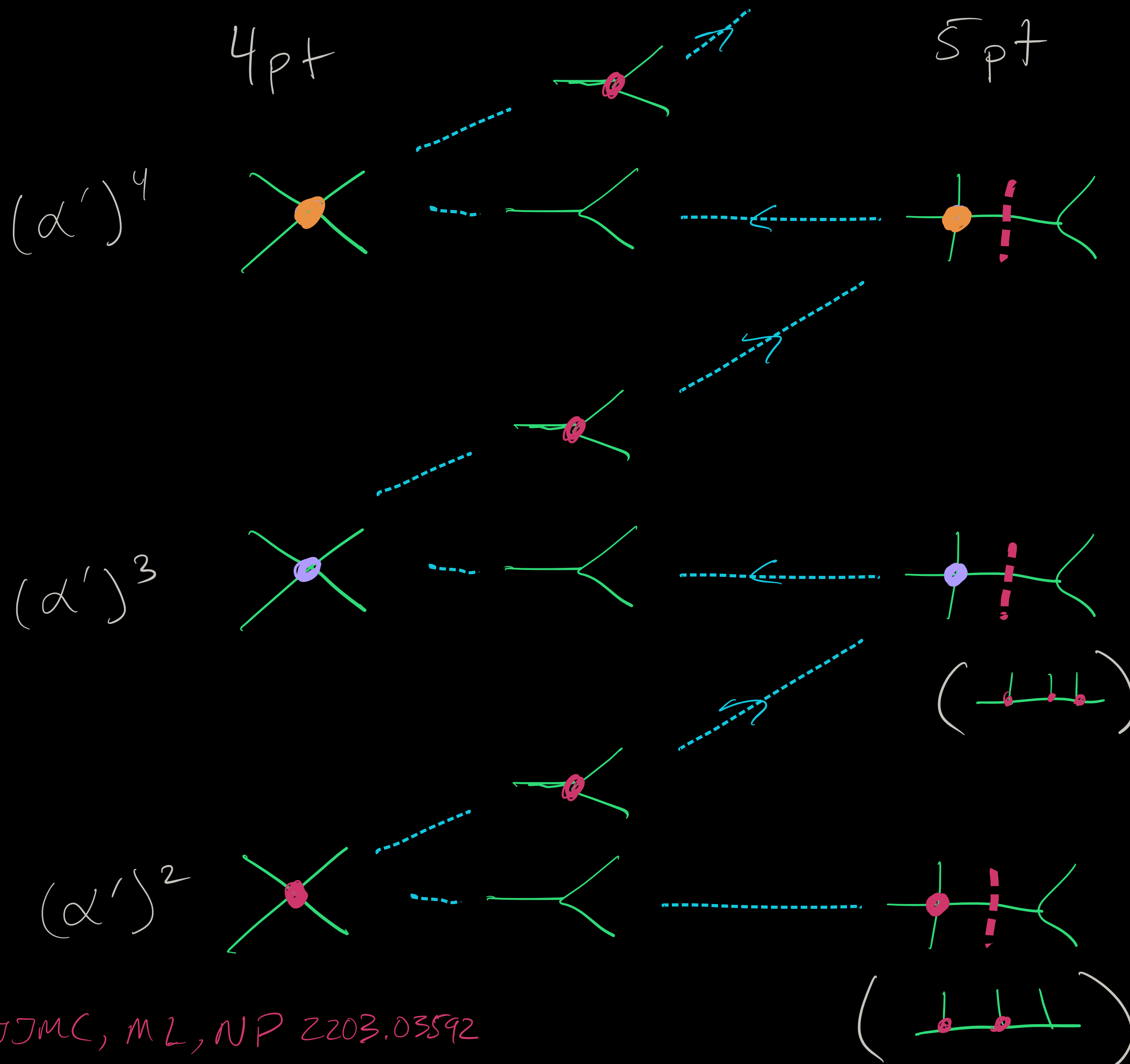
Needs 4pt $tr(F^4)$ ct.

So is $\mathrm{tr}F^2 + \alpha' \mathrm{tr}F^3 + \dots$

Double Copy Consistent

with a finite # of CT?

Apparently not



JJMC, ML, NP 2203.03592

Requiring C/K &

Factorization even

Between 4 & 5 point

INDUCES A TOWER

ALL THE WAY TO THE

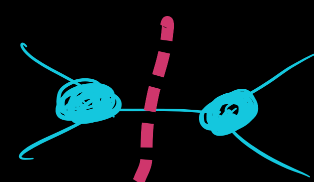
UV

How can you tell?

4pt Ansatz:

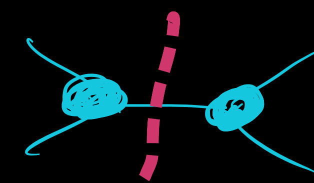
$$n^{4pt} = \sum_{i \in I}^{8} \sum_{x,y}^{2x+3y<MAX} c^{i}_{xy}\, \sigma_2^{x} \sigma_3^{y}\, n_i$$

fix on cuts to 3 points



Everything is either precluded, set to α' or 1 (YM coupl'g), or free (could be set to ∅)

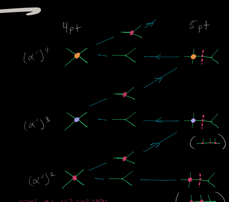
$$A = A^{YM} + \alpha' A^{YM+F^3} + \alpha'^2 A^{(F^3)^2+F^4} +$$



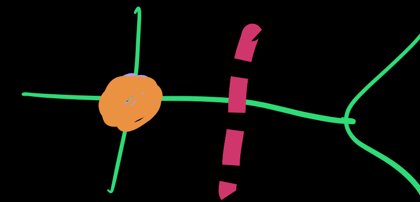
$$\alpha'^3 \left(c_3 \left(A^{D^2F^4} + \zeta_2 A^{YM+F^3} \right) + \tilde{c}_3 \left(\zeta_3 A^{YM} \right) \right) +$$

$$\alpha'^4 \left[c_4 \left(A^{((DF)^2)^2} + \zeta_2 A^{(F^3)^2+F^4} \right) + \overset{\checkmark}{c_4} A^{(DF^2)^2}_2 + \tilde{c}_4 c_3 A^{YM+F^3} \right]$$

c unconstrained – could be zero

unless  forces particular values

Which it does !



$$C_3 = 1$$

$$\tilde{C}_4 = 1 + \tilde{C}_3$$

$$C_4 = \check{C}_4 = 1$$

$$\Longrightarrow$$

$$n = n^{(DF)^2 + YM} + \mathcal{O}(\alpha'^5) + \tilde{C}_3 \, \alpha'^3 \zeta_3 \left(n^{YM} + \alpha' n^{F^3} + \dots \right)$$

$$\mathcal{N}=4\ SG + \text{c.t.} = \mathcal{N}=4\ sYM \otimes \left[\left(YM + \alpha' \operatorname{tr}(F^3) + \alpha'^2 \operatorname{tr}(F^4) + \cdots \right. \right.$$

(resums?)

$$= [DF^2 + YM] + \tilde{c}_3 \alpha'^3 (YM + \alpha' \operatorname{tr}(F^3) + \cdots) \check{G}_3 + \cdots \Big]$$

= Berkovitz Witten SG

or Heterotic String.

Calculated through α'^4

Proof & Loop Level results await!

(A few words about DF^2)

$$DF^2 \otimes sYM = CSG$$

Johanson, Nohle '17

$$(DF^2 + YM) \otimes sYM = \text{Weyl-Eistein}$$

Johansson, Mogull, Teng '18

$$(DF^2 + YM) \otimes Z = {}^{*}\text{Bosonic String}$$

$$(DF^2 + YM)^{sV} \otimes sYM = {}^{*}\text{gravitons in Heterotic String}$$

$$(X)^{sV}_{c} = X_a \otimes^{ab} sV(Z_{bc})$$

*Azvedo, Chiodaroli, Johansson, Schlotterer '18

Next Steps

- Composition to build vector BB?
- Proof that $\mathrm{tr}\,F^3$ closes to $(DF)^2 + YM$
- Effect of tower on explicit loop-level calculation in $\mathcal{N}=4$ SG + ...
- Loop-level composition?
- KLT composition at Tree Level?
- Lot's to Do!

[Gen KLT: Chi, Elvang, Herderschee, Jones, Paranjape 2106.12600]

Summary:

- color-kinematics lets us bootstrap ≡ GI. 2010.1345 2108.06798 2112.05178
- composition lets us build & classify EFT operators for gauge & gravity theories with SMALL set of Building Blocks 1910.12850 2104.08370

- $\mathcal{N}=4$ SG may have a non-local color-dual fate: 2203.03592
  - Berkovits Witten CSG
  - Heterotic String