CLASSICAL BLACK HOLE SCATTERING FROM A WORLDLINE QFT

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Based on joint work with

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2010:02865, *JHEP* 02 (2021) 048

2101.12688, *PRL* 126 (2021) 20

2106.10256, *PRL* 128 (2022) 1

2109.04465, *JHEP 01 (2022) 027*

2109.10345, PRD 105 *(2022) 2*

* 2201.07778, *PRL* 128 *(2022) 14*

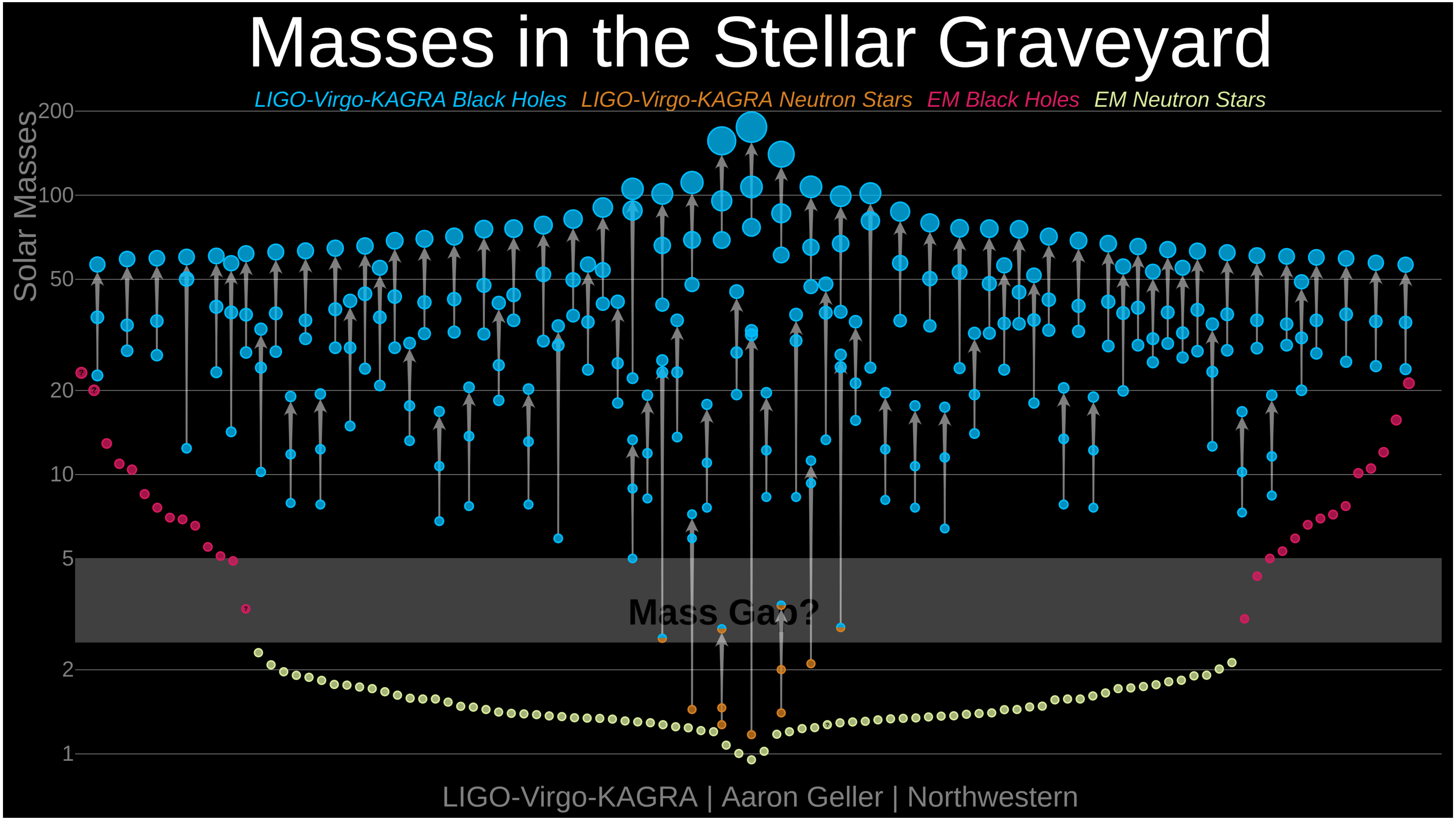
2206.next week

SAGEX Scattering Amplitudes: from Geometry to Experiment

RTG 2575: Rethinking Quantum Field Theory

SAGEX Closing meeting, 23/06/22

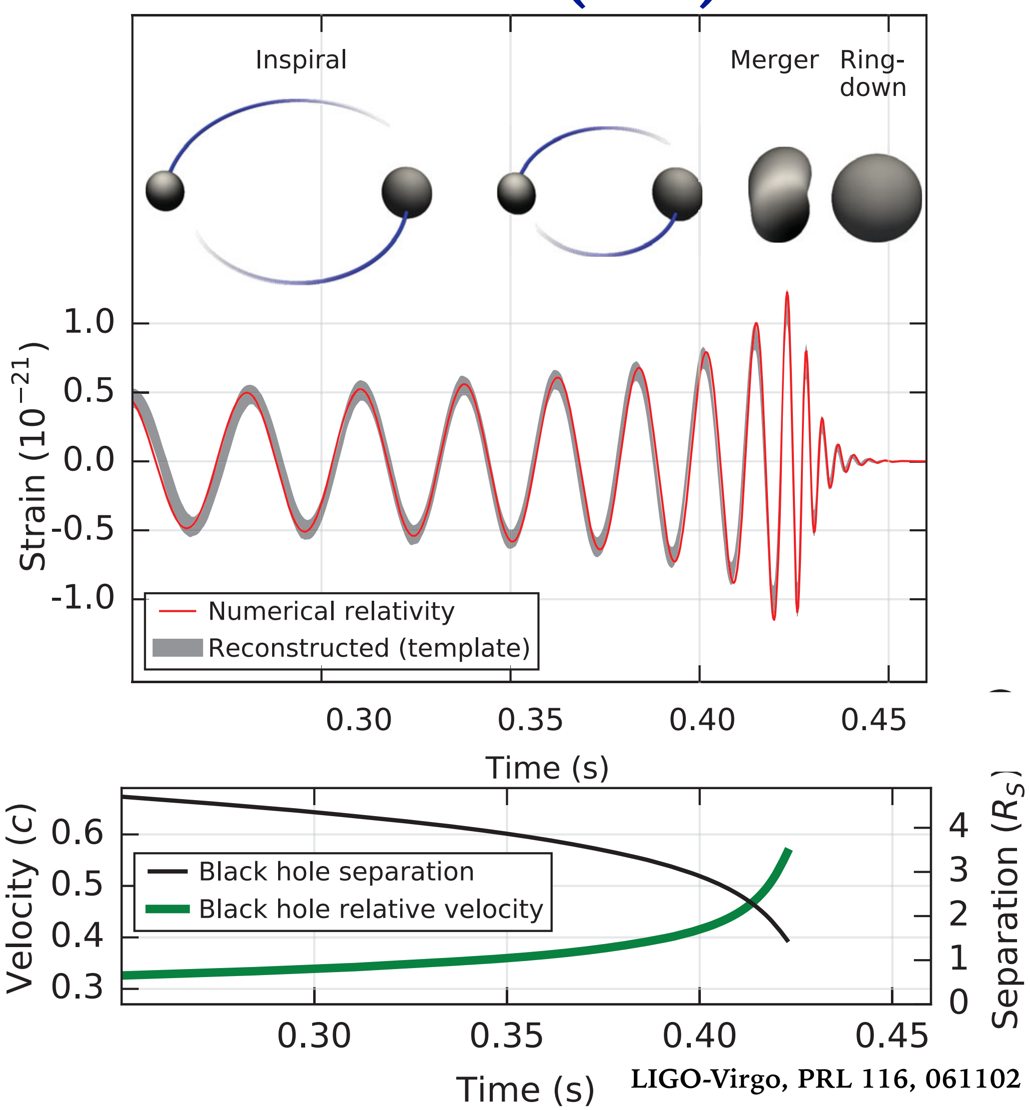
GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA



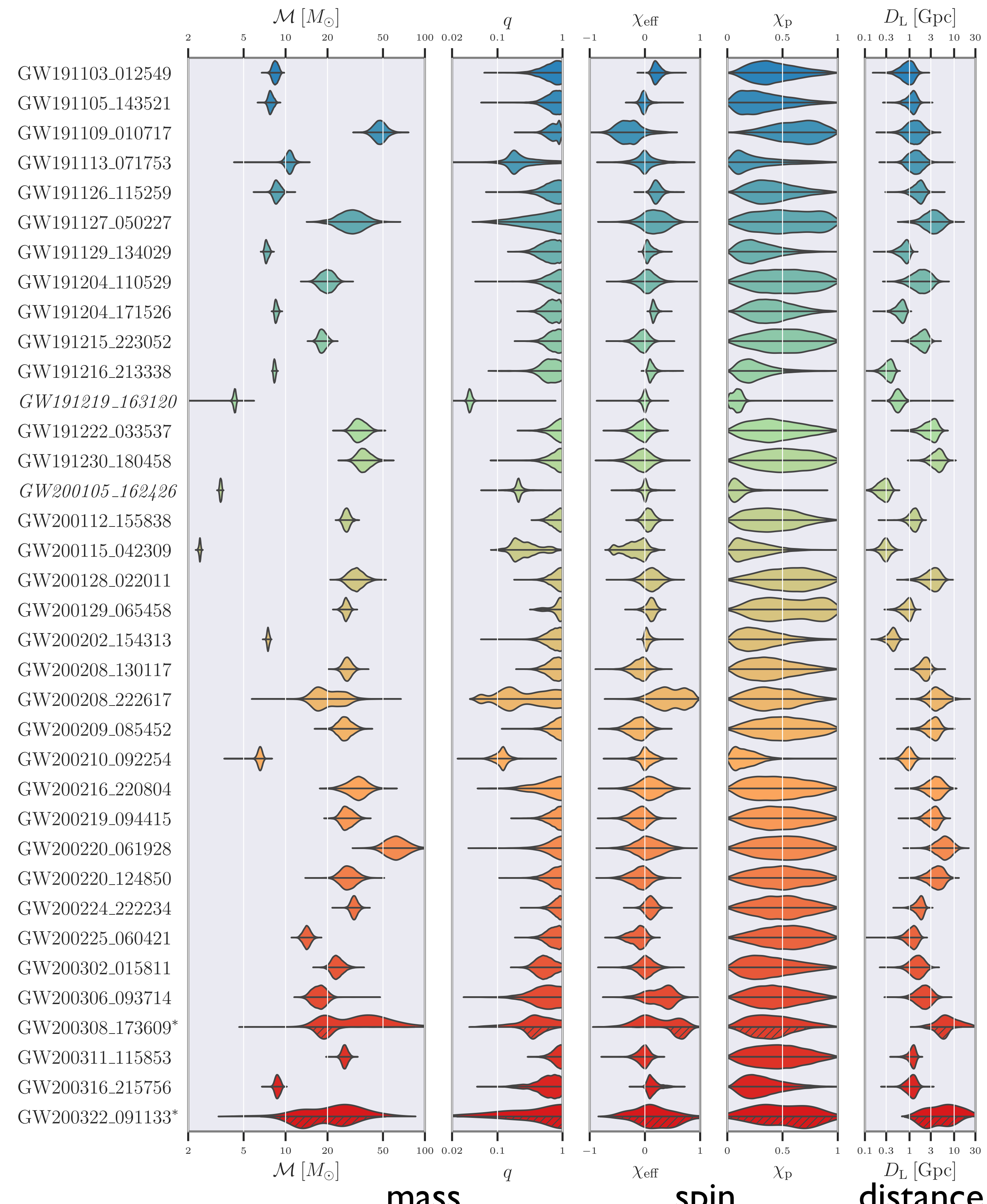
Following GW150914: To date 90 binary mergers detected by LIGO-Virgo-Karga Collaboration

GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA

Binary mergers of black holes (BHs) and neutron stars (NS)



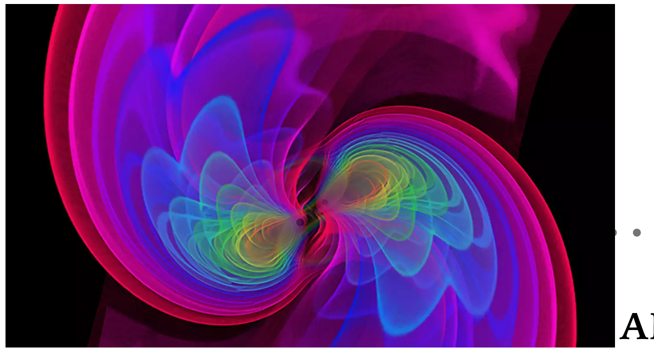
LIGO-Virgo, PRL 116, 061102 (2016)



LVG collaboration arXiv:2111.03606

Measurements of binary parameters: Masses, Spins, Distance

PHYSICS CASES


AEI

- 3rd generation of GW observatories (Einstein Telescope; Advanced LIGO, LISA) to start in 2030’s.
- Highly increased sensitivity expected: Need for high precision theory predictions

Astrophysics:

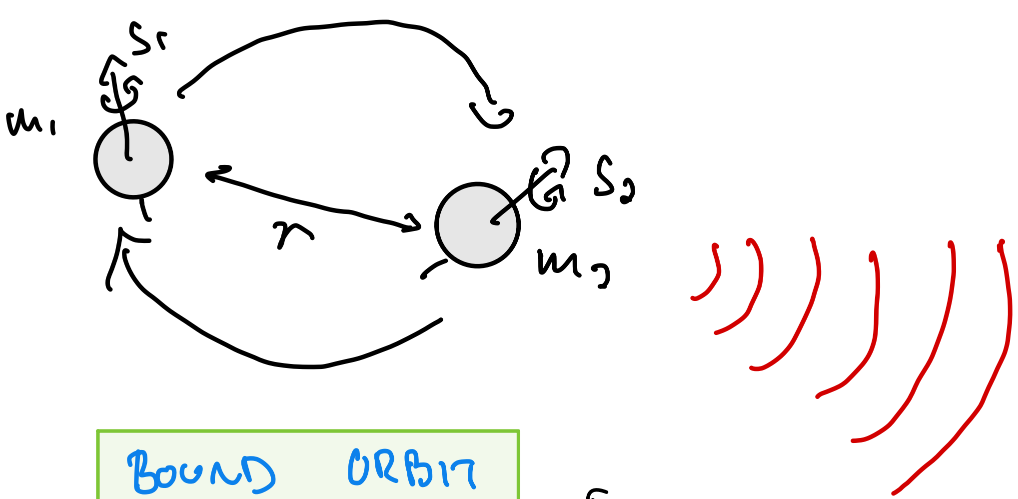
- Black hole formation & evolution
- Neutron star properties: Equation of state, strong interacting matter
- Multi-messenger astronomy
- New astrophysical sources of GW

Fundamental physics:

- Precision tests of (strong field) GR
- New physics signals? Modifications of GR, Higher curvature, Dark Matter…

Gravitational 2 body problem

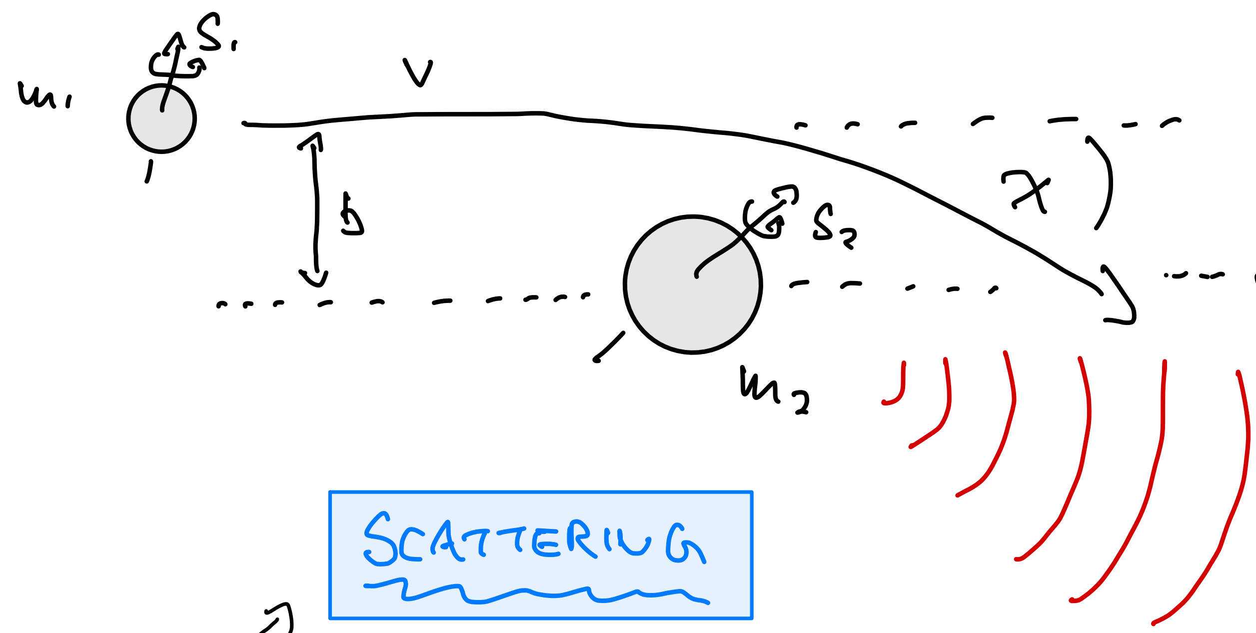
[BLACK HOLES, NEUTRON STARS OR STARS]



BOUND ORBIT

INSPIRALING PHASE

VS.



SCATTERING

WELL SEPERATED $|b| \gg r \sim GM$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa \cdot h_{\mu\nu}$$

PERTURBATION THEORY

κ: "POST-MINKOWSKIAN" EXPANSION (WEAK FIELD)

κ & v: "POST-NEWTONIAN" EXPANSION (SLOW VELOCITY & WEAK FIELD)

$$\frac{\kappa^2 m}{r} \sim v^2 \quad \text{(VIRIAL THM)}$$

⇒ USE perturbative quantum field thy techniques ?

THE 2-BODY PROBLEM IN GR: TRADITIONAL APPROACH

$$S = -\sum_{i=1}^{2} m_i \int dz_i \left[\sqrt{-g_{\mu\nu}(x)\, \dot{x}_i^{\mu}(z_i)\, \dot{x}_i^{\nu}(z_i)} + \frac{2}{\kappa^2} \int d^4x \sqrt{-g}\, R + \begin{array}{l}\text{finite size}\\ \text{\& spin}\\ \text{corrections}\end{array} \right]$$

POINT PARTICLE APPROXIMATION

BULK GRAVITY

① EQUATIONS OF MOTION:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 4\pi G\, T_{\mu\nu} \qquad \ddot{x}^{\mu} - \Gamma^{\mu}{}_{\nu\rho}\, \dot{x}^{\nu} \dot{x}^{\rho} = 0$$

(EINSTEIN) (GEODESIC)

② SOLVE ITERATIVELY IN κ:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} \kappa^n h_{\mu\nu}^{(n)}(x) \qquad x_i^{\mu}(z) = b_i^{\mu} + v_i^{\mu} z + \sum_{n=1}^{\infty} \kappa^n z_{i,(n)}^{\mu}(z)$$

③ CONSTRUCT OBSERVABLES:

FAR FIELD WAVEFORM

retarded time $u = t - r$

$$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(u, \theta, \varphi)}{r} + \mathcal{O}(1/r^2)$$

"IMPULSE" (CHANGE OF MOMENTUM):

$$\Delta p_i^{\mu} = m_i \left[\dot{x}_i^{\mu}(t=+\infty) - \dot{x}_i^{\mu}(t=-\infty) \right]$$

Do this using QFT!

USING QFT TECHNIQUES TO SOLVE CLASSICAL FIELD EQUATIONS

CONSIDER SCALAR FIELD THY AS PROXY:

$$S[\phi; Q] = \frac{1}{2}\int d^4x \left[(\partial_\mu \phi)^2 + m^2\phi^2\right] + S_{int}[\phi; Q]$$

Q: PHYSICAL SOURCE OR BACKGROUND

GOAL: (PERTURBATIVE) SOLUTION OF E.O.M.:

$$\left.\frac{\delta S[\phi, Q]}{\delta \phi}\right|_{\phi=\phi_{class}(x)} \doteq 0$$

QFT: GENERATING FUNCTIONAL

$$e^{\frac{i}{\hbar}W[J]} = \int [D\phi] \exp\left\{\frac{i}{\hbar}S[\phi; Q] + \frac{i}{\hbar}\int d^4x\, J(x)\,\phi(x)\right\}$$

ONE-POINT FUNCTION

$$\langle \hat{\phi}_H(x) \rangle_{IN-OUT} = \left.\frac{\delta W[J]}{\delta J(x)}\right|_{J=0}$$

EFFECTIVE ACTION:

(LEGENDRE-TRANSFORM)

$$S_{eff}[\phi] = \frac{i}{\hbar}\int d^4x\, J(x)\,\phi(x) - W[J]$$

ONE-POINT FUNCTION & E.O.M.

① EFFECTIVE E.O.M. ARE SOLVED BY ONE-POINT FUNCTION :

$$\left.\frac{\delta S_{eff}[\phi]}{\delta \phi(x)}\right|_{\phi(x) = \langle \hat{\phi}_H(x) \rangle} = 0$$

② TREE-LEVEL ⇒ CLASSICAL ACTION : $S_{eff}[\phi] = S[\phi; Q] + \mathcal{O}(\hbar)$

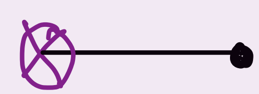
⇒ TREE-LEVEL (FEYNMAN-DIAGRAMMATIC) EVALUATION OF $\langle \hat{\phi}_H \rangle$ YIELDS SOLUTION TO CLASSICAL E.O.M.

CAUSALITY:

EXAMPLE :

$$S[\phi] = \frac{1}{2} \int d^4x \left[(\partial_\mu \phi)^2 + m^2 \phi(x)^2 + Q(x)\, \phi(x) \right]$$

IN-IN ONE POINT FUNCTION :

$$\langle \hat{\phi}_H(x) \rangle_{\text{IN-OUT}} =$$  x $$= \int d^4y \; G_{FEYN}(x-y)\; Q(y)$$

Q

SOLVES E.O.M BUT WE WANT RETARDED PROPAGATOR !

IN-OUT FORMALISM: STANDARD PATH INTEGRAL

[Galley,Tiglio]

INTERACTION PICTURE:

STATE EVOLVES WITH $\hat{H}_{INT}$,
OPERATORS WITH $\hat{H}_0$

$$|\psi(t)\rangle = \mathcal{U}(t,-\infty)\,|\psi\rangle$$

$$\mathcal{U}(T,T') = \mathcal{T}\exp\left[\frac{i}{\hbar}\int_T^{T'} dt \int d^3x\, \hat{H}_{INT}[\phi_I(\vec{x},t)]\right]$$

HEISENBERG PICTURE:

OPERATORS INERT, STATE EVOLVE WITH $\hat{H}_0 + \hat{H}_{INT}$

RELATION:

$$\hat{\phi}_I(t,\vec{x}) = \mathcal{U}(t,-\infty)\,\hat{\phi}_H(t,\vec{x})\,\mathcal{U}(-\infty,t)$$

PATH INTEGRAL REPRESENTATION

$|0\rangle$: GROUNDSTATE AT $T=-\infty$

$$\langle 0|\,\mathcal{U}_J(\infty,-\infty)\,|0\rangle = \int [D\phi]\exp\left\{\frac{i}{\hbar}S[\phi;Q] + \frac{i}{\hbar}\int d^4x\, J(x)\,\phi(x)\right\} = e^{\frac{i}{\hbar}W[J]}$$

$$\langle\hat{\phi}_H(t,\vec{x})\rangle_{IN\text{-}OUT} = \left.\frac{\delta W[J]}{\delta J(t,\vec{x})}\right|_{J=0} = \langle 0|\,\mathcal{U}(\infty,t)\,\hat{\phi}_I(t,\vec{x})\,\mathcal{U}(t,-\infty)\,|0\rangle$$

$$= \langle 0|\,\mathcal{U}(\infty,-\infty)\,\hat{\phi}_H(t,\vec{x})\,|0\rangle = {}_{OUT}\langle 0|\hat{\phi}_H(x)|0\rangle_{IN}$$

IN-IN (SCHWINGER-KELDYSH) FORMALISM

[Galley, Tiglio]

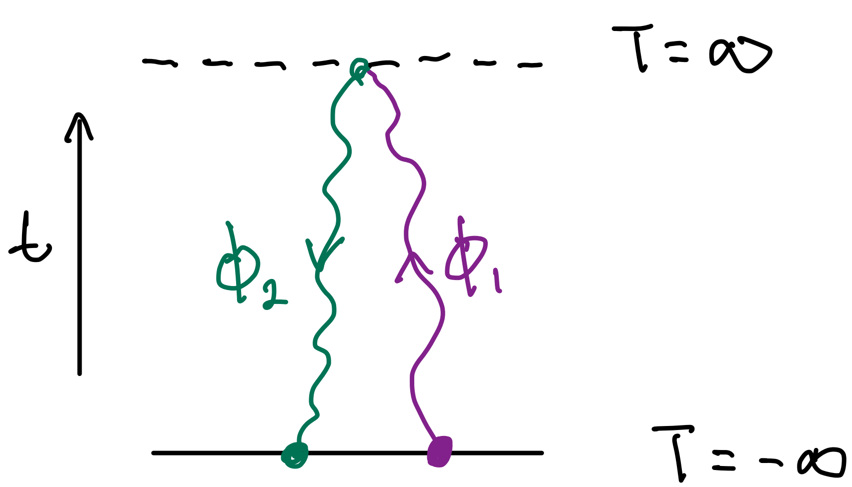
IN-OUT (STANDARD) FORMALISM YIELDS $\langle \hat\phi_H(x)\rangle_{\text{IN-OUT}} = {}_{\text{OUT}}\langle 0|\hat\phi_H(x)|0\rangle_{\text{IN}}$ BUT WANT

$$\langle \hat\phi_H(x)\rangle_{\text{IN-IN}} := {}_{\text{IN}}\langle 0|\hat\phi_H(x)|0\rangle_{\text{IN}} = \langle 0|\hat{U}(-\infty,t)\,\hat\phi_I(t,\vec{x})\,\hat{U}(t,-\infty)|0\rangle$$

NEED TWO TIME EVOLUTION OPERATORS $\Rightarrow$ DOUBLE FIELDS IN PATH-INTEGRAL

$$e^{\frac{i}{\hbar}W[J_1,J_2]} = \langle 0|\hat{U}_{J_2}(-\infty,\infty)\,\hat{U}_{J_1}(\infty,-\infty)|0\rangle$$

$$= \int D\phi_1 D\phi_2 \exp\left\{\frac{i}{\hbar}\left(S[\phi_1]-S[\phi_2]+\int d^4x\, J_1(x)\phi_1(x)-J_2(x)\phi_2(x)\right)\right\}$$



BOUNDARY CONDITIONS:

$$\phi_1(T=\infty,\vec{x}) = \phi_2(T=\infty,\vec{x})$$

$$\phi_1(T=-\infty,\vec{x}) = \phi_2(T=-\infty,\vec{x}) = 0$$

$$\boxed{\langle \Phi_H(x)\rangle_{\text{IN-IN}} = \left.\frac{\delta W[J_1,J_2]}{\delta J_1(x)}\right|_{J_i=0}}$$

KELDYSH BASIS

$\phi_+ = \frac{1}{2}(\phi_1 + \phi_2)$ $\qquad$ $\phi_- = \phi_1 - \phi_2$

(SAME FOR $J_\pm$)

THIS YIELDS

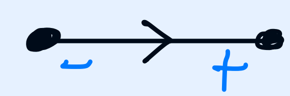
$$e^{\frac{i}{\hbar}W[J_+,J_-]} = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- \exp\left\{\frac{i}{\hbar}\left(S[\phi_+ + \tfrac{1}{2}\phi_-] - S[\phi_+ - \tfrac{1}{2}\phi_-] + \int d^4x \,(J_+\phi_- + J_-\phi_+)\right)\right\}$$

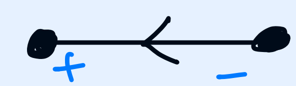
PROPAGATOR MATRIX FROM FREE PART :

$$\Rightarrow D^{ab}(x,y) = \begin{array}{cc} & \begin{array}{cc} + & \qquad\qquad - \end{array} \\ \begin{array}{c} + \\ - \end{array} & \begin{pmatrix} 0 & D_{adv}(x,y) \\ D_{ret}(x,y) & \frac{i}{2} D_H(x,y) \end{pmatrix} \end{array}$$

RETARDED PROPAGATOR

$\langle \{\phi(x), \phi(y)\} \rangle$ IRRELEVANT @ TREE-LEVEL

$D_{ret}(k) =$  $= \frac{i}{(k^0 + i\varepsilon)^2 - \vec{k}^2}$

$D_{adv}(k) =$  $= \frac{-i}{(k^0 - i\varepsilon)^2 - \vec{k}^2}$

VERTICES FROM

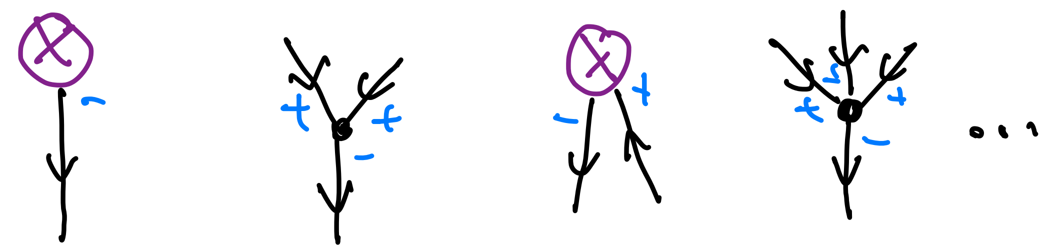
$$S_{INT}[\phi_+ + \tfrac{1}{2}\phi_-] - S_{INT}[\phi_+ - \tfrac{1}{2}\phi_-] = \phi_- \left(\frac{\delta S_{INT}(\phi)}{\delta \phi}\right)_{\phi \to \phi_+} + O(\phi_-^3)$$

$\Rightarrow$ $\boxed{\text{ONLY ODD NUMBER OF } \phi_- \text{ LEGS}}$

ONE-POINT FUNCTIONS @ TREE-LEVEL

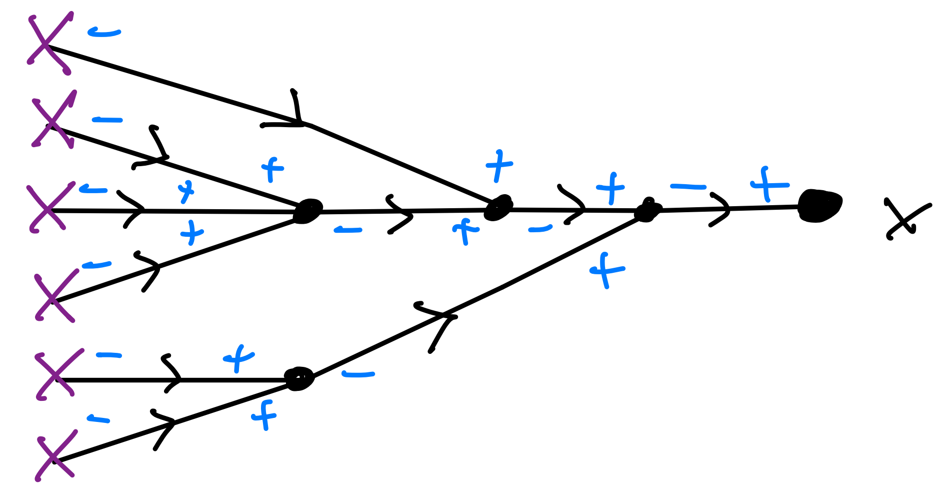
$$S_{INT}[\phi; Q] \xrightarrow[\text{IN-IN}]{} \phi_- \left(\frac{\delta S_{INT}[\phi, Q]}{\delta \phi} \right)_{\phi \to \phi_+} + O(\phi_-^3)$$

VERTICES:



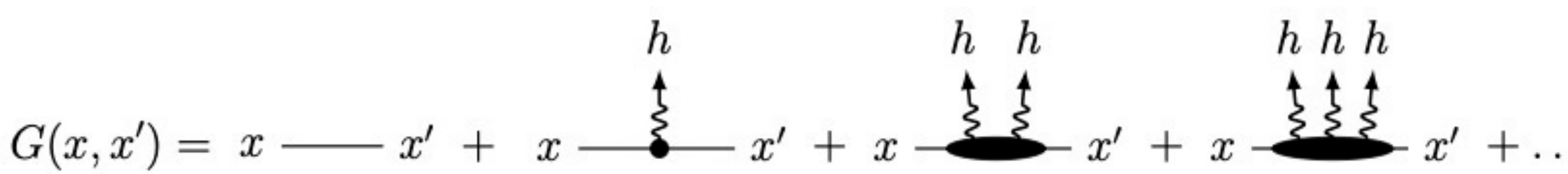
ONE-POINT FCT. ⇒

$$\langle \hat{\phi}_H(x) \rangle_{\text{IN-IN}} =$$



$$\boxed{\text{ONLY RETARDED PROPAGATORS CONTRIBUTE}}$$

WORLDLINE QUANTUM FIELD THEORY



WORLDLINE EFFECTIVE FIELD THEORY

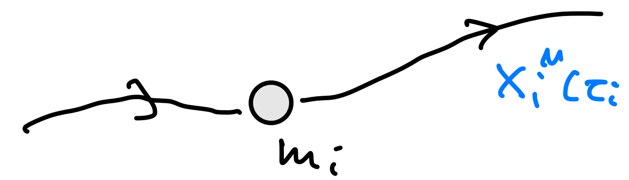
[Goldberger,Rothstein] [Porto,Källin] [Foffa,Sturani]

- MODEL BHs/NSs AS POINT PARTICLES:

$$S_p = -\sum_{i=1}^{2} m_i \int_{-\infty}^{\infty} d\tau_i \sqrt{g_{\mu\nu}\, \dot{x}_i^{\mu}\, \dot{x}_i^{\nu}}$$

BETTER: INTRODUCE EINBEIN $e(\tau)$:

$$S_p = -\frac{m}{2}\int d\tau \left(e^{-1} g_{\mu\nu}\, \dot{x}^{\mu} \dot{x}^{\nu} + e \right)$$



ALGEBRAIC E.O.M. YIELD $e^2 = g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}$ $\Rightarrow$ PROPER TIME GAUGE $e = 1 \Leftrightarrow \dot{x}^2 = 1$.

- ## INCLUSION OF FINITE SIZE / TIDAL EFFECTS

$$S_p = -\frac{m}{2}\int d\tau \Big(g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} + c_1 R\, \dot{x}^2 + c_2 R_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} + c_{E^2}\left(R_{\mu\alpha\nu\beta}\dot{x}^{\alpha}\dot{x}^{\alpha}\right)^2 + c_{B^2}\left(R^{*}_{\mu\alpha\nu\beta}\dot{x}^{\alpha}\dot{x}^{\alpha}\right)^2 + \dots$$

- ## COUPLE TO GRAVITY

$$S_G = \frac{2}{k^2}\int d^4x \sqrt{-g}\, R + S_{g.f.}$$

WEAK GRAVITATIONAL FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + k \cdot h_{\mu\nu}$$

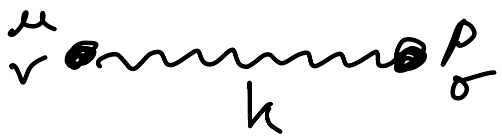
WORLDLINE QFT: FLUCTUATE GRAVITON & WORLDLINE

OBJECTIVE: FOCUS ON OBSERVABLES !

[Jakobsen, Mogull, JP, Steinhoff]

$$S = -2m_{Pl}^2 \int d^4x \sqrt{-g} R - \sum_i \frac{m_i}{2} \int d\tau_i \, g_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu \quad \left\} \begin{array}{l} g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa \, h_{\mu\nu}(x) \\ x_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + \underbrace{z_i^\mu(\tau_i)}_{\text{QUANTUM FIELDS}} \end{array} \right.$$

Graviton propagator in de Donder gauge



$$= i \frac{P_{\mu\nu;\rho\sigma}}{(k^0 + i\varepsilon)^2 - \vec{k}^2} \qquad P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

Worldline fluctuation propagator:

$$= -\frac{i}{m} \frac{\eta^{\mu\nu}}{(\omega + i\varepsilon)^2}$$

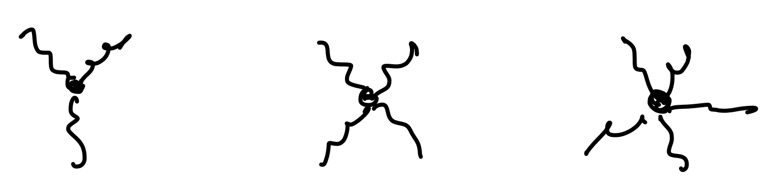
N.B.: $i\varepsilon$ prescription is crucial here!

For classical physics want retarded prop.

⇒ IN-IN FORMALISM

[Schwinger, Keldysh]

Graviton interactions:

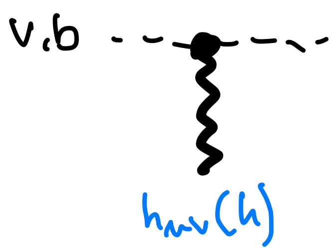


$+ \ldots$

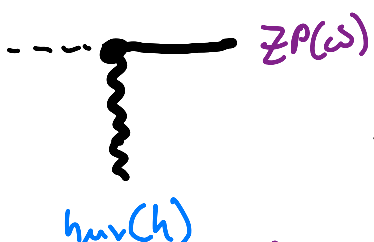
Worldline Interactions

EMERGE FROM $h_{\mu\nu}[X(\tau)]\,\dot{X}^\mu(\tau)\,\dot{X}^\nu(\tau)$ WITH $X_i^\mu(\tau_i) = \underbrace{b_i^\mu + \tau_i v_i^\mu}_{\text{"}Q\text{"}} + \underbrace{z_i^\mu(\tau_i)}_{\text{"}\phi\text{"}}$

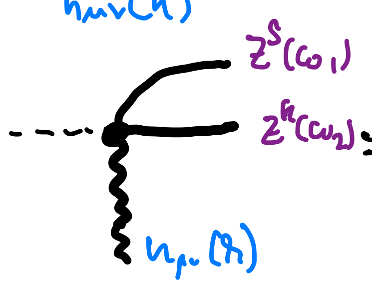
IN MOMENTUM SPACE



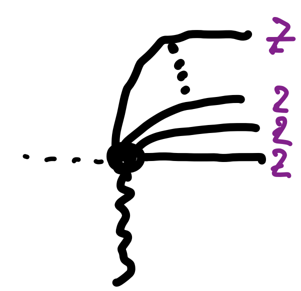
$$= -imK\, e^{ik\cdot b}\,\delta(k\cdot v)\,v^\mu v^\nu$$



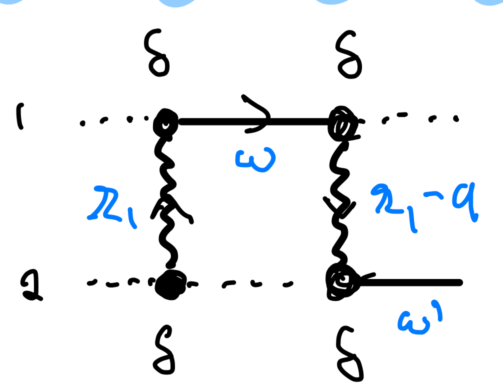
$$= mK\, e^{ik\cdot b}\,\delta(k\cdot v+\omega)\left(2\omega\, v^{(\mu}\delta^{\nu)}_\rho + v^\mu v^\nu k_\rho\right)$$



$= \ldots$ and higher!



TREE LEVEL WQFT GRAPHS YIELD LOOP-LEVEL FEYNMAN INTEGRALS

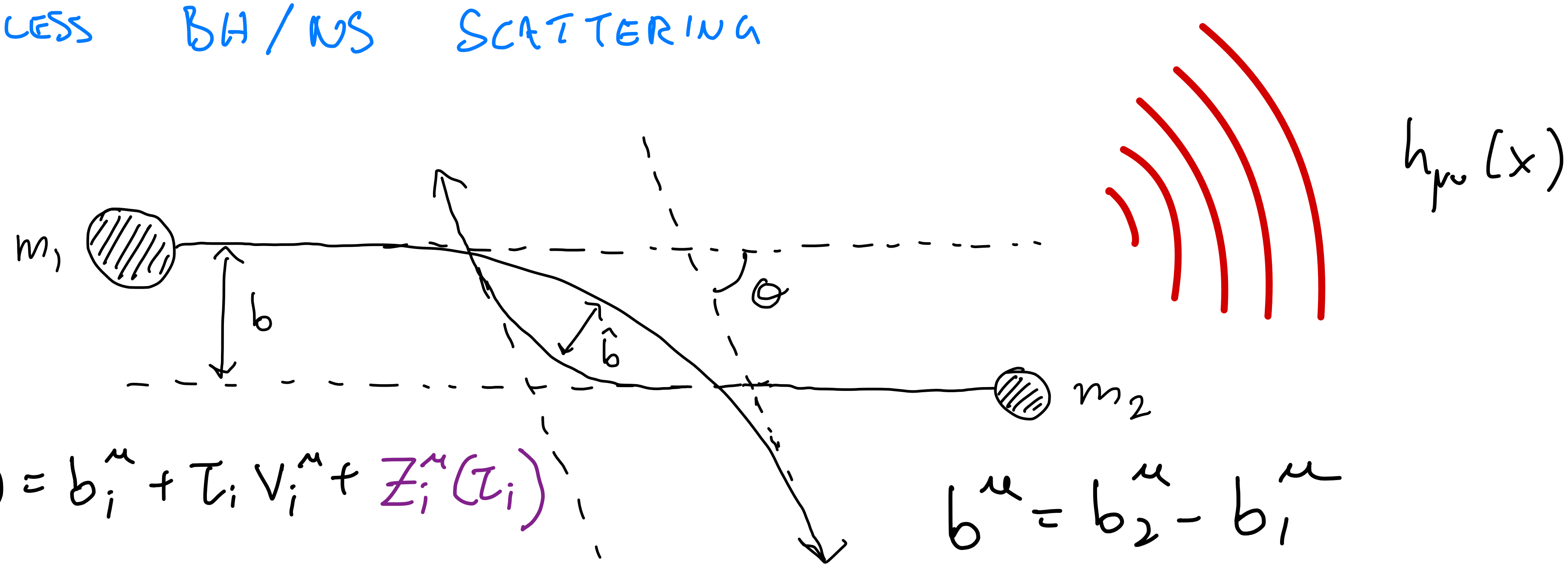


$$\hat{=} \int d^4\ell_1 \int d\omega\; \delta^4(\ldots)\ldots = \delta(q\cdot v_1)\,\delta(q\cdot v_2) \int d^4\ell_1\, \delta(\ell_1\cdot v_1)\ldots$$

1-LOOP

WQFT OBSERVABLES: ONE-POINT FUNCTIONS

SPIN-LESS BH / NS SCATTERING



$$X_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + Z_i^\mu(\tau_i)$$

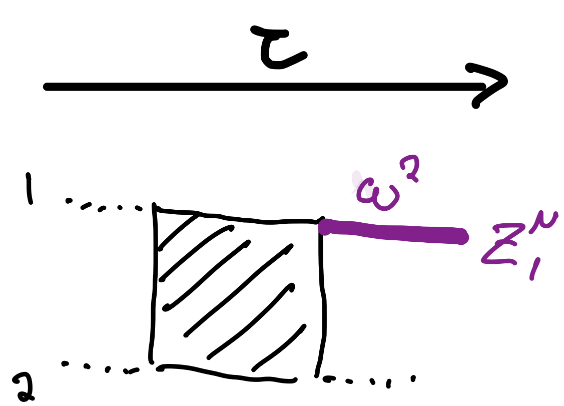
$$b^\mu = b_2^\mu - b_1^\mu$$

1 IMPULSE: CHANGE OF MOMENTUM

$$\Delta p_i^\mu = m_i \dot{X}(\tau)\Big|_{\tau=-\infty}^{\tau=\infty} = m_i \int_{-\infty}^{\infty} d\tau \, \langle \ddot{X}_i^\mu(\tau) \rangle_{WQFT} = m_i \int_{-\infty}^{\infty} d\tau \, \frac{d^2}{d\tau^2} \langle Z_i^\mu(\tau) \rangle_{WQFT}$$

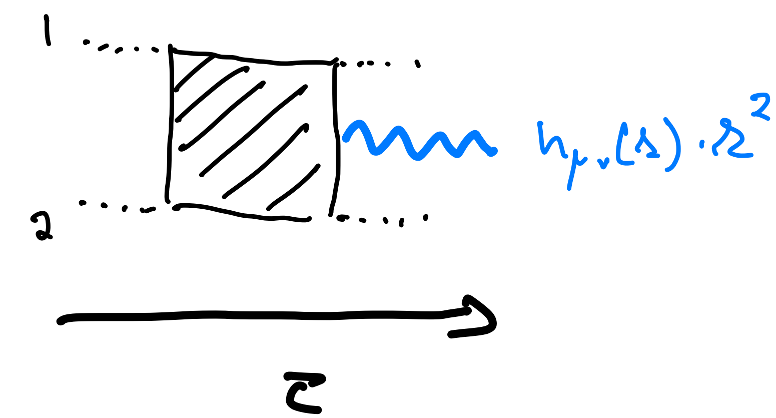
F.T.

$$= -m_i \, \omega^2 \langle Z_i^\mu(\omega) \rangle_{WQFT} \Big|_{\omega=0}$$



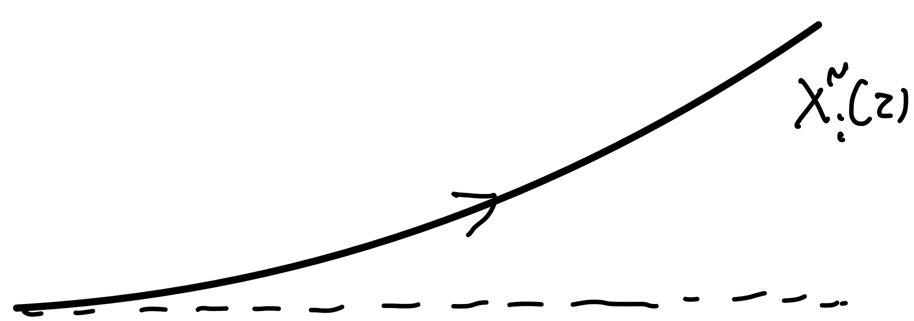
② EMITTED WAVEFORM: "BREMSSTRAHLUNG"

$$\mathcal{T}_{\mu\nu}^{\mathrm{T.T.}} = k^2 \left\langle h_{\mu\nu}(k) \right\rangle_{\mathrm{WQFT}}$$



③ TRAJECTORY ?

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + \int d\omega\, e^{i\omega\cdot\tau} \left\langle z_i^\mu(\omega) \right\rangle_{\mathrm{WQFT}}$$



PUTTING SPIN ON THE WORLDLINE:

{ "SUSY IN THE SKY WITH GRAVITONS"
[Jakobsen,Mogull,JP,Steinhoff]

Represent spin with Grassmann-odd ψ^a vectors

$$\frac{D\psi^a}{D\tau} = \dot{\psi}^a + i\dot{x}^\mu \omega_\mu{}^{ab}\psi_b$$

$$S = -m\int d\tau \left[\frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + i\bar{\psi}_a\frac{D\psi^a}{D\tau} + \frac{1}{2}R_{abcd}\bar{\psi}^a\psi^b\bar{\psi}^c\psi^d\right]$$ } valid up to $O(S^2)$

The spin tensor $S^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}p_\rho a_\sigma$ (a^ν: PAULI-LUBANSKI VECTOR)

$$S^{\mu\nu} = -2i\bar{\psi}^{[\mu}\psi^{\nu]}$$

$\{\bar{\psi}^\mu, \psi^\nu\}_{P.B.} = -i\eta^{\mu\nu}$ (1^{st}-order formalism)

$\Rightarrow \{S^{\mu\nu}, S^{\rho\sigma}\}_{P.B.} = \eta^{\mu\rho}S^{\nu\sigma} + \eta^{\nu\sigma}S^{\mu\rho} - \eta^{\nu\rho}S^{\mu\sigma} - \eta^{\mu\sigma}S^{\nu\rho}$

$\mathcal{N}=2$ SUSY implies four conserved supercharges:

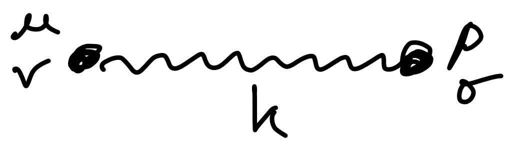
$$\underbrace{p^2}_{\text{energy}}, \quad \underbrace{p\cdot\psi, \quad p\cdot\bar{\psi}}_{\Rightarrow\ \text{SSC}\ p_\mu S^{\mu\nu}=0}, \quad \underbrace{\psi\cdot\bar{\psi}}_{\text{spin length}}$$

$$\underbrace{S_E = -m\int d\tau\, C_E E_{ab}\bar{\psi}^a\psi^b\bar{\psi}\cdot\psi}_{\text{finite-size correction}}$$

Classical EoMs $\Rightarrow$ Mathisson-Papapetrou-Dixon eq$^{\underline{n}}$s @ $O(S^2)$

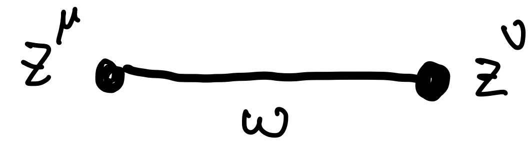
Spinning WQFT Feynman rules

Graviton propagator

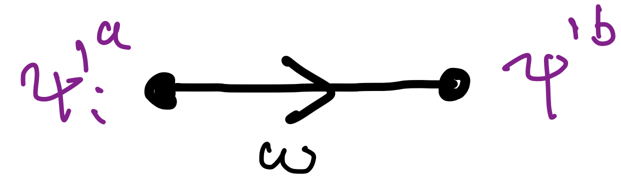


$$= i \frac{P_{\mu\nu;\rho\sigma}}{(k^0+i\epsilon)^2-\vec{k}^2}$$

Worldline fluctuation propagator:



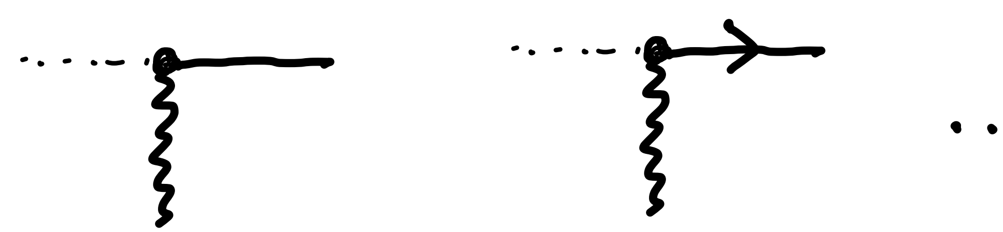
$$= -\frac{i}{m}\frac{\eta^{\mu\nu}}{(\omega+i\epsilon)^2}$$



$$= i\,\frac{\eta^{ab}}{\omega+i\epsilon}$$

Worldline interactions

$\sim v^2 + Sv + S^2$ $\qquad \sim S^2$ $\qquad \ldots$



$\sim v^2 + Sv + S^2$ $\qquad \sim v + S$ $\qquad \ldots$

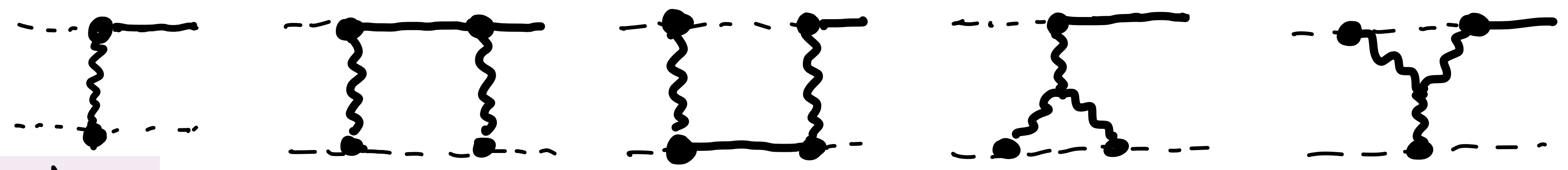
$\sim v^2 + Sv + v^2$ $\qquad \sim S^2$ $\qquad \ldots$

$\sim v^2 + Sv + v^2$ $\qquad \sim v + S$ $\qquad \ldots$

OBSERVABLES @ NLO

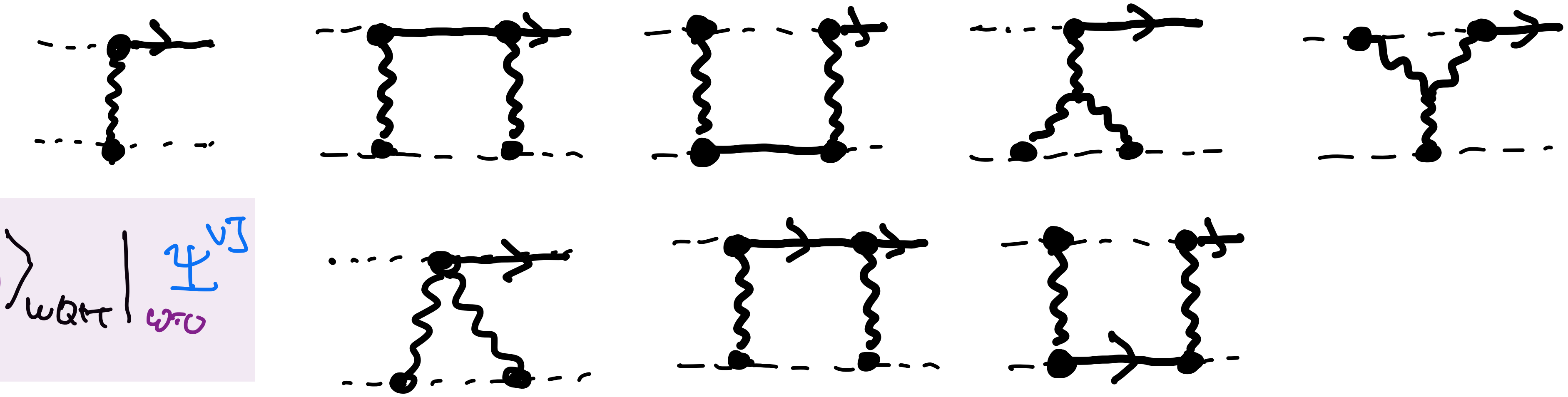
1PM 2PM

- **DEFLECTION**



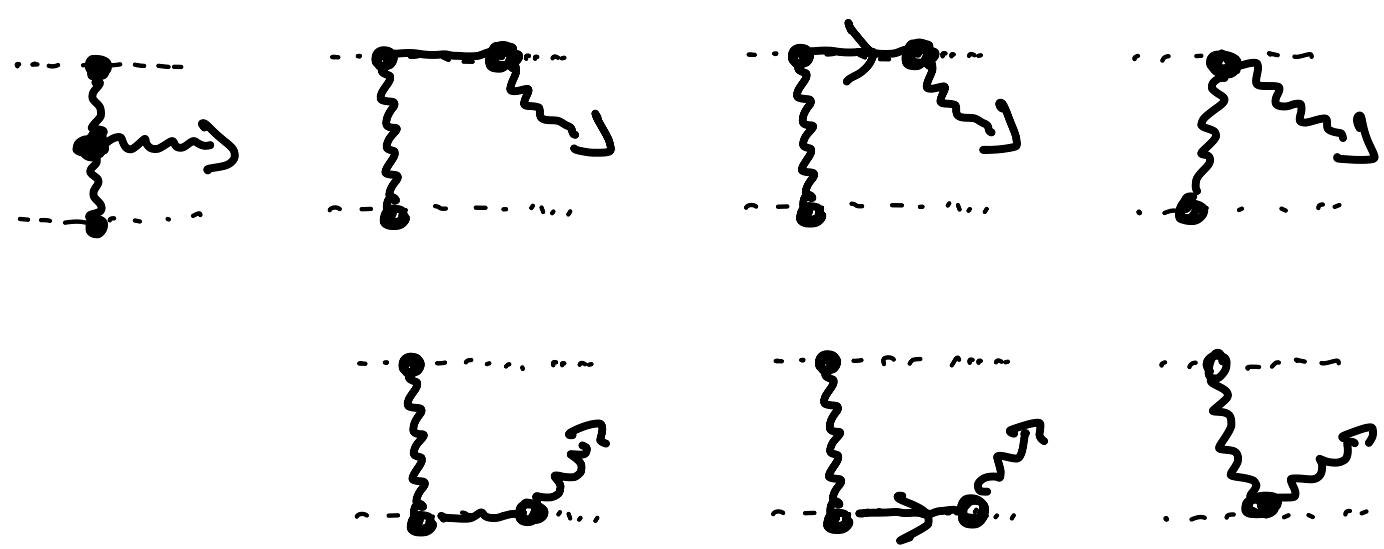
$$\Delta p_1^\mu = -m_1 \omega^2 \langle z_1^\mu(\omega) \rangle_{WQFT} \Big|_{\omega=0}$$

- **SPIN KICK**



$$\Delta S_1^{\mu\nu} = -2i\omega \langle \bar{\psi}^{[\mu}(\omega) \rangle_{WQFT} \Big|_{\omega=0} \Psi^{\nu]}$$

- **BREMSSTRAHLUNG**



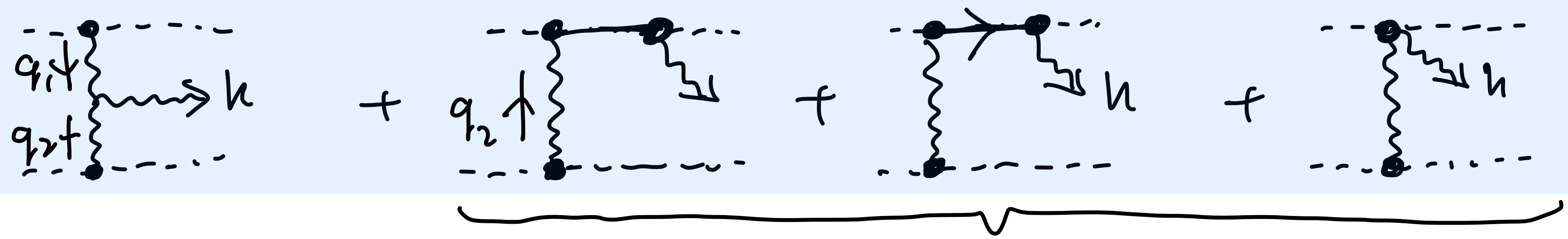
$$-i\kappa^2 \langle h^{\mu\nu}(k) \rangle_{WQFT}$$

Spinning Waveform

[Jakobsen, GM, Plefka, Steinhoff '21]

Sum on diagrams with an outgoing graviton. Integrate on internal lines:

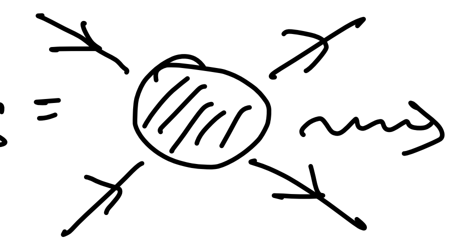
$\langle h_{\mu\nu}(k)\rangle =$



$+ (1 \leftrightarrow 2)$

We obtain the time-domain waveform for large $|\underline{x}| = r$. This requires integrating on the outgoing energy:

$M_s =$



$$\frac{f_{+,\times}(u,\hat{\underline{x}})}{r} = \frac{4G}{r} \int_\Omega e^{-ik\cdot x}\, \varepsilon^{\mu\nu}_{+,\times} \langle h_{\mu\nu}(k=\Omega p)\rangle \quad \Big\}\; p^\mu = (1, \hat{\underline{x}})$$

where $k^\mu = \Omega p^\mu$, $p^\mu = (1, \hat{\underline{x}})$ points to the observer.

$k\cdot x = \Omega p\cdot x = \Omega(t-r) = \Omega u$

$u = t - r =$ retarded time

Two components: $+/\times$ polarizations.

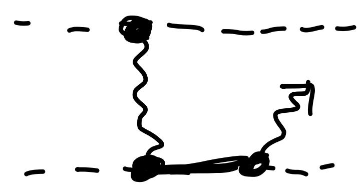
$$f_{+,\times}(m_1, m_2, u, \underbrace{\Theta, \phi}_{\hat{\underline{x}}}, \underbrace{v}_{\gamma}, |b|)$$

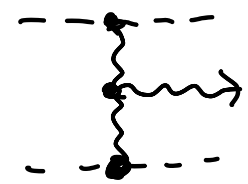
Non-spinning

Integrated waveform

first computed by [Kovacs,Thorne `75] in 4 long papers !

$$\frac{f^{(2)}}{m_1 m_2} = 4\pi \int_q e^{i q\cdot \tilde{b}} \Bigg(\underbrace{\frac{N_\mu q^\mu}{q^2 (q\cdot \hat{e}_1 - i\epsilon)}} + \underbrace{\frac{M_{\mu\nu} q^\mu q^\nu}{q^2 (q^2 + q\cdot L \cdot q)}} \Bigg)$$





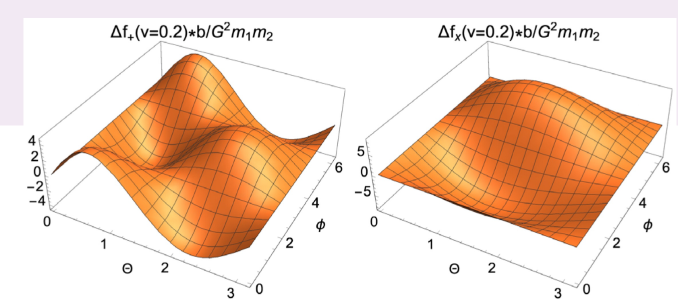
Performing these integrals yields time-domain waveform :

VIDEO

$$\boxed{\frac{f^{(2)}}{m_1 m_2} = \frac{\hat{e}_1 \cdot N}{\sqrt{b^2+\tau^2}} - \frac{b\cdot N}{b^2}\left(1 + \frac{\tau}{\sqrt{b^2+\tau^2}}\right) + \frac{2M^{ij}}{\Delta(G)}\left[\frac{(G_0+\alpha G_1)A^{ij} - (G_1+\alpha G_2)B^{ij}}{\sqrt{G(\alpha)}}\right]_{\alpha=0}^{\alpha=1}}$$

Wave memory

$$\frac{f^{(2)}(u=+\infty) - f^{(2)}(u=-\infty)}{m_1 m_2} = \frac{4(2\gamma^2-1)\, \varepsilon\cdot v_1 \left(2\, b\cdot\varepsilon\, \varrho\cdot v_1 - b\cdot\varrho\, \varepsilon\cdot v_1\right)}{b^2 \sqrt{\gamma^2-1}\, (\varrho\cdot v_1)^2} + 1 \leftrightarrow 2$$



$\gamma = v_1\cdot v_2$ $\quad b = b_2 - b_1$ $\quad \varrho = (1, \hat{x})$

ε : polarization

Deflections

$$\Delta p_i^\mu = -m_i\,\omega^2 \left\langle Z_i^\mu(\omega)\right\rangle_{\text{WQFT}}\Big|_{\omega=0}$$

Graphs with single outgoing worldline excitation Z_i^μ

1PM: 

outgoing line, $\omega=0$

2PM: 

Δp_1^μ

WE ONLY COMPUTE TREE-LEVEL GRAPHS

($\hbar=0$)

Integration gives (w/o spin)

$$\Delta p_1^\mu = \frac{G m_1 m_2 b^\mu}{b^2}\left(\frac{2(2\gamma^2-1)}{\sqrt{\gamma^2-1}} + \frac{3\pi}{4}\frac{(5\gamma^2-1)}{\sqrt{\gamma^2-1}}\frac{G(m_1+m_2)}{b}\right) + O(G^3)$$

$\gamma = v_1\cdot v_2$

1PM [Mogull,JP,Steinhoff]

2PM [Jakobsen,Mogull,JP,Steinhoff]

3PM [Jakobsen,Mogull]

AGREES WITH WEFT & AMPLITUDE APPROACHES [Källin,Porto][Bern et al][Brandhuber et al][Bjerrum-Bohr er al]

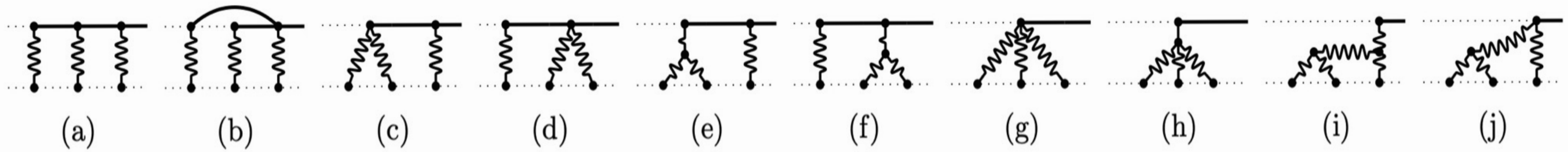
Conservative 3PM S^2

[GM, Jakobsen '22]

Calculated observables directly

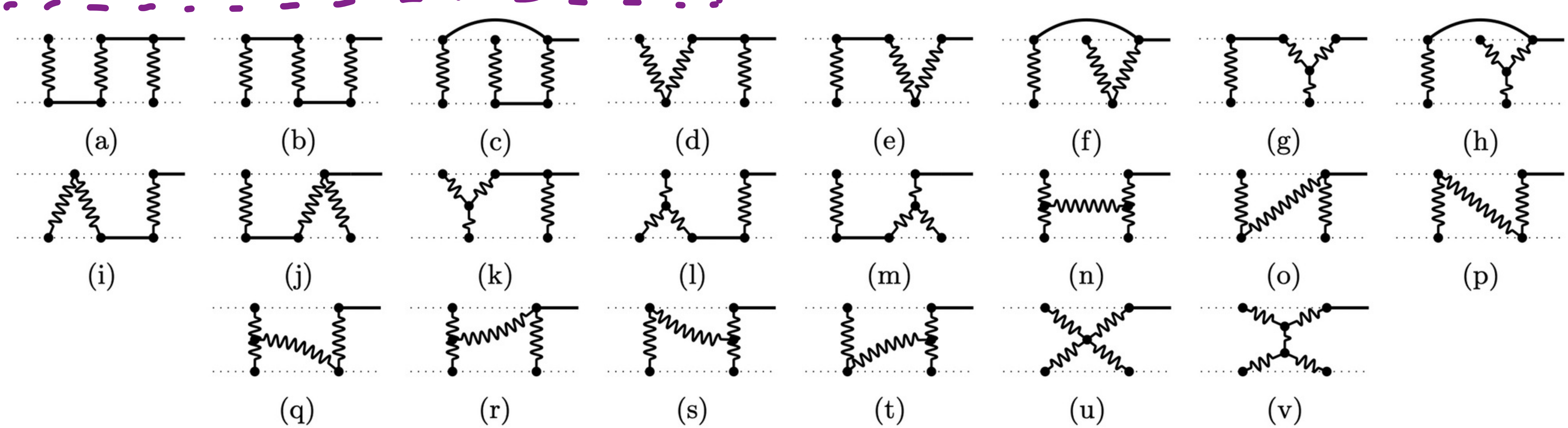
$$\Delta p_i^\mu = m_i \int_{-\infty}^{\infty} d\tau \left\langle \frac{d^2 x_i^\mu(\tau)}{d\tau^2} \right\rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega=0}$$

$$\Delta \mathcal{S}_i^\mu = \int_{-\infty}^{\infty} d\tau \left\langle \frac{d \mathcal{S}_i^\mu(\tau)}{d\tau} \right\rangle = i\omega \langle \mathcal{S}_i'^\mu(\omega) \rangle \Big|_{\omega=0}$$



Test Body $m_1 \ll m_2$

Comparable Mass (Potential Region)



STATE-OF-THE-ART IN WQFT

- DEFLECTION & SPIN KICK UP TO SPIN^2:

  1PM & 2PM [Jakobsen,Mogull,Plefka,Steinhoff]

  3PM [Jakobsen,Mogull] (CONSERVATIVE) WITH RADIATION REACTION [unpublished]

- WAVEFORM @ LO WITH UP TO SPIN^2 [Jakobsen,Mogull,Plefka,Steinhoff]

  WITH LEADING TIDAL EFFECTS [unpublished with Sauer]

  [Mougiakos,Riva,Vernizzi]

- DEFLECTION WITH LEADING TIDAL EFFECTS @ 3PM [unpublished with Sauer]

- DOUBLE COPY STRUCTURE [Shi,Plefka]

- LIGHT-BENDING [Bastianelli,Comberiati,de la Cruz]

INTEGRATION TECHNOLOGY @ 3PM & BEYOND

- ☐ BOTTLENECK IS NOT GENERATION OF INTEGRANDS
- ☐ ALL INTEGRALS WITH RETARDED PROPAGATORS
- ☐ IMPORT HIGH-ENERGY PHYSICS TECHNOLOGY:

  1. INTEGRATION-BY-PARTS REDUCTION TO MASTER INTEGRALS (WITH REDUCED SYMMETRIES!)
  2. DIFF. EQUATION TECHNIQUE TO INTEGRATE MASTERS
  3. BOUNDARY CONDITIONS (STATIC LIMIT) [IF RELEVANT]

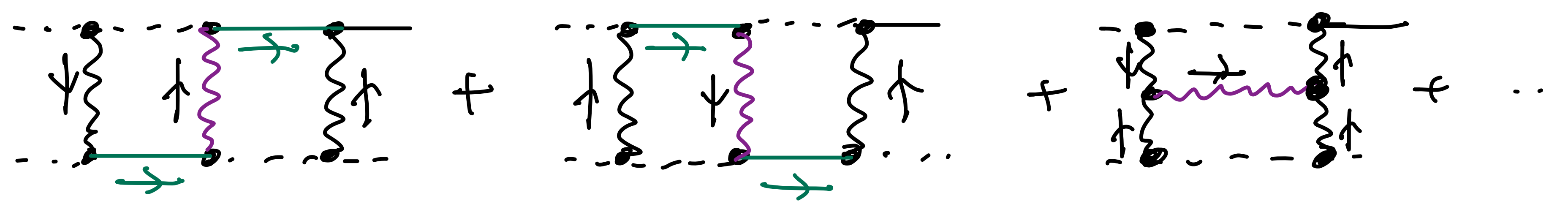
→ SAME SET OF MASTERS WITH SPIN & TIDALS @ 3PM

→ SCALES WELL FOR HIGHER-LOOP PM ORDERS

INTEGRATION TECHNOLOGY @ 3PM

3PM DEFLECTION, ONLY RETARDED PROPAGATORS ARISE:

$$\Delta p_1^{\mu} = $$



INTEGRAL FAMILY:

$$I_{n_1 n_2 n_3 n_4 n_5 n_6 n_7} :=$$

$$\int d^d \ell_1 \, d^d \ell_2 \, \frac{\delta(\ell_1 \cdot v_2)\, \delta(\ell_2 \cdot v_1)}{\underbrace{(\ell_1 \cdot v_1 \pm i\varepsilon)^{n_1} (\ell_2 \cdot v_2 \pm i\varepsilon)^{n_2}}_{\text{active worldline prop.}} \underbrace{\left((\ell_1+\ell_2-q)^2 \pm i\varepsilon \,\mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)^{n_3}}_{\text{active graviton propagators.}} (\ell_1^2)^{n_4} (\ell_2^2)^{n_5} ((\ell_1-q)^2)^{n_6} ((\ell_2-q)^2)^{n_7}}$$

INTEGRALS ARE (PSEUDO)-REAL IN PHYSICAL $\gamma = v_1 \cdot v_2$ REGION

$$\boxed{I^*_{n_1, n_2, \dots, n_7} = (-1)^{n_1+n_2} I_{n_1, n_2, \dots, n_7}} \Big\} \text{ when } 1 < \gamma < 1$$

Contrast with Feynman integrals, real for $\underline{\underline{-1 < \gamma < 1}}$.

Performing Retarded Integrals

USE STATE-OF-THE-ART INTEGRATION TECHNOLOGY:

IBP, DIFF. EQUATIONS & METHOD OF REGIONS ADAPTED TO RETARDED PROPAGATORS!

$$I^{(\sigma_1;\sigma_2;\sigma_3)}_{n_1 n_2 \cdots n_7} := \int_{\ell_1 \ell_2} \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_1)}{(\ell_1\cdot v_1+\sigma_1 i\varepsilon)^{n_1}(\ell_2\cdot v_2+\sigma_2 i\varepsilon)^{n_2}\left((\ell_1+\ell_2-q)^2+i\varepsilon\sigma_3\,\mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)^{n_3}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$

$$\frac{\delta^{(n)}(\omega)}{(-1)^n n!} = \frac{i}{(\omega+i\varepsilon)^{n+1}} - \frac{i}{(\omega-i\varepsilon)^{n+1}}$$

} treat $\delta(\omega)$ as a propagator from perspective of IBPs

$i\varepsilon$ RELEVANT FOR SYMMETRIES IN IBP REDUCTION. HERE: 3 FAMILIES

$$I^{(++ +)}_{n_1 n_2 \cdots n_7} \qquad I^{(--+)}_{n_1 n_2 \cdots n_7} \qquad I^{(+-+)}_{n_1 n_2 \cdots n_7}$$

System of DEs in $x = \gamma - \sqrt{\gamma^2-1}$ takes canonical form:

$$\frac{d\vec{I}}{dx} = \varepsilon\left(\frac{A}{x} + \frac{B_+}{1+x} - \frac{B_-}{1-x}\right)\vec{I}$$

} $\vec{I} = \vec{I}^{(0)} + \varepsilon \vec{I}^{(1)} + O(\varepsilon^2)$

Method of Regions

Fix boundary conditions to leading order in the static limit $v \to 0$.
Behavior characterized by one graviton:

$$k^{pot} = (k^0, \vec{k}) \sim (v, 1)$$
$$k^{rad} = (h^0, \vec{h}) \sim (v, v)$$

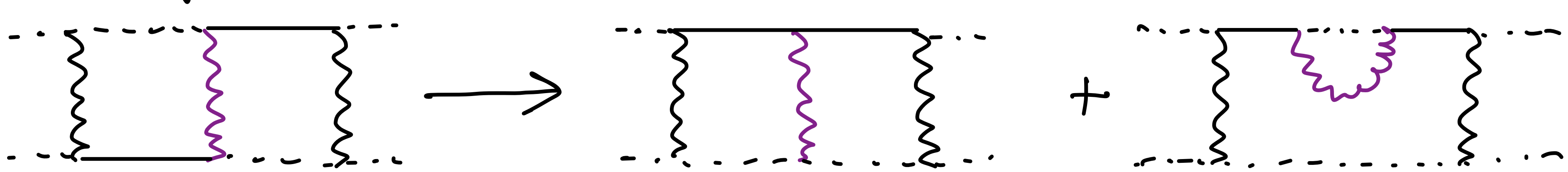
$$I^{(\sigma_1\sigma_2\sigma_3)}_{n_1 n_2 \cdots n_7} = I^{(\sigma_1\sigma_2\sigma_3)\,pot}_{n_1 n_2 \cdots n_7} + I^{(\sigma_1\sigma_2\sigma_3)\,rad}_{n_1 n_2 \cdots n_7}$$

Expand integrand in v, assuming all other loop momenta are potential.
Reduce to simpler integrals with manifest dependence on $\gamma = (1-v^2)^{-1/2}$.

$$I^{(\sigma_1\sigma_2\sigma_3)\,pot}_{n_1 n_2 \cdots n_7} = \int_{\ell_1 \ell_2} \frac{\delta(\ell_1 \cdot v_2)\,\delta(\ell_2 \cdot v_2)}{(\ell_1 \cdot v_1 + \sigma_1 i\varepsilon)^{n_1}(\ell_2 \cdot v_1 + \sigma_2 i\varepsilon)^{n_2}((\ell_1+\ell_2-q)^2)^{n_3}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$
$$+ \mathcal{O}(v^{2-n_1-n_2})$$

$$I^{(\sigma_1\sigma_2\sigma_3)\,rad}_{n_1 n_2 \cdots n_7} = \int_{\ell, h} \frac{\delta((h-\ell)\cdot v_1)\,\delta(\ell \cdot v_2)}{(\ell \cdot v_1 + \sigma_1 i\varepsilon)^{n_1}(\ell \cdot v_1 + \sigma_2 i\varepsilon)^{n_2}(h^2 + \sigma_3\,\mathrm{sgn}(h^0)\, i\varepsilon)^{n_3}(\ell^2)^{n_4+n_7}((\ell-q)^2)^{n_5+n_6}}$$
$$+ \mathcal{O}(v^{D+1-n_1-n_2-2n_3})$$

Intuitively...



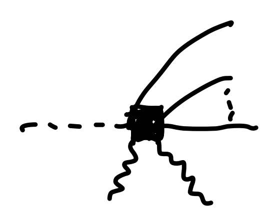
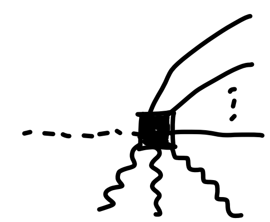
Tidal effects - work with Benjamin Sauer

Consider a simple extension to the non-spinning theory:

$$S_{pp} + S_{tidal} = m \int d\tau \left[-\tfrac{1}{2} g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} + C_{E^2} E_{\mu\nu} E^{\mu\nu} + C_{B^2} B_{\mu\nu} B^{\mu\nu} \right]$$

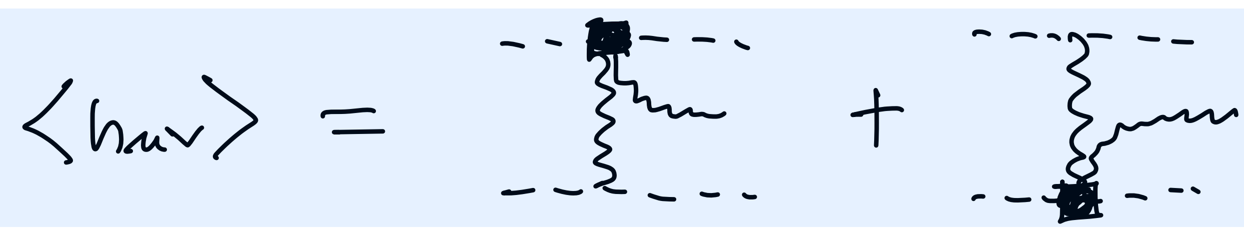
$$E_{\mu\nu} = R_{\mu\alpha\nu\beta} \dot{x}^{\alpha} \dot{x}^{\beta}, \quad B_{\mu\nu} = R^{*}_{\mu\alpha\nu\beta} \dot{x}^{\alpha} \dot{x}^{\beta}, \quad R^{*}_{\mu\alpha\nu\beta} = \tfrac{1}{2} \varepsilon_{\nu\beta\rho\sigma} R_{\mu\alpha}{}^{\rho\sigma}$$

Gives rise to new kinds of vertices:

$\left.\begin{matrix} C_{E^2} \\ C_{B^2} \end{matrix}\right\}$ quadrupole Love no's

We begin with the waveform, consists of 2 diagrams:



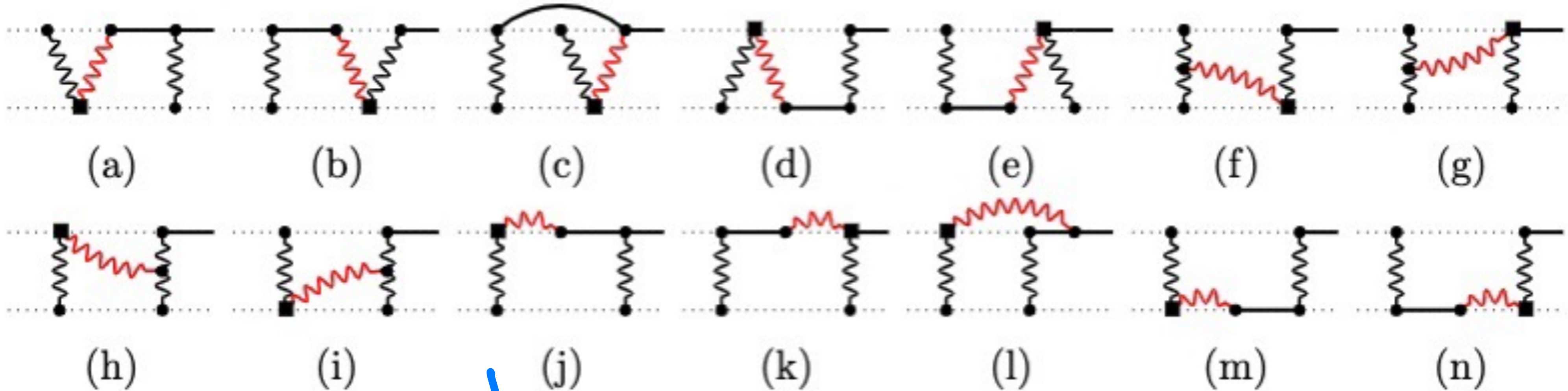
Lack of diagrams with a propagating worldline mode $\Rightarrow$ vanishing wave memory

$$\Delta f_{tidal}(\hat{x}) := f_{tidal}(t=+\infty, \hat{x}) - f_{tidal}(t=-\infty, \hat{x}) = O(G^3)$$

$\Rightarrow J^{rad} \sim P^{rad} \sim O(G^3) \quad \Rightarrow \quad \Theta_{rad, tidal} \sim O(G^4)$

Tidal effects (2)

To compute Δp_1, similar diagrams to spinning calculation:



"mushrooms"

Final result takes a convenient schematic form:

$$\Delta p^{\mu}_{1,\mathrm{cons}} = p_\infty \sin \Theta_{\mathrm{cons}} \frac{b^\mu}{|b|} + (\cos \Theta_{\mathrm{cons}} - 1) \frac{m_1 m_2}{E^2}\left[(\gamma m_1 + m_2) v_1^\mu - (\gamma m_2 + m_1) v_2^\mu\right]$$

$$\Delta p^{\mu}_{1,\mathrm{rad}} = \underbrace{p_\infty \sin \Theta_{\mathrm{rad}} \frac{b^\mu}{|b|}}_{\text{real integrals}} + \underbrace{\frac{P_{\mathrm{rad}} \cdot v_2}{\gamma^2 - 1} (v_2^\mu - \gamma v_1^\mu)}_{\text{imaginary integrals}}$$

Confirmed: $\Theta_{\mathrm{rad}} = 0$, Θ_{cons} has finite high-energy $\gamma \to \infty$ limit
P^μ_{rad} agrees with result from squaring waveform (PN expansion)

SUMMARY

- WQFT HIGHLY EFFICIENT FOR CLASSICAL SCATTERING:
  - FOCUS ON OBSERVABLES BY "QUANTIZING" WORLDLINE D.O.F.
  - ONLY COMPUTE TREE-DIAGRAMS (NO "SUPER-CLASSICAL" CONTRIBUTIONS)
  - ALL PROPAGATORS RETARDED: NO "SPECIAL" TREATMENTS OF CONSERVATIVE & RADIATION-REACTED CONTRIBUTIONS
- SPIN CARRIED BY GRASSMANN VECTORS ON THE WORLDLINE (à la STRING THEORY)

OUTLOOK

- RELATION TO SELF-FORCE APPROACH?
- BOUND ORBITS ?
- HIGHER ORDERS IN SPIN ?
- OBSERVABLES @ 4PM ?

THANK YOU !