Amplituhedron-Like Geometries and the Product of Amplitudes

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 764850

- Super amplitudes integrand in planar $\mathcal{N} = 4$ SYM can be computed from the Amplituhedron.

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- Product of two amplitudes

$$A_n^{\mathrm{N}^{k'}\mathrm{MHV}}(l')A_n^{\mathrm{N}^{k-k'}\mathrm{MHV}}(l-l')/(A_{n\ \mathrm{tree}}^{\mathrm{MHV}})^2 \quad \longleftrightarrow \quad ?$$

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- Amplituhedron-like geometries

$$? \quad \longleftrightarrow \quad \mathcal{A}_{n,k,l}^{(k',l')}$$

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- We have formulated a new conjecture

$$A_n^{\mathrm{N}^{k'}\mathrm{MHV}}(l')A_n^{\mathrm{N}^{n-4-k'}\mathrm{MHV}}(l-l')/(A_{n\ \mathrm{tree}}^{\mathrm{MHV}})^2 \quad \longleftrightarrow \quad \mathcal{A}_{n,n-4,l}^{(k',l')}$$

The n-point super amplitude

$$A_n = A_n^{\text{MHV}} + A_n^{\text{NMHV}} + A_n^{\text{N}^2\text{MHV}} + \cdots + A_n^{\text{N}^{n-2}\text{MHV}}$$

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The **N^KMHV sector** has degree $4(k+2)$ in the η variables and factorizes as

$$A_n^{\text{N}^k\text{MHV}} = A_n^{\text{MHV}_{\text{tree}}} A_{k,n}$$

$A_{k,n}$ has a degree $4k$ in the η's.

Super-correlators/super-amplitudes duality

$$\left(\prod_{i=1}^{n} D_i^4\right) G_n = \langle \mathcal{O}(x_1, \theta_1) \cdots \mathcal{O}(x_n, \theta_n) \rangle$$

$$\lim_{x_{i,i+1}^2 \to 0} G_n / G_{n;0}^{(0)} = (\mathcal{A}_n / \mathcal{A}_n^{\text{MHV tree}})^2$$

[Eden, Heslop, Korchemsky, Sokatchev](2011)

The super amplitude squared is literally the product of the super amplitude with itself

$$A_n A_n = \sum_{k}^{n-4} A_n^{\mathrm{N}^k\mathrm{MHV}} \sum_{k'}^{n-4} A_n^{\mathrm{N}^{k'}\mathrm{MHV}}$$

Like for the super amplitude we can define k sectors invariant under supersymmetry

$$(A^2)_{k,n} = \sum_{k'=0}^{k} A_n^{\mathrm{N}^{k'}\mathrm{MHV}} A_n^{\mathrm{N}^{k-k'}\mathrm{MHV}}$$

The $A_{5,1}$ amplitude in terms of super momentum twistors (Z_i, χ_i) reads

$$A_{5,1} = R[1,2,3,4,5] = \frac{\prod_{I=1}^{4}(\langle 1234\rangle \chi_5^I + \text{cyclic})}{\langle 1234\rangle \langle 2345\rangle \langle 3451\rangle \langle 4512\rangle \langle 5123\rangle}.$$

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All $A_{n,k}$ are in one to one correspondence with differential top forms. For example

$$\tilde{A}_{5,1} = \frac{\langle 12345\rangle^4 \langle Y \mathrm{d}^4 Y\rangle}{\langle Y1234\rangle \langle Y2345\rangle \langle Y3451\rangle \langle Y4512\rangle \langle Y5123\rangle}$$

where $\langle ijklm\rangle = \det(Z_i, Z_j, Z_k, Z_l, Z_m)$ and $Y = (Y^1, Y^2, Y^3, Y^4, Y^5)$

More generally

$$A_{k,n} \quad \leftrightarrow \quad \text{Top dimensional form from } Gr(k, k+4) \text{ to } \mathbb{R} \text{ or } \mathbb{C}$$

For example for $\overline{MHV}$

$$A_{n-4,n} = \frac{\langle 12 \cdots n \rangle^4 \prod_i^k \langle Y \mathrm{d}^4 Y_i \rangle}{\langle Y1234 \rangle \langle Y2345 \rangle \cdots \langle Yn123 \rangle},$$

where Y is a k-plane $Y^{I_1 \cdots I_n} = Y_1^{I_1} \cdots Y_n^{I_n}$

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\begin{aligned}
&(A_{6,1})^2 = ([12345] + [12356] + [13456])^2 = \\
&= 2([12345][12356] + [12356][13456] + [12345][13456]) = \\
&= A_{6,2} \frac{(\langle Y1245\rangle \langle Y2361\rangle \langle Y3456\rangle + \text{cyclic})}{\prod_{i=1}^{3} \langle Yi(i+1)(i+3)(i+4)\rangle}
\end{aligned}
$$

The Amplituhedron

The $A_{n,k,l}$ is defined as the plane $Y \in Gr(k, k+4)$ and $AB \in Y^{\perp}$ satisfying

$$\langle Yii+1jj+1\rangle > 0, \qquad \langle YABii+1\rangle > 0,$$
$$(-1)^{k}\langle Y1nii+1\rangle > 0, \qquad (-1)^{k+2}\langle Y(AB)_{p}1n\rangle > 0,$$

which correspond to the proper poles of the amplitudes. Moreover the strings

$$S_Y := \{\langle Y1234\rangle, \cdots, (-1)^{k}\langle Y123n\rangle\},$$
$$S_{AB} := \{\langle YAB12\rangle, \cdots, (-1)^{k+2}\langle YAB1n\rangle\},$$

have exactly $f_Y = k$ and $f_{AB} = k+2$ sign flips respectively.

The Amplituhedron-Like Geometries

The $A_{n,k,l}^{(f_Y,n_m)}$ amplituhedron-like geometry is defined as the plane Y and l 2-planes $(AB)_p$ satisfying

$$\langle Yii+1jj+1\rangle > 0, \qquad \langle Y(AB)_p ii+1\rangle > 0,$$
$$(-1)^{f_Y}\langle Y1nii+1\rangle > 0, \qquad (-1)^{f_{(AB)_p}}\langle Y(AB)_p 1n\rangle > 0,$$

which correspond to the proper poles of the amplitudes. Moreover the strings

$$S_Y := \{\langle Y1234\rangle, \cdots, (-1)^{f_Y}\langle Y123n\rangle\},$$
$$S_{(AB)_p} := \{\langle Y(AB)_p 12\rangle, \cdots, (-1)^{f_{(AB)_p}}\langle Y(AB)_p 1n\rangle\},$$

where have S_Y f_Y and n_m $S_{(AB)_p}$ has f_Y+2 sign flips. We proved that $f_{(AB)_p} = f_Y, f_Y+2$.

Evidences

$$A_n^{\mathrm{N}^{k'}\mathrm{MHV}}(l')A_n^{\mathrm{N}^{n-4-k'}\mathrm{MHV}}(l-l')/(A_{n\ \mathrm{tree}}^{\mathrm{MHV}})^2 \quad \longleftrightarrow \quad A_{n,n-4,l}^{(k',l')}$$

- The number of inequivalent geometries and the number of products of amplitudes match.

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For $k = n - 4$ we have the equivalence relation (Parity invariance)

$$A_{n,n-4,l}^{(k',l')} \simeq A_{n,n-4,l}^{(k-k',l-l')}$$

And some are degenerate

$$f_Y \leq k, \quad f_{AB} = f_Y, f_Y + 2$$

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- Explicit computation up to $k \leq 3$
- Proof for $n = 4$ at all loop order.

Beyond the Maximal case

For $k < n - 4$ the configuration the Z's becomes non trivial.
The number cyclic invariant geometries and the number of products of amplitudes ($[(k + 1)/2]$) match!

Summary

We have conjectured and provided evidence for the correspondence

$$A_n^{\mathrm{N}^{k'}\mathrm{MHV}}(l')A_n^{\mathrm{N}^{n-4-k'}\mathrm{MHV}}(l-l')/(A_{n\ \mathrm{tree}}^{\mathrm{MHV}})^2 \quad \longleftrightarrow \quad A_{n,n-4,l}^{(k',l')},$$

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Outlooks

- Perform more explicit computations. In particular, does $A_{n,1}^{(0)}$ correspond to $(NMHV)^2$?
- Computes maximal residues trough the geometry to bootstrap correlators.
- Are the product of amplitudes Yangian invariant?