The 2PM three-body gravitational interaction

Canxin Shi

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Based on an ongoing work with: Jan Plefka, Florian Löbbert and Tianheng Wang

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Introduction:

- History of the three-body interaction:

  0PN(Post-Newtonian): Newton

  1PN: Einstein-Hoffmann-Infeld(1938), Eddington-Clark(1938), Landau-Lifshitz

- Our approach:

  Post-Minkowskian(PM) Effective Field Theory(EFT) [Kälin-Porto: 2006.01184,2007.04977]

  PN expansion to recover Landau-Lifshitz

  Reduced to two-body interaction

EFT action

- Three point particles coupled to Einstein gravity

$$S = S_{\mathrm{EH}} + S_{\mathrm{gf}} + S_{\mathrm{pp}}$$

$$S_{\mathrm{pp}} = -\sum_{i=1}^{3} m_i \int d\tau_i \sqrt{g_{\mu\nu} u_i^\mu u_i^\nu} \quad = \sum_{i=1}^{3} \int d\tau_i \left(-p_{i,\mu}\dot{x}_i^\mu + \lambda_i \left[g^{\mu\nu} p_{i,\mu} p_{i,\nu} - m_i^2\right]\right)$$

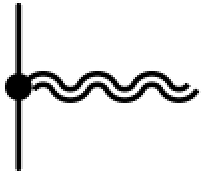
- We expand the inverse metric around Minkowski space as

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu}$$

- The effective action

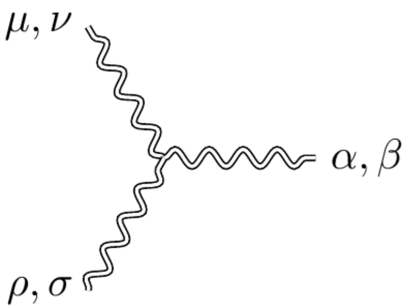
$$e^{iS_{\mathrm{eff}}} = \int \mathcal{D}h_{\mu\nu} e^{iS_{\mathrm{EH}} + iS_{\mathrm{gf}} + iS_{\mathrm{pp}}}$$

- Feynman rules



$= -i\kappa\, \lambda(\tau) p_\mu(\tau) p_\nu(\tau)$

μ, ν  ρ, σ $= \dfrac{iP^{\mu\nu\rho\sigma}}{2} D_{ij}$



$$P^{\mu\nu\rho\sigma} = \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \frac{2}{D-2}\eta^{\mu\nu}\eta^{\rho\sigma}. \qquad D_{ij} = D(x_i - x_j)$$

- Since we only care the conservative sector, only the real part of D_{ij} is needed

$$\bar{D}_{12} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon} e^{ik\cdot x_{12}} = \frac{1}{4\pi^2} \frac{i}{x_{12}^2 - i\epsilon}$$
$$= \frac{1}{4\pi^2}\left(-\pi\delta(x_{12}^2) + \frac{i}{x_{12}^2} \right),$$

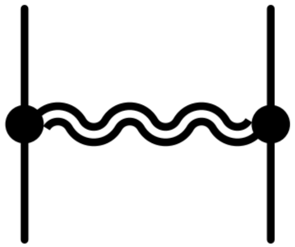
PM Potential

- Effective action

$$S_{\text{eff}} = S^{\text{free}} + \kappa^2 S^{1\text{PM}} + \kappa^4 S^{2\text{PM}} \qquad S^{\text{free}} = \sum_{i=1}^{3} \int d\tau_i \left[-p_i \cdot \dot{x}_i + \lambda_i \left(p_i^2 - m_i^2 \right) \right]$$

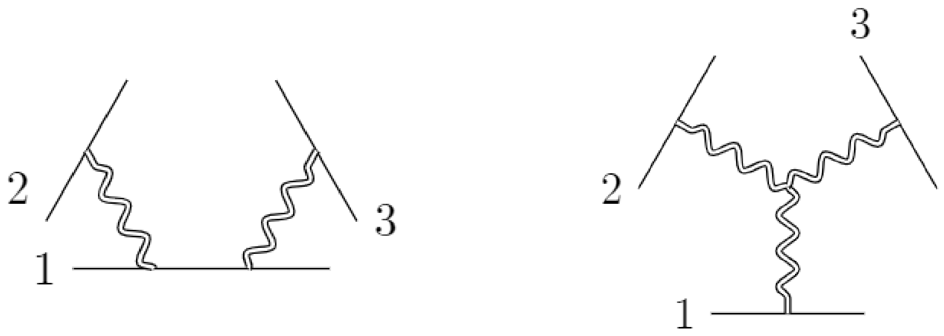
- 1PM

$$\kappa^2 S^{1\text{PM}} = -\frac{1}{2}\kappa^2 \sum_i \sum_{j \neq i} \int d\tau_i d\tau_j \lambda_i \lambda_j \left[(p_i \cdot p_j)^2 - \frac{1}{2} p_i^2 p_j^2 \right] D_{ij}$$



This agrees with Kälin-Porto[2006.01184] if it's restricted to two bodies.

- 2PM: There are only two topologies contributing



Adding all combinations we have

$$\kappa^4 S^{2\mathrm{PM}} = \frac{\kappa^4}{6} {\sum_{i,j,k}}' \int d\tau d\tau' d\tau'' \lambda_i(\tau)\lambda_j(\tau')\lambda_k(\tau'') \\ P\left(p_i(\tau), p_j(\tau'), p_k(\tau''), \frac{\partial}{\partial x_i^\mu(\tau)}, \frac{\partial}{\partial x_j^\mu(\tau')}, \frac{\partial}{\partial x_k^\mu(\tau'')} \right) \boxed{\int dx^4 D_{ix} D'_{jx} D''_{kx}}$$

(the three-body integral)

$$P\left(p_1, p_2, p_3, \frac{\partial}{\partial x_1^\mu}, \frac{\partial}{\partial x_2^\mu}, \frac{\partial}{\partial x_3^\mu}\right) = -\Big[\left((p_1\cdot p_2)(p_1\cdot p_3)(p_2\cdot p_3) - \tfrac{1}{2}(p_1\cdot p_3)^2 p_2^2 - \tfrac{1}{2}(p_1\cdot p_2)^2 p_3^2 + \tfrac{1}{4} p_1^2 p_2^2 p_3^2 \right) \left(\frac{\partial}{\partial x_1^\mu}\right)^2 \\ + \left((p_1\cdot p_2)^2 p_3^\mu p_3^\nu - \tfrac{1}{2} p_1^2 p_2^2 p_3^\mu p_3^\nu + 2(p_1\cdot p_3)(p_2\cdot p_3) p_2^\mu p_1^\nu \right) \frac{\partial}{\partial x_1^\mu} \frac{\partial}{\partial x_2^\nu} \Big] + (\text{cyclic})$$

where the sum ${\sum_{i,j,k}}'$ is over $i, j, k = 1, 2, 3$ excluding $i = j = k$.

1PN three-body expansion

- Expand the effective action with c^{-1} gives (fixing $\tau_i = t_i$)

$$D_{ij} = -\frac{\delta(t_i - t_j)}{4\pi r_{ij}} + O(c^{-2}), \qquad p_i^\mu \frac{\partial}{\partial x_j^\mu} \sim O(c^{-1})$$

$$E_i^{(0)} \sim m_i + \frac{1}{c^2}\frac{\mathbf{p}_i^2}{2m_i} + O(c^{-4}), \qquad \mathbf{p}_i \sim \frac{\mathbf{p}_i}{c} + O(c^{-2}),$$

- We can compute the 1PN static term

$$S^{\mathrm{1PNstatic}} = -\frac{1}{2}\sum_i \sum_{j\neq i} \sum_{k\neq i} \int dt \frac{G^2 m_i m_j m_k}{r_{ij} r_{ik}}$$

which agrees with Landau-Lifshitz

2-body case

- Our approach is similar to Kälin-Porto [2006.01184], though we are in the second-order formalism while they are in the first-order.
- We computed the momentum impulse and found it agree with Kälin-Porto's result.

$$\Delta p_1^\mu = -\frac{G m_1 m_2 b^\mu}{|b^2|}\left(\frac{2\left(2\gamma^2-1\right)}{\sqrt{\gamma^2-1}} + \frac{3\pi}{4}\frac{\left(5\gamma^2-1\right)}{\sqrt{\gamma^2-1}}\frac{GM}{|b^2|^{1/2}}\right) + 2\frac{m_1 m_2\left(2\gamma^2-1\right)^2}{(\gamma^2-1)^2}\frac{G^2}{|b^2|}\left((\gamma m_2 + m_1)u_2^\mu - (\gamma m_1 + m_2)u_1^\mu\right)$$

[2006.01184]

Conclusion

- We compute the 2PM three-body potential from EFT.
- The result is consistent with literatures both in the PN limit and in the two-body case.

- Ongoing: the three-body integral, the equation of motion…