Differential equations for phase-space integrals and Cutkosky rules

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SAGEX review
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SAGEX
Scattering Amplitudes: from Geometry to Experiment

HELMHOLTZ
SPITZENFORSCHUNG FÜR GROSSE HERAUSFORDERUNGEN

Academic background, training in the SAGEX network

My background

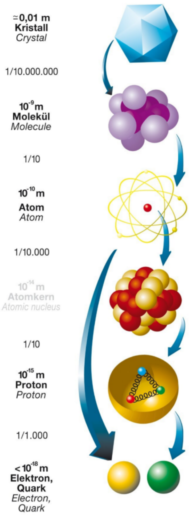
- MSc in Physics, *Univeristy of Torino*, Italy (2017)
- *Diploma* in Composition, *Conservatorio of Torino*, Italy (2015)
- BSc in Physics, *Univeristy of Torino*, Italy (2011)

Training in the SAGEX network

- Schools and conferences: CAPP 2019, SAGEX school on gauge and string theory, RADCOR 2019.
- Teaching: preparation of study material on QFT for an online learning platform.
- Other training: DESY & HU seminars, German language course, soft skills as per SAGEX curriculum.
- Planned secondment: RISC Software GmbH (April 1 - June 30, 2020).
- Outreach: collaboration to build an exhibition, poster session for the general public (upcoming).

Motivation

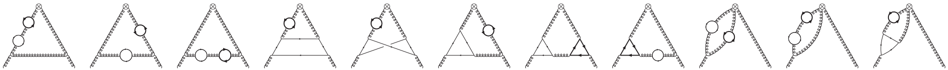
- Scattering experiments allow the study of the structure of microscopic particles.
- Knowledge about the structure of the proton is necessary for precision results at colliders such as the LHC.
- Historical importance of Deep inelastic scattering: discovery of quarks and gluons, asymptotic freedom $\rightarrow$ perturbative QCD.
- Precision calculations in DIS involve a class of multi-loop Feynman integrals.



Goals of the project

- To study the methods for evaluating these integrals and the mathematical structures arising in the process.
- The computation of quantities of use to phenomenology: two-mass contributions to the polarized operator matrix elements $\Delta A_{gg,Q}^{(3)}$ and $\Delta A_{Qq}^{\mathrm{PS},(3)}$.
- Development of Fortran code for numerical applications.

Two-mass contribution to the polarized OME $\Delta A_{gg,Q}^{(3)}$



- Work in progress in collaboration with the group at RISC.
- **Goals**: the analytic computation of the 2-mass OME $\Delta A_{gg,Q}^{(3)}$ in N-space and in x-space.
- **Methodology**: Feynman parametrization, Mellin-Barnes decomposition, residue theorem $\rightarrow$ formulation in terms of nested sums.
  Treatment of γ_5 in the Larin scheme.
- **Mathematical structures**:
  1. in N-space, classes of sums, e.g.

$$S_{\{a_1,b_1,c_1\}\dots\{a_n,b_n,c_n\}}(s_1,\dots,s_n;N) = \sum_{k=1}^{N} \frac{s_1^k}{(a_1k+b_1)^{c_1}} S_{\{a_2,b_2,c_2\}\dots\{a_n,b_n,c_n\}}(s_1,\dots,s_n;k), \quad S_\emptyset = 1$$

  2. in x-space, iterated integrals

$$G\left[\left\{g(x),\vec{h}(x)\right\};z\right] = \int_0^z dy\, g(y) G\left[\left\{\vec{h}(x)\right\};y\right]$$

- As an example, we show the $\mathcal{O}(\varepsilon^0)$ part of the result for the first diagram:

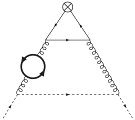
$$
\begin{aligned}
D_{1,\varepsilon^0} = & -2\frac{1-(-1)^N}{2} C_A T_F^2 \Bigg\{ \frac{2\log(\eta)P_2}{27N(1+N)^3} - \frac{4P_3}{729N(1+N)^4} - \frac{\log^2(\eta)P_1}{18N(1+N)^2} - \frac{64(4+N)\log^3(\eta)}{27N(1+N)} + \\
& \Bigg[\frac{32(4+N)\big(25+48N+29N^2\big)}{81N(1+N)^3} + \frac{32(4+N)(2+5N)\log(\eta)}{27N(1+N)^2} + \frac{32(4+N)\log^2(\eta)}{9N(1+N)} \\
& + \frac{32(4+N)}{27N(1+N)} S_2\Bigg] S_1 + \Bigg[-\frac{32(4+N)(2+5N)}{81N(1+N)^2} - \frac{16(4+N)\log(\eta)}{9N(1+N)}\Bigg] S_1{}^2 \\
& + \frac{32(4+N)}{81N(1+N)} S_1{}^3 + \Bigg[-\frac{32(4+N)(2+5N)}{81N(1+N)^2} - \frac{16(4+N)\log(\eta)}{9N(1+N)}\Bigg] S_2 \\
& + \frac{64(4+N)}{81N(1+N)} S_3 + \frac{32(4+N)}{9N(1+N)} \sum_{i_1=1}^{\infty} \frac{\eta^{i_1}}{i_1^3} + \frac{16(4+N)\log^2(\eta)}{9N(1+N)} \sum_{i_1=1}^{\infty} \frac{\eta^{i_1}}{i_1} \\
& - \frac{4(1+\eta)\big(5+22\eta+5\eta^2\big)(4+N)}{9\eta N(1+N)} \sum_{i_1=1}^{\infty} \frac{\eta^{i_1}}{\big(1+2i_1\big)^3} + \frac{2(1+\eta)\big(5+22\eta+5\eta^2\big)(4+N)\log(\eta)}{9\eta\ N(1+N)} \\
& \times \sum_{i_1=1}^{\infty} \frac{\eta^{i_1}}{\big(1+2i_1\big)^2} - \frac{(1+\eta)\big(5+22\eta+5\eta^2\big)(4+N)\log^2(\eta)}{18\ \eta N(1+N)} \sum_{i_1=1}^{\infty} \frac{\eta^{i_1}}{1+2i_1} + \Bigg[-\frac{32(4+N)(2+5N)}{27N(1+N)^2} \\
& + \frac{32(4+N)}{9N(1+N)} S_1 - \frac{16(4+N)\log(\eta)}{3N(1+N)}\Bigg] \zeta_2 + \frac{64(4+N)}{27N(1+N)} \zeta_3 \\
& - \frac{32(4+N)\log(\eta\ )}{9N(1+N)} \sum_{i_1=1}^{\infty} \frac{\eta^{i_1}}{i_1^2} \Bigg\}
\end{aligned}
$$

$$\eta = \frac{m_c^2}{m_b^2}$$

- A detailed numerical investigation of the magnitude will be performed; expected size 20-40% of the single-mass effect, as in the unpolarized case.

Two-mass contribution to the polarized OME $\Delta A_{Qq}^{\mathrm{PS},(3)}$

- **Goal**: analytic calculation in terms of iterated integrals in x-space.



$$
\begin{aligned}
\tilde{a}_{Qq}^{(3),\mathrm{PS}}(x) \; = \; & C_F T_F^2 \Big\{ R_0(m_1, m_2, x) + \big(\theta(\eta_- - x) + \theta(x - \eta_+)\big) x\, g_0(\eta, x) \\
& + \theta(\eta_+ - x)\theta(x - \eta_-)\left[x\, f_0(\eta, x) - \int_{\eta_-}^{x} dy \left(f_1(\eta, y) + \frac{x}{y} f_3(\eta, y) \right) \right] \\
& + \theta(\eta_- - x) \int_{x}^{\eta_-} dy \left(g_1(\eta, y) + \frac{x}{y} g_3(\eta, y) \right) - \theta(x - \eta_+) \int_{\eta_+}^{x} dy \left(g_1(\eta, y) + \frac{x}{y} g_3(\eta, y) \right) \\
& + x\, h_0(\eta, x) + \int_{x}^{1} dy \left(h_1(\eta, y) + \frac{x}{y} h_3(\eta, y) \right) + \theta(\eta_+ - x) \int_{\eta_-}^{\eta_+} dy \left(f_1(\eta, y) + \frac{x}{y} f_3(\eta, y) \right) \\
& + \int_{\eta_+}^{1} dy \left(g_1(\eta, y) + \frac{x}{y} g_3(\eta, y) \right) \Big\}.
\end{aligned}
$$

where the functions $f_i(x,\eta)$, $g_i(x,\eta)$, $h_i(x,\eta)$ depend on the same class of iterated integrals as in the unpolarized case, e.g. $G\left(\left\{\frac{\sqrt{1-4\tau}}{\tau}, \frac{1}{\tau}\right\}; \frac{x(1-x)}{\eta}\right)$

Scheme-invariant evolution of structure functions

- **Motivation**: study of scaling violation in DIS $\rightarrow$ pathway to measuring α_s.
- Progress so far: FORTRAN implementation of solutions to the AP equations to NNLO in Mellin space.
- Fortran implementation of the analytic continuation of harmonic sums in the complex plane.

Thank you for your attention!