Scheme-invariant evolution of Deep-inelastic Structure Functions at NNLO and N^3LO

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1. Scheme-invariant evolution
2. Direct extraction of α_s from the scaling violation of structure functions
3. Aspects of the code

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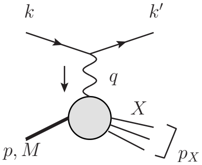
Deep inelastic scattering



$$Q^2 = -q^2 > 0, \quad \nu = p\cdot q/M, \quad x = Q^2/2p\cdot q$$

$$Q^2 \to \infty, \quad x \to \text{const (Bjorken limit)}$$

$$\frac{d\sigma}{dx\,dQ^2} \propto L^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(x,Q^2) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right) W_2(x,Q^2)$$

Structure functions:

$$F_2(x,Q^2) = \frac{\nu}{2M} W_2(x,Q^2),$$

$$F_L(x,Q^2) = -W_1(x,Q^2) + (1+\nu^2/Q^2)\, W_2(x,Q^2)$$

F_i are observables.

Deep inelastic scattering

$$M\left[F_i(x,Q^2)\right](N) = \int_0^1 dx\, x^{N-1} F_i(x,Q^2) = C^{(N)}(Q^2)\cdot f_i^{(N)}(Q^2)$$

$C(x,Q^2)$ *Wilson coefficient*, $f_i(x,Q^2)$ *parton densities*

- The $f_i(x,Q_0^2)$ cannot be computed perturbatively.
- Their Q^2 dependence can be computed perturbatively, however.

$$C^{(N)}(Q^2) = c_1^{(N)} a_s(Q^2) + c_2^{(N)} a_s^2(Q^2) + \cdots$$

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2} f^{NS\,(N)}(Q^2) = P^{NS,(N)}(Q^2)\, f^{NS\,(N)}(Q^2)$$

$$\frac{d}{d\log Q^2}\left(f^{S\,(N)}(Q^2)\right)_i = \left(P^{S\,(N)}(Q^2)\right)_{ij}\left(f^{S\,(N)}(Q^2)\right)_j$$

$$\frac{d}{d\log Q^2} a_s(Q^2) = -\sum_{k=0}^{\infty} \beta_k a_s^{k+2}(Q^2), \qquad a_s = \alpha_s/(4\pi)$$

$P_{ij}(x,Q^2)$ - *splitting function*

The approach

- We want to solve analytically[1]

$$F_2^{NS}(x,Q^2) = E(Q^2,Q_0^2)\, F_2^{NS}(Q_0^2)$$

and

$$\begin{pmatrix} F_2^S(x,Q^2) \\ \partial_t F_2^S(x,Q^2) \end{pmatrix} = K(Q^2,Q_0^2) \begin{pmatrix} F_2^S(x,Q_0^2) \\ \partial_t F_2^S(x,Q_0^2) \end{pmatrix}$$

$$t = -\frac{2}{\beta_0} \log \frac{a_s(Q^2)}{a_s(Q_0^2)}$$

- $E(Q^2,Q_0^2)$ and $K(Q^2,Q_0^2)$ are the scheme-invariant evolution operators.

[1] **See also: J. Blümlein, V. Ravindran, W.L. van Neerven, Nucl. Phys. B586 (2000) 349; J. Blümlein and A. Guffanti, Nucl.Phys.Proc.Suppl. 152 (2006) 87.**

The approach

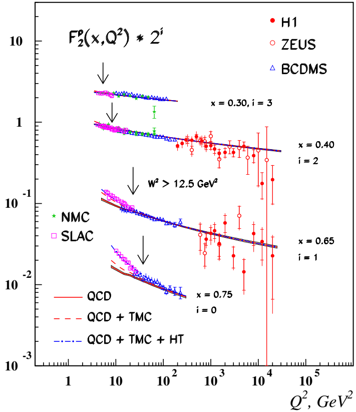
Example – non-singlet evolution operator

$$E^{NS}(Q^2,Q_0^2) = \left(\frac{a_s}{a_0}\right)^{-\frac{P_0}{\beta_0}} \Bigg\{ 1 + (a_s - a_0)\left[c_1 + \frac{\beta_1 P_0}{\beta_0^2} - \frac{P_1}{\beta_0}\right] + \frac{1}{2}(a_s - a_0)^2 \left[c_1 + \frac{\beta_1 P_0}{\beta_0^2} - \frac{P_1}{\beta_0}\right]^2$$
$$+ (a_s^2 - a_0^2)\left[-\frac{c_1^2}{2} + c_2 + h_2 - \left(\frac{\beta_1^2}{2\beta_0^3} - \frac{\beta_2}{2\beta_0^2}\right)P_0 + \frac{\beta_1 P_1}{2\beta_0^2} - \frac{P_2}{2\beta_0}\right]$$
$$+ \frac{1}{6}(a_s - a_0)^3 \left[c_1 + \frac{\beta_1 P_0}{\beta_0^2} - \frac{P_1}{\beta_0}\right]^3 + \frac{1}{3}(a_s^3 - a_0^3)\Bigg[c_1^3 - 3c_1(c_2 + h_2) + 3(c_3 + h_3)$$
$$+ \left(\frac{\beta_1^3}{\beta_0^4} - \frac{2\beta_1\beta_2}{\beta_0^3} + \frac{\beta_3}{\beta_0^2}\right)P_0 - \left(\frac{\beta_1^2}{\beta_0^3} - \frac{\beta_2}{\beta_0^2}\right)P_1 + \frac{\beta_1 P_2}{\beta_0^2} - \frac{P_3}{\beta_0}\Bigg] + \frac{1}{2}(a_s - a_0)(a_s^2 - a_0^2)$$
$$\left[c_1 + \frac{\beta_1 P_0}{\beta_0^2} - \frac{P_1}{\beta_0}\right]\left[-c_1^2 + 2(c_2 + h_2) - \left(\frac{\beta_1^2}{\beta_0^3} - \frac{\beta_2}{\beta_0^2}\right)P_0 + \frac{\beta_1 P_1}{\beta_0^2} - \frac{P_2}{\beta_0}\right] + \cdots \Bigg\}$$

- Measure $\alpha_s(M_Z)$ from the scaling violation of the structure functions of deep-inelastic scattering directly.
- The measurement is widely free of systematic and theory errors because the quantities involved depend on α_s only, given precision input for m_c and m_b.
- No fit of the parton densities,
- Measure the input distributions $F_2^S(x,Q_0^2)$, $\partial_t F_2^S(x,Q_0^2)$ experimentally.

The approach

J. Blümlein, H. Böttcher, A. Guffanti,
Nucl.Phys.B774:182-207 (2007)



- All ingredients are known in the NS case to N^3LO:
- P^{NS} up to 3 loops,
  C^{NS} up to 3 loops,
  Moments of P_3^{NS} and excellent bounds: N^3LO.
- Correspondingly, singlet case: NNLO.

$W^2 \geq 12.5$ GeV^2

- One goal is also to produce a software library that can ultimately work with experimental data.
- It will also contain the asymptotic heavy flavor effects for the first time, which are sufficient for $Q^2 > 25$ GeV^2.

The challenges

- The evolution kernels are given by harmonic sums

$$S_{k_1,k_2,\ldots,k_m}(N) = \sum_{n_1=1}^{N} \frac{(\mathrm{sign}(k_1))^{n_1}}{n_1^{|k_1|}} \sum_{n_2=1}^{n_1} \frac{(\mathrm{sign}(k_2))^{n_2}}{n_2^{|k_2|}} \cdots \sum_{n_m=1}^{n_{m-1}} \frac{(\mathrm{sign}(k_m))^{n_m}}{n_m^{|k_m|}}$$

- Need systematic way to compute harmonic sums, e.g.

$$S_{-2,3}(N) = \sum_{n_1=1}^{N} \frac{(-1)^{n_1}}{n_1^2} \sum_{n_2=1}^{n_1} \frac{1}{n_2^3}$$

  and their analytic continuation[2] for complex N.
- Use factorial series for $|N| \to \infty$ and recurrences for $N \to N-1$.
- Factors of $(-1)^N \to$ project on even N (unpolarized case) or odd N (polarized case).
- Perform inverse Mellin transformation numerically as last step (one contour integral).

[2] J. Blümlein, Comput.Phys.Commun. 180 (2009) 2218.

Current status of the code

- The code can currently compute harmonic sums up to weight 5 numerically for complex N and perform the Mellin inversion.
- The coded splitting functions and Wilson coefficients were checked against the results in x space obtained with the package HarmonicSums[3] .

Next steps:

- Testing of the evolution over parametrized input structure functions.
- Testing the heavy flavor corrections.
- Error propagation of physical input distributions.
- Application to experimental data to extract α_s.

[3] J. Vermaseren, Int. J. Mod. Phys. A14 (1999) 2037; J. Blümlein and S. Kurth, Phys. Rev. D60 (1999) 014018; J. Ablinger et al. J. Math. Phys. 54 (2013) 082301; 52 (2011) 102301; 55 (2014) 112301; J. Ablinger arXiv:1305.0687[math-ph]; PoS (RADCOR2017) 001; arXiv:1011.1176[math-ph]; PoS(LL2014)019.

Next Steps in the project:

- Calculation of the small x limit of massive 3-loop operator matrix elements.
- Renormalization of massive 3-loop polarized operator matrix and associated analytic loop calculations.