Amplituhedron and Squared Amplituhedron

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$N = 4$ Super Yang Mills

The field content of the theory correspond to

- 1 gluon $G = (G^+, G^-)$, helicity $h = 1$
- 4 fermions $\lambda = (\lambda^a, \lambda^{abc})$ $h = 1/2$
- 6 scalars $S^{[ab]}$, with $a, b = 1, 2, 3, 4$

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On-shell states can be packaged by the on-shell supermultiplet

$$O = G^+ + \eta_a \lambda^a - \frac{1}{2!}\eta_a \eta_b S^{ab} - \frac{1}{3!}\eta_a \eta_b \eta_c \lambda^{abc} + \eta_1 \eta_2 \eta_3 \eta_4 G^-$$

$N = 4$ Super Yang Mills

Amplitudes can be packaged in superamplitudes

$$A_n(x, \eta) = \langle O(p_1, \eta_1) O(p_2, \eta_2) \cdots O(p_n, \eta_n) \rangle$$

The n-particle superamplitude is a $4n$ order polynomial in $\eta_1^a, ..., \eta_n^a$. The polynomial coefficients are ordinary amplitudes.

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$N^K MHV$ Amplitudes

The $n-$point YM **N$^{\mathbf{K}}$MHV amplitude** is defined as

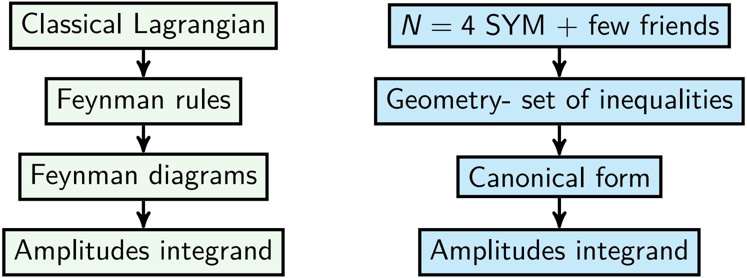
$$A_{k,n} = \langle G_1^- \cdots G_{k+2}^- G_{k+3}^+ \cdots G_n^+ \rangle$$

The N$^{\mathbf{K}}$MHV sector is the set of amplitudes connected to the $N^k MHV$ amplitude via supersymmetry.

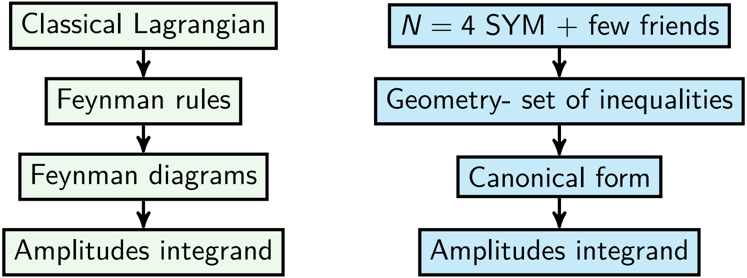
$$\begin{aligned} A_4^{\mathrm{MHV}} = & A_4[g^- g^- g^+ g^+](\eta_1)^4(\eta_2^4) + \\ & + A_4[g^- \lambda^{123} \lambda^4 g^+](\eta_1)^4(\eta_{21}\eta_{22}\eta_{23})(\eta_{34}) + \cdots , \end{aligned}$$

where $(\eta_1)^4 = \eta_{11}\eta_{12}\eta_{13}\eta_{14}$.

Amplitudes without Feynman diagrams

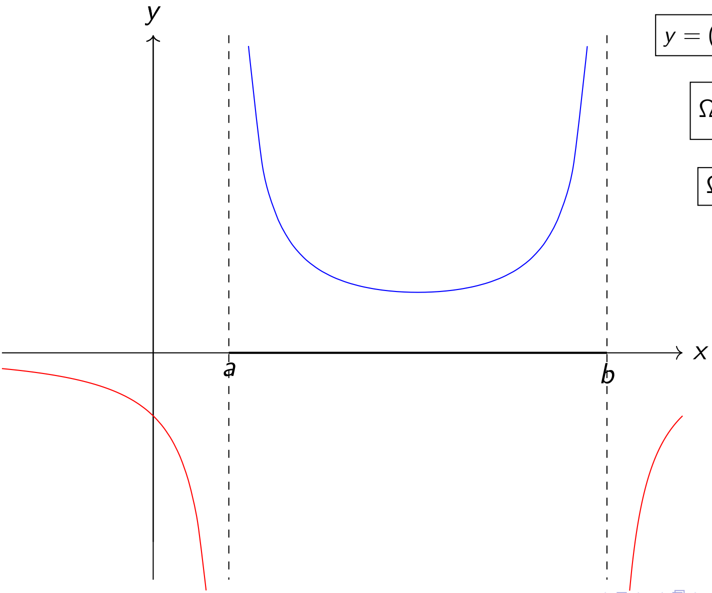


Amplitudes without Feynman diagrams



The Amplituhedron calculates amplitudes to all loops but only in the planar limit.

The segment Canonical form



$$y = (x-a)^{-1} - (x-b)^{-1}$$

$$\Omega_{ab} = d\log\left(\frac{x-a}{x-b}\right)$$

$$\Omega_{ab} + \Omega_{bc} = \Omega_{ac}$$

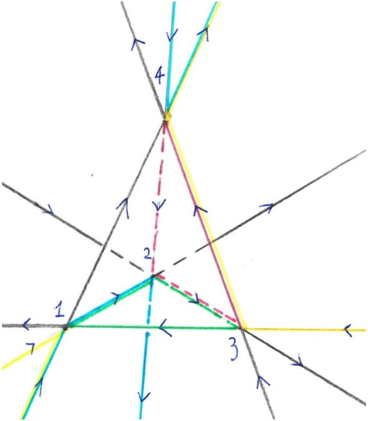
Tree level Amplituhedron Geometries

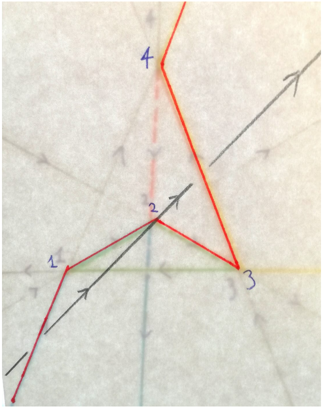
Given n external momenta p_i we can associate (not uniquely) n variables Z_i in an auxiliary $4+k$ dimensional projective space.

We choose Z_i to define an oriented polytope in $4+k-1$ dimension.

The Amplituhedron geometry is given by all the (k-1)-planes intersecting the polytope in a precise way.

4 points 1 Loop MHV





$A_{L=1}[g^- g^- g^+ g^+]$. The geometry is given by the oriented lines crossing blue, green, red, yellow

The Squared Amplituhedron

The square of a super amplitude is

$$A_n^2 = \left(\sum_{k=0}^{n-4} A^{\mathrm{N}^k\mathrm{MHV}}\right)^2 = A^{\mathrm{MHV}} A^{\mathrm{MHV}} + \cdots + A^{\mathrm{N}^i\mathrm{MHV}} A^{\mathrm{N}^j\mathrm{MHV}} + \\ + \text{all combination}$$

The Squared Amplituhedron

The square of a super amplitude is

$$\mathcal{A}_n^2 = \left(\sum_{k=0}^{n-4} \mathcal{A}^{\text{N}^k\text{MHV}} \right)^2 = \mathcal{A}^{\text{MHV}}\mathcal{A}^{\text{MHV}} + \cdots + \mathcal{A}^{\text{N}^i\text{MHV}}\mathcal{A}^{\text{N}^j\text{MHV}} +$$

$$+ \text{ all combination}$$

We define the Squared Amplituhedron geometry as the union of all the possible geometries with well defined flipping number (topological invariant). The canonical form of the Squared Amplituhedron is conjectured to give

$$\mathcal{A}_{n,k}^2 = \sum_{k'=0}^{k} \mathcal{A}_n^{\text{N}^{k'}\text{MHV}} \mathcal{A}_n^{\text{N}^{k-k'}\text{MHV}}$$

Thank you