# RATIONAL TERMS FROM GENERALISED UNITARITY

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SAGEX
Scattering Amplitudes: from Geometry to Experiment

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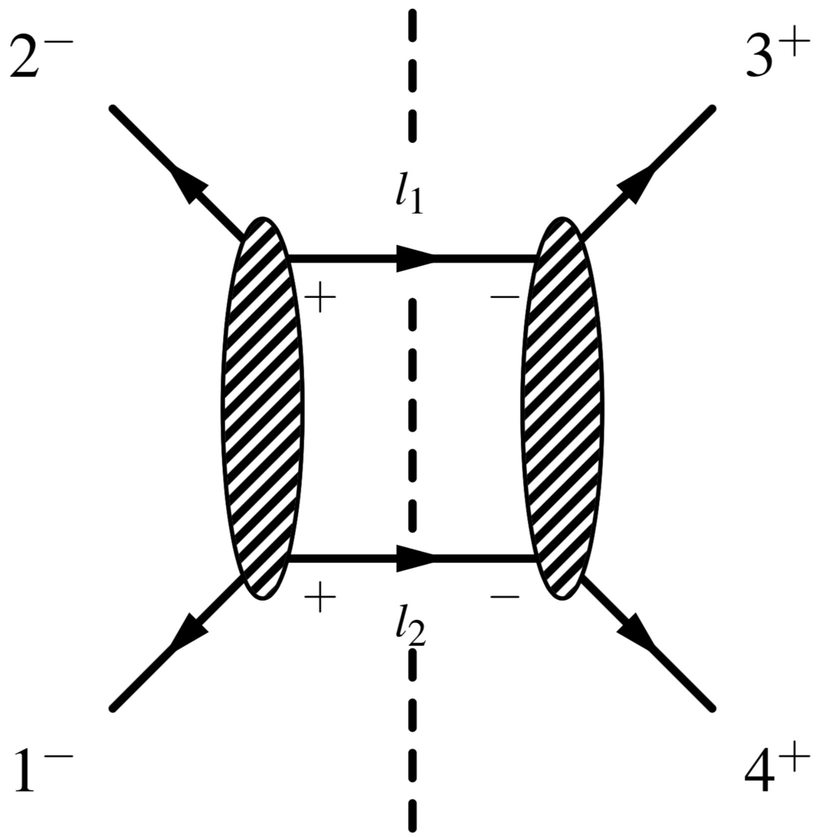
# UNITARITY

$$S^\dagger S = \mathbf{1} \qquad S = \mathbf{1} + iT$$

$$-i\left(T - T^\dagger\right) = TT^\dagger \qquad 2\,\mathrm{Im}T_{ij}^{1-\mathrm{loop}} = \int \mathrm{d}\mu T_{i\mu}^{\mathrm{tree}} T_{\mu j}^{\mathrm{tree}}$$

$$\frac{1}{p^2 + i\epsilon} = P\left(\frac{1}{P^2}\right) - i\pi\delta\left(p^2\right)$$

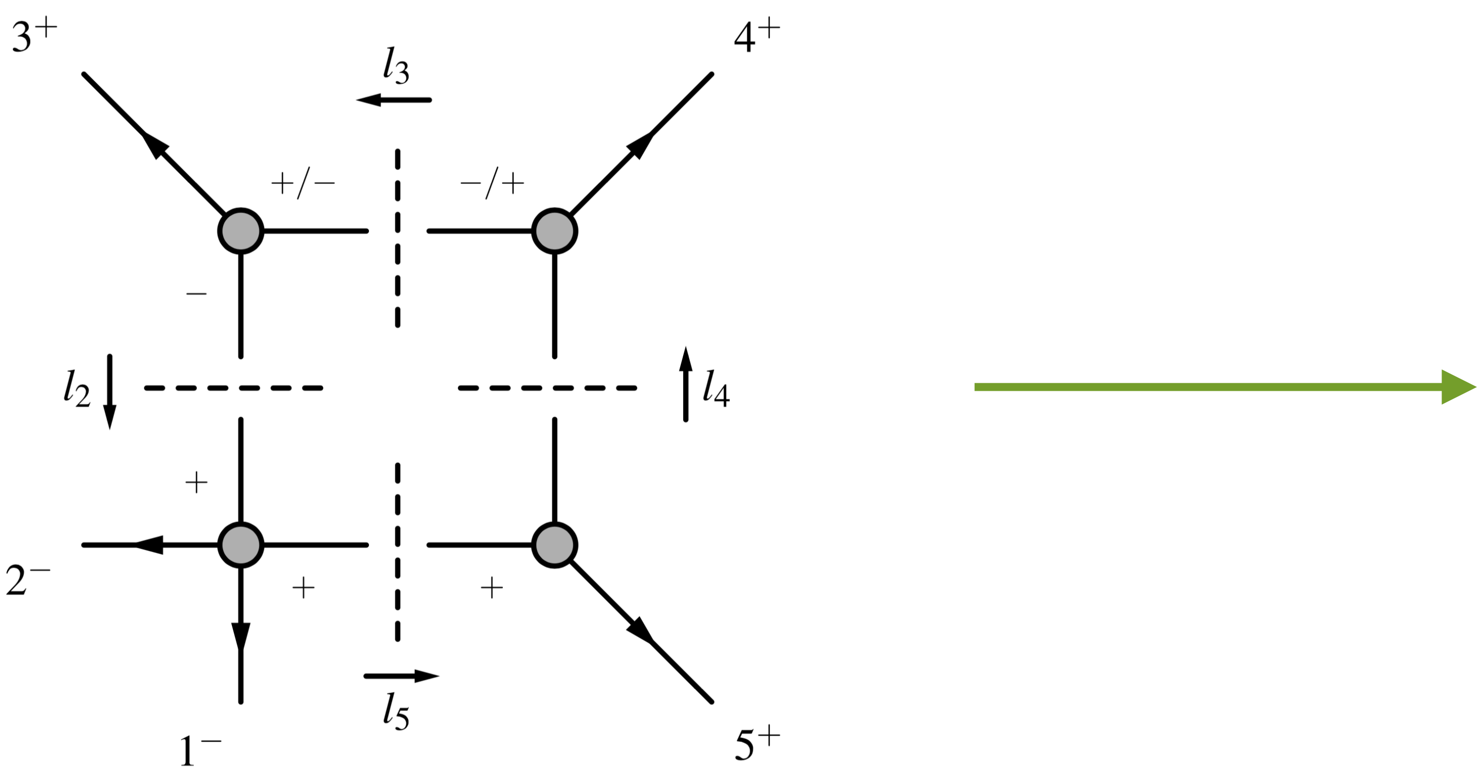



$$\frac{1}{p^2 + i\epsilon} \longrightarrow -\,i\pi\delta^+\left(p^2\right)$$

$$\text{Numerator} \longrightarrow \sum \varepsilon^* \varepsilon$$

# GENERALISED UNITARITY

$$A_n^{1-\text{loop}}(l) = \sum \frac{d_{i_1 i_2 i_3 i_4}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum \frac{c_{i_1 i_2 i_3}}{d_{i_1} d_{i_2} d_{i_3}} + \sum \frac{b_{i_1 i_2}}{d_{i_1} d_{i_2}} + \sum \frac{a_{i_1}}{d_{i_1}} + \text{rational terms}$$




$d_{i_1 i_2 i_3 i_4}$

# 6D GENERALISED UNITARITY

(W. T. Giele, Z. Kunszt, K. Melnikov; Z. Bern, J. J. Carrasco, T. Dennen, Y. Huang, H. Ita; S. Davies)

$$A_{(D,D_s)}\left(\{p_i\},\{J_i\}\right) = \int \frac{\mathrm{d}^D l}{i\pi^{D/2}} \frac{\mathcal{N}^{(D_s)}\left(\{p_i\},\{J_i\}\right)}{d_1 d_2 \cdots d_n}$$

$$\mathcal{N}^{(D_s)}\left(l\right) = \mathcal{N}_0\left(l\right) + \left(D_s - 4\right)\mathcal{N}_1\left(l\right)$$

Gluon scattering in QCD

$$A^{\mathrm{FDH}} = A_{(D,D_s=6)} - 2A^{\mathrm{scal}}_{(D)}$$

This technique gives  the full D-dimensional integral (including rational terms)

# 6D SPINOR HELICITY FORMALISM

(C. Cheung, D. O'Connell; T. Dennen, Y. Huang, W. Siegel)

## 4D

$$\mathscr{L} = SO(1,3) \simeq SL(2,\mathbb{C})$$

$$L = \widetilde{SO}(2) \simeq U(1)$$

$$\lambda_\alpha, \tilde{\lambda}_{\dot{\alpha}}$$

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$

$$\langle \lambda_1 \lambda_2 \rangle = \epsilon^{\alpha\beta} \lambda_{1\beta} \lambda_{2\alpha}$$

$$[\lambda_1 \lambda_2] = \epsilon^{\dot{\alpha}\dot{\beta}} \lambda_{1\dot{\alpha}} \lambda_{2\dot{\beta}}$$

## 6D

$$\mathscr{L} = SO(1,5) \simeq SU^*(4) \simeq SL(2,\mathbb{H})$$

$$L = \widetilde{SO}(4) \simeq SU(2) \times SU(2)$$

$$\lambda_a^A, \tilde{\lambda}_{\dot{a}A}$$

$$p^{AB} = \lambda_a^A \lambda_b^B \epsilon^{ab}$$

$$\langle \lambda_{1a} \tilde{\lambda}_{2\dot{a}}] = \lambda_{1a}^A \tilde{\lambda}_{2\dot{a}A}$$

$$\langle \lambda_{1a} \lambda_{2b} \lambda_{3c} \lambda_{4d} \rangle = \epsilon_{ABCD} \lambda_a^A \lambda_b^B \lambda_c^C \lambda_d^D$$

# 4D QUANTITIES FROM 6D

## Lorentz group

$$\sigma^{\tilde{\mu}} = \begin{pmatrix} 0 & \sigma_4^{\tilde{\mu}\alpha}{}_{\dot{\alpha}} \\ -\sigma_4^{\tilde{\mu}\dot{\alpha}}{}_{\alpha} & 0 \end{pmatrix} \qquad \lambda_a^A = \begin{pmatrix} -\frac{m}{\langle\lambda\mu\rangle}\mu_\alpha & \lambda_\alpha \\ \tilde{\lambda}^{\dot{\alpha}} & \frac{\tilde{m}}{[\mu\lambda]}\tilde{\mu}^{\dot{\alpha}} \end{pmatrix}$$

$$\delta_B^A \longrightarrow \delta_\alpha^\beta + \delta_{\dot{\beta}}^{\dot{\alpha}} \qquad \epsilon_{ABCD} \longrightarrow \sum \epsilon^{\alpha\beta}\epsilon_{\dot{\gamma}\dot{\delta}}$$

## Little group indices

$$\varepsilon^\mu_{a\dot{a}}(p,k) = \frac{1}{\sqrt{2}}\left(\langle k_b p^{\dot{a}}]\right)^{-1}\langle p_a \sigma^\mu k_b\rangle$$

- $a = 1, \dot{a} = \dot{1} \longrightarrow \varepsilon_+^{\tilde{\mu}}$
- $a = 1, \dot{a} = \dot{2}, \longrightarrow$ scalar
- $a = 2, \dot{a} = \dot{2} \longrightarrow \varepsilon_-^{\tilde{\mu}}$

# WHY 6D UNITARITY?

$$\left\langle g_1^+ g_2^+ \ldots g_m^+ \middle| \mathrm{Tr}F^n\left(q^2\right) \middle| \Omega \right\rangle \text{ in QCD where } \mathrm{Tr}F^n = F^{\mu_1}{}_{\mu_2} F^{\mu_2}{}_{\mu_3} \cdots F^{\mu_n}{}_{\mu_1}$$